Fundamentals of macro axial gradient index optical design and engineering

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Abstract. Homogeneous lens material is characterized by an index of refraction and a point on the glass map $n_d = f(\upsilon_d)$. Gradient refractive index (GRIN) lenses have a spatially varying index and dispersion and are represented by a line on the glass map. GRIN lenses open the door to a wide variety of optical design applications incorporating entire lenses of axial gradient refractive material (macro-AGRIN). Axial gradient material essentially gives biaspheric behavior to lenses with spherical surfaces and exhibits a controlled gradient in both index and dispersion. Thus, the applications for this material range from simple singlet lens used for imaging laser light, in which spherical aberration is eliminated, to complex multielement lens systems, where improved overall performance is desired. The fusion/diffusion process that produces this material is surprisingly simple, repeatable, and applicable to mass production. The advantages of AGRIN technology coupled with the recent advances in material development and its accessibility in commercially available lens design programs provides optical designers with the opportunity to push the performance of optical systems farther than with conventional optics. [S0091-3286(97)00406-6]

Subject terms: optical materials; optical systems; gradient index; axial gradient refractive index; gradient refractive index; radial gradient refractive index; macro axial gradient, radial gradient; optical design; glass lines; unique optical systems; high performance; asphere.

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1 Introduction

Traditional optical systems are based on the use of homogeneous refractive materials with spherical surfaces. A spherical surfaced lens will produce images with intrinsic optical aberrations. Basic optical principles lead to the use of multiple elements in a lens system to provide imagery across a required field with acceptably low aberration content. Fabrication of spherical surfaces can be carried out using several traditional methods to extremely high accuracy and smoothness.

Improved images with reduced aberrations can be obtained by using aspheric surfaces. The production and testing of nonspherical surfaces is more complex and considerably more expensive than for spherical surfaces. Generally, aspheric surfaces will cost about 10 times that of a spherical surface. Nevertheless, cost efficient mass production of optical elements with aspheric surfaces can be accomplished with plastics or glasses, usually to acceptable accuracy for some applications. While mass production of molded aspherics can be cost competitive, it is subject to extremely high startup costs, limited material selection and small diameters.

Axial gradient refractive index (AGRIN) offers an approach that would build specific aspheric effects into the refractive properties of the material while permitting the use of traditional spherical surface finishing methods. The refractive power at each point on a surface is determined by Snell's law, given in Eq. (1), where θ is the angle of incidence the ray makes

$$n \sin \theta = n' \sin \theta' \tag{1}$$

with the surface normal, θ' is the angle after refraction, n is the index of refraction of the incident medium and n' is the index of the refractive medium. The shape of the surface determines the progressive change in angle of incidence across the aperture that leads to image formation. This same relation determines the intrinsic aberration that will be produced by the surface. Additional control of the ray passage through a surface can be gained by using a material in which the index of refraction varies with the location on the surface and/or within the medium. If the refractive index varies along the optical axis, it is known as an axial gradient. If the index varies with radius, it is known as a radial gradient.

2 Axial Gradients

The term macro-AGRIN refers to a solid piece of axial gradient optical material with relatively large diameter, thickness and change in index (Δn), and is used interchangeably with the term AGRIN throughout this paper.

Macro-AGRIN is made via a high temperature fusion/diffusion process described later in this text that turns a series of discrete glasses on the glass map into glass lines (see Fig. 1). An axial gradient is typically defined by a polynomial function in z, the distance along the optical axis. A common expression for this type of gradient is

$$n_0 = n_{00} + n_{01}z + n_{02}z^2 + n_{03}z^3 + \dots \quad (2)$$

The most significant way an axial gradient affects aberrations is by the variation of index over the curved surface of the lens. Snell's law states that there are only two factors that determine how light is bent when it hits a glass surface. The first factor is the angle of incidence θ of the ray on the surface, and the second factor is the index of refraction n' at the point of contact. An aspheric surface controls aberrations by changing the localized radius of curvature (surface normal) on the lens surface, thus changing the angle of incidence for a particular ray. An axial gradient lens affects aberrations by leaving the surface spherical and changing the index of refraction through the material. When a curved surface is put on a plane parallel piece of this material, the gradient is exposed because the curve of the surface exposes different depths z in the material. This results in a changing index value tracking radially outward along the curved surface. A lens made from a solid block of axial gradient material has two surfaces exposing the gradient (front and back), thereby giving each lens the equivalent of two aspheric surfaces. The contribution to aberration at the surface of a lens, due to a gradient material is referred to as a *surface* term. In addition to the two surface terms, there is a *transfer* term as the wavefront propagates through the lens, between the two surfaces. In this region, the light travels in a curve as the wavefront encounters a continuous change in index and dispersion, identified by the Abbe number,

$$\nu_d = \frac{n_d - 1}{n_F - n_C}. \quad (3)$$

It is important to think in terms of wavefronts, as opposed to rays, when understanding how light travels through a gradient material. A wavefront always bends *toward* the high index, where light travels slower, similar to a marching band turning a corner.

Adding up the contributions from the two surface terms and the transfer term can give AGRIN an advantage over aspheric lenses. In addition, the spherical surfaces of lenses made of AGRIN are self generating, easy to test, plate fit to high accuracy and no more expensive to fabricate than conventional surfaces. The effect on aberrations from a macro-AGRIN lens can be adjusted by changing the lens shape factor or gradient profile[1] [Eq. (2)]. A multielement lens system can benefit from a gradient index lens by relaxing the requirements of the other lenses. The gradient material essentially acts like a ''work sponge,'' soaking up a portion of the work that would have previously been distributed among all the elements. Designing functionality into an optical material helps relax optical systems and paves the way to improved performance. Improved performance can be in the guise of smaller, cheaper, sharper, lighter or stronger products.

The importance of gradient index materials have been realized for over 40 years.[2–4] Extensive theoretical exploration and conceptual designs can be found in many scientific articles.[5–8] The poor reputation of GRIN material, however, comes from stagnation in the material development. Small size (up to 3 mm), poor repeatability, and high cost are associated with GRIN materials. Four major different processes, i.e., ion exchange, sol-gel, chemical vapor deposition (CVD), and high temperature fusion/diffusion are currently used to produce GRIN materials. Recognizable commercial products are Selfoc™ produced by Nippon Sheet Glass via an ion-exchange process, graded-index optical fibers via the CVD process and axial gradient laser singlets via fusion/diffusion. Theoretically, radial GRIN is more versatile than AGRIN because the radial refractive index gradient directly contributes to the optical power, Petzval and spherical aberration of the optical system. However, by the same token, this makes the performance of the radial GRIN lens more sensitive to manufacturing variations and repeatability in the gradient index profile than AGRIN lenses. A radial gradient lens will directly affect the first and higher order properties of an optical system because the gradient is perpendicular to the wavefront propagation. Repeatability is an important issue with radial GRIN.

The contribution to aberration correction from the transfer term of a thin axial gradient lens is secondary in that the gradient does not directly contribute to the optical power or aberrations of the lens, but rather, it is the exposure of the gradient along a curved surface that is responsible for most of the aberration correction. The gradient in this case is essentially parallel to the wavefront propagation, so the transfer term is not as dominant as it is with radial gradient. Therefore, the AGRIN lens performance is more tolerant to small manufacturing variations in the profile compared to the radial GRIN lens.

3 Fusion/Diffusion Process (Macro-AGRIN)

A process to produce solid blocks of AGRIN is based on controlled fusion/diffusion of different glasses at a high temperature. The process, developed by Hagerty and Pulsifer,[9] starts by identifying a specific family of glass compositions that span the desired range in optical index and dispersion. AGRIN glass is created by fusing together a stack of discrete molten glass layers, where each constituent layer has a distinctive composition and desired optical property. After a controlled diffusion process at high temperature (see Fig. 2), the glass layers fuse and diffuse into one piece of gradient index glass with a smooth variation of optical properties throughout. Prescribable index profiles (linear, quadratic and cubic), in lead and crown glass compositions have been achieved in large diameters, thickness and index variation.[10–13] The initial step function index profile of the layers is eliminated via the diffusion of constituent atoms within and across the layer boundaries. This process results in a smooth variation of index. By varying the index and thickness of each layer, and by controlling diffusion temperature and time, AGRIN can be produced for arbitrary monotonic index profiles. The profiles are typically monotonic because of the separation of the constituent molten glasses based on density. High temperature diffusion (1100 °C) yields high diffusion rates for all com-

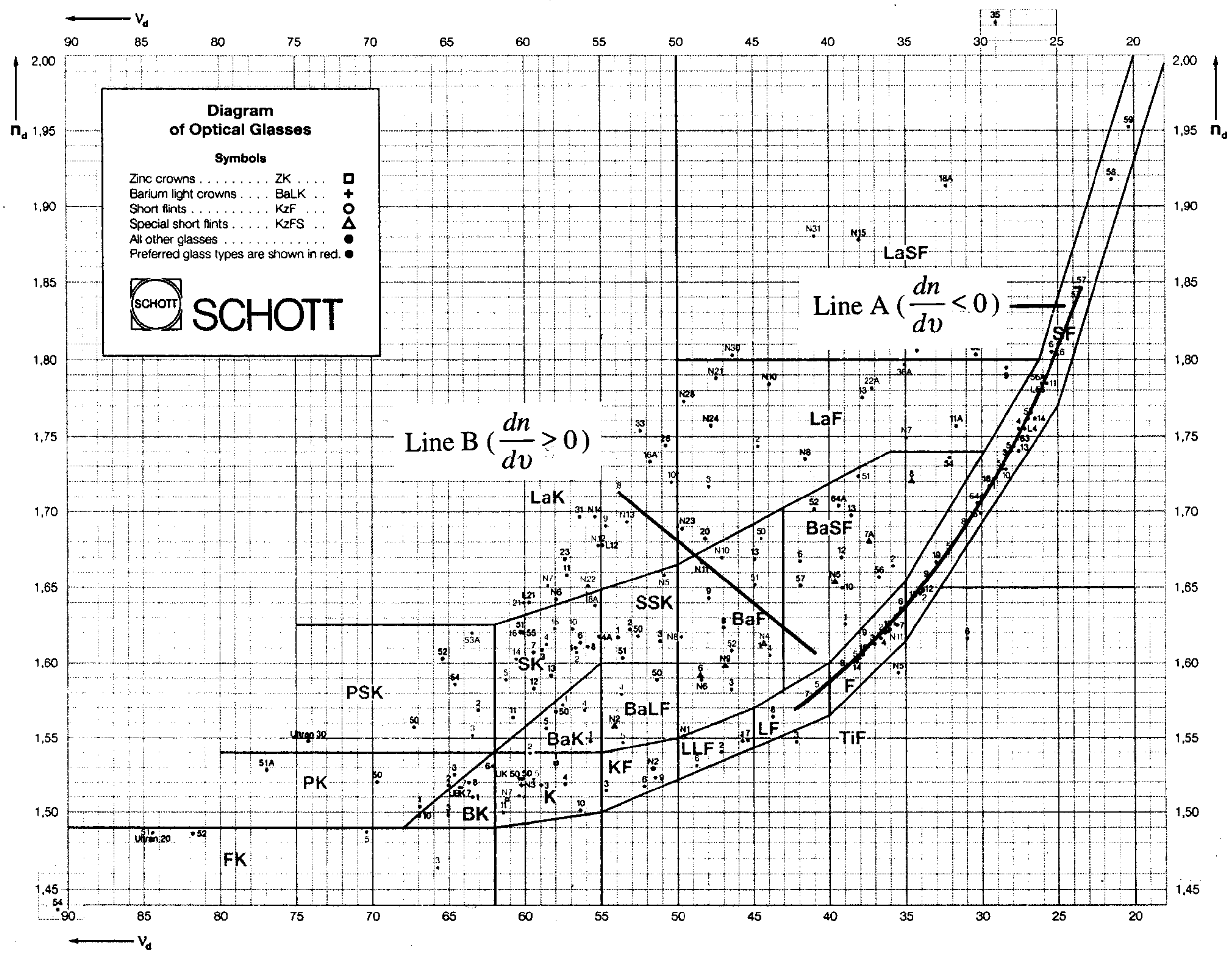

Fig. 1 Glass map showing refractive index as a function of Abbe number.

ponents in the glass, therefore, a wide variety of glass families can be used to produce AGRIN with different index/dispersion relationships $[n_d = f(\upsilon_d)]$. Profiles with steep slopes (dn/dz), experience more stress than profiles with small slopes. This stress is due to mismatches in the coefficient of thermal expansion of the constituent glasses after diffusion. The resultant stress manifests itself as residual asymmetric radial terms in the AGRIN material, limiting the useful aperture over which an optical quality wavefront can be obtained. Steeper sloped gradients have smaller useful diameter than shallower sloped gradients because of the residual stress associated with a more rapid change in material composition.

Two key factors of the fusion/diffusion process are (1) the selection of base glass compositions and (2) the diffusion process. The glass compositions must be compatible with each other, so that the compositional changes due to the diffusion between glass layers will not cause undesirable property changes, such as thermal expansion, phase separation and devitrification. The step function-like index profile between layers is eliminated via the diffusion of constituent atoms within and between the layers during the process; this eventually results in smooth variation in index as well as other physical and material properties. The process is illustrated in Fig. 2.

4 Diffusion Software

To prescribe the index profile, it is necessary to (1) establish the relation between glass properties, such as refractive index, dispersion, thermal expansion coefficients, glass transition temperature, and glass composition, and (2) understand and simulate the diffusion process. The design problem for prescribing the refractive index profile is to find the desirable glass family and an achievable initial index distribution $n(z,0)$ that will yield the desired index profile $n(z,t)$ to the accuracy required after diffusion at temperature T for a time t. In the fabrication methods already described, the index of each layer can be fixed for a given sample batch; the design variables are then the thickness of each layer, the diffusion temperature T and time t. Diffusion simulation software[14,15] determine these process variables quickly. The diffusion model is based on multicomponent interdiffusion theory. All constituents of

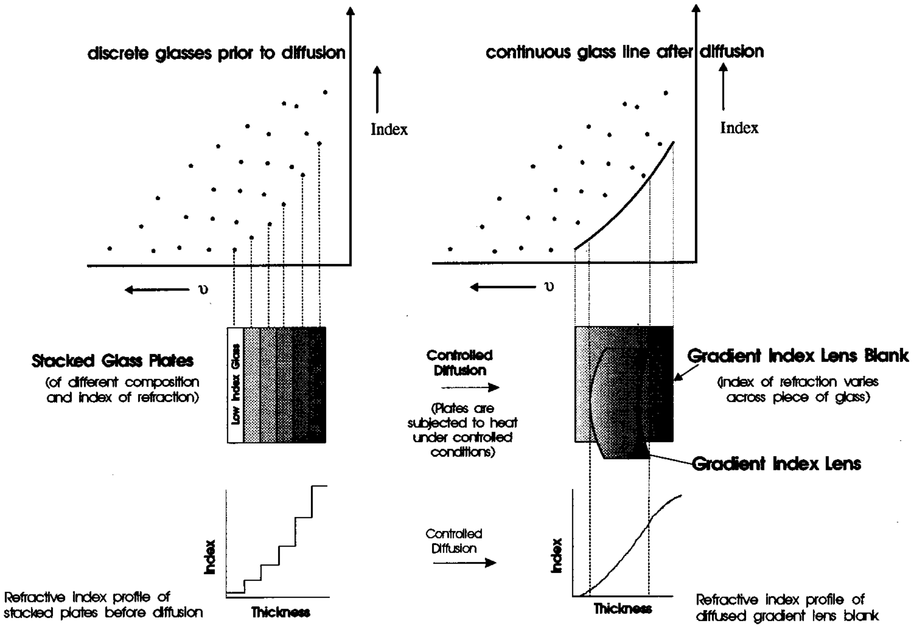

Fig. 2 Fusion/diffusion process for axial gradient material fabrication.

the glass participate in diffusion at high temperature. The diffusion constants of these constituents are also functions of their elemental concentrations.

The change in the index distribution during the diffusion process was simulated using the diffusion software and is illustrated in Fig. 3 for a seven-layer sample. It shows that diffusion at the high index region is much faster than at the low index side. This is expected because of the higher lead densities in the higher index material. After being diffused at high temperature for the required time, the index profile becomes smooth and has a linear index gradient region between 1.580 and 1.750. Using the parameters established by the diffusion software, the calculated index profiles are in excellent agreement with the produced index profiles, demonstrating the effectiveness of the software in prescribing the index profile for specific optical designs.[10–12] Figure 4 plots the calculated index profile along with the measured data of the produced profile. The results from different diffusion experiments also show excellent repeatability in both the index profile and optical quality.

The unique aspect of this process for producing macro gradient index glass is that layers of glass possessing different physical properties can be fused together to produce a repeatable predetermined index gradient. The controlled diffusion of glass components such as Pb, Ba and La, provides a gradual transition in properties from layer to layer.

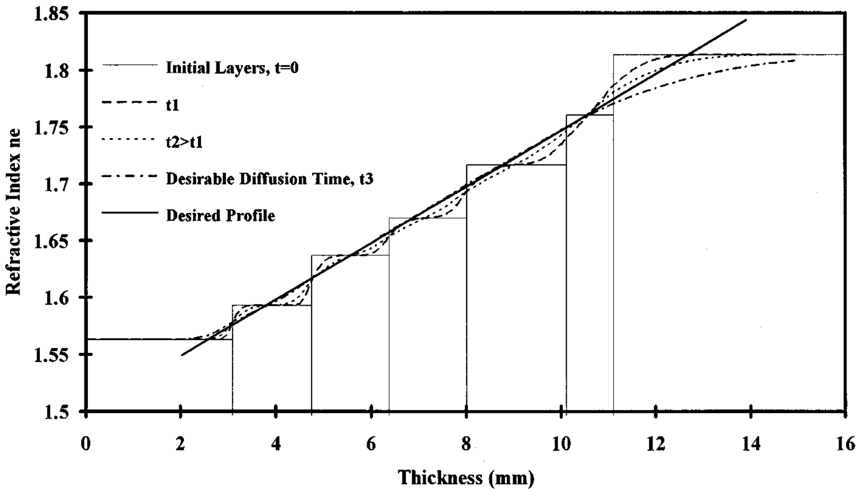

Fig. 3 Diffusion simulation for seven-layer profile as a function of diffusion time *t* at 1000 °C.

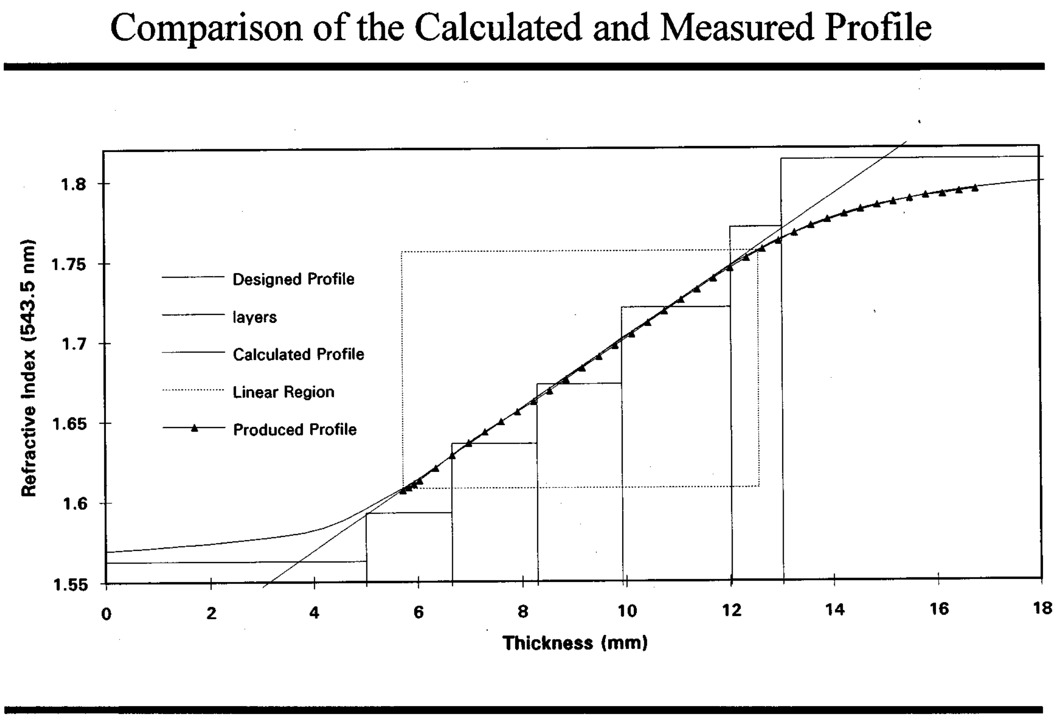

Fig. 4 Comparison of the produced index profile with the profile calculated from diffusion software.

Carefully chosen glasses enable this high temperature diffusion process to produce gradient index glass having macro size, high Δn and good wavefront quality.

5 Optical Design

Macro-AGRIN offers optical designers with a unique opportunity to incorporate lenses made from solid blocks of gradient material into their optical systems. Gradient materials are described by profiles. For axial gradients, the profile is a plot of the index of refraction at a reference wavelength (typically 0.58756 μm), as a function of axial position within the blank [Eq. (2)]. Profiles can be for either increasing or decreasing index as a function of position. The slope is negative for a decreasing index and positive for an increasing index. Some fundamental insights into the behavior of lenses made from this material follow, along with a process to define and model the dispersive properties of the gradient.

5.1 *Linear Versus Nonlinear Profile*

New issues have emerged while designing optical systems with macro-AGRIN materials. For instance, it has been found that nonlinear profiles are more advantageous than linear profiles because the relative slope of the gradient over each lens surface can be different.[1] Figure 5(a) shows calculated third order spherical aberration, (SA3) of an $F/3$ singlet lens, plotted as a function of lens shape factor X, for a linear gradient range of $-0.4 \leqslant \Delta n \leqslant 0.4$. The different curves represent different slopes, or Δn's. For a lens with the object at infinity, minimum SA3 occurs near a shape factor, or bending, of $+1.0$, which corresponds to a convex first surface and a flat rear surface. To correct an $F/3$ singlet for SA3 using a linear gradient, Δn must be between -0.03 and $+0.25$. Once that condition is met, there exists a solution for zero SA3. The most prominent aspect of this plot however, is that the curves tend to pivot about a point that has a shape factor of zero (equiconvex/convex). The phenomena that we call the pivot point has been observed for all cases analyzed, including root mean square (rms) spot size, coma, astigmatism and distortion. The pivot point also exists for spherical type gradient geometry's (but shifts as a function of the exact radius of the isoindicial surfaces). In essence, the existence of the pivot point implies that a monotonic linear profile throughout the material has no effect in controlling SA3 when the shape factor of the lens is near zero. Wang hints at this in his dissertation.[16] For an equiconvex lens with a linear axial gradient, the net effect of the gradient on spherical aberration is zero because the two spherical surface curvatures, of equal and opposite sign, have the same gradient slope over the saggita z (sag), given by

$$z=\frac{cr^2}{1+(1-c^2r^2)^{1/2}}. \tag{4}$$

The contribution to spherical aberration from one gradient surface is canceled by the other gradient surface. For a lens bending $-1 \leqslant X \leqslant +1$, a linear axial gradient lens' optical performance is compromised by one of the surfaces. Meniscus lenses, however, have the property that the gradient contribution to spherical aberration is additive on both surfaces because their curvatures are of the same sign.

The existence of the transfer term, and the fact that the marginal ray heights are different on each surface for a

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