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Impedance of Hydraulic Fractures: Its Measurement and Use for Estimating Fracture Closure Pressure and Dimensions

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ABSTRACT

The growth of a hydraulic fracture increases the period of free oscillations in a well. Simultaneously, the decay rate of free oscillations decreases. The properties of forced oscillations in a well also change during fracture growth. All of these effects result from the changing impedance of the hydraulic fracture that intersects the well. Fracture impedance can be determined directly by measuring the ratio of downhole pressure and flow oscillations, or determined indirectly from wellhead measurements using impedance transfer functions. Because impedance is a function of fracture dimensions and the elasticity of the surrounding rock, impedance analysis offers a promising new approach for evaluating fracture geometry. Because oscillatory flow conditions occur continuously a hydraulic-fracturing treatment, data collection is simple and economical, adding to the attractiveness of this technique.

INTRODUCTION

This paper introduces ***impedance analysis*** as a tool for fracture diagnostics. Impedance analysis is based on the dynamics of wave propagation in a well and the effect the hydraulic fracture has on oscillatory pressures and flows.

Impedance analysis is a logical extension of the two pressure analysis techniques currently used for evaluation of hydraulic fractures. The first, pressure transient analysis, is based on the solution of a diffusion equation derived from Darcy's law and the principle of conservation of mass.[1] In this method gradual pressure changes resulting from fluid flow through the pores of the fracture and formation are measured and used for estimating fracture size and permeability. The second pressure analysis method[2] is also derived from the principle of mass conservation and considers gradual pressure changes associated with the elasticity of an inflating fracture. Neither of these approaches considers the inertial component of fluid flow, an effect important in the study of wave propagation and reflection. Inertial forces are accounted for by invoking the principle of conservation of momentum.[3] This principle, along with that of conservation of mass, forms the basis of the study of oscillatory pressure and flow in wells and other conduits.

This paper begins with a definition of impedance and then presents several field examples of oscillatory pressure changes resulting from the changing impedances of hydraulic fractures. Reasons for these changes are subsequently derived using impedance analysis techniques.[3,4] To illustrate the relationship between fracture impedance and fracture dimensions, we then construct a hydraulic model of a fracture intersecting the bottom of a well. The properties of the fracture are combined in two lumped parameters, a flow resistance and a capacitance, which determine the impedance at the well-fracture interface. These parameters can be expressed in terms of fracture dimensions and the elastic properties of the surrounding rock.

It is not our purpose in this paper to provide a definitive recipe for measurement of fracture dimensions based on impedance analysis. We hope instead to illustrate the potential of the method and provide a framework for its further development.

CONCEPT OF HYDRAULIC IMPEDANCE

Imagine that a specialized tool is placed at the bottom of a well beside a low-permeability zone about to be fractured. This tool is able to precisely measure very small changes of both pressure and flow as injection rates are increased. In addition, the tool can measure *oscillatory* pressures and flows resulting from the reciprocating action of the pistons in the fracturing pumps. When injection begins, the pumps force fluid into the well, although flow into the formation is not possible since breakdown has not occurred. At the same time, the pressure begins to rise because the pumps are compressing the fluid in the well. Our specialized tool would therefore measure large pressure oscillations (in addition to large static pressures) but would measure zero flow. The corresponding ratio of pressure to flow would be infinite.

Continuing this thought experiment, we know that formation breakdown will occur when the downhole pressure becomes great enough to overcome both the rock strength and the minimum in situ compressive stress at the treatment depth. If we were to again measure pressure and flow after fracture growth has begun, we would expect static and oscillatory pressures to be less than before, while flow would be greater because fluid is now moving from the well into the fracture. The ratio of pressure to flow would thus be less than in the pre-fracturing case.

As the fracture continues to grow, we would expect the relative values of downhole pressure and downhole flow to continue to change. Because fracture growth is accompanied by an increase in the cross-sectional area of the fracture where it intersects the wellbore, the ease with which fluid can flow into or out of the fracture should increase. At the same time, the pressure gradient required to maintain that flow should decrease. Since fracture growth is accompanied by an increase in the fluid stored in the fracture, the quantity of fluid contained in a single flow oscillation should become a smaller and smaller fraction of total fracture volume. The fracture thus behaves as a large capacitor becoming more and more effective at holding downhole pressure constant as its size increases. In the limiting case, a very large fracture would behave as a constant pressure boundary, although it is questionable whether this case is ever attained in practice. We therefore expect the downhole pressure oscillations associated with oscillatory flow to diminish as the fracture grows.

As anticipated from the above discussion, our specialized pressure-and-flow-measurement tool should detect a decreasing ratio of oscillatory pressure to oscillatory flow as the fracture grows. If we knew how to analyze the pressure-to-flow ratio, we could use it as a means of interpreting fracture dimensions.

In hydraulics, the ratio of oscillatory pressure (or hydraulic head) to oscillatory flow is called the ***hydraulic impedance***, Z.[3] The impedance is a complex number defined by the amplitude, frequency, and phase of the pressure and flow oscillations at a point. It is a function of the physical properties of the piping system and fluid. Impedance can be written in terms of oscillatory head H and flow Q as

$$Z = \frac{He^{i\omega(t+\phi)}}{Qe^{i\omega t}} = \frac{H}{Q}e^{i\omega\phi} \qquad (1)$$

where ω is the circular frequency in radians per second, t is time in seconds, ϕ is the phase difference between the head and flow oscillations, and $i = \sqrt{-1}$. The relationship between head H and pressure P is $P = \rho g H$ where ρ is fluid mass density and g is gravitational acceleration. Useful relationships between frequency ω, frequency f (hertz) and period T (seconds) are: $f = \omega/2\pi$ and $T = 2\pi/\omega = 1/f$.

Another concept that will be valuable in our subsequent analyses is that of ***characteristic impedance*** Z_c. The characteristic impedance can be considered as a hydraulic impedance that describes the proportionality between head and flow moving in one direction only.[3] In an infinite frictionless conduit, the phase difference between head and flow oscillations is either 0 or π/ω, depending on whether the flow is moving in a positive or negative direction. The imaginary term in the expression for impedance (Eq. 1) vanishes and the characteristic impedance assumes a purely real value that can be shown to be[3]

$$Z_c = \frac{a}{gA} \qquad (2)$$

where a is the acoustic wavespeed in the conduit and A is the cross-sectional area of the conduit.

FREE AND FORCED OSCILLATIONS

In the analysis of impedance in hydraulic systems, it is convenient to distinguish between ***free oscillations*** and ***forced oscillations***. The latter is also referred to as ***steady-oscillatory*** behavior. In the forced oscillation of a fluid system, all oscillations are at the frequency of the forcing function. During a hydraulic fracturing treatment, forcing is provided by the reciprocating action of the pumps that inject fluid down the treatment well. The frequency of forcing is determined by the frequency of the piston strokes and higher-order harmonics. In contrast, free oscillations result from an initial, temporary excitation, such as the sudden removal of fluid from a pressurized well by valving, or the sudden opening of a hydraulic fracture at breakdown. Upon removal of the excitation, the oscillations attenuate as a result of natural physical damping in the system. The frequency of free oscillations is determined by the wavespeed of the fluid, the lengths of the system elements, and the physical properties of the system boundaries.

Both free and forced oscillations occur throughout a typical hydraulic-fracturing treatment. Steady pumping results in a condition of forced oscillation, whereas free oscillations are caused by suddenly starting or stopping the pumping and by numerous other disturbances that naturally occur during pumping. The same theoretical framework is used to evaluate both free and forced oscillations. In the former, the frequencies of interest are one or more of the natural frequencies of the system. In the latter, the frequencies of the forcing functions are used.

FIELD OBSERVATIONS

The configuration of surface and downhole pipe (tubing, casing, etc.) remains constant during a hydraulic-fracturing treatment, whereas the geometry of the fracture changes continuously as it is being created.

Changes in oscillatory behavior observed under conditions of uniform excitation and constant fluid properties should therefore be related to the changing geometry of the one variable in the system: the hydraulic fracture. Three recent experiments provide examples of the effects of fracture growth on wellbore pressure oscillations.

In the first experiment (Figure 1), a transient condition was initiated by rapidly removing a small volume of fluid ($<$ 10 liters) by abruptly valving at the wellhead. The well was cased to a total depth of 1589 m. There were 18 casing perforations between 1448 and 1395 m in the production zone whose permeability was several microdarcies. A packer was set in the 16-cm I.D. casing at a depth of 1296 m. Tubing from the wellhead to the packer had an I.D. of 6.2 cm. The viscosity of fluid in the well was approximately 80 *cp*[5]. Figure 1a shows free oscillations measured at the wellhead prior to fracturing. The period of these oscillations was $2L/a$, where L is well depth and a is sonic wavespeed in the fluid in the well, about 1400 m/sec. Figure 1b illustrates free oscillations at the wellhead recorded a few minutes after the completion of the 320,000-liter fracturing treatment. The period of these oscillations was approximately double the pre-fracturing case. Doubling of the period of free oscillations has also been reported by Anderson and Stahl[6] who performed tests on three wells in the 1960s.

The second experiment was conducted in a 330-m deep test well at Mounds, Oklahoma that was cased (0.126-m I.D.) to a depth of 311 m. Below this depth was an open-hole completion in the Skinner sandstone (porosity $\approx$ 20%, permeability $\approx$ 20 millidarcies). All injections were down the casing with no tubing in the well. The two pressure records shown in Figure 2 were made with water in the well after the sandstone had been hydraulically fractured. The well was shut in at these times and the pressure was declining as a result of leakoff into the formation. The first oscillations were recorded when the wellbore pressure was about 0.4 MPa above the statically determined fracture closure pressure. The oscillations continued for several cycles before damping out (Figure 2a). In contrast, they damped out almost immediately after excitation at fracture closure pressure (Figure 2b). Subsequently, free oscillations were initiated above fracture closure pressure after different volumes of water had been injected into the already-created fracture. In every case, greater volumes were characterized by reduced rates of attenuation of the oscillations (Figure 3). Plots of peak-to-peak amplitude versus time (Figure 4) clearly illustrate this effect.

The third experiment was conducted in the same Mounds, Oklahoma test well with tubing run to the bottom of the casing and an open annulus. Pressure oscillations were measured at the top of the annulus, on the tubing at the wellhead, and on the treatment line near the two pump trucks used to pump the fracturing fluid (Figure 5). Pressure records made during proppant injection indicated that the ratio of oscillatory annulus pressure to oscillatory wellhead pressure declined as pumping progressed (Figure 6a and 6b). This observation is consistent with the expected effect of a fracture behaving as a large capacitor if we make the assumption that annulus pressure oscillations are directly proportional to pressure oscillations at the fracture orifice. We also observed that the relative phase of pressure oscillations on the annulus fell further behind the phase of oscillations at the wellhead and pump trucks as proppant injection progressed (Figure 7).

INTERPRETATION OF PRESSURE AND FLOW OSCILLATIONS

In this section, a hydraulic well-fracture model is developed and used to derive expressions for fracture impedance and the frequency and decay of free oscillations. We subsequently show how impedances derived from pressure and flow measurements can be used to evaluate fracture closure and dimensions.

Impedance Analysis

We can illustrate the effect of fracture growth on wellbore pressure oscillations using a simple model of a fracture intersecting the bottom of a well. In this model, the physical properties of the fracture are lumped into two parameters: the flow resistance R_f at the fracture-well interface and the hydraulic capacitance, or storage of the fracture, C_f (Figure 8). The R_f and C_f elements are combined in series to reflect the fact that flow into the fracture must first overcome a resistance before fracture capacitance can be increased.

A change of hydraulic head ΔH_f in a fracture gives rise to a change of fracture volume ΔV_f. We define the ratio of volume change to head change as the capacitance of the fracture. Sneddon[7] derived the relationship between internal pressure and opening of an oblate-ellipsoidal (penny-shaped) fracture in an infinite elastic medium. Following his results, we can write fracture capacitance as

$$C_f = \frac{\Delta V_f}{\Delta H_f} = 8\pi g h^2(1-\nu)/3\mu \qquad (3)$$

where h is fracture radius, ν Poisson's ratio and μ the shear modulus of the medium. Because capacitance is proportional to the cube of the fracture radius, it increases rapidly as the fracture grows. When the radius exceeds a few meters, flow oscillations of a few liters per second should produce pressure oscillations at the well-fracture interface of no more than a few hundredths of a megapascal (i.e., several psi) (Figure 9). The larger the fracture, the more effective it is at maintaining itself at relatively constant pressure during periods of pressure and flow oscillations in the wellbore.

The fracture resistance R_f is the proportionality constant relating a change of flow into or out of the fracture to a corresponding change of hydraulic head:

$$\Delta H_f = R_f \Delta Q_f \qquad (4)$$

We wish to analyze the impedance in our model well (Figure 8) and its relationship to fracture growth, as expressed by the R_f and C_f parameters. For forced-oscillation conditions, a reciprocating pump at the wellhead generates sinusoidal flow at a given frequency. For free-oscillation conditions, excitation is by a sudden temporary flow change. The tubing or casing leading from the wellhead to the fracture is modeled as a frictionless pipe of length L. The characteristic impedance Z_c of the well is given by equation (2) and the propagation constant γ is given by[4]

$$\gamma = \frac{i\omega}{a} \quad . \tag{5}$$

Pressure transducers are connected to the top and bottom of the well to measure oscillatory pressure behavior. In addition, we assume that there is a sensor at the bottom of the well that measures flow oscillations into and out of the fracture. The bottom of the well is characterized by a lumped impedance Z_f that is a function of the fracture constants R_f and C_f:

$$Z_f = R_f + \frac{1}{i\omega C_f} \quad . \tag{6}$$

During a forced-oscillation test, the fracture impedance can be determined in the following manner: A sinusoidal fluid flow of known frequency ω and known magnitude is set up at the wellhead. Once steady state is reached, the downhole pressure sensor will show a sinusoidal oscillation of the same frequency as the wellhead source but of a different magnitude. Similarly, the downhole flow oscillation will be at the same frequency but different magnitude. The magnitudes and phases of the downhole pressure and flow oscillations are recorded. Since flow and pressure have been measured, the magnitude of the fracture impedance can be determined by dividing these two quantities. Once this has been done, the frequency of the flow source is changed and the whole process repeated. In this manner, the magnitude of the fracture impedance can be determined as a function of frequency.

In many fracturing jobs, downhole pressure and flow measurements are not available but wellhead measurements are. In these situations, the downhole (fracture) impedance must be determined by applying an impedance transformation to the wellhead impedance. If the magnitude and phases of wellhead pressure and flow have been measured, the complex-valued wellhead impedance is easily determined by vector division. Transformation from wellhead impedance Z_w to downhole (fracture) impedance Z_f is accomplished through the transformation[4]:

$$Z_f = Z_c \cdot \frac{1 + \left(\frac{Z_w - Z_c}{Z_w + Z_c}\right) e^{-2\gamma L}}{1 - \left(\frac{Z_w - Z_c}{Z_w + Z_c}\right) e^{-2\gamma L}} \quad . \tag{7}$$

The impedance transformation can be carried out at several different source frequencies to give downhole impedance as a function of frequency.

We now turn our attention to the case of free-oscillation testing. Our goal is to derive an equation that expresses the fracture impedance in terms of the frequency and rate of decay of the free oscillations. It is a well known result from steady-state Laplace analysis that the character of free oscillations is determined by the singularities of the impedance with respect to frequency.[4] It can be shown that the wellhead impedance is

$$Z_w = Z_c \cdot \frac{1 + \Gamma_f e^{2\gamma L}}{1 - \Gamma_f e^{2\gamma L}} \tag{8}$$

where Γ_f is the downhole reflection coefficient given by

$$\Gamma_f = \frac{Z_f - Z_c}{Z_f + Z_c} \quad . \tag{9}$$

Replacing $i\omega$ by s and using the definition of γ, it can be shown that $2\gamma L = sT_d$ where $s = \alpha + i\omega$ is complex frequency and $T_d = 2L/a$ is the two-way travel time up and down the wellbore. The poles (singularities) of Eq. (8) occur at values of s for which the denominator goes to zero, i.e.,

$$e^{-sT_d} = \Gamma_f \quad . \tag{10}$$

To study the effect of fracture resistance on the rate of decay of free oscillations, we assume a purely real impedance of $Z_f = R_f$ and a frictionless well of characteristic impedance Z_c. The downhole reflection coefficient is then real and given by

$$\Gamma_f = \frac{R_f - Z_c}{R_f + Z_c} \quad . \tag{11}$$

In terms of this reflection coefficient, there are multiple values of s for which equality (10) is met. These are

$$\omega = \text{imaginary}[s] = \frac{n\pi}{T_d} \tag{12}$$

and

$$\alpha = \text{real}[s] = \frac{-\ln|\Gamma_f|}{T_d} \tag{13}$$

where n is odd for $\Gamma_f < 0$ and n is even for $\Gamma_f > 0$. The imaginary part of s determines the frequency of the free oscillations while the real part of s determines the rate of decay. More specifically, the fundamental natural frequency is

$$f = \omega/2\pi = \begin{cases} a/4L & \Gamma_f < 0 \\ a/2L & \Gamma_f > 0 \end{cases} \quad . \tag{14}$$

The system oscillates at odd harmonics for fracture resistances below the characteristic impedance of the wellbore ($\Gamma_f < 0$) and at even harmonics for fracture resistances above the characteristic impedance ($\Gamma_f > 0$). This behavior was observed in our field tests described earlier (Figures 1 and 3). The time constant of the free-oscillation decay is

$$\tau = \frac{1}{|\alpha|} = \frac{|\Gamma_f|}{1-|\Gamma_f|} T_d \quad . \qquad (15)$$

Notice that τ approaches infinity (zero decay) as the magnitude of the reflection coefficient approaches one. That is, the rate of decay becomes extremely slow as the fracture impedance approaches either zero (no fracture) or infinity (completely open fracture). Also notice that the time constant τ goes to zero as the reflection coefficient goes to zero. That is, free oscillation will not occur when the fracture is open to a point where the fracture impedance and the characteristic impedance of the wellbore are equal. This explains the effect observed in Fig. 2b. The theoretical effect of a purely resistive fracture impedances on free oscillations in a frictionless well is shown in Figure 10.

To study the effect of fracture capacitance on the frequency of free oscillation, assume a purely capacitive fracture impedance of the form $Z_f = 1/C_f\, s$ The downhole reflection coefficient of Eq. (9) then becomes

$$\Gamma_f = \frac{1-Z_c C_f s}{1+Z_c C_f s} \quad . \qquad (16)$$

For a frictionless well and capacitive fracture, the singularities of Eq. (8) must occur at purely imaginary values of s. Thus we can assume that $s = i\omega$ and find the values of ω for which equality (10) is met. This turns out to give an equation of the form

$$Z_c C_f \omega = \tan(\omega T_d /2) \quad . \qquad (17)$$

Equation (17) was evaluated for several values of C_f to generate the plot of natural frequency versus fracture capacitance shown in Figure 11. A value for the characteristic well impedance was taken from the Mounds well: $Z_c = 11{,}250$ sec/m^2. Notice that as C_f varies from 0 to ∞, the natural frequency of the free oscillations shifts from $a/2L$ (an even harmonic) down to $a/4L$ (an odd harmonic). Most of the shift is accomplished when fracture capacitances are between about 10^{-6} and 10^{-4} m^2.

Fracture Closure and Dimensions

As shown above, the amplitude and decay of pressure oscillations in the wellbore are strongly dependent on the resistive characteristics of the hydraulic fracture. If fracture capacitance is large or excitation frequency is high (Eq. 6), the selection of a purely real (resistive) fracture impedance can be justified. By equating fracture resistance to the characteristic impedance of a frictionless conduit, we can then define a procedure for deriving fracture size estimates. Characteristic impedance of a frictionless fracture is now given by:

$$Z_f = R_f = \frac{a}{gA} \quad . \qquad (18)$$

where A is the area of the fracture where it intersects the well and a is the wavespeed in the fracture. Fracture wavespeed can be derived from the following expression[3]

$$a = \left[\frac{K/\rho}{1+(K/A)(\Delta A/\Delta P)} \right]^{1/2} \qquad (19)$$

where $\Delta A/\Delta P$ is the area change corresponding to a fluid pressure change ΔP. In a stiff conduit, such as a cased well, the denominator is very close to 1 and the wavespeed is close to K/ρ, that of a perfectly rigid pipe. For water, this limiting wavespeed is about 1485 m/sec. Wavespeeds in the two test wells discussed previously were measured at about 1400 m/sec.

For a very compliant conduit such as a hydraulic fracture, the denominator in Eq. (19) is large with respect to unity and the wavespeed in the conduit is very slow. In this case Eq. (19) simplifies to

$$a = \left[(\Delta P\ A)/(\rho \Delta A) \right]^{1/2} \quad . \qquad (20)$$

We can estimate wavespeeds in hydraulic fractures and their relationship to fracture dimensions by considering the expansion of a penny-shaped fracture resulting from changes of internal pressure. The change of area of a cross-section drawn through the center of such a fracture is[7]

$$\Delta A = 2\Delta P h^2 (1-\nu)/\mu \qquad (21)$$

Substituting (21) into (20) and using the formula for the area of an ellipse, $A = \pi b h$, the following expression for wavespeed results:

$$a = \left[\frac{\pi b\ \mu}{2\rho h\,(1-\nu)} \right]^{1/2} \qquad (22)$$

where b is half-width of the ellipse. Evaluation of this equation reveals that wavespeeds in the fracture can be extremely slow with respect to those in the well (Figure 12).

We can now define the fracture impedance in terms of fracture dimensions and elastic properties. Assuming that the fracture has an elliptical cross section where it intersects the well and that it is bi-winged so that it intersects the well on two sides, we can use Eqs. (18) and (22) to write

$$R_f = \left(\frac{\mu}{8g^2 \rho\pi(1-\nu)} \right) \left(\frac{1}{bh^3} \right) \qquad (23)$$

where R_f is the characteristic impedance of a frictionless penny-shaped fracture. The corresponding expression for the impedance of an infinitely long (two-dimensional) fracture can be derived using the formula for opening of such a fracture under internal pressure[8] and is

$$R_{f\,(2\text{-}D)} = \left(\frac{\mu}{4g^2 \rho\pi^2(1-\nu)} \right) \left(\frac{1}{bh^3} \right) \quad . \qquad (24)$$

In Figure 13 we have plotted curves of impedance versus fracture half-width for penny-shaped fractures of several different radii (solid lines). One curve (dashed line) is the impedance of an infinitely long fracture with a half-height of 6.10 m. Figure 13 illustrates a way in which changes of fracture impedance may help track fracture growth. Assume that the fracture begins to grow

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