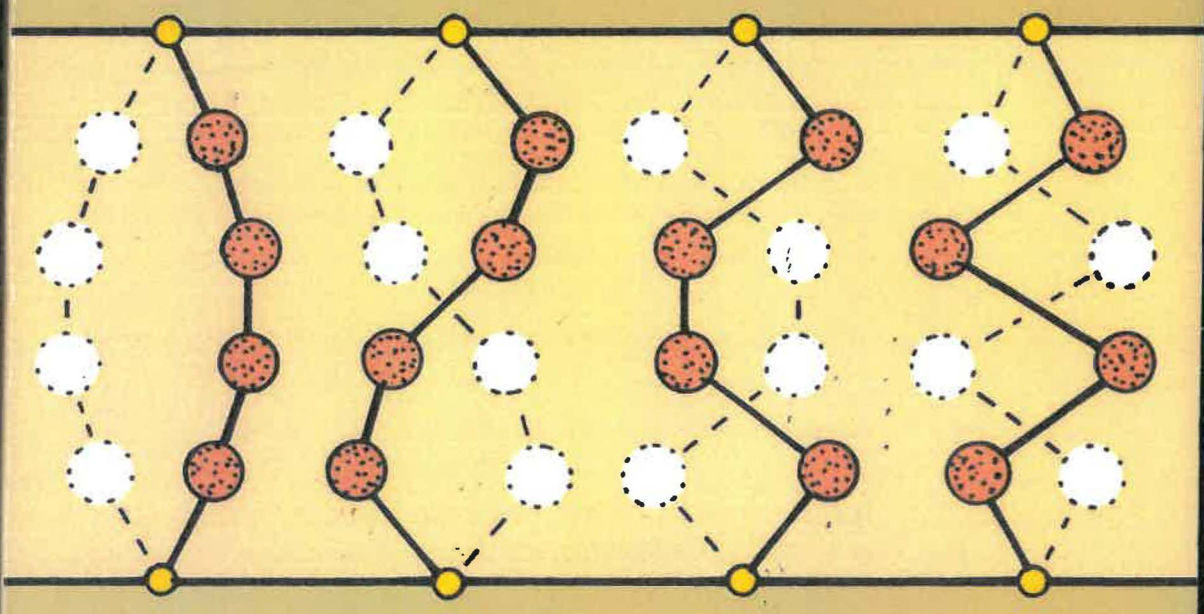
FUNDAMENTALS OF MUSICAL ACOUSTICS

Second, Revised Edition



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notice some of the remarkable implications of the first of these observations.

5.5. Sounds Having Whole-Number Frequency Ratios

Let us imagine that we have available to us a hypothetical string which, when plucked or struck, vibrates in a family of characteristic damped sinusoidal oscillations whose frequencies are arranged in an *exact* whole-number relation; that is, Q is exactly 2P, R = 3P, S = 4P, and so on.

Digression on the Numerical Labeling of Natural Frequencies.
We can express this whole-number relationship between oscillation frequencies very compactly as follows. If we use the letter n to stand for any one *of the integers—that is, n = 1, or 2, or 3, etc.—and if the characteristic frequencies are given the serially numbered names* f_1, f_2, f_3 *instead of our alphabetical names, then the nth one of these frequencies can be referred to as* f_n. *The desired integer relation between the successive string frequencies can be written in a mathematically tidy fashion as follows:*

$$f_n = nf_1$$

One reads this mathematical sentence thus: "f sub-n is equal to n times f sub-1," meaning that the nth frequency is n times as large as the first one in the set.

In the language of chapter 2, we can say that the repetition rate for any one of our idealized string's sinusoidal oscillations is a whole number times the repetition rate associated with its lowest frequency oscillation. Let us look into what happens when account is taken of the fact that the string is actually vibrating with a whole set of integrally related frequencies.

Suppose that for conceptual simplicity we assign an imaginary drummer to each characteristic oscillation of our string, giving him the job of tapping with a repetition rate equal to that measured for his "own" string oscillation. The whole-number relation between the string frequencies then requires that the drummer assigned to keep time with the second characteristic oscillation should beat twice as fast as drummer number 1. Similarly drummer 3 taps three times as fast

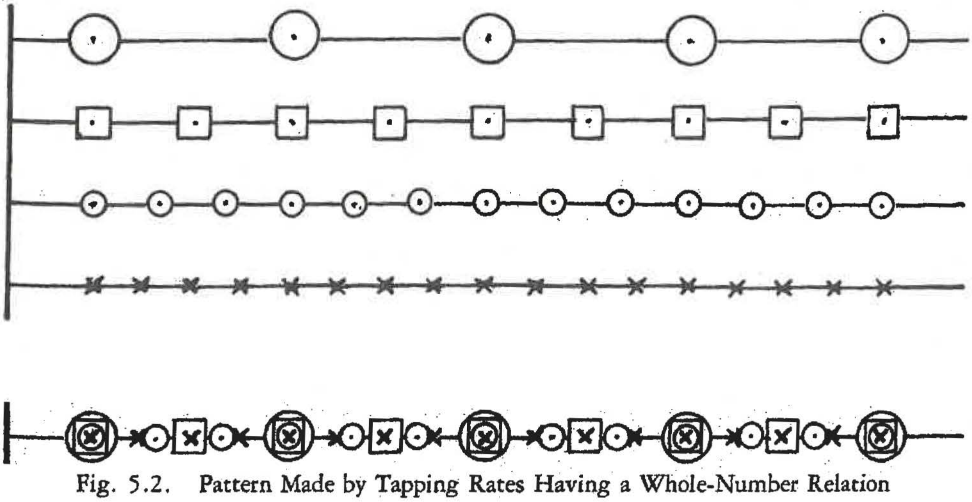

Fig. 5.2. Pattern Made by Tapping Rates Having a Whole-Number Relation

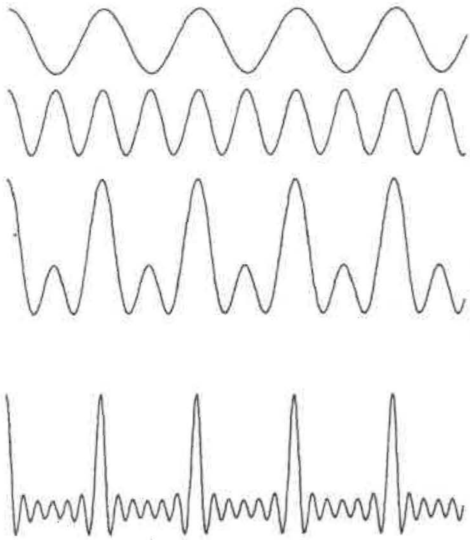

Fig. 5.4. Combination of Sinusoids Having a Whole-Number Frequency Relation

as drummer 1, and so on. The upper four lines of figure 5.2 show the timing of the successive taps produced by the first four of our set of drummers. The bottom line of the diagram shows the resulting rhythmic pattern that one would hear. Every drummer strikes in unison with the blows of drummer 1, giving a strongly marked beat, and drummers 2, 4, ... strike at the midpoints between these accented taps, giving a somewhat less accented tap. The important thing to notice is that the repetition rate of the *complete* rhythmic pattern produced by the composite set of tappings is exactly the same as that of the lowest frequency member (see sec. 2.3, "Repetition Rates of Rhythmic Patterns"). Musicians should not find this idea hard to understand if they compare my explanation above with what they would expect from a rhythmic pattern written out as in figure 5.3.

Let us look now at some examples using sinusoidal disturbances instead of drumbeats. The top two parts of figure 5.4 show sinusoids whose frequencies differ by a factor of two. If our simplified string could be excited by some means that sets into motion only the first two of its characteristic oscillations, then the oscilloscope picture produced from a microphone in its neighborhood would look

Fig. 5.3.

something like the curve shown in the third part of the figure. This curve is produced by the addition of the two curves immediately above it. Notice that the repetition time of the somewhat spiky composite curve (and hence its repetition rate) is exactly that of the f_1 component at the top of the diagram. The bottom part of the figure shows the result of combining additional sinusoids, so that the curve is that belonging to the sum of the first six oscillations in our specially chosen set.

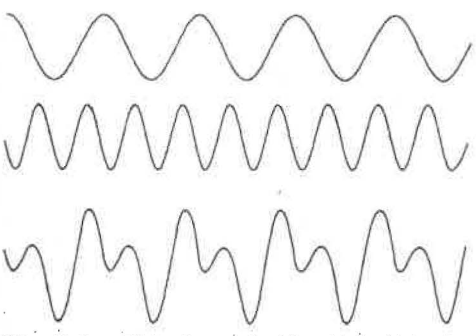

Fig. 5.5. This figure is identical with the upper three sections of figure 5.4 except that the second component has been displaced. Note that the repetition time is unaffected by this change.

Figure 5.5 shows a slightly modified version of the upper three sections of figure 5.4. This time the f_2 component is "slid over" in time so that it no longer has every second upward excursion coincident with every upward excursion of the f_1 component. We notice that the summation of these two oscillations gives a resultant pattern whose shape is different from the one obtained before, but once again we see that the repetition time is equal to that of the lowest frequency (f_1) oscillation.

Adding components whose frequencies are in whole-number relationships has shown us something that will prove to be very important to our understanding not only of the physical basis of tone color but also of the special relationships between notes which underlie formal music all over the world. Let us set down some of the properties of the class of sounds that would be made by our hypothetical strings.

1. No matter what the strength of excitation of the various oscillations, the repetition rate for the whole signal as it reaches a microphone (or our ears) would be exactly that of the lowest frequency sinusoidal component that is characteristic of the string.

2. Because the net repetition rate of the vibration is independent of how or where the string is struck, one would always get the same perceived pitch sensation for the string sound. This means that *the pitch is unambiguous.*

Digression: Sounds with Only Even Harmonics.

In the strictest of logic, one might ask about a possible inadequacy of item 1 above. Imagine an ingenious excitation method that fails to excite the odd-numbered oscillations, so that only f_2, f_4, f_6, . . . are present. These may be written out as follows:

$$f_2 = 2f_1 = 1 \times (2f_1)$$
$$f_4 = 4f_1 = 2 \times (2f_1)$$
$$f_6 = 6f_1 = 3 \times (2f_1)$$
etc.

This shows that our new set of frequency components is itself constructed out of integer multiples of a new basic frequency whose value is ($2f_1$). The repetition rate is therefore doubled, and the whole game begins again. We would perceive this altered sound as having a pitch one octave higher than the normally excited one.

As a practical matter, it is not particularly difficult to arrange peculiar excitations of the sort described in the preceding paragraph, and if one were *to meet such a situation it could easily be recognized as such with the help of simple auxiliary experiments. One would need only to pluck or strike the string at random spots once or twice in order to find out the true nature of the string.*

There is something intellectually very attractive about the apparent simplicity of sounds made up of components having integer frequency ratios, and it is easy to devise lengthy numerological games based on their presumed properties. Before we fall into this trap, however, it would be advisable to find out whether such sounds can in fact be generated. If such sounds can be generated, we then must ask whether our ears and nervous system deal with them in a way that corresponds at all with experiencing the sounds from real strings. The first question can be answered affirmatively in two ways:

1. A truly uniform slender string of suitable material, stretched tightly enough between sufficiently rigid supports, will produce sounds whose components have frequencies that are in very nearly perfect integer relation. The sounds from such a string differ only subtly from those produced by a string vibrating under less formalistic conditions. That is, nothing drastic happens to the perceived sound as long as the string has *nearly* integer frequency relations.

2. We find that there is a large class of familiar sound sources that normally produce sounds whose frequency components are found to be related in the *precisely* whole-number manner that we postulated for our hypothetical strings. Examples of sources of this kind are very common. The human voice is the most familiar one, while the woodwind and brass instruments join with the violin family to provide orchestral examples. These diverse sound sources have one common element in their nature that sets them apart from the bells, chimes, and strings we have considered so far. Instead of simply ringing (and decaying away) in response to an impulsive stimulus, all of these instruments are capable of producing *sustained* sounds. They are devices that are capable of converting the steady flow of air from a man's lungs, or the steady motion of the bow in his hand, into the oscillatory vibrations which give rise to the sound we hear. We shall see in a later chapter that only under very special circumstances can such devices be persuaded to maintain steady oscillations whose frequency components are *not* in an exact whole-number relation to the basic repetition rate.

It turns out that the vast majority of our musical listening experiences are with sounds whose frequency components are in exact whole-number relation, or very nearly so. It is not surprising, then, that the formal structure of music (wherever it has developed over the world) is strongly influenced by the properties of sounds each of which has whole-number relations among its components. We also find that many subtleties in music arise through the slight *in*harmonicities which are present in the tones of some instruments.

This book has opened with an investigation of impulsive and heterogeneous sounds from struck objects, not only because of the simplicity of initial exposition but also as a means for underlining the special nature of the sound-producers that man has selected for his musical activities. It is time therefore to return to the sounds of bells and chimes in order to compare them with the sounds of plucked or struck strings.

5.6. The Pitch of Chimes and Bells: *Hints of Pattern Recognition*

We have found that the characteristic frequencies that make up any one sound from any one of the commoner orchestral instruments are arranged as exact (or very nearly exact) integer multiples of a certain basic frequency. It is this basic frequency component that determines the repetition rate of the sound we hear and also, as we have learned, its musical pitch. Let us use this knowledge to help ourselves gain some understanding of the way in which we assign pitches to chimes and bells, whose characteristic frequencies do not arrange themselves in whole-number relationships.

Digression on Terminology: Some Partials Are Harmonic.

It will save a great deal of circumlocution if we provide ourselves with some terminology carefully chosen for the description of the various components making up the sound we are dealing with. First of all, in any sound made up of sinusoidal components, we will continue to assign identifying letters from the latter part of the alphabet, or serial numbers, assigning them according to their order, beginning at the lowest one. That is, we will call these frequencies P, Q, R, . . . or f_1, f_2, f_3, *. . . Sometimes it will be useful to refer to these components as the* partials *of the sound in question. When this word is used, we will understand that no particular relationship is to be assumed between the frequencies of these partials; their frequencies may or may not have a whole-number relationship. These components will still be referred to by their serial numbers as* first partial *(referring to the component labeled P or* f_1*),* second partial *(also known as Q or* f_2*), etc.*

We turn now to the special case of sounds in which the frequencies of the various sinusoidal partials are whole-number multiples of some basic repetition rate. The sinusoidal component whose frequency matches that of the repetition rate will be referred to as the fundamental *component, and its frequency as the* fundamental frequency. *It is often referred to also as the* first harmonic. *The partial whose frequency is exactly double that of the fundamental will be said to have a frequency which is the* second harmonic *of the fundamental frequency. Similarly we will say that sinusoidal oscillations running at three times the fundamental frequency are vibrating at the* third harmonic *of the fundamental frequency.*

We will have to be very strict in our terminology or endless confusion can result. The word harmonic *is to be used* only *when we mean to imply an exact whole-number frequency relationship. To help make things clear, we may notice that the partials of a guitar string have frequencies which are very nearly, but not exactly, harmonics of the frequency of the first (lowest) partial.*

We learned earlier in this chapter that musically experienced people won't necessarily agree on what pitch to assign to the sound of a grandfather clock chime. In the context of our present understanding of musical sounds, we may wonder whether the frequencies of the chimes' partials can be recognized by our nervous system as belonging to two differently organized sequences of harmonics. That is, can we find hints of a series of harmonics whose fundamental corresponds to the approximate F_3 that some listeners hear? Similarly, can we detect signs of a harmonic series whose fundamental implies the pitch just above the C_5 perceived by others? In our earlier investigation of this sound we recognized that the second partial has a frequency consistent with one of these pitch assignments while the two closely spaced partials (which were labeled R_a and R_b) are associated with the other one. Our earlier difficulty stemmed from our inability to dispose of all the other partials making up the tone; could these be members of harmonic series based on the assigned pitches?

Figure 5.6 shows the frequencies of all the partials up through f_4 (S) laid out as dots along a frequency scale. Above the frequency axis of this diagram we see a pair of arrows located at frequencies corre-

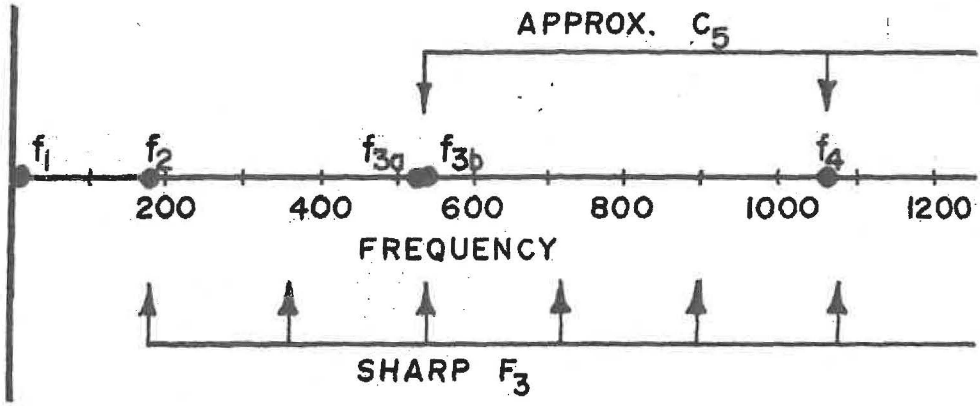

Fig. 5.6. Assignment of Pitches to Sound from a Clock Chime

sponding to a fundamental, belonging to the note C_5, and its second harmonic. The fundamental arrow is pointing at the pair of Rs, while the arrow for the second harmonic points almost exactly at the measured S. It seems possible, then, that our ears can seize on the relationship of these two strong components and accept them jointly as the two lowest members (fundamental and second harmonic) of a set of partials belonging to a sound whose pitch is near C_5.

Below the frequency axis we find in similar fashion a set of arrows indicating the frequencies making up the set (fundamental and its harmonics) belonging to the sharp-pitched F_3 which we associated with the measured f_2 (which was labeled Q earlier). This time we find that the arrows corresponding to the fundamental, the third, and the sixth harmonics point very nearly at the dots indicating the measured components Q, R, and S.

Our search for integer relations among the frequency components of a struck chime rod has been reasonably successful, in that it gives results that seem consistent with the *hypothesis* that our ears assign pitch (when possible) on the basis of any whole-number sequences they can find.

We turn our attention next to the bell sounds, to see whether they give any support to our hypothesis that pitch is assigned on the basis of approximately whole-number frequency relationships. The individual lines of figure 5.7 show the frequencies of the first five partials for the first five bells in the Terling Peal, laid out by means of dots on a frequency axis in exactly the same way as was done for the chime rod. The dashed vertical lines appearing on the diagram indicate the fundamental repetition frequency and its harmonics belonging to a reference sound whose pitch matches that of the bells as made uniform, by a variable-speed recording device.

Inspection of the line corresponding to bell 1 shows that the first partial (marked P) has a frequency quite close to that assumed for the fundamental. Furthermore, we see that partials 4 and 5 agree extremely well with harmonics 3 and 4 of the pitch reference tone. We note that partials 2 and 3 do not seem to agree with any member of the reference harmonic series.

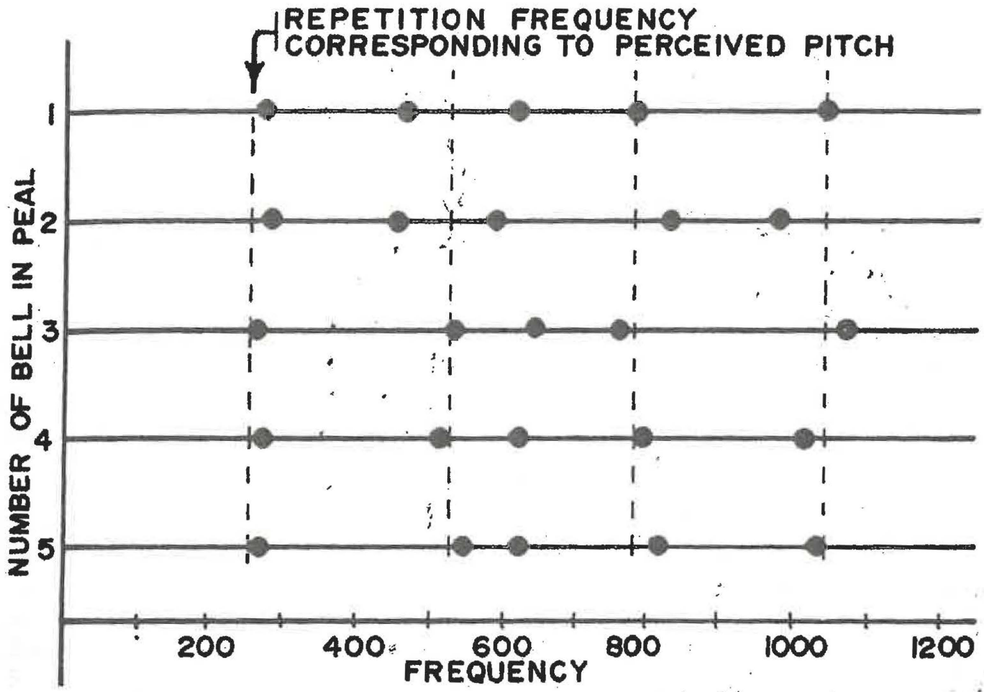

Fig. 5.7. Frequency Components of Bells Adjusted to the Same Pitch

Skipping now to bells 3, 4, and 5, we find that partials 1, 2, 4, and 5 agree quite well with the fundamental and harmonics 2, 3, and 4 belonging to our pitch reference. Partial 3 never seems to fit in. Bell 2 does not show such a clearcut relation, although the frequencies of partials 1 and 4 are roughly equal to those of the fundamental and third harmonic of our reference sound. Interestingly enough, most listeners feel quite uneasy about assigning pitch to this bell, even though they find no difficulty with the other ones.

Looking over the data we come to realize that for a bell to have a reasonably well-defined pitch (so that it can be matched with a normal sort of tone having harmonic partials), our ears do not demand any particular set of component frequencies from it. That is, our ears do not demand that the same (only approximately harmonic) partials serve identically as the "pointers" in the sounds for all the bells. All that is required is a sufficient number of sufficiently consistent clues. The frequencies of the skillet clang listed on p. 43 are similar to these bell sounds in that they are not harmonically related.

5.7. Another Pitch Assignment Phenomenon: *The Effect of Suppressing Upper or Lower Partials*

In the previous section of this chapter we found ourselves thinking about the ways in which our ears respond to sounds fed to them from bells and chimes. We noticed that the act of assigning pitch to