$y)\sin(x+y)+(x-y)(\sin y-\sin x)$ is always ≥ 0 for x and y in this region. Thus $f(x,y)\leq 4$ and $d_B^2\leq 4$ for all $h_1, h_2 \leq 1.5$.

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Channel Coding with Multilevel/Phase Signals

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***Abstract*—A coding technique is described which improves error performance of synchronous data links without sacrificing data rate or requiring more bandwidth. This is achieved by channel coding with expanded sets of multilevel/phase signals in a manner which increases free Euclidean distance. Soft maximum—likelihood (ML) decoding using the Viterbi algorithm is assumed. Following a discussion of channel capacity, simple hand-designed trellis codes are presented for 8 phase-shift keying (PSK) and 16 quadrature amplitude-shift keying (QASK) modulation. These simple codes achieve coding gains in the order of 3–4 dB. It is then shown that the codes can be interpreted as binary convolutional codes with a mapping of coded bits into channel signals, which we call "mapping by set partitioning." Based on a new distance measure between binary code sequences which efficiently lower-bounds the Euclidean distance between the corresponding channel signal sequences, a search procedure for more powerful codes is developed. Codes with coding gains up to 6 dB are obtained for a variety of multilevel/phase modulation schemes. Simulation results are presented and an example of carrier-phase tracking is discussed.**

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I. Introduction

IN CHANNEL CODING of the "algebraic" coding type, one is traditionally concerned with a discrete channel provided by some given modulation and hard-quantizing demodulation technique. Usually, inputs and outputs of the channel are binary. The ability to detect and/or correct errors can only be provided by the additional transmission of redundant bits, and thus by lowering the effective information rate per transmission bandwidth.

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In addition, hard amplitude or phase decisions made in the demodulator prior to final decoding cause an irreversible loss of information. In the binary case, this amounts to loss equivalent to approximately 2 dB in signal-to-noise ratio (SNR).

In this paper, we take the viewpoint of "probabilistic" coding and decoding and regard channel coding and modulation as an entity [1]. Comparisons are strictly made on the basis of equal data rate and bandwidth. A possibility for redundant coding can therefore only be created by using larger sets of channel signals than required for nonredundant (uncoded) transmission. Maximum-likelihood (ML) soft decoding of the unquantized demodulator outputs is exclusively assumed, thus avoiding loss of information prior to final decoding. This implies that codes for multilevel/phase signals should be designed to achieve maximum free Euclidean distance rather than Hamming distance. For coded 2-amplitude modulation (AM) and 4-phase-shift keying (PSK) modulation, this was never a problem because in this case, binary Hamming distance (HD) and Euclidean distance (ED) are equivalent. The situation is different, however, if signal sets are expanded beyond two signals in one modulation dimension.

The investigations leading to this paper started some time ago with the heuristic design of simple trellis codes for 8-PSK modulation conveying two bits of information per modulation interval. When soft ML-decoded by the well-known Viterbi algorithm (VA) [2], coding gains of 3–4 dB were found over conventional uncoded 4-PSK modulation. The investigations were then extended to other modulation forms. First results were presented in [3]. A much better understanding of the subject was later obtained by interpreting the hand-designed codes as binary convolutional codes with a mapping of coded bits into multilevel/phase channel signals called "mapping by set partitioning."

Related work was reported in [4]–[8]. The approaches taken in [6]–[8] aim at constant-envelope modulation which is in contrast to the present paper. In [9], a comparison of various "bandwidth-efficient" modulation techniques by computer simulation is presented which includes codes of this paper.

In Section II, we investigate the potential gains in terms of channel capacity obtained by introducing more signal levels and/or phases. The results are similar to those of Wozencraft and Jacobs [10] on the exponential bound parameter R_0, and suggest that for coded modulation, it will be sufficient to use twice the number of channel signals than for uncoded modulation. In Section III, heuristically designed trellis codes for coded 8-PSK and 16-QASK modulation are presented and the concept of mapping by set partitioning is introduced. The codes are interpreted in Section IV as binary convolutional codes of rate $R = m/(m+1)$ with the above mapping of coded bits into channel signals. Preference is given to realizations in the form of systematic encoders with feedback. The mapping rule allows the definition of a new distance measure that can easily be applied to binary code sequences and efficiently lower-bounds the ED between the corresponding channel-signal sequences. Based on this distance measure, a search procedure for more powerful codes is developed in Section V, and codes with coding gains up to 6 dB are obtained for a larger variety of coded one- and two-dimensional modulation schemes. In Section VI simulation results are presented. Finally in Section VII, the problem of carrier-phase tracking, which can play an important role in the practical application of coded two-dimensional modulation schemes, is discussed for one specific case of coded 8-PSK modulation.

II. Channel Capacity of Multilevel/Phase Modulation Channels

Before addressing the code-design problem with expanded channel-signal sets, it is appropriate to examine in terms of channel capacity the limits to performance gains which may thereby be achieved. Because of our intended use of soft ML-decoding in the receiver, we must study modulation channels with discrete-valued multilevel/phase input and continuous-valued output. One- and two-dimensional modulation is considered and intersymbol interference-free signaling over bandlimited channels with additive white Gaussian noise (AWGN) is assumed. With perfect timing and carrier-phase synchronization, we sample at time $nT + \tau_s$ where T is the modulation interval and τ_s the appropriate sampling phase. The output of the modulation channel becomes

$$z_n = a_n + w_n, \tag{1}$$

where a_n denotes a real- or complex-valued discrete channel signal transmitted at modulation time nT, and w_n is an independent normally distributed noise sample with zero mean and variance σ^2 along each dimension. The average SNR is defined as

$$\mathrm{SNR} = \frac{E\{|a_n^2|\}}{E\{|w_n^2|\}} = \begin{Bmatrix} E\{|a_n^2|\}/\sigma^2 \cdots \text{(a) one-dimensional modulation} \\ E\{|a_n^2|\}/2\sigma^2 \cdots \text{(b) two-dimensional modulation} \end{Bmatrix}. \tag{2}$$

Fig. 1 illustrates the channel-signal sets considered in this paper. Normalized average signal power $E\{|a_n^2|\} = 1$ is assumed.

Extension of the well-known formula for the capacity of a discrete memoryless channel [11] to the case of continuous-valued output yields

$$C = \max_{Q(0)\cdots Q(N-1)} \sum_{k=0}^{N-1} Q(k) \int_{-\infty}^{+\infty} p(z/a^k) \cdot \log_2 \left\{ \frac{p(z/a^k)}{\sum_{i=0}^{N-1} Q(i) p(z/a^i)} \right\} dz \tag{3}$$

in bit/T. N is the number of discrete channel input signals

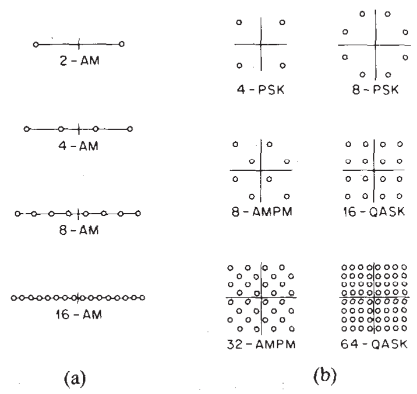

Fig. 1. Channel-signal sets considered in this paper. (a) One-dimensional modulation. (b) Two-dimensional modulation.

$\{a^0 \cdots a^{N-1}\}$ and $Q(k)$ denotes the *a priori* probability associated with a^k. Because of AWGN, we can substitute in (3)

$$P(z/a^k) = \exp\left[-|z - a^k|^2/2\sigma^2\right] \cdot \begin{Bmatrix} (2\pi\sigma^2)^{-1/2} & \cdots & (a) \\ (2\pi\sigma^2)^{-1} & \cdots & (b) \end{Bmatrix}. \quad (4)$$

With the further assumption that only codes with equiprobable occurrence of channel input signals are of interest, the maximization over the $Q(k)$ in (3) can be omitted. Equation (3) can now be written in the form

$$C^*_{Q(k)=1/N} = \log_2(N) - \frac{1}{N} \cdot \sum_{k=0}^{N-1} E\left\{\log_2 \sum_{i=0}^{N-1} \exp\left[-\frac{|a^k + w - a^i|^2 - |w|^2}{2\sigma^2}\right]\right\}. \quad (5)$$

In (5) we have integration replaced by expectation over the normally distributed noise variable w which is real with variance σ^2 for (a), and complex with variance $2\sigma^2$ for (b). Using a Gaussian random number generator, C^* has been evaluated by Monte Carlo averaging of (5). In Figs. 2(a) and 2(b), C^* is plotted as a function of SNR for the signal sets depicted in Fig. 1. The value of SNR at which in uncoded transmission symbol-error probability $Pr(e) = 10^{-5}$ is achieved is also indicated.

In order to interpret Figs. 2(a) and 2(b), we consider as an example transmission of 2 bit/T by uncoded 4-PSK modulation where $Pr(e) = 10^{-5}$ occurs at SNR = 12.9 dB. If the number of channel signals is doubled, e.g., by choosing 8-PSK modulation, error-free transmission of 2 bit/T is theoretically possible already at SNR = 5.9 dB (assuming unlimited coding and decoding effort). Beyond this—with no constraint on the number of signal levels/phases except average signal power—only 1.2 dB can further be gained. Similar proportions hold for the other modulation schemes. It can be concluded that by doubling the number of channel signals, almost all is gained in terms of channel capacity that is achievable by

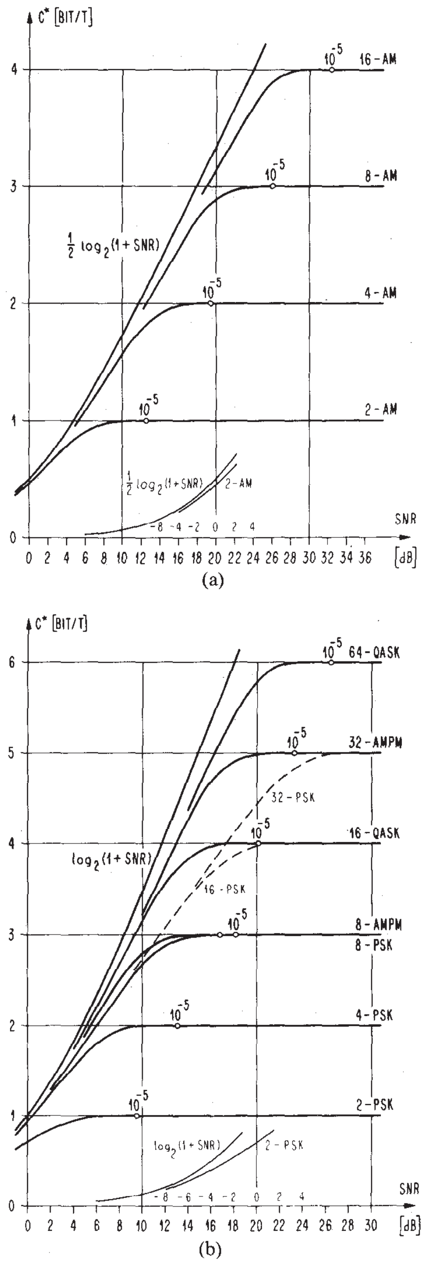

Fig. 2. Channel capacity C^* of bandlimited AWGN channels with discrete-valued input and continuous-valued output. a) One-dimensional modulation. b) Two-dimensional modulation.

signal-set expansion if at given SNR satisfactory error performance can no longer be achieved by uncoded modulation.

III. Simple Trellis Codes

Coding gains can be realized either by block coding or by state-oriented trellis coding. There exists also the possibility of concatenation, e.g., by assigning short block-code words to state transitions in a trellis structure. Note that the choice of a signal set for two-dimensional modulation

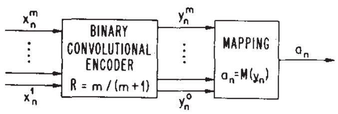

Fig. 3. Multilevel/phase encoder structure.

corresponds already to a simple form of block coding. In this paper however we do not further pursue the block coding aspect because the richer structure and omission of block boundaries together with the availability of Viterbi ML-decoding algorithm make trellis codes appear to us more attractive for the present coding problem.

Following the arguments of Section II, in order to improve error performance, m bit$/T$ must be transmitted in redundantly coded form by a set of 2^{m+1} channel signals. The coding expert will easily conclude that this may be accomplished by expanding the binary data sequence by suitable convolutional encoding with rate $R = m/(m+1)$, and subsequent mapping of groups of $m+1$ bits into the larger set of channel signals. The encoder structure is shown in Fig. 3. With $d(a_n, a'_n)$ denoting the ED between channel signals a_n and a'_n, the encoder should be designed to achieve maximum free ED:

$$d_{\text{free}} = \min_{\{a_n\} \neq \{a'_n\}} \left[\sum_n d^2(a_n, a'_n) \right]^{1/2} \qquad (6)$$

between all pairs of channel-signal sequences $\{a_n\}$ and $\{a'_n\}$ which the encoder can produce. If soft ML-decoding is applied, the error-event probability will approach asymptotically at high SNR the lower bound [2]

$$\Pr(e) \gtrsim N(d_{\text{free}}) \cdot Q(d_{\text{free}}/2\sigma). \qquad (7)$$

Here $N(d_{\text{free}})$ denotes the (average) multiplicity of error events with distance d_{free}, and $Q(\cdot)$ is the Gaussian error-probability function.

For transmission at 2 bit$/T$ by coded 8-PSK modulation it has been suggested [4], [5] to use known $R = 2/3$ binary convolutional codes with maximum free HD for given constraint length [12], and Gray coding as a mapping function. Yet there are problems with this approach. Firstly, the Gray code mapping does not monotonically translate larger HD into larger ED and secondly, permutations of the binary outputs of the convolutional encoder will have an unknown, perhaps significant influence on the ED structure of the resulting 8-PSK codes. Neither does the approach seem to be extendable to all modulation forms of Fig. 1.

We will therefore pursue a different design method which aims more directly at maximizing free ED. The approach is based on a mapping rule called "mapping by set partitioning." This mapping follows from successive partitioning of a channel-signal set into subsets with increasing minimum distances $\Delta_0 < \Delta_1 < \Delta_2 \cdots$ between the signals of these subsets. The concept is illustrated in Figs. 4 and 5 for 8-PSK and 16-QASK modulation, respectively, and is applicable to all modulation forms of Fig. 1.

Before addressing the systematic search for convolutional codes for the encoder suggested by Fig. 3, we discuss

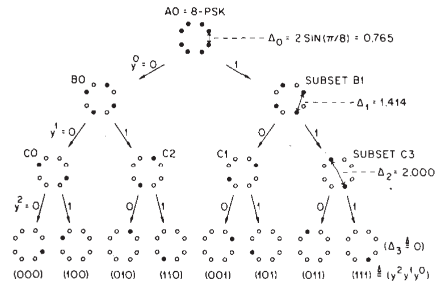

Fig. 4. Partitioning of 8-PSK channel signals into subsets with increasing minimum subset distances ($\Delta_0 < \Delta_1 < \Delta_2$; $E\{|a_n^2|\} = 1$).

in the remainder of this section the heuristic construction of simple but already very effective codes. This does not require knowledge of convolutional codes and will establish an intuitive basis for the development of more powerful codes later in the paper. Work leading to this paper progressed also in this order.

We regard an encoder simply as a finite-state machine with a given number of states and specified state transitions. If m bits are to be encoded per modulation interval T, there must be 2^m possible transitions from each state to a successor state. More than one transition may occur between pairs of states, and for obvious reasons only regular structures are of interest. After selecting a suitable trellis state-transition diagram, the remaining task consists of assigning channel signals from an extended set of 2^{m+1} signals to the transitions such as to achieve maximum free ED. For codes with up to eight states, this could be accomplished "by hand," and a 16-state code could still be found with the aid of a computer program that checked free ED.

The heuristic design of 8-PSK codes for coded transmission of 2 bit$/T$ will be discussed in more detail. Uncoded 4-PSK modulation is regarded as a reference system. As shown in Fig. 6, we can view uncoded 4-PSK as coding with a trivial one-state trellis and four "parallel" transitions, to which are assigned from the 8-PSK signal set four signals with largest minimum distance among them, i.e., the signals of subset $B0$ (or $B1$). Next consider a two-state trellis. The first code illustrated in Fig. 7 was easily found. Signals of subsets $B0$ and $B1$ are assigned to the transitions originating from the first and the second state, respectively, which guarantees that free ED is at least as large as for uncoded 4-PSK modulation. However with two states, it is not possible to have the same property also for transitions joining into one state, and hence the gain in free ED over 4-PSK remains limited to 1.1 dB.

The other 8-PSK codes depicted in Fig. 7 with 4, 8, and 16 states, and gains in free ED of 3 dB, 3.6 dB, and 4.1 dB, respectively, required more effort. Nevertheless after considerable experimentation with various trellis structures and channel-signal assignments, we were convinced that these codes are optimum for the given number of states. The following rules for assigning channel signals were applied:

1) all 8-PSK signals should occur with equal frequency

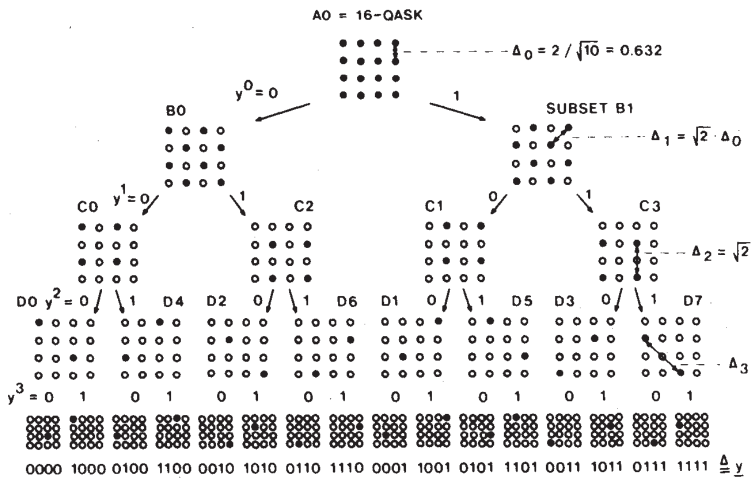

Fig. 5. Partitioning of 16-QASK channel signals into subsets with increasing minimum subset distances ($\Delta_0 < \Delta_1 < \Delta_2 < \Delta_3$; $E\{|a_n^2|\} = 1$).

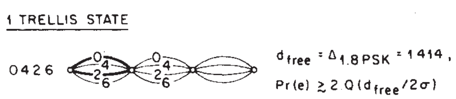

Fig. 6. Uncoded 4-PSK modulation, 2 bit/T.

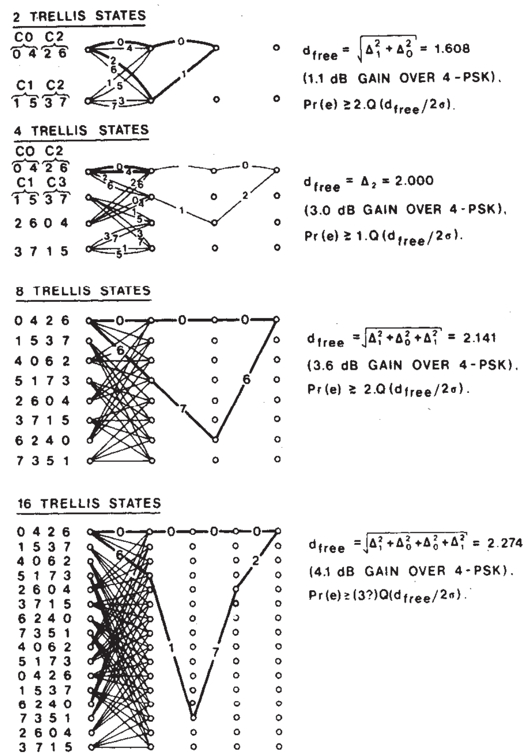

Fig. 7. Coded 8-PSK modulation, 2 bit/T.

and with a fair amount of regularity and symmetry,

2) transitions originating from the same state receive signals either from subset $B0$ or $B1$,
3) transitions joining in the same state receive signals either from subset $B0$ or $B1$,
4) parallel transitions receive signals either from subset $C0$ or $C1$ or $C2$ or $C3$.

Rule 1) reflects the intuition that good codes should exhibit regular structures, and rules 2), 3), and 4) guarantee that the ED associated with all single and multiple signal-error events exceeds the free ED of uncoded 4-PSK modulation by at least 3 dB. We have seen that with only two states, rules 2) and 3) cannot be simultaneously satisfied. Since this is not unique to coded 8-PSK, we shall not be further interested in two-state codes.

Note that parallel transitions imply that single signal-error events can occur. This limits achievable free ED to the minimum distance in the subsets of signals assigned to parallel transitions. On the other hand, parallel transitions reduce the "connectivity" in the trellis and thus allow extension of the minimum length of multiple signal-error events. With four states, the trade-off still worked in favor of parallel transitions; the best 4-state 8-PSK code gains 3 dB over 4-PSK, with single signal errors being most likely, whereas codes with distinct transitions to all successor states remained inferior because rules 2) and 3) could not be satisfied simultaneously. With eight and more states, only trellis structures with distinct transitions can be of interest because otherwise free ED gains would remain limited to 3 dB.

The ideas can also be applied to the other modulation forms. As a further example, we consider transmission of 3 bit/T by coded 16-QASK modulation. Uncoded 8-PSK or 8-AMPM modulation is regarded as a reference system. From the preceding discussion and the partitioning of the 16-QASK signal set into subsets shown in Fig. 5, codes follow easily. In the 8-PSK codes of Fig. 7, the 8-PSK signals must only be replaced by the corresponding 16-QASK signal subsets $D0$–$D7$ of Fig. 5. An 8-state 16-QASK code obtained in this manner is presented in Fig. 8. The reader must be cautioned, however, about this approach. A similarly obtained 16-state 16-QASK code turned out to

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