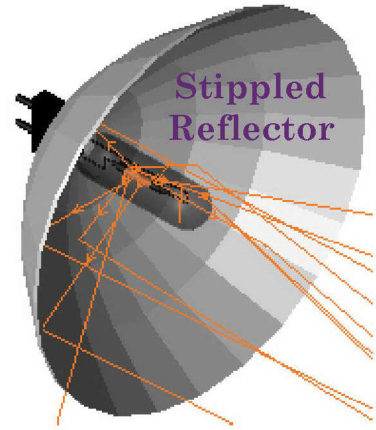
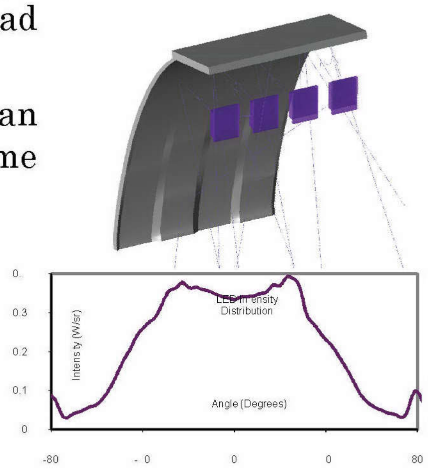
Faceted Reflector Design

Essentially there are two design procedures for **faceted reflectors**: those based on the **tailored-edge-ray** method and those that provide a **stippled illumination pattern**. Stippling means that the target irradiance distribution is created from the overlap of the light from different segments of the reflector. This washes out any structure that could be imaged from the sources, such as a filament and its supports. Thus, the designer builds a basic reflector shape, such as parabolic, and then replaces the one smooth reflector with a series of flat, areal segments. This type of faceted reflector can be found in LCD and overhead projectors.





Tailored-edge-ray reflectors can also use this effect with some added benefits:

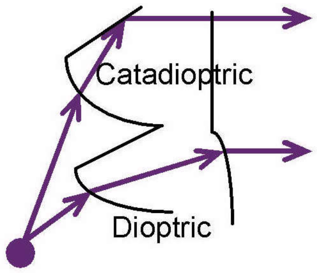
- Energy conservation restrictions mean the reflectors grow large, but faceting allows the shape to be "restarted" to minimize the overall volume.
- Facets can individually address different por-tions of the desired target distribution.
- Tolerancing is improved since various allowances can be incorporated as a function of segment position.

These reflectors are typical in the automotive headlight industry and are increasingly used in other applications. This example (LED) shows the utility of faceting. The LEDs' intensity distribution pattern, along with uniformity at the target, gives the reflector shape shown here.

Advanced Nonimaging Optic Design

There are a number of advanced nonimaging design algorithms, such as **nonimaging Fresnel lens design**, **nonedge-ray design**, and **simultaneous multiple surfaces method (SMS)**. Nonimaging Fresnel lens design is used in lighthouses, solar concentrators, traffic lights, and automotive lamps. The in-expensive, small-volume optics are thin dielectrics, plastic or glass, with two types of Fresnel elements:

- **Catadioptric:** uses two refractions and one TIR to bend the light in the desired direction.
- **Dioptric:** uses two refractions to bend the light in the desired direction.



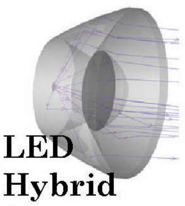
Dioptrics are used toward the lens center while catadioptrics are used once Fresnel losses become large. The TIR condition reduces the angles of incidence on the two refractive surfaces.

Nonedge-ray design follows the equations of tailored-edge ray design but adds two additional factors:

- System performance criteria drive optimization; and
- Multiple extended-size sources are allowed.

This design method trades between system performance and transfer efficiency from the source to the absorber. It is used in multiple small-source applications, such as LED lighting and diode-laser pumping.

SMS provides for multiple ray paths from the source to points on the to-be-generated optical surfaces of the device. Refraction, reflection, and TIR are used in conjunction to generate the multiple surfaces and provide the optimal output angular spread from the optic. SMS is part of a family of optics called **hybrid optics** that use many different optical phenomena for their operation. A primary example is the pseudo-collimating lenses used for high-brightness LEDs.


LED Hybrid Optic

Displays—Overview

A multitude of existing **displays** incorporate different illumination strategies to provide a lit screen. Optical display technologies include **backlighting**, **projection**, and **organic LED (OLED)**.

Backlit displays use large liquid crystal (LC) modules that are lit from the rear by small sources coupled to a TIR element that spans the extent of the screen. The TIR is frustrated by structures placed on a surface of this element. Sources used in backlit displays include **cold-cathode fluorescent lamps (CCFL)** and LEDs, in which the ejected light proceeds through many additional layers, including polarizers, the LC, and diffusers. Additional layers may include a **brightness enhancement film (BEF)**, which recirculates ejected light until it is in the desired angular range. The next few pages describe the components of a backlit display in more detail.

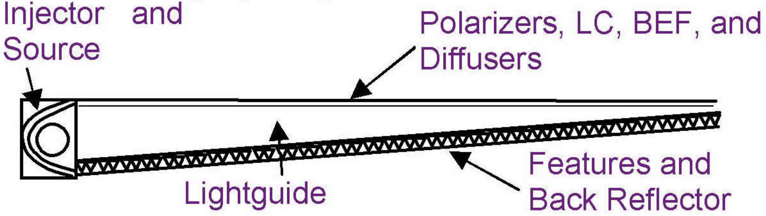
Projection displays use smaller **SLMs** in different spectral ranges to multiplex a full-color image. The illumination components include a broadband source (e.g., a narrow-gap arc lamp or LED), a reflector to capture the emitted radiation, lenslet arrays (often called fly's eyes), and dichroic filters to separate the light into the desired spectral ranges (typically red, green, and blue). There are both **front-projection displays** and **rear-projection displays**. Front-projection displays use distinct spectral channels to illuminate the screen; however, this increases cost and can reduce tolerances. Rear-projection displays fold the system in order to maintain a smaller display depth. Projection displays are discussed in more detail later.

Unlike backlit and projection displays, **OLED displays** deposit pixel emitters onto a substrate. These emitters provide both the illumination and display information, so the design demands for the illumination engineer are negligible. OLED modules can be used in projection displays.

Backlit Display Components

Standard components of a **backlit LCD** include:

- **Source:** Typically CCFL, LED, or **electroluminescent (EL)**.
- **Injector:** A specular or diffuse reflector that captures and injects the light into the lightguide.
- **Lightguide:** A dielectric, typically acrylic, that captures the injected light via TIR. Features are placed on the backside of the lightguide to break the TIR condition. The lightguide is also wedged using decreasing thickness with increasing distance from the injector.
- **Features:** Paint patterns or geometric structures to frustrate the TIR. The density and/or depth of the features increases with distance from the injector to provide uniform illumination over the screen. The geometric structures can be holes (extending into the lightguide) or bumps (extending out of the lightguide).
- **Back reflector:** A diffuse or specular reflector placed below the features to capture and recirculate any light that is emitted from the lightguide backside.
- **Polarizers:** Two crossed linear polarizers placed on the display output side with an LC placed in between.
- **Liquid crystal:** Sandwiched between the two crossed linear polarizers to rotate the polarization by 90 deg for a pixel that has information content. Closely placed pixels provide for the color content (e.g., three pixels to provide red, green, and blue).
- **Diffusers:** Sometimes placed on the output side of the lightguide to provide better angular uniformity from the display.
- **Brightness enhancement film:** A microstructure, such as a prism, to select a desired angular output range while the higher angular content is recirculated to increase display brightness.



Backlit Display: Source and Injector

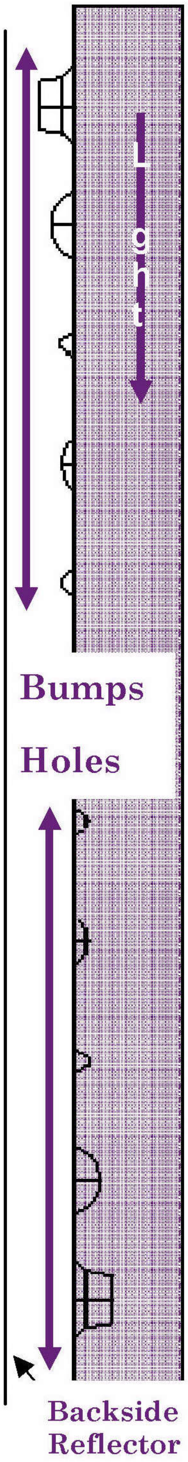
Small sources are preferable for backlights to reduce the overall display volume. The source can be located to the side of a lightguide or placed directly behind the polarizers and LC. The former allows for thin displays at the expense of lightguide complexity; the latter increases the depth due to the removal of the lightguide, and careful design is required to provide uniform luminance. Three standard sources are used: **CCFLs**, **LEDs**, or **EL** films. CCFLs are small-diameter lamps that run the length of one or more sides of the display. For LED backlights, multiple LEDs are used to provide the required luminance level. They can be white-light emitters, a combination of multiple colors (e.g., red, green, and blue), or a combination of color-emitting LEDs and white-light emitters. EL films provide single-color background displays with information shown in black. Examples include watch faces and automotive dashboards. They work by passing current through the EL material, which then emits spatially uniform Lambertian light. EL backlights have no need for a lightguide because the EL is mated to the back of the LC-polarizer module.

An **injector** is standard for any type of backlighting scheme. For a backlight whose source is located to the side of a lightguide, either diffuse or specular reflectors are placed around the source to better capture the emitted radiation. Standard shapes for a CCFL include spherical, parabolic, and elliptical troughs. For LEDs, dielectric (especially acrylic) couplers akin to the hybrid optics presented are used. The output aperture of the injector is mated to the input aperture of the lightguide. For backlighting without a lightguide, reflectors are often placed around the sources to assist in directing the light and to provide uniform luminance from the display. The simplest case is the **lightbox**, which is a highly reflecting, diffuse material placed around the sources over the extent of the screen backside. Lightboxes are analogous to integrating spheres.

Backlit Display: Lightguides, Features, Reflectors

Plastics are best suited for **backlight lightguides** because they can take advantage of **injection molding**. The thickness at the injector end of the lightguide depends on the screen size, with larger displays requiring thicker lightguides. The lightguide is thinned with increasing distance from the injector. This thinning assists with ejection of the trapped TIR light, because as the lightguide cross-sectional area decreases, the conservation of étendue demands an increase in the angular extent.

Structure or **features** are added to the backside of the lightguide, i.e., on the wedged surfaceside because the backside has more distance for the light to spread over the spatial extent of the display. Initially, paint patterns were used to cause the ejection; however, the paint must undergo a separate and costly process, and the paint spots provide little direct control of the resulting angular distribution. For this reason, replicated geometrical structures are added during injection molding either as **holes**, which extend into the lightguide, or **bumps**, which extend out of the lightguide. Geometrical shapes, as shown in the figure, include hip roofs, spheres, and ellipsoids. The density and/or depth of these features increases with distance from the injector. The design of such feature patterns is from the **diffusion equation**, followed by optimization for improved performance.

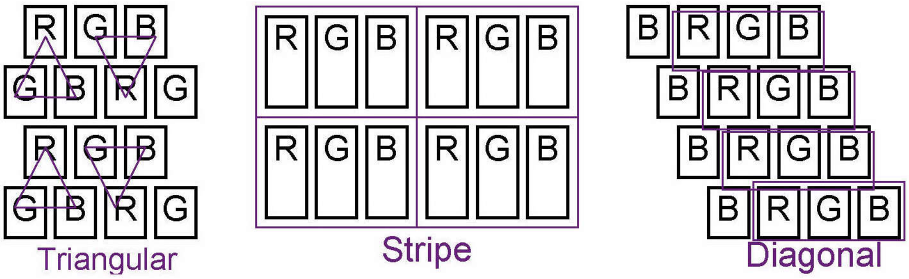


Light leaked through the back feature-side of the lightguide is caught with a reflector, diffuse, or specular, which is placed below the lightguide. The **backside reflector** provides recirculation and better efficiency.

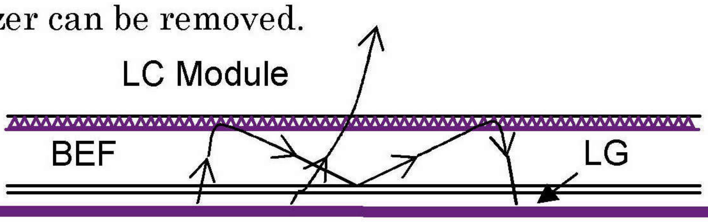
Backlit Display: Polarizers, LC, and BEF

There are a number of additional components that comprise an LCD, including two **linear polarizers** and the **twisted-nematic LC module**. The first polarizer passes linear polarization at one orientation, while the second passes linear polarization orthogonal to the first. The LC is sandwiched in between these two polarizers, and in a pixel's transmissive state, it rotates the light that exits the first polarizer by 90 deg. A pixel in a nontransmissive state absorbs the incident radiation. The LC module also has glass substrates on both sides of the LC, and on each glass substrate there are transmissive **indium-tin oxide (ITO)** electrodes. A **spectral filter mask** is inserted to provide color output from the display. Typically, a three-color mask is used, where neighboring subpixels pass red, green, or blue. The combination of these pixels forms a display through resolution considerations of the viewer. There are a number of **color-pixel patterns** including:

- **Triangular** or **delta:** better for motion pictures;
- **Stripes:** better for television; and
- **Diagonal:** better for motion pictures.



A **BEF**, a replicated structure of microprisms, recirculates emitted light until it is in the desired angular range. The BEF is situated just below the polarizers and LC. A **dual brightness enhancement film (DBEF)** incorporates the polarization into the optic such that the first linear polarizer can be removed.



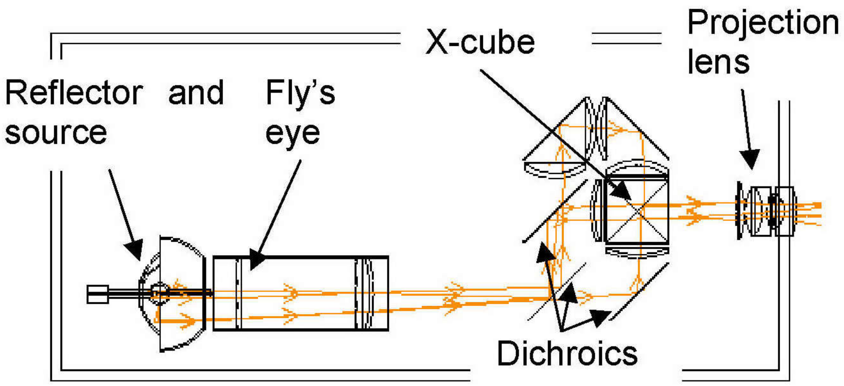
Projection Displays

Projection displays typically use three channels— red, green, and blue—to develop the object to be displayed by the projection lens. Each of the channels uses a **spatial light modulator (SLM)** to generate this object. One-channel systems use color filter wheels to temporally generate the scene. There are essentially three options for the SLM:

- Transmissive LCs akin to those used in backlighting;
- **Digital light processing (DLP)** modules, which incorporate millions of micromirrors over their surface area; or
- Reflective LCs, such as **liquid crystal on silicon, (LCoS)**, which integrate the LC with the circuitry.

The SLMs are microdisplays that use magnification from the projection lens to generate the screen image. An X-cube combines the three spectral channels, and the resulting "object" is projected onto the screen.

The illumination components of a projection display include the source, a reflector, and fly's eye lenses and/or straight lightpipes. The source is typically a narrow-gap arc or even LEDs. The reflector (conic, edge-ray, or faceted) is specular, and it captures most of the source emission. The fly's eye provides better spatial uniformity over the SLMs by creating several images of the source. The lightpipe mixes the light to provide better spatial uniformity. The overall illumination system is typically arranged in a Köhler scheme to hide the source structure.



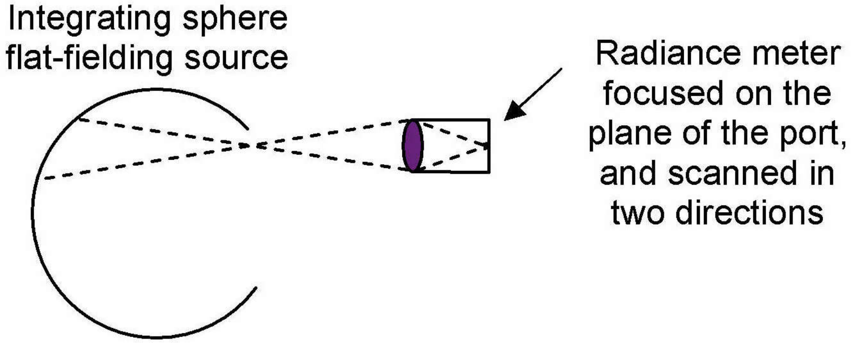
Mapping Flat-Fielding Sources

High-performance camera systems such as airborne and satellite cameras generally go through a process known as **flat-fielding**. The camera is presented with a large-sized extended light source that has nearly perfectly uniform radiance. Since the flat-field source is uniform, any pixel-to-pixel nonuniformities in the camera are inherent to the camera and can be remedied with image processing.

Generally, flat-fielding sources are realized by internally illuminated integrating spheres. Spheres with exit ports of about a 50-cm diameter are common. Ports of over a meter in diameter are sometimes needed depending on the aperture of a single large camera or the combined apertures of an array of smaller cameras. Radiance uniformities of 98% or 99% or better are the norm.

To verify that the flat-fielding sources have been designed properly and that there are no deficiencies in their manufacture, they are mapped for radiance uniformity. The mapping is done with a radiance meter, which is often photopically filtered for no other reason than commercial availability and the desire to band-limit the silicon detector to a region of good sensitivity.

The radiance meter is operated either in a collimated mode or is focused on a small spot in the plane of the exit port. Keeping the viewing direction constant, the meter is scanned in two directions to create a radiance map of the source.



Goniophotometers

Light sources designed to produce useable irradiance (automobile headlamps, roadway luminaires, and interior lighting fixtures) as well as those designed to produce useable intensity (automobile tail lights, traffic signals, aircraft and marine running lights) are all characterized by **goniophotometers**—devices used to measure the directional distribution of light from sources.

A goniophotometer consists of a small detector placed at a distance from the source where **intensity** is meaningful, (i.e., the inverse-square law applies). Except for highly collimated sources such as searchlights, a distance from the source of five to ten times the largest dimension of the source is usually sufficient. The lamp or detector (or a combination of the two) is moved to map the intensity distribution of the source.

Goniophotometers are classified as type A, B, or C depending on how they are constructed. This can be confusing because, in addition to three types of physical construction, there are three variations of spherical coordinates for reporting data that are also called types A, B, and C. These usually, but not always, match the type of goniophotometer used. Details of the three coordinate systems are shown on the next page.

Types A and B goniophotometers are similar in that the luminaire is mounted on a device with horizontal and vertical axes and a distant fixed detector.

Type C goniophotometers move the detector around the luminaire on a horizontal axis and rotate the luminaire on a vertical axis. Sometimes, for large luminaires, involving large distances, the detector is fixed and a large high-quality mirror moves on a horizontal axis, directing the light to the detector.

Type C goniophotometers are necessary for measuring lamps that are sensitive to the burning position.

Types A, B, C Goniometer Coordinate Systems

All are spherical coordinate systems.

Type A spherical coordinates:

Polar axis: vertical
Label on vertical angles: Y
Label on horizontal angles: X
Range of Y: –90 (nadir) to +90 (zenith)
Range of X: –180 (left, from luminaire) to +180
Straight ahead: $Y = 0$, $X = 0$
Primary uses: optical systems, automotive lighting

Type B spherical coordinates:

Polar axis: horizontal
Label on vertical angles: V
Label on horizontal angles: H
Range of V: –180 to +180
Range of X: –90 (left, from luminaire) to +90
Straight ahead: $V = 0$, $H = 0$
Primary uses: floodlights

Type C spherical coordinates:

Polar axis: vertical
Label on vertical angles: V
Label on horizontal angles: L (lateral)
Range of V: 0 (nadir) to 180 (zenith)
Range of L: 0 (along primary axis of luminaire) to 360
Straight down: $Y = 0$, $X = 0$
Primary uses: indoor lighting, roadway lighting

“Snapshot” Goniophotometers

Conventional **goniophotometers** take a long time to produce an **intensity** mapping. In addition, they must have precise motion control to achieve the desired angular resolution. As such, they are well suited for characterizing luminaire designs, but not really useful for quality control or sorting of LEDs, for example. For these applications, several versions of rapid “snapshot” goniophotometers have been developed:

- Rapid-scan goniophotometers
- Multiple-detector goniophotometers
- Tapered fiber bundle goniophotometers
- Camera-based goniophotometers

Rapid-scan goniophotometers are small devices used to characterize LEDs and the output of optical fibers. They operate on similar principles to the conventional type C goniophotometers, but motions are much faster, making measurements in seconds rather than minutes.

Multiple-detector goniophotometers place numerous discrete detectors in the intensity field of interest and capture the entire intensity distribution at one time. The angular resolution is restricted to the spacing of the detectors.

A tapered fiber optic bundle can be manufactured with one concave spherical face with all the fibers directed toward the source. At the other end of the bundle, the fibers can be aligned with the pixels of a detector array. The detector array captures the entire intensity distribution at one time.

Camera-based goniophotometers place a diffuse reflecting surface (flat or concave) at an appropriate distance from the source, and view the light reflected from the surface with an imaging photometer that, together with the reflecting surface, is calibrated to capture the entire intensity distribution in one “snapshot.”

Software Modeling Discussion

Outside the laboratory, software programs are used to model, optimize, and tolerance optical systems. Two types of codes exist in the optical design arena: **lens design codes** and **optics analysis codes**. The former are used primarily to design the lenses used in optical systems. They include robust analysis tools such as point spread function graphs, spot diagrams, and modular transfer function curves; optimization tools to improve upon the performance of the imaging system; and tolerancing tools to ensure manufacturability. Increasingly, these lens design codes include **nonsequential ray tracing**. Nonsequential ray tracing is required for a number of illumination systems, especially those based on nonimaging optics. In standard lens design, rays follow a prescribed sequence of optical interfaces. Thus, the traced rays know the sequence of surface intercepts, which reduces the computation load since the algorithm does not need to determine which surface is struck next by a ray.

Optics analysis codes are based around nonsequential ray tracing such that computation time must be spent to determine which surfaces are struck by each ray. Nonsequential ray tracing is inherently slower than sequential ray tracing. Analysis codes are further broken down into two geometry types: **surface-based geometry** and **solid-based geometry**. Surface-based codes require the user to generate each surface, assigning the optical properties on the two sides of each interface. Solid-based codes develop enclosed objects that allow the user to assign volume-based properties such as the type of material (e.g., BK7) and surface-based properties (e.g., a silver mirror).

Optical design codes incorporate more **computer-aided design (CAD)** into their capabilities. This feature allows the codes to import mechanical design formats such as IGES and STEP. Certain industries such as the automotive and architectural industries have specialized codes. The list of codes is extensive and always changing.

Role of Light in Architecture

The illumination of buildings is a design process aimed at orchestrating light for the user's well-being. The layering and patterning of light is considered successful when complex physiological and psychological responses are satisfied. Such responses are centrally conditioned by vision: the medium through which information and perceptions about a given space are recorded and interpreted. Economics and energy efficiency play a critical role in design decisions, but the satisfaction of vision requirements is of overriding importance.

The characteristic features of an architectural space only come to life with light. Hence, no light no architecture. At the same time, light is not neutral: The way it is arranged gives a particular appreciation of the space and generates specific emotive and aesthetic responses.

The electric illumination of an architectural space is simply the result of transmitted or reflected light emanating from distant and immediate surrounding surfaces. Therefore, the lighting designer can influence the interface between light and matter to meet these visual requirements and sensations. Hence, only with a proper understanding of physiological and psychological factors and a familiarity with available technologies can lighting decisions be made for proper effect.

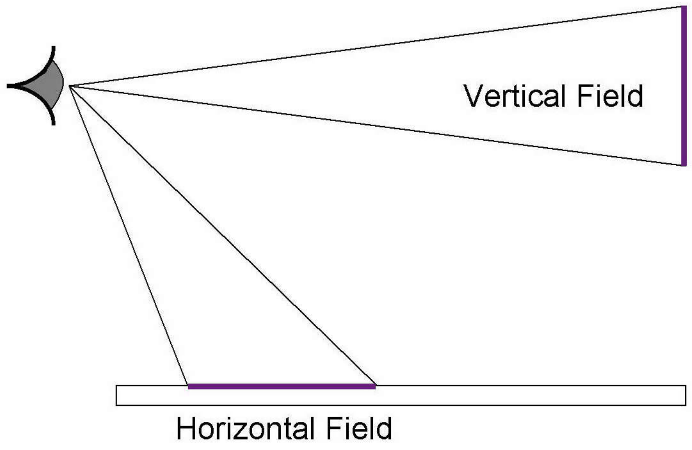
Despite some setbacks in the 1970s caused by an advocacy for windowless buildings to save energy, light available from the sun and sky has regained the attention of lighting designers for the many benefits it brings to users. When available and well controlled, daylight is by far the preferred source of illumination. Today, the common design approach combines the contribution of both electric and natural lights for increased work productivity, and reduced absenteeism or visual fatigue.

Eye Adaptation and Visual Fields

Eye adaptation to the visual environment is the eye's response and sensitivity to the ambient light level as the person moves from one environment to the next, such as walking from the bright and sunny outdoors to the dark indoors. If the difference between the two light levels is extreme, the person may feel like he or she has moved into a totally black environment. Slowly, the sensitivity of the eye attunes itself to the dark environment and details become increasingly distinguished. It takes 20 to 30 minutes for the eyes to completely adapt to a dark environment and grasp the details. Conversely, eyes adapt to a sunny environment in 2 to 3 minutes.

Transient adaptation is the ability of the visual system to adapt in short intervals to the different luminances prevailing in a fixed visual field, for instance, when looking through a bright window and down to a desk. Due to such variations, the iris constantly adjusts the aperture to control the light entering the eye. Large variations between luminances in a scene are considered detrimental to visual comfort and lead to eye fatigue.

Visual fields refer to the direction of the eyes' line of sight. When looking down, the viewer apprehends a **horizontal field**, and when looking up, a **vertical field**.

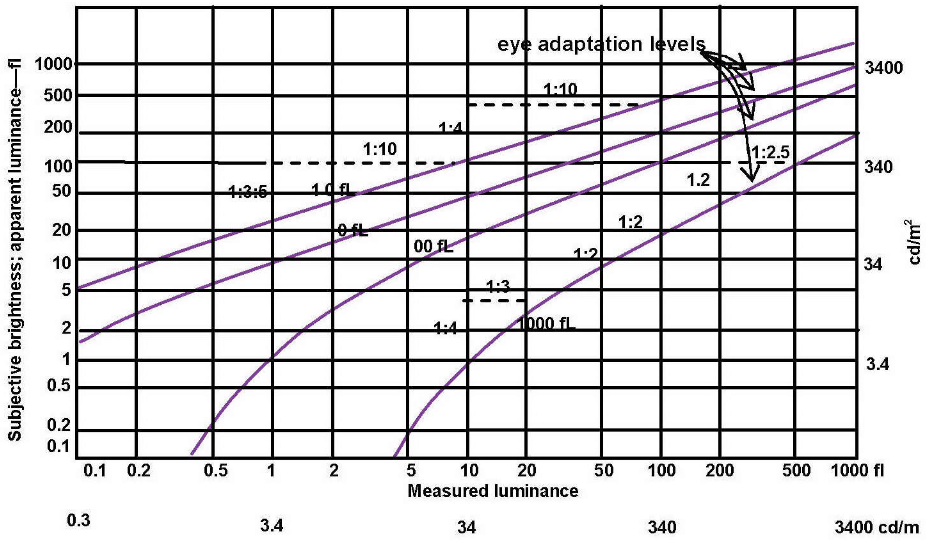


Visual Fields

Apparent Brightness

Vision is stimulated by brightness mapped on the retina as a byproduct of light reflected from an opaque surface or transmitted through a transparent medium (the glass bulb of a lamp, for example). A distinction must be made, however, between **photometric brightness** or luminance and **apparent brightness**. Luminance or photometric brightness is calibrated in relation to the eye's sensitivity to various wavelengths, while apparent brightness is perceived in the context of the ambient light level to which the eye is adapted. Hence, the brightness of an object relative to the retinal image is by no means a complete specification of its visual appearance. This may be understood by considering the blinding brightness of a car's headlights during the night that is barely perceivable during the day, even though a light meter will register the same photometric brightness.

At an ambient surrounding of 3.4 cd/m² (1 fL), a measured luminance of 34 cd/m² (10 fL) appears to be 340 cd/m² (100 fL). At a low ambient level, the difference perceived between two surfaces is also reduced from a difference of 1:10 to 1:4.



Subjective brightness versus measured luminance. (Reprinted with permission from Stein and Reynolds, copyright Wiley & Sons, 2005.)

Lighting Design—Layering of Light

For both energy conservation and visual variety, lighting design is implemented in layers to properly distribute light throughout the architectural space.

The **horizontal ambient layer** is maintained to 1/3 to 2/3 the task illumination level. Lower bound levels (1/3) for horizontal ambient light may be appropriate for a museum or boutique store to emphasize a display. Upper bound levels (2/3) are more relaxing for most casual activities where a 25- to 35-fc ambient light level is sufficient and relates well to tasks requiring 50 to 60 fc.

The **vertical ambient layer** is critical in keeping vertical tasks glare free, such as washout of video display terminals (VDTs). In addition, when people look away from a task, the line of sight is then the vertical average luminance from the walls and ceiling. **Wall washing** and **grazing** are some of the techniques used to reinforce the sensation of spaciousness, clarity, and pleasantness.

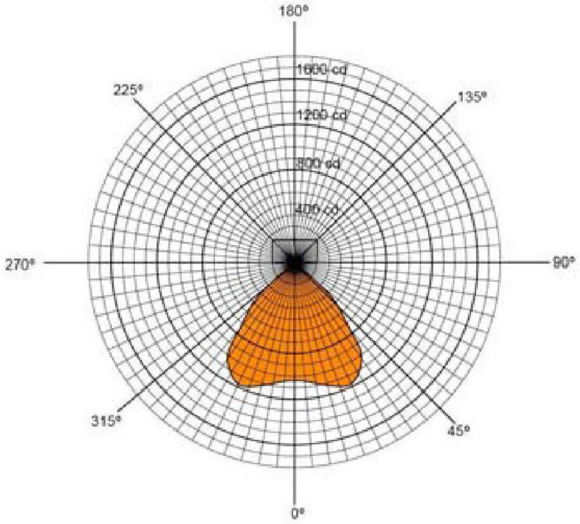
A **task layer** supplements the ambient illumination to fulfill lighting requirements for critical activities. Energy is saved by (1) locating the source near the task to provide the light level recommended by the Illuminating Engineering Society of North America (IESNA), (2) reducing ambient light levels, and (3) turning off the task light when not in use. The scene presents varied lighting instead of the monotonous atmosphere, resulting from the **general illumination approach**.

The **accent** or **focal layer** gives the space its identity and mood by highlighting or spotlighting certain architectural elements and objects, such as paintings, sculptures, and landscapes. **Downlighting**, **accent lighting**, and **backlighting** are some techniques used to produce such effects on various elements in the space.

The **ornamental layer** introduces elements that add sparkle to the space with effects similar to those of Christmas lights. Chandeliers, candles, and sconces can be considered for this purpose.

Photometric Report and VCP

Manufacturers provide a **photometric report** that details the optical performance and characteristic light distribution patterns of a luminaire. The **candela distribution curve (CDC)**, presented in either polar (figure below) or rectilinear plots and in tables, shows the luminous intensity distribution measured at different angles, from 0 to 360 deg in increments of 5 deg. Using the plot below, the luminous intensity can be found for a specific direction. Rectangular luminaires (2 x 4 or 1 x 4) require candela distribution curves in at least three planes: crosswise, longitudinal, and 45 deg. These luminous intensities can quickly reveal the potential for glare.



Candela distribution curve.

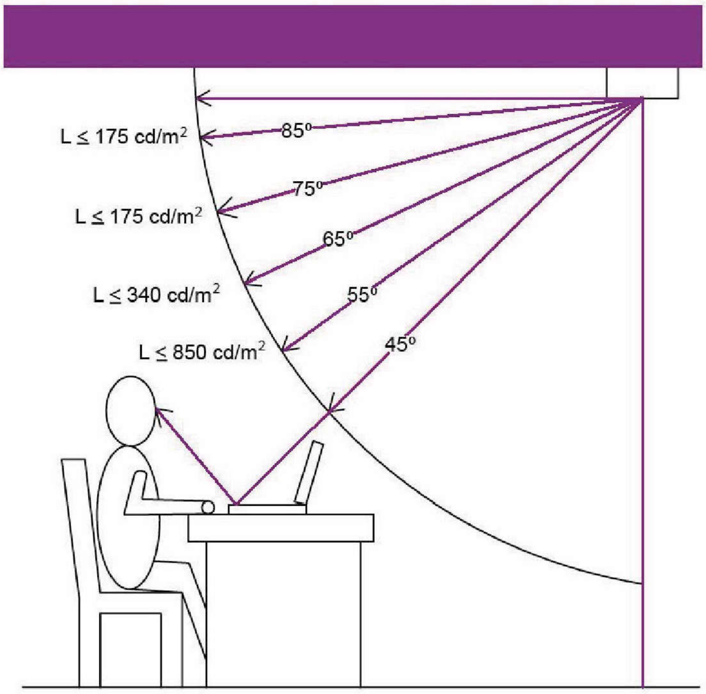
The **visual comfort probability (VCP)**, a rating system for evaluating direct discomfort glare, is expressed as the percent of occupants of a space who will be bothered by direct glare. Standard data provided for a luminaire specification include tables of its VCP ratings for various room geometries, based on IESNA standard conditions. These include a uniformly distributed illumination level of 1000 lux (~100 fc), luminaire height, observer position, and room surface reflectances (ceiling, 80%; walls, 50%; and floor, 20%). In general, a minimum VCP of 70 is the established limit for the viable use of a luminaire.

The Layered Approach

Spacing criteria (SC) to achieve uniform ambient illumination is the ratio of the spacing (S) distance between the respective axis of parallel luminaires and their mounting height (MH), i.e., SC = S/MH. For a rectangular luminaire, SC is given along both axes, lengthwise and crosswise. The distance between walls and adjacent light fixtures is set at no more than one-half S.

The **coefficient of utilization (CU)** specifies the proportion of lumens that reach the workplane from the fixture for given room geometries and surface reflectances. The CU gives some indication of the luminaire's efficiency.

When located in the field of view, bright light sources can cause discomfort and disability glare. However, the severity of glare depends on the angle at which the luminaire is seen. The *IESNA Handbook* provides as maxima the following **luminances from a direct luminaire** according to the view or cut-off angle:

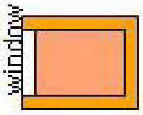


Maximum luminance according to view angle.

Daylight Factor

Windows and skylights admit daylight as free illumination, and the viewers have visual contact with the outside. However, only a portion of outdoor light is received inside a building. The sky's condition, along with the size, placement, and orientation of the window(s) opening, the glazing type and transmittance, and shading as well as room proportions, affect the quantity and quality of received light.

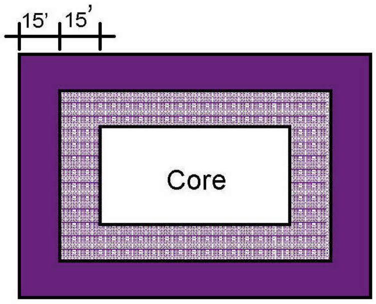
The **daylight factor (DF)** accounts for these parameters and is used to determine the percentage of outside light that can be received inside a room with a specific configuration. DF is the ratio of interior illuminance at a given point on a given plane (usually the work plane) to the exterior illuminance under minimal light conditions of an overcast sky, e.g., the CIE overcast sky distribution. For parallelepipedic-room buildings with windows on one side only, a minimum DF of 2% is generally recommended throughout the work plane.



The DF general equation is

$$DF = Ei/Eo,$$

where *Ei* and *Eo* are the indoor and outdoor horizontal illuminances, respectively.



Daylight should be allowed to reach through side windows for the tasks performed at the rear of the space. Shelves in side windows can help reflect light deep into interior spaces and shade the vision portion of the window.

Office tasks can be lit at a depth of up to 15 feet from the window. The next 15 feet may need to be supplemented by electric light. Beyond 30 feet, little or no daylight is available unless the space has windows on opposite sides. Evenly distributed roof skylights provide uniform light but for one story only, assuming proper glazing transmittance and sunlight control.

Daylight Strategies

IESNA RP-5-99 Recommended Practice of **daylighting** emphasizes the following illumination practices:

1. Block direct sunlight in the vicinity of tasks. Blocking direct sunlight at the window (e.g., with louvers) is the first step toward glare control by allowing only light from the sky and that reflected from the ground to pass through the window glass.
2. Design windows to minimize direct glare. East, west, or south facing windows can have too much glare if excessive sunlight strikes the glass. With 100,000 lux of light available on the outside on a sunny day, a glass pane, with typically only 2% transmittance, can reach a luminance up to 2000 cd/m2, which exceeds the tolerated average of 850 cd/m2. Internal blinds (structurally stable external shading devices with adjustable louvers) can block direct sunlight and reduce the luminance of the window or skylight. And glazing that has a visible light transmittance of 25% can be an acceptable trade-off of daylight availability, the view to the outside, and minimize glare.
3. Zone electric lighting for daylight responsive control. The electric light distribution system should be zoned according to daylight availability inside an open-plan office. Daylight zoning depends on the room configuration, sky condition, and solar exposure. Large open-plan offices are often subdivided into the perimeter zones, the intermediate zone, and the core zones based on daylight availability as indicated in item 1.
4. Provide responsive lighting controls. Controls are at the heart of efficient electric light operation and daylight harvesting, specifically to accommodate the time-dependent electric light demand. The variables governing control strategies include the space layout, configuration, orientation, the occupancy patterns, lighting usage, and daylight availability. Controls include tuning to reduce electric power while still meeting each user's needs, and adaptive compensation to lower the light levels at night.

Nighttime Visibility Criteria

The eye is capable of adapting to a wide range of light levels but not at the same time. To function well, it must be adapted to the prevailing light conditions. As previously indicated for daytime conditions, our eyes use **photopic vision**, which utilizes the eye's cones and the center of the visual field. The eye works differently when it is adapted to low light levels. Under very dark, moonlit conditions, our eyes use **scotopic vision**, which primarily utilizes the eye's rods, resulting in greater acuity in the peripheral visual field.

For **nighttime visibility** in most urban and suburban environments, our eyes use **mesopic vision**, which is a combination of both photopic and scotopic. In nighttime environments, the goal of the lighting design is to keep the eye adapted to mesopic or scotopic vision, and not to introduce high light levels that will create an imbalance in the visual field and cause the eye to try to use photopic vision. Recent research indicates that light sources rich in blue and green (metal halide or fluorescent) improve peripheral mesopic vision, clarity, and depth of field better than sources rich in red and yellow, such as incandescent and high-pressure sodium.

Extreme **glare** leads to loss of visibility. Glare is caused by a high luminance ratio between the glare source and the prevailing light conditions to which the eye is adapted. In other words, insufficiently shielded light sources generate direct glare. The following measures reduce the luminance ratio and control nighttime glare:

- **Uniform light distribution** in a visual scene and brightness ratios kept to 1:5 between average and maximum luminance;
- **Reduction of light levels** and **source brightness** using fixtures with low wattage;
- **Shielding the light source** and **locating fixtures** to avoid glare. Fixtures near the property line should have "house-side shielding" to prevent glare to residential neighbors.

Recommended Illuminance for Façades

Façade illumination aims to reproduce at night a building's aesthetic and formal characteristics that are perceived during the day for the purpose of attracting attention and creating a good impression. To this effect, **floodlighting** is one technique employed, which treats a building as a giant piece of sculpture for visual display. Luminaires are typically mounted in close proximity to buildings and are aimed to illuminate the structure. Lighting the building from the top down reduces stray uplight, and precisely aimed fixtures minimize trespass light.

Effective illumination of facades is a complex and subjective task. Results depend heavily upon surrounding light levels, the surface finish of the intended target, the spectral color distribution of the lamp source, mounting location allowances, and viewers' perceptions.

The following table lists the IESNA's recommended illuminance levels for the floodlighting of buildings and monuments.

Area Description	Target Surface Finish	Average Target Illuminance
Bright	Light	5 (50)
Bright	Medium light	7(70)
Bright	Dark	10(100)
Dark	Light	2(20)
Dark	Medium light	3(30)
Dark	Medium dark	4(40)
Dark	Dark	5(50)

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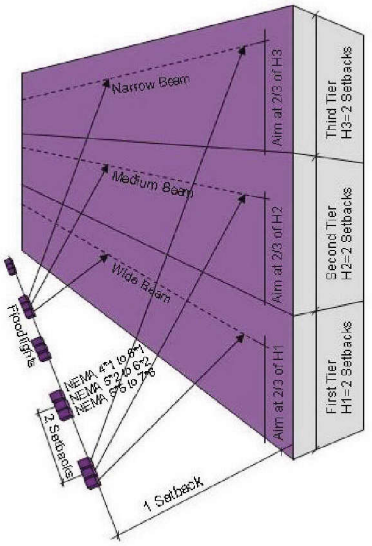
Façade Floodlighting for Uniform Illumination



Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.

Floodlighting fixtures can be mounted at ground level, or on stands and poles. They can also be attached to the building itself or to adjacent structures. The key lights are set up for a modeling effect but should be combined with other color sources to soften the strong effects of shadows.

Floodlight categories are **narrow beam** (types 1, 2, 3), **medium beam** (types 4, 5) and **wide beam** (types 6, 7) (IESNA RP-33). The further away the luminaire is from the facade, the narrower the light beam must be. Aiming and positioning ground-mounted floodlights for uniform illumination depend first on the available **setback** in relation to the building height. If the height is 2 times the setback dimension, the center of a “wide beam” floodlight aimed at 2/3 the height of the building is recommended. If a building is 30 feet high, the recommended aiming point is 20 feet high. Floodlight **spacing** along the facade should not exceed 2 times the setback distance. If the setback is 22.5 feet, the floodlights should be placed no more than 45 feet apart, with the first floodlight at ½ to 1 of the setback dimension. As the building height increases to 4 times the setback, a medium-beam floodlight with the same aiming elevation is recommended. Buildings with up to 6 times the setback require more narrow-beam floodlights. Thus, one location on the ground may hold multiple floodlights, each aimed at different building elevations. The illumination from the ground of façades with more than 6 times the setback is not recommended due to the difficulty of achieving uniformity.

Illumination of Outdoor Areas

Lighting **building entries** at night provides (1) vertical illumination to comfortably light people's faces, and (2) horizontal illumination to light the pathway and any changes in the light level. Such a pool of light comes from a mounting position high on the building, on a pedestrian-scaled post lantern, or on the underside of a canopy.

Emergency egress doors are provided with lighting on the outside of the door threshold and extended for a distance at least equal to the width of the door opening.

Softscape lighting is for private yards, patios, parks, gardens, boulevards, entry markers, and other natural features such as water. They are softly illuminated and emit a minimum of glare, contrast, or spill light to the neighbors. Some techniques used to light trees to achieve the desired effect are **frontlighting** to highlight details, texture, and color; **backlighting** to show form and separate the plant from the background; **sidelighting** to emphasize plant texture and create shadows; **uplighting** to make branches glow; and **downlighting** for accent details, colors, and texture. The illumination of tree trunks along with canopies helps anchor them to the landscape.

Hardscape lighting is for outdoor sculptures, fountains, or vertical displays. A 3D sculpture is illuminated from two directions to provide highlights and soften shadows. The key light is focused on the mass of the sculpture with light added to relieve shadows.

Stairs and **ramps** are hazardous in low light, so contrast is essential for their safe use. Illuminated handrails, step lights, or small fixtures in the balustrade provide light differentiation between the step risers and threads. Other techniques to complement light effects are coloring of the step nosing and color differentiation between threads and risers.

Walkways, **sidewalks**, and **bikeways** are illuminated at levels recommended by the IESNA with lights placed to provide visual information.

CX-0693

Special Considerations for Outdoor Fixtures

Controls using astronomical time switches and/or photosensors are deployed to ensure that exterior lighting is not operated when sufficient daylight is available or during nighttime except those fixtures for security.

Special considerations must be given to proper installation of luminaires exposed to the outdoor environment. If installed on ground they must have the "**wet location rated**" label and if placed under canopies but still exposed to the elements, they must be "**damp location rated**." In addition, durable with vandal-resistant components and regularly maintained luminaires to minimize dirt accumulation or to prevent obstruction by grass, leaves, mud, and other debris, ensure steady operation of exterior lighting. Separate security fixtures are exclusively used to provide low light levels for security cameras (.01 footcandle). The table below summarizes the illuminance and luminance ratios for various outdoor areas.

Outdoor space type	Horiz. Avg. Illuminance (fc)	Vert. Avg. Illuminance (fc)	Lum. Ratio
Building entrance (Active/Inactive)	5.0 / 3.0	3.0 / 3.0	
Emergency lighting: Egress Path	1		
Roadside sidewalks & Type A bikeways: commercial, intermediate, residential areas	1; 0.6; 0.2	2.2; 1.1; 0.5	4:l to 5:1
Walkways distant from roadside & Type B bikeways	0.5	0.5	4:l to 5:1
General parking and pedestrian areas	3.6; 2.4; 0.8		4:l to 5:1
Loading docks	10	3	
Storage yards,active-inactive	10	3	

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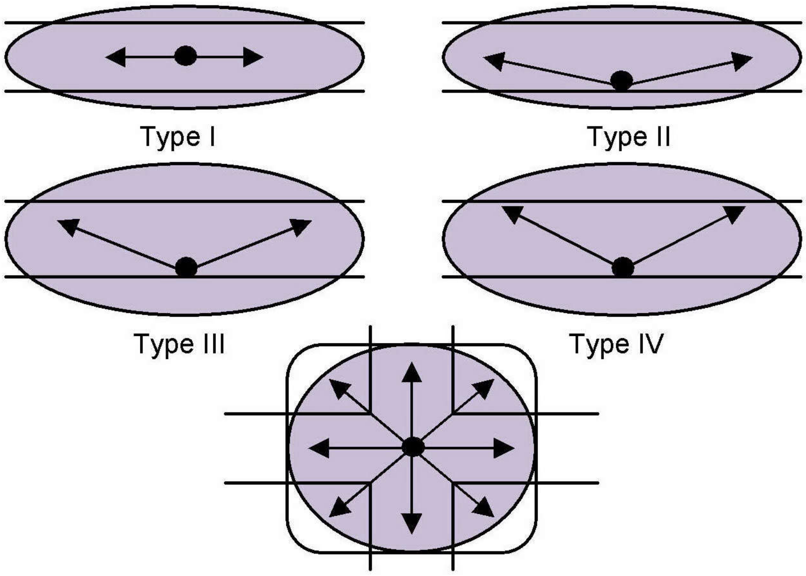
Outdoor Luminaire—Transverse Light Distribution

Luminaires' **beam pattern distributions** are classified by IESNA according to transverse and lateral projections. (see the next two figures). Five types, shown below, are illustrated according to the maximum candlepower and the trace of the half-maximum value. A **luminaire's transverse reach** is expressed in MH units: **type I** (1 MH), **type II** (1.75 MH), **type III** (2.75 MH), **type IV** (6 MH), and **type V** (symmetric distribution in four quadrants). Type V is usually best at the center of parking lots. Type IV or forward-throw distribution is best for wide, multilane roads and parking lot perimeters. Type I has a long and narrow distribution that can be applied to narrow roadways, walkways, or bike paths. It also can be located at or near the center of a pathway, approximately two MHs in width, or used as overhead lighting in areas such as parking lots, plazas, courtyards, and along walkways. Types II and III distribute light to one side of the light source. These luminaires should generally be used for street lighting to direct light to the street side of the lamp but not shining into the building side.



Type I

Type II

Type III

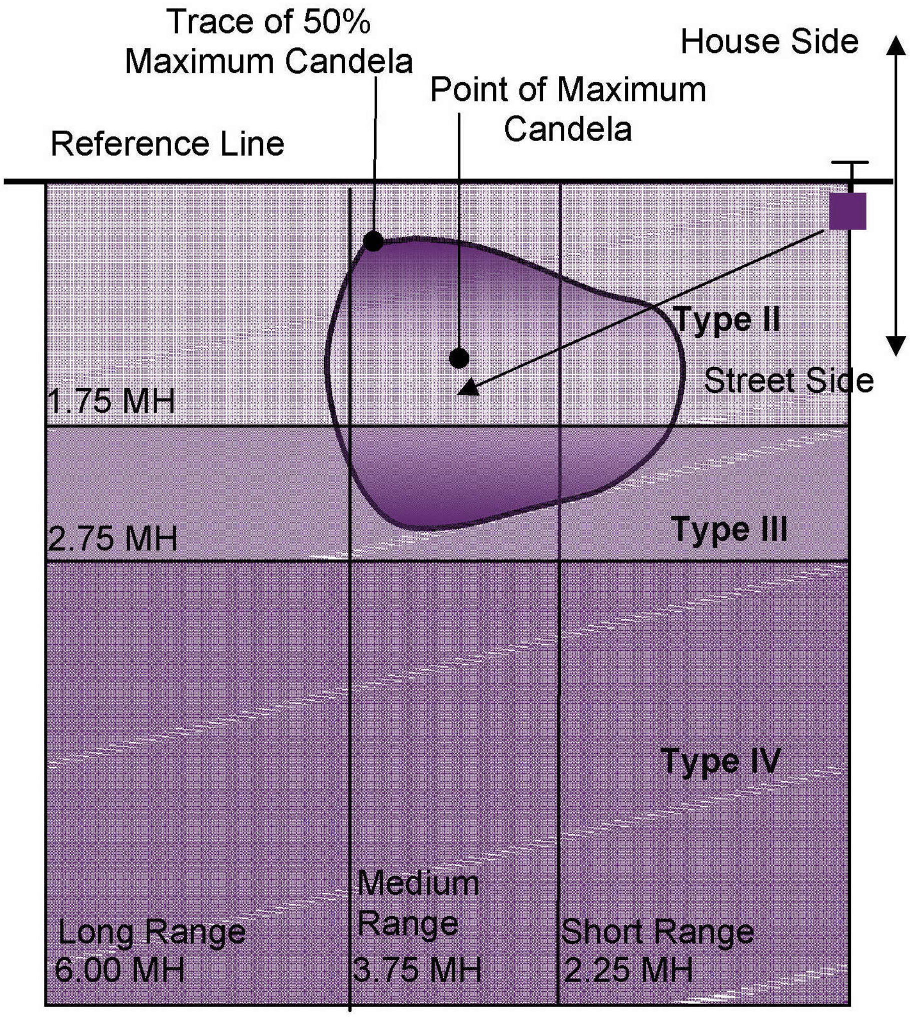
Type IV

Type V

Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.

Outdoor Luminaire—Lateral Light Distribution

Fixtures for roadway and parking applications are further classified as **short**, **medium**, or **long lateral distribution**. This classification relates the types of fixtures, the spacing between them according to the point of maximum candelas, and the MH. For a short-range lateral throw, the maximum luminaire spacing is generally less than 4.5 times the MH. A medium throw allows a maximum spacing of generally less than 7.5 times the MH, and a long throw is generally less than 12 times the MH (see figure below).



Type III distribution of a luminaire. Half-maximum candelas trace falls within 2.75MH. (Adapted from *IESNA Handbook*, 9th Edition, copyright IESNA, 2000.)

Criteria For Roadway Lighting

Three principal criteria are used to design major roadway lighting systems: **luminance**, **illuminance**, and the newer concept of **small target visibility (STV)**. The illuminance **inverse square law calculations** are well known. It was found, however, that illuminance levels do not correlate well with visibility or driver performance. IESNA Standard RP-8-00 addresses one shortcoming of the illuminance method by adding a maximum **veiling luminance ratio (VLR)** that is specifically intended to limit glare from a luminaire. The luminance determination is necessary to calculate the VLR. Luminance describes the reflected light from the pavement as seen when driving, so evaluating the quality of a lighting system by how it looks at night is actually the same as evaluating its luminance.

In reference to the figure on the next page, luminance at point *P* is determined as the sum of contributions from all *n* luminaires:

$$L_P = \Sigma r(\beta_i, \gamma_i) I(\varphi_i, \gamma_i)/10{,}000\ h^2,$$

where r is the **reflectance coefficient** at angles β and γ.

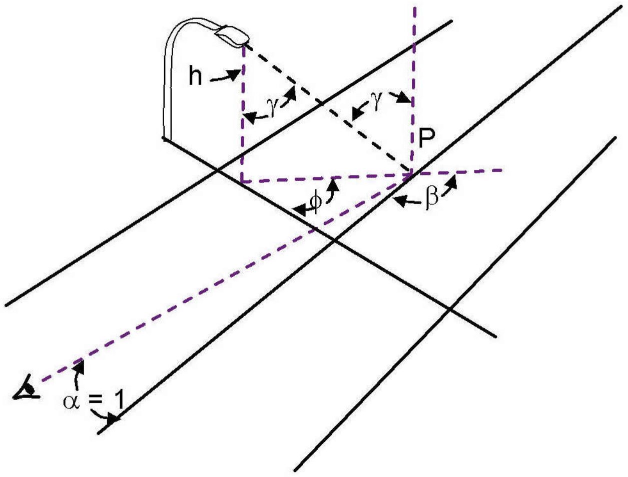
The veiling luminance, *Lv*, and the VLR are also necessary to limit the glare effect. The *Lv* can be determined as follows:

$$L_v = \Sigma\ 10\ E_v / (\theta^2 + 1.5\ \theta),$$

where: L_v is the veiling luminance at the observer's location, in cd/m^2; E_v is the vertical illuminance on the plane of the observer's eye; and θ is the angle between the line of sight and the luminaires in degrees.

Values recommended for luminance and VLR are found in the 9th Edition of the *IESNA Handbook.*

Small Target Visibility



Single fixture for luminance determination. (Adapted from *IESNA Hand-book*, 9th Edition, copyright IESNA, 2000.)

The **STV** method was developed to account for the contrast that must be present to allow drivers moving at high speed to quickly detect hazards and react to them. Indeed, a roadway lighting system may provide a high and uniform road surface luminance, yet the visibility threshold may be low due to the absence of contrast. Three luminance components influence the visibility of a target: the target luminance itself, the luminance of the background, and the veiling luminance or glare. Given these three luminances, all of which can be calculated, the visibility of each target in the array can be determined in terms of the **visibility level (VL)**. VL is the ratio of target contrast to the contrast of a similar target at threshold, a measure of visibility that has been widely used.

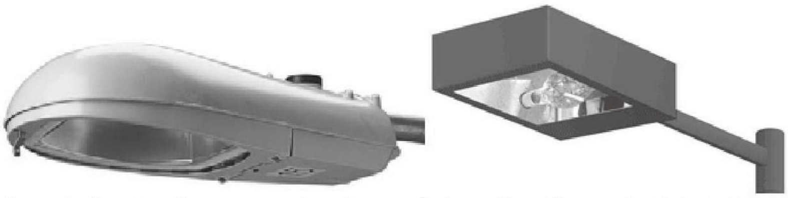
STV predicts the visibility of a standard object (18 × 18 cm) located on the roadway at a specific distance from the driver, and accounts for the contrast between the standard target and its background by considering the driver's age, viewing time, pavement reflectance, and glare from the luminaire. The larger the STV number, the more visible the object. For more details, go to IESNA RP-8-00.

Recommended Roadway Luminaires

After the desired minimum illuminance and the pole height are initially set based on light spill, road cross-section, pavement type, and roadway category (*IESNA Handbook*), a few luminaires and sources can be tested to achieve a design that meets the recommended light level, uniformity, and acceptable glare. Common layouts include luminaires on one side of the road, luminaires on both sides, or luminaires in a center median. One-side and median configurations often have the additional advantage of requiring less wire and conduit, resulting in lower construction costs.

Starting with any of the five previously described types of luminaires is a convenient way to facilitate selection according to roadway width and light spill control. **Full cutoff** luminaires should be specified wherever possible to prevent light pollution. Typically, Types I, II, and III are appropriate for narrow roadways, while type IV is proper for multilane roads. Lateral-medium-throw luminaires are preferred over short-throw types because fewer poles are required and over long-throw types because then **semicutoffs** are not needed. Combining transversal and longitudinal distributions helps the designer select luminaires for even light distribution based on roadway widths and pole spacing.

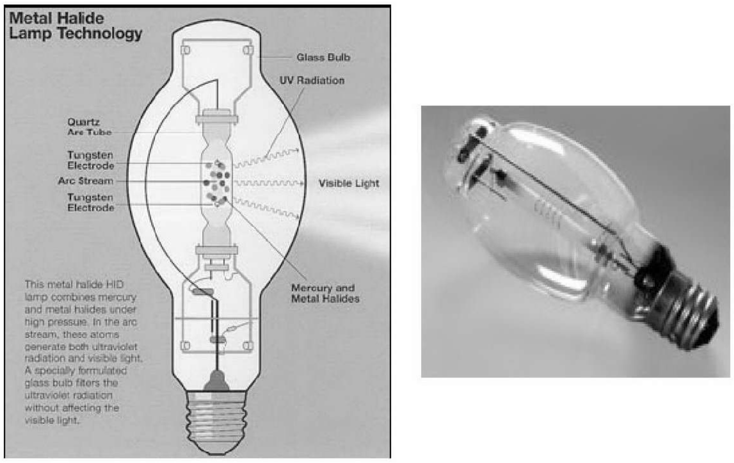
The architecture of such luminaires comes in many shapes. Two are shown below. The most prevalent is the "**cobra head**" luminaire, typically mounted on a 6-ft arm. This fixture with a flat lens is recommended to minimize disability glare and light trespass. Reflectors may be formed or faceted aluminum. The **rectilinear** "**shoebox**" luminaire, designed as a full cutoff, is also popular. The reflector in this design is usually larger than that in a cobra head to achieve better optical control.



(Reprinted with permission of Acuity Brands Lighting.)

Recommended Lamps for Roadway Luminaires

Metal halide, **high-pressure sodium**, and **induction lamps** are the most common lamps, ranging from 100 to 400 W depending on the MH and roadway cross-section. Metal halide is used where color rendition is a concern or white light is desired, but this lamp also emits light in the blue-green portion of the spectrum, which enhances the driver's peripheral vision. A **light-loss factor (LLF)** accounts for the depreciation in lamp output and luminaire performance over time. Typical LLFs are 0.60 to 0.70 for **high-pressure sodium** lamps and 0.45 to 0.55 for **metal halide lamps**. Note that the LLF varies depending on environmental conditions and maintenance procedures.



Left: metal halide lamp. Above: 400W high pressure sodium BT-37. (Images reprinted with permission from Sylvania.)

The isofootcandle procedure, not covered here, can be used to determine luminaire spacing based on needed illuminance and uniformity. With the help of software programs, many iterations can be performed quickly to compare various lighting systems and determine the safest and most efficient solution.

Equation Summary

Basic Quantities in Illumination

$$\text{photopic luminous flux} = 683\int \Phi_\lambda(\lambda)\cdot V(\lambda)\cdot d\lambda$$

Color

Light source color:

$$X = \int S_\lambda(\lambda)\cdot \bar{x}(\lambda)\cdot d\lambda \qquad x = \frac{X}{X+Y+Z}$$

$$Y = \int S_\lambda(\lambda)\cdot \bar{y}(\lambda)\cdot d\lambda \qquad y = \frac{Y}{X+Y+Z}$$

$$Z = \int S_\lambda(\lambda)\cdot \bar{z}(\lambda)\cdot d\lambda$$

$$u = 4X/(X+15Y+3Z) = 4x/(-2x+12y+3)$$

$$v = 6Y/(X+15Y+3Z) = 6y/(-2x+12y+3)$$

$$u' = 4X/(X+15Y+3Z) = 4x/(-2x+12y+3)$$

$$v' = 9Y/(X+15Y+3Z) = 9y/(-2x+12y+3)$$

Color temperature and CCT:

$$(MK)^{-1} = {}^{10^6}\!\!/_{CCT}$$

Surface color:

$$X = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{x}(\lambda)\cdot d\lambda$$

$$Y = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{y}(\lambda)\cdot d\lambda$$

$$Z = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{z}(\lambda)\cdot d\lambda$$

$$k = \frac{100}{\int S_\lambda(\lambda)\cdot \bar{y}(\lambda)\cdot d\lambda}$$

Equation Summary (cont.)

Color of fluorescent surfaces:

$$X = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{x}(\lambda)$$

$$Y = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{y}(\lambda)$$

$$Z = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{z}(\lambda)$$

$$k = 100 \Big/ \sum_{\lambda=380}^{780} S_{\lambda}(\lambda)\, \bar{y}(\lambda)$$

Illumination Properties of Materials

BRDF and reflectance factor:

BRDF=L/E R=BRDF·π

Harvey/*ABg* scatter model:

$$B = \left|\beta_{rolloff}\right|^{g} \qquad \mathrm{BSDF} = \frac{A}{B + \left|\beta - \beta_0\right|^{g}}$$

Retroreflectors:

$$R_I = \frac{I}{E_\perp} \qquad R_A = \frac{R_I}{A} \qquad R_L = \frac{R_A}{\cos v} = \frac{L}{E_\perp}$$

Illumination Transfer

Known intensity:

$$E = \frac{I \cos\xi}{d^2} \qquad \Phi_i = I\omega \qquad L = \frac{I}{A_r \cos\theta}$$

Equation Summary (cont.)

Known flux:

$$I = \Phi_i / \omega \qquad E = \Phi_i / A_i \qquad L = \frac{\Phi_i}{\omega\, A_r \cos\theta} = \frac{\Phi_i}{\Omega\, A_r}$$

Known radiance:

$$E_0 = \frac{\Phi_i}{A_i} = \frac{L\, A_r\, \Omega}{A_i} = \frac{L\, A_r \cos\theta \cos\xi}{d^2} = L\ \ \Omega_i$$

Known irradiance:

$$I = \frac{E \cdot d^2}{\cos\xi} \qquad \Phi = E \cdot A_i \qquad L = \frac{R \cdot E}{\pi}$$

Relationship of configuration factor to projected solid angle and of form factor to average projected solid angle:

$$C = \Omega / \pi \qquad F_{a\,to\,b} = \bar{\Omega}_{a\,to\,b} / \pi$$

Projected solid angle of circular area from on-axis point:

$$\Omega = \frac{\pi\, r^2}{r^2 + d^2} = \pi \sin^2\theta$$

Projected solid angle of circular area from off-axis point:

$$\Omega = \frac{\pi}{2}\left(1 - \frac{1 + \tan^2\delta - \tan^2\theta}{\left[\tan^4\delta + \left(2\tan^2\delta\right)\left(1 - \tan^2\theta\right) + \sec^4\theta\right]^{1/2}}\right)$$

$$\Omega = \frac{\pi}{2}\left(1 - \frac{1 + \left(d/x\right)^2 - \left(r/x\right)^2}{\left[\left\{1 + \left(d/x\right)^2 + \left(r/x\right)^2\right\}^2 - 4\left(r/x\right)^2\right]^{1/2}}\right)$$

Equation Summary (cont.)

Average projected solid angle from one circular area to another, with both areas parallel and centered on the same axis:

$$\bar{\Omega}_{r\,to\,i} = \frac{\pi}{2}\left[1+\frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2} - \left\{\left(1+\frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2}\right)^2 - 4\left(\frac{r_i}{r_r}\right)^2\right\}^{\frac{1}{2}}\right]$$

Cosine fourth and increase factor:

$$E_i = \pi\, L_r \sin^2\theta \cdot \cos^4\delta \cdot F'$$

ω, Ω, NA, and *f*/# for a circular cone:

$$\omega = 2\pi(1-\cos\theta) \qquad \Omega = \pi\sin^2\theta$$

$$NA = n\cdot\sin\theta \qquad f/\# = 1/2\sin\theta$$

Illumination in Imaging Systems

Object and image radiance:

$$\frac{L_i}{n_i^2} = \tau\cdot\frac{L_o}{n_o^2}$$

Image irradiance off axis:

$$E_i = \pi\,\tau L_o \sin^2\theta\cdot\cos^4\delta\cdot F'$$

Image irradiance on axis:

$$E_{i0} = \pi\,\tau L_o \sin^2\theta = \frac{\pi\,\tau\,L_0}{4(f/\#)^2} = \frac{\pi\,L_0}{4(T/\#)^2}$$

$$= \pi\,\tau\,L_0\,NA^2 = \tau\,L_0\,\Omega$$

Image flux:

$$\Phi_i = \tau\,L_o\,a_i\Omega_i = \tau\,L_o\,a_p\Omega_p$$

Equation Summary (cont.)

Illumination in Nonimaging Systems

Generalized étendue:

$$\mathcal{E} = n^2 \iint_{\text{aperture}} \cos\theta \, dA_s \, d\omega$$

$$\Phi = \iint_{\text{aperture}} L(\mathbf{r}, \hat{\mathbf{a}}) \cos\theta \, dA_s \, d\omega,$$

$$\Phi = L_s \iint_{\text{aperture}} \cos\theta \, dA_s \, d\omega$$
$$= \frac{L_s \mathcal{E}}{n^2}$$

Concentration:

$$C = A/A'$$

$$C_{2D} = \frac{a}{a'} = \frac{n' \sin\theta'}{n \sin\theta} \qquad C_{3D} = \frac{A}{A'} = \left(\frac{n' \sin\theta'}{n \sin\theta}\right)^2$$

$$C_{\text{2D,opt}} = \frac{n'}{n \sin\theta_a} \qquad C_{\text{3D,opt}} = \left(\frac{n'}{n \sin\theta_a}\right)^2$$

Skew invariant:

$$f_{\text{skew}}(s) = \frac{d\mathcal{E}(s)}{ds},$$
$$\text{where } s = r_{\min} k_t$$

Fibers, Lightpipes, and Lightguides

Maximum acceptance angle:

$$\sin\theta_{\max} = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

Numerical aperture:

$$\text{NA} = n_0 \sin\theta_{\max} = \sqrt{n_1^2 - n_2^2}$$

Equation Summary (cont.)

Étendue:

$$\text{E'tendue} = \frac{\pi^2}{4} d^2 NA^2$$

Tapered single fiber:

$$a_i \cdot \mathrm{NA}_i^{\,2} \approx a_o \cdot \mathrm{NA}_o^{\,2}$$

Tapered bundle:

$$\mathrm{NA}_o = \mathrm{NA}_i = \mathrm{NA}_{\text{fiber}}$$
$$a_o \cdot pf_o = a_i \cdot pf_i$$

Uniform Illumination

Bent lightpipes:

$$R = r_2/r_1 = 1 + t/r_1 \le n$$

Integrating sphere radiance:

$$L = \frac{\Phi}{\pi \cdot A_s} \cdot M \qquad M = \frac{1}{1-\bar{\rho}}$$

$$\theta(\phi) = \arctan\left[\tan\theta_1 + \int_{\phi_1}^{\phi} I_{\text{src}}(v)\,dv\right]$$

$$r(\phi) = r_1 \exp\left[\int_{\phi_1}^{\phi} \tan\left(\frac{s-\theta(s)}{2}\right) ds\right]$$

Nonimaging Compound Concentrators

Preservation of étendue:

$$a_i \Omega_i = a_o \Omega_o$$

Maximum concentration ratio:

$$\frac{a_i}{a_o}(\max) = \frac{1}{\sin^2\theta_i}$$

Equation Summary (cont.)

Diameter of CPC used as a collimator:

$$\frac{d_o}{d_i} = \frac{1}{\sin\theta_{\max}}$$

Length of CPC:

$$\text{Length} = \frac{d_o + d_i}{2\tan\theta_{\max}}$$

Equation for surface of CPC:

$$\begin{aligned}&(r\cos\theta_{\max} + z\sin\theta_{\max})^2\\ &+r\,d_s(1+\sin\theta_{\max})^2\\ &-z\,d_s\cos\theta_{\max}(2+\sin\theta_{\max})\\ &-\frac{d_s^2}{4}(1+\sin\theta_{\max})(3+\sin\theta_{\max})\\ &=0,\end{aligned}$$

where:
r is the radius of the cone, perpendicular to the axis,
z is the axial position measured from the small end,
d_s is the diameter of the small end, and
$\theta_{\max}$ is the maximum field angle at the large end.

Tailored edge ray design:

$$\theta(\phi) = \arctan\left[\tan\theta_1 + \int_{\phi_1}^{\phi} I_{src}(v)\,dv\right]$$

$$r(\phi) = r_1\exp\left\{\int_{\phi_1}^{\phi}\tan\left[\frac{s-\theta(s)}{2}\right]ds\right\}$$

CIE Illuminants A and D65

λ (nm)	CIE Ill. A Tungsten at 2856 K	CIE Ill. D65 Sunlight
300	0.930483	0.03410
305	1.128210	1.66430
310	1.357690	3.29450
315	1.622190	11.76520
320	1.925080	20.23600
325	2.269800	28.64470
330	2.659810	37.05350
335	3.098610	38.50110
340	3.589680	39.94880
345	4.136480	42.43020
350	4.742380	44.91170
355	5.410700	45.77500
360	6.144620	46.63830
365	6.947200	49.36370
370	7.821350	52.08910
375	8.769800	51.03230
380	9.795100	49.97550
385	10.899600	52.31180
390	12.085300	54.64820
395	13.354300	68.70150
400	14.708000	82.75490
405	16.148000	87.12040
410	17.675300	91.48600
415	19.290700	92.45890
420	20.995000	93.43180
425	22.788300	90.05700
430	24.670900	86.68230
435	26.642500	95.77360
440	28.702700	104.86500
445	30.850800	110.93600
450	33.085900	117.00800
455	35.406800	117.41000
460	37.812100	117.81200
465	40.300200	116.33600
470	42.869300	114.86100