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# Optical performance of axial gradient and aspheric surface lenses: study and analysis

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## ABSTRACT

Using the caustic merit function for plane waves incident upon a singlet lens, a comparison between the optical performance of the lens using an axial gradient index with spherical surfaces or aspheric surfaces with constant index material has been investigated. Results indicate that the use of an inhomogeneous medium for the lens material is more effective in the controlling aberrations than the use of aspheric surfaces for a similar lens.

## 1. INTRODUCTION

In lens design, the aberrations of an optical system can be controlled by using of aspheric surfaces<sup>1</sup> or using an inhomogeneous medium<sup>2</sup> between the surfaces of the optical system. Sands<sup>3</sup> has shown that the contributions of an axial gradient index (GRIN) to the third order aberrations of an optical system are equivalent to those of aspheric surfaces. However, it has been expected that inhomogeneous media would be more effective in some cases for controlling aberrations than the use of aspheric surfaces when higher order aberrations are considered. Attempts<sup>4</sup> have been made to produce axial GRIN lens designs which have a comparable optical performance with an aspheric surfaces lens. These efforts have been hampered by the lack of optical design software which is capable of calculating the fifth and higher order aberration coefficients for the GRIN lens. As a result, the lens designer has not been able to study the effect of using GRIN materials on aberrations correction and balancing.

In this paper, a comparison between the effectiveness of axial GRIN materials and aspheric surfaces for controlling aberrations has been studied for the case of plane waves incident upon a singlet lens. Using the caustic merit function (CMF),<sup>5</sup> two lens designs, which have the same back focal length as that of an initial lens, are produced. The first design is an axial GRIN lens with spherical surfaces, and the second design is an aspheric surfaces lens with constant index media. The optical performance of each design has been evaluated as a function of the lens parameters to arrive at an optimum design that has a minimum value of the average value of the CMF.

Starting with the parameters of the initial lens, the refractive index of the lens material is varied linearly along the optic axis of the lens while the other parameters (the radii of curvature of the surfaces and the back focal length) are kept constant. Then, values of the CMF are averaged over the entire entrance pupil and the average CMF is evaluated as a function of the GRIN parameter to reach the optimum axial GRIN design. Similarly, the optimum design of an aspheric lens has been obtained by variation of the aspherical deformation coefficients<sup>6</sup> of the spherical surfaces of the initial lens while the other parameters (refractive index, thickness, and the back focal length) are kept constant. In a computational program, all the deformation coefficients<sup>6</sup> of both surfaces of the lens are varied until a minimum value of the average CMF is achieved. This study has not addressed the issue of whether these designs represent global minima of the CMF and other optical merit functions.

The optical performance of the initial lens (homogeneous lens), the axial GRIN lens, the aspheric surfaces lens, and the axial GRIN lens given in Ref. [4] has been evaluated and compared for a wide range of field angles, using the average CMF and the RMS<sup>7</sup> of blur circle radius. Also, for these lenses, the ray aberrations are evaluated and compared over the range of the entrance pupil radius by using the CMF.

## 2. THEORETICAL ANALYSIS

In general, tracing a single ray through an axial GRIN lens or an aspheric lens can be used to derive a formula for evaluating the irradiance or energy flux density<sup>8,9</sup> along the path of this ray. Then, this formula can be employed to evaluate the caustic surface configurations at the focal plane of such a lens. The mathematical procedures for the derivation of the flux flow formula and the CMF are summarized below.

### 2.1. Ray Trace Procedures

#### 2.1.1. Axial GRIN Lens

Consider a ray incident at the boundary surface,  $S_1$ , of an axial GRIN lens (see Fig. 1), which has a refractive index profile as

$$n(z) = n_0(1 - \alpha z), \quad (1)$$

where  $n_0$  is the refractive index at the vertex of the first surface of the lens,  $\alpha$  is the gradient index parameter, and  $z$ -axis is the optic axis of the lens. Then, the solution of the ray equation<sup>10</sup> in the axial GRIN medium gives the ray path which is expressed as

$$x = x_0 + p_0 \int_{z_0}^z \frac{dz}{l}, \quad (2)$$

$$y = y_0 + q_0 \int_{z_0}^z \frac{dz}{l}, \quad (3)$$

where  $p_0$  and  $q_0$  are the optical direction cosines of the ray as it leaves the surface  $S_1$  and  $(x_0, y_0, z_0)$  are the initial coordinates of the incident point. Since  $n$  is a function of  $z$  alone, the optical direction cosines of the refracted ray through the axial GRIN medium,  $p$  and  $q$ , are invariant along the path of this ray. That is,  $p = p_0$  and  $q = q_0$ . Also, the optical direction of this ray in the  $z$  direction,  $l$ , can be expressed as

$$l = (n^2(z) - p^2 - q^2)^{1/2} = (n^2(z) - p_0^2 - q_0^2)^{1/2}. \quad (4)$$

Combining Eqs. 1-4 and carrying out the integration gives

$$x = x_0 + (p_0/n_0\alpha) \ln Q, \quad (5)$$

$$y = y_0 + (q_0/n_0\alpha) \ln Q, \quad (6)$$

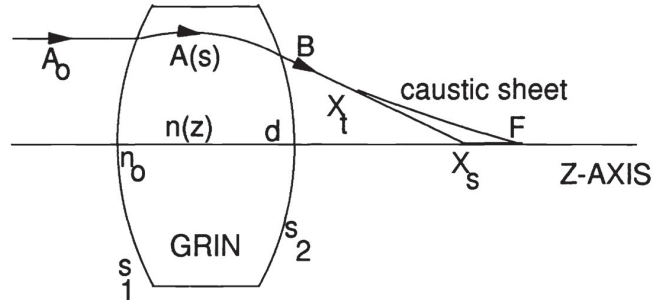


Figure 1: Schematic diagram for a ray trace in an axial GRIN lens.

where

$$\begin{aligned}
 Q &= (2\sqrt{z^2 + bz + c + 2z + b}) / (2\sqrt{z_0^2 + bz_0 + c + 2z_0 + b}), \\
 c &= (n_0^2 - p_0^2 - q_0^2) / (n_0 \alpha)^2, \\
 b &= -2/\alpha.
 \end{aligned}$$

The intersection point of the ray with the second surface of the lens,  $S_2$ , and the optical direction cosines of this ray,  $\vec{A}(s)$ , just before this boundary, are specified by the use of a numerical ray trace technique.<sup>11</sup> Then, the optical directional cosines of the emergent ray, as it leaves the second surface are specified by the use of the laws of refraction.<sup>12</sup>

### 2.1.2. Aspheric Surface Lens

Consider a rotationally symmetric aspheric surface of an equation which is expressed as<sup>13</sup>

$$\vec{X}(s, \phi) = \vec{i} \cos \phi + \vec{j} \sin \phi + \vec{k} Z(s), \quad (7)$$

where

$$Z(s) = \frac{cs^2}{1 + \sqrt{1 - (cs)^2}} + e s^4 + f s^6 + g s^8 + h s^{10} + O(s^{12}),$$

$s^2 = x^2 + y^2$ , and  $c$  is the vertex radius of curvature,  $\phi$  is the polar coordinate angle measured from the positive  $x$ -axis, and the numerical coefficients ( $e$ ,  $f$ ,  $g$ , and  $h$ ) are the aspherical deformation constants, and the term  $O(s^{12})$  stands for the rest of the terms in the series which are neglected in these numerical calculations.

In this study, only meridional rays are considered. Then, following the mathematical description given in Ref. [13], (see Eqs. 6-20, Chap. 4), the incident ray is brought fairly close to the point where the ray actually crosses

the aspheric surface  $S_1$  (see Fig. 4-2, Ref. [13]). Next, one considers the refraction process at this point and uses Eqs. 21-39 in Ref. [13] to compute the optical direction cosines of the refracted ray as it leaves the  $S_1$  toward the second surface  $S_2$ . Similarly, the same procedures are repeated to determine the intersection point of the ray with  $S_2$  and the optical direction cosines of the emergent ray from this surface.

## **2.2 Caustic Merit Function**

The configurations of the caustic surfaces<sup>8</sup> in the image plane of a singlet lens can be specified throughout the use of the flux flow equation<sup>9</sup> which represents the illuminance associated with each traced ray through such a lens. For an axial GRIN lens and an aspheric surfaces lens, the mathematical descriptions for the derivation of the CMF are summarized below

### **2.2.1. Axial GRIN Lens**

The input to the flux density formula (see Eq. 28 in Ref. [9]) is obtained from the initial direction of the incident rays, the equations of the lens surfaces, the equations of the geodesic in the axial GRIN lens, and the intersection points of the traced rays with the surfaces of the lens (see Fig. 1). Since the caustic surfaces represents the loci of singularities of the flux density,<sup>14</sup> then equating the denominator of Eq. 28 in Ref. [9] to zero gives the distances along the emergent ray,  $r_t$  and  $r_s$ , from the intersection point of this ray with  $S_2$  to the caustic points,  $\vec{X}_t$  and  $\vec{X}_s$ , in the tangential and sagittal planes respectively (see Fig. 1):

$$\vec{X}_t = \vec{X}(2) + r_t \vec{B}, \quad (8)$$

$$\vec{X}_s = \vec{X}(2) + r_s \vec{B}, \quad (9)$$

where  $\vec{B}$  is a unit vector along the emergent ray and  $\vec{X}(2)$  is the position vector of the intersection point of this ray with the second surface of the lens. Following the notation given in Ref. [5], the CMF is expressed as

$$CMF = |\vec{X}_t - \vec{X}_f| + |\vec{X}_s - \vec{X}_f|, \quad (10)$$

where  $\vec{X}_f$  is the position vector of the paraxial image.

### **2.2.2. Aspheric Surfaces Lens**

To derive the CMF for a meridional ray which is traced through an aspheric surfaces lens, the mathematical description given in Ref. [15] has been used to specify the values of  $r_t$  and  $r_s$ . Then, the use of Eqs. 8-10 specifies the value of CMF.

## **3. Results and Analysis**

The parameters of both optimized designs, the axial GRIN and the aspheric surfaces lens, and a comparison between the optical performance of these designs with that of the initial lens are presented below.



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