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Physics and Music

The Science of Musical Sound

To Beverly

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Chapter Eight

HARMONICS AND WAVE COMBINATIONS

There are an infinite number of musical tones that can be produced with instruments and the human voice, and yet each one can be described by specifying the intensity or loudness, the frequency or pitch, and the waveform or timbre. From the standpoint of music, the waveform or timbre of any complex tone is all-important and can be described by specifying the relative amplitudes and phases of all the different frequencies of which it is composed. We will study these concepts in this chapter.

8.1 Wave analysis

If a wave is generated by simple harmonic motion, it will be a ***sinusoidal*** or a ***sine wave.*** See Section 3.2 and Figure 3–3. A sine wave is indicative of one well-defined and definite frequency. The analysis of most musical tones shows that they are composed of a number of such components of various frequencies called **partials.** The process of adding these components to produce any complex vibration or wave is called **synthesis.** The converse of this process, breaking down any complex vibration or wave into its components, is called **analysis.**

Figure 8–1 represents two common graph forms for the same sound. Diagram (a) is a time graph representing the vibrations of a source emitting sound waves. Diagram (b) is a distance graph, or wave graph, representing the contour of the waves traveling to the right through the air with a velocity V.

Diagram (a) also represents a time graph of the vibrations of the eardrum, or a microphone diaphragm, detecting the sound.

If a wave graph (b) were drawn traveling to the left instead of to the right, it would look exactly like graph (a). Graph (a) is just the mirror image of graph (b), and vice versa. Since all three graphical representations look alike, it makes little difference which one is drawn to represent a given sound.

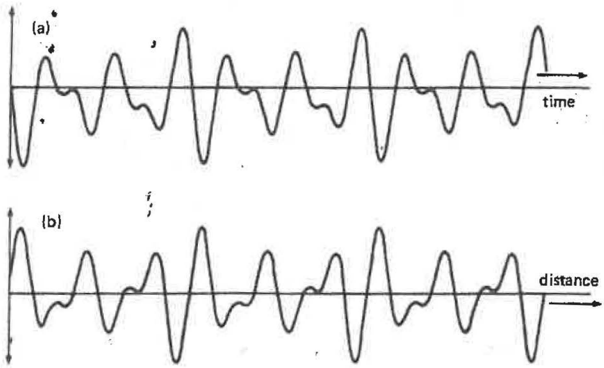

FIGURE 8-1
(a) Time graph of the vibrations of a musical source of sound or of the vibrations imposed on the eardrum by incident sound waves. (b) A wave graph of the same sound as the waves travel with a speed *V* to the right.

8.2 Partials and harmonics

The simplest waveform is a sine wave, usually drawn as a time graph of simple harmonic motion. See Figures 2-4, 2-5, and 2-6. A time graph of one of the prongs of a tuning fork, the waves transmitted through the air to an observer, and the vibration the waves impose upon the eardrum serve as good examples of this. See Figure 3-5. Any vibrating body that rapidly executes simple harmonic motion in air emits a sinusoidal sound wave. This sound wave is referred to as a pure tone, although the aural perception of even a pure tone is impure (see Section 15.2). Actually, nearly all tones produced by musical instruments, and other sources in general, are not pure tones but mixtures of pure-tone frequencies called **partials**. The lowest such frequency is called the **fundamental**. All partials higher in frequency than the fundamental are referred to as **upper partials**, or **overtones**.

In special cases, the frequencies of these overtones are exact multiples of the fundamental and are called **harmonics**. If we designate the frequency of any fundamental by f, all higher harmonics are designated by $2f$, $3f$, $4f$, $5f$, and so on. If, for example, we select a fundamental frequency of 200 Hz and call it the first harmonic, it and its higher harmonics are given by

First harmonic: $1f = 200$ Hz
Second harmonic: $2f = 400$ Hz

Third harmonic: $3f = 600$ Hz
Fourth harmonic: $4f = 800$ Hz

and so forth.

If singing voices, or different musical instruments, sound notes of the same pitch and loudness, we recognize the pitch as that of the fundamental, but the timbre or quality of each note differs from the others by virtue of the relative amplitudes of its partials. In most cases, particularly with the percussion instruments, the upper partials (overtones) are not exact multiples of the fundamental frequency. Such an overtone is called an **inharmonic partial**, and the combined tone is often unpleasant.

Most musical tones are composed of harmonics. In fact, the entire musical scale, as played by most musicians today, is based on a scale of harmonics. (See Chapter 14.) With these principles in mind, we begin our study with the combination of two pure tones, combine them, and find their *resultant waveform.*

8.3 Two pure tones in unison

If two pure tones of the same frequency are sounded simultaneously, and both waves arrive at the listener's ears, the resultant vibrations will have the same frequency. Such sources are said to be vibrating in unison. This is illustrated in Figure 8–2 by the combination of two SHMs, each having a frequency of 833 Hz and a period of 12×10^{-4} s but with different amplitudes, $a_1 = 8 \times 10^{-7}$ m and $a_2 = 6 \times 10^{-7}$ m, respectively. Vibration (a) has an initial phase angle $\phi_0 = 0°$, and vibration (b) has an initial phase angle $\phi_0 = +90°$. See Section 2.5 and Figure 2–6.

Since the frequencies are equal, the graph points p_1 and p_2 move around their circles of reference in the same time, always keeping the same phase angle difference of 90° between them. As a consequence, their resultant amplitude A always has the same magnitude of 10×10^{-7} m and an initial phase angle of $\phi_0 = 37°$. The amplitudes a_1 and a_2 are *added vectorially* in the left-hand side of diagram (c).

Each of the graph points p_1 and p_2, as well as the *resultant* graph point P, is seen to move once around its respective circle in the same time, and the SHMs along the y-axis trace out sinusoids with the period T. It will be observed that the vertical lines from 0 to 12 show that, at all points in time, the vertical displacements of curve (c) are always equal to the sum of the displacements of curves (a) and (b). The three time graphs are superposed in Figure 8–3. We conclude from this result that the combination of two SHMs of the same frequency will always give rise to a resultant SHM of the same frequency, but with a resultant amplitude that depends upon the two amplitudes and their phase angle difference.

This same principle is illustrated for two vibrations of the same frequenstructure of the same note played by two violinists, for example, will not be identical, for various reasons. No two instruments are exactly alike structurally, and no two musicians will bow their strings in exactly the same way. While the harmonic structures will all be slightly different, each one will, of course, sound like a violin. The sound spectrum of each note will have a fundamental, as well as the appropriate harmonics, but will vary slightly from one instrument to another.

We have seen in the previous section that the fundamentals of a group of violins will not have the same phase angles and that in general they will be random. It is also reasonable to assume that all musicians will not produce exactly the same frequency. This means that beat notes of different frequencies will be produced between fundamentals, between second harmonics, between third harmonics, and so on, and these will make the overall waveform from the group of violins more complex. The sound quality produced by the combined frequencies from a number of instruments of the same kind, playing the same note, is called the chorus effect. Although the primary purpose of using a number of violins in the string section of a symphony orchestra, for example, is to obtain a loudness balance with the other orchestral instruments, the chorus effect contributes to the overall richness of the musical sound.

8.6 Composition of first and second harmonics

Let us assume that a musical instrument sounds a tone in which the first and second harmonics, and no others, are present. The same resultant vibrations at the ear can be produced by sounding of one of the pure tones by one in-

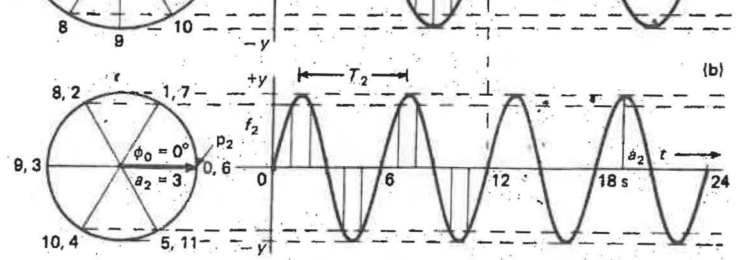

FIGURE 8–5 Time graphs for the generation of two SHMs with initial phase angles $\phi_0 = 0°$: (a) frequency f and amplitude 4, (b) frequency $2f$ and amplitude 3.

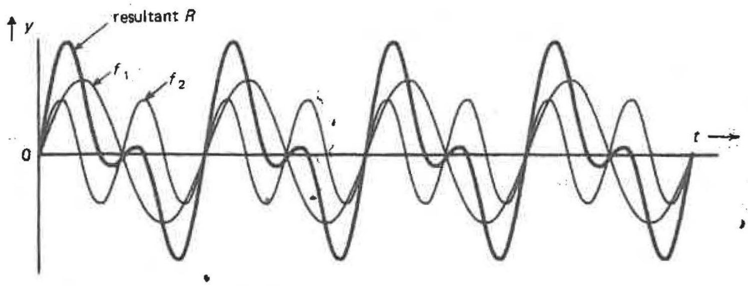

FIGURE 8-6 Composition of the two SHMs in Figure 8-5 showing the resultant *R* in relation to the amplitudes a_1 and a_2 of the separate components.

strument and the other pure tone by a separate instrument. These two SHMs, with frequencies in the ratio of 1 to 2 and initial phase angles both zero, are given graphically in Figure 8-5. The sum of the two displacements f_1 and f_2 (light lines) is shown by the resultant vibration *R* (heavy line) in Figure 8-6. Since f_2 has twice the frequency of f_1, the graph points p_2 and p_1 (on the circles

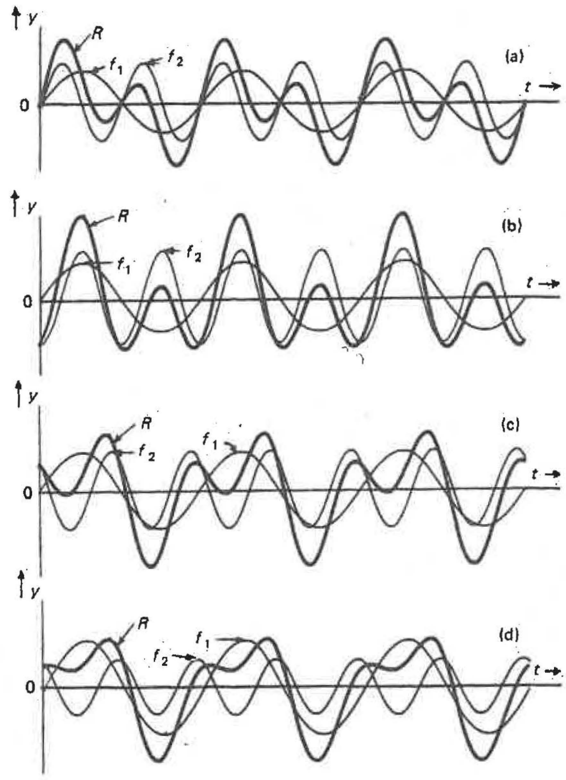

FIGURE 8-7 Time graphs combining the first and second harmonics of a fundamental frequency f_1 to form a resultant. Both frequencies f_1 and f_2 have (a) the same initial phase angles but different amplitudes, (b) different initial phase angles and different amplitudes, (c) different initial phase angles but equal amplitudes, and (d) different initial phase angles and different amplitudes. All four *resultants* (heavy lines) have different shapes but reveal the same two frequencies f_1 and f_2.

of reference) rotate with frequencies in the ratio 2 to 1. The second harmonic makes two vibrations for every one of the first harmonic. For this example, the periods are assumed to be 12×10^{-4} s and 6×10^{-4} s, corresponding to frequencies of 833 Hz and 1666 Hz, respectively. The resultant R in Figure 8–6 is obtained by adding the vertical displacements of f_1 and f_2 at each instant of time and drawing a smooth curve through them.

If the relative amplitudes are changed without changing the initial phase angles, we obtain curves of a different shape. Changing the relative amplitudes and the initial phase angles also changes the resultant curve. Typical graphs with such changes are shown in Figure 8–7. It should be pointed out that these are but a few of the infinite number of resultant vibration patterns that can be drawn. See Figure 15–5 for others.

8.7 Two, three, and four harmonics

Suppose we sound a pure tone of any given frequency, and then, one after another, we add the second, third, and fourth harmonics. The quality of each combination will depend upon the relative amplitudes, while the resultant vibration pattern becomes progressively more complex and, in many cases, more pleasant to hear. (Two consonant notes sounded together are called a *dyad,* three notes a *triad,* and four notes a *tetrad.*)

As an example, let us choose a first harmonic, or fundamental, of 833 Hz, followed by the second, third, and fourth harmonics. Let the relative amplitudes of the four harmonics be $a_1 = 8$, $a_2 = 6$, $a_3 = 4$, and $a_4 = 6 \times 10^{-7}$ m, and let the initial phase angles be $\phi_1 = 90°$, $\phi_2 = 45°$, $\phi_3 = -90°$, and $\phi_4 = -45°$. Graphs of these combinations are given in Figure 8–8. It can be seen that, as harmonics are added, the resultant vibration curve becomes more and more complex, and, in general, the tone becomes richer in quality.

8.8 Wave generation

The separation of any sound into its various components can be accomplished by mechanical or electronic devices called analyzers, and any set of components can be recombined to produce the original sound by similar mechanical or electrical devices called synthesizers.[2] In 1622 the French mathematician Fourier showed that it was possible to break down any complex periodic curve into a series of sinusoids whose frequencies are harmonically related. Stated another way, any periodic waveform can be constructed by combining a sufficient number of sine waves. This is called Fourier's theorem. This means that any periodic sound wave of arbitrary waveform will act acoustically as a combination of pure tones. While we will not go into the mathematics, we will graphically add, or synthesize, a number of SHMs to form several special vibration forms used by electronic engineers in the development of oscilloscopes and television receivers, by audio engineers in their development of electronic music devices for special effects, and by manufacturers in producing musical instruments of various kinds.

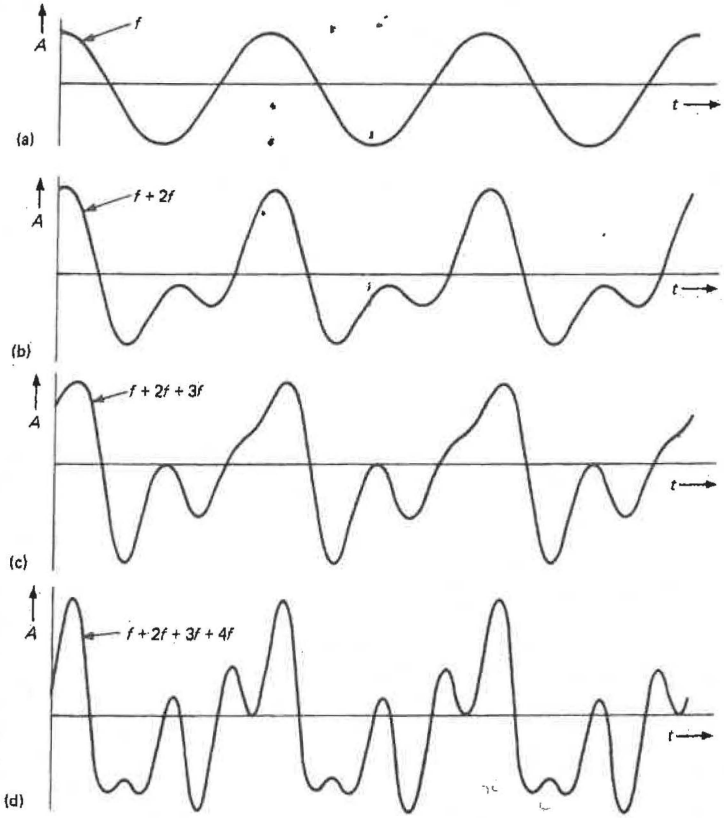

FIGURE 8-8 Time graphs for the addition of the first four harmonics of a given fundamental frequency *f*. Vibration modes for (a) the fundamental alone, (b) the first and second harmonics together, (c) the sum of the first, second, and third harmonics together, and (d) all four harmonics together.

The four simplest waveforms, or vibration forms, in common use in synthesizers today are called (a) sine waves, (b) sawtooth waves, (c) square waves, and (d) triangular waves. See Figure 8-9. Diagrams (b) and (d) belong to a family of straight-line forms called ramp waves. All four of these waveforms can be produced with relatively simple electronic circuits. Since the analysis of complex waves can be broken down into sine waves, and the synthesis of a number of sine waves (harmonics) can be compounded to produce complex waveforms, we can apply Fourier analysis—that is, a series of