Field Guide to

Illumination

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John E. Greivenkamp, Series Editor

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Introduction to the Series

Welcome to the *SPIE Field Guides*—a series of publications written directly for the practicing engineer or scientist. Many textbooks and professional reference books cover optical principles and techniques in depth. The aim of the *SPIE Field Guides* is to distill this information, providing readers with a handy desk or briefcase reference that provides basic, essential information about optical principles, techniques, or phenomena, including definitions and descriptions, key equations, illustrations, application examples, design considerations, and additional resources. A significant effort will be made to provide a consistent notation and style between volumes in the series.

Each *SPIE Field Guide* addresses a major field of optical science and technology. The concept of these *Field Guides* is a format-intensive presentation based on figures and equations supplemented by concise explanations. In most cases, this modular approach places a single topic on a page, and provides full coverage of that topic on that page. Highlights, insights, and rules of thumb are displayed in sidebars to the main text. The appendices at the end of each *Field Guide* provide additional information such as related material outside the main scope of the volume, key mathematical relationships, and alternative methods. While complete in their coverage, the concise presentation may not be appropriate for those new to the field.

The *SPIE Field Guides* are intended to be living documents. The modular page-based presentation format allows them to be easily updated and expanded. We are interested in your suggestions for new *Field Guide* topics as well as what material should be added to an individual volume to make these *Field Guides* more useful to you.

John E. Greivenkamp, *Series Editor*
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Field Guide to Illumination

In writing this *Field Guide* to Illumination, the first task was to decide what topics to include. Illumination tends to mean different things to different people. Certainly any subject matter under the purview of the CIE, Commission Internationale de l'Eclairage (the International Commission on Illumination) or the Illuminating Engineering Society of North America (IESNA) must be considered. Some particular areas pertaining to imaging systems and nonimaging optics are potentially overlooked. Thus, we chose to address a number of topics that fall under the following three categories: imaging system illumination, nonimaging optics for illumination, and architectural illumination, which all call upon principles of radiometry and photometry. Although this is not a guide to radiometry, enough information on the subject is included to make this manual a self-contained document. Additionally, those optical properties of materials that are pertinent to illumination, such as surface color, scattering, and retroreflection are described.

The content in this Field Guide starts with traditional illumination in imaging systems, followed by the recent advances in computer-aided design of high efficiency nonimaging illumination optics, along with the modern source models that support these techniques. Sections on the illumination of visual displays are included.

There was not enough room for a complete treatment of architectural illumination, but some important topics are included at the end of this *Field Guide* such as indoor and outdoor architectural illumination.

The notation and terminology are consistent throughout this *Guide*, but we do not lose sight of the fact that they may not be consistent in the field. Examples of alternate notation and terminology are presented.

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Glossary

α	Absorptance
α	Observation angle (in retroreflection)
a_i	Input area to a compound concentrator
a_o	Output area of a compound concentrator
A	Absorbance
A_i	Illuminated area
A_r	Radiating area
a_x	Area of plane *x*
$a\Omega$	Throughput, étendue
C	Concentration ratio
CCT	Correlated color temperature
CIE	International Commission on Illumination
CRI	Color rendering index
d_i	Diameter of input aperture to a CPC
d_o	Diameter of output aperture of a CPC
d_s	Diameter of small aperture of a CPC
$D(\mu,\lambda)$	Donaldson matrix
E	Irradiance
$E_\perp$	Illuminance normal to the illumination
E_0	Axial irradiance
E_e	Edge irradiance
E_i	Image irradiance
E_{i0}	Axial image irradiance
E_λ	Spectral irradiance
$f/\#$	F-number
$f/\#_w$	Working F-number
F'	Increase factor
$F_{a \text{ to } b}$	Form factor from *a* to *b*
f_{skew}	Skew invariant
I	Intensity
$I_{\text{LED A}}$	Averaged LED intensity, CIE condition A
$I_{\text{LED B}}$	Averaged LED intensity, CIE condition B
I_λ	Spectral intensity
L	Radiance
L*,a*,b*	CIE 1976 (L*a*b*) color space; CIELAB
L*,u*,v*	CIE 1976 (L*u*v*) color space; CIELUV
L_i	Image radiance
L_o	Object radiance
L_λ	Spectral radiance

Glossary (cont.)

M	Integrating sphere multiplier
m	Lateral image magnification
n	Index of refraction
NA	Numerical aperture
OD	Optical density
p_e	Purity
pf	Packing fraction
psa	Projected solid angle
R	Reflectance factor
R_A	Coefficient of retroreflection
R_a	General color rendering index
R_I	Coefficient of retroreflected luminous intensity
R_L	Coefficient of retroreflected luminance
$S_\lambda(\lambda)$	Spectral density of a light source
T	Transmittance factor
$T/\#$	T-number
u, v	CIE 1960 UCS chromaticity coordinates
u', v'	CIE 1976 UCS chromaticity coordinates
$V(\lambda)$	Photopic luminous efficiency
$V'(\lambda)$	Scotopic luminous efficiency
$W^* U^* V^*$	CIE 1964 uniform space coordinates
x, y	CIE 1931 chromaticity coordinates
X, Y, Z	CIE tristimulus values
$\bar{x}, \bar{y}, \bar{z}$	CIE color matching functions
β	Entrance angle (in retroreflection)
ξ	Generalized étendue
λ	Wavelength, emission wavelength
$\lambda_{0.5m}$	Center wavelength (for LED)
λ_c	Centroid wavelength (for LED)
λ_d	Dominant wavelength
λ_p	Peak wavelength (for LED)
μ	Excitation wavelength
ρ	Reflectance
$\bar{\rho}$	Average reflectance
τ	Transmittance
υ	Viewing angle (in retroreflection)
Φ	Flux
Φ_λ	Spectral flux
Ω	Projected solid angle (psa)

Glossary (cont.)

ω	Solid angle
$\overline{\Omega}_{a \text{ to } b}$	Average projected solid angle from a to b
Ω_i	Input psa to a compound concentrator
Ω_o	Output psa from a compound concentrator
Ω_x	Projected solid angle viewed from plane x
θ_i	Input half-angle of compound concentrator
θ_o	Output half-angle from compound concentrator
$\theta_{\max}$	Maximum output half-angle from CPC

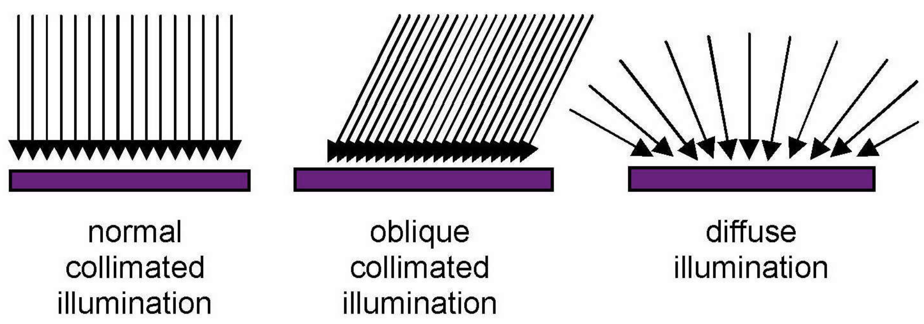
Flux and Irradiance

In examining terminology for illumination, it is useful to separate the spatial considerations from the spectral concerns. In many cases, the spatial and spectral issues are independent and can be separated without losing any generality. In other cases, the spatial and spectral issues cannot be separated physically, but it is useful to separate them conceptually. The commonly used spatial quantities are **flux**, **irradiance**, **intensity**, and **radiance**.

Flux, Φ, is the optical power or rate of flow of radiant energy.

Irradiance, E, is the flux per unit area striking a surface. Occasionally, the flux per unit area leaving a surface, called **exitance**, M, is important. However, the geometry is the same as for irradiance, so it will not be treated separately here. Furthermore, when exitance is used, it is often the flux leaving a nonphysical surface such as the exit port of an integrating sphere or the real image in an imaging system, where it is identical to the irradiance onto the surface.

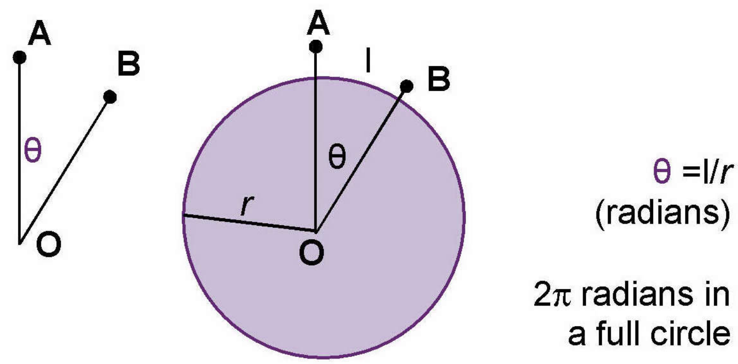
The irradiance quantity itself says absolutely nothing about the directionality of the flux. For example, if the three cases in the figure below all have the same flux per unit area striking the surface, then they all have the same irradiance. Because of this ambiguity, specifications for illumination systems often qualify the irradiance quantity with an added description of the desired directional properties.



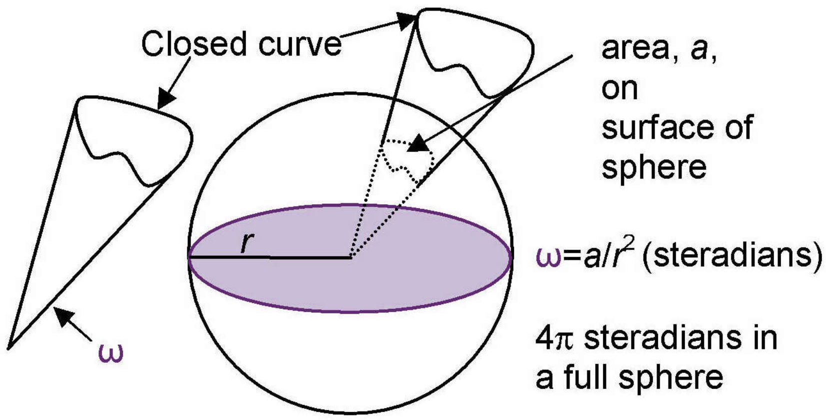
Solid Angle

The definition of intensity involves the concept of a **solid angle**. A solid angle is a 3D angular volume that is defined analogously to the definition of a plane angle in two dimensions.

A **plane angle**, θ, made up of the lines from two points meeting at a vertex, is defined by the arc length of a circle subtended by the lines and by the radius of that circle, as shown below. The dimensionless unit of plane angle is the **radian**, with 2π radians in a full circle.



A solid angle, ω, made up of all the lines from a closed curve meeting at a vertex, is defined by the surface area of a sphere subtended by the lines and by the radius of that sphere, as shown below. The dimensionless unit of solid angle is the **steradian**, with 4π steradians in a full sphere.



Intensity, Radiance, and Projected Solid Angle

Intensity, *I*, is the flux per unit **solid angle**. It is the amount of flux from a point source contained in a small angular volume. A source can be considered a point source for this application if the irradiance falls off as the **inverse square** of the distance from the source. Intensity, for a given source, can vary with direction.

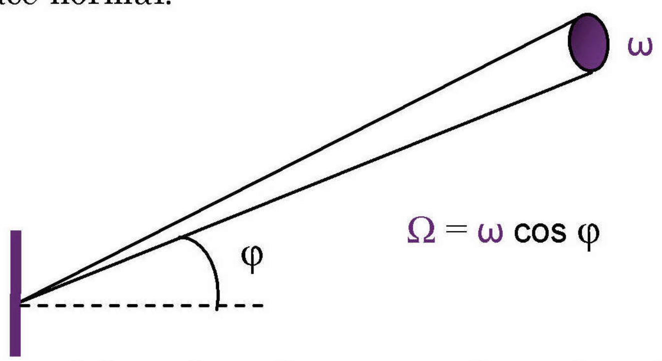
> The term "intensity" is used in many disciplines, some even closely related to optics, to mean things other than flux per unit solid angle. Use caution and rely on context to determine the meaning of the word in a particular situation.

Radiance, *L*, applies to extended sources and surfaces. It is the flux per unit solid angle per unit **projected area** of the source or surface. The projected area is the projection of the area onto a surface normal to the direction of view and is equal to the actual area times the cosine of the angle between the surface normal and the direction of view. Radiance can vary with position on a surface, and like intensity, it can vary with direction. A source or surface with constant radiance in all directions is called Lambertian. A **Lambertian source** or surface has intensity that varies with the cosine of the angle with the surface normal.

In many cases, the angle of view changes over the extent of the receiver. These cases require an alternate definition of radiance: radiance is the flux per unit area per unit **projected solid angle**. (In fact, this is the more general definition and covers the simpler case where the entire surface of the extended source is at essentially the same angle as the direction of view.)

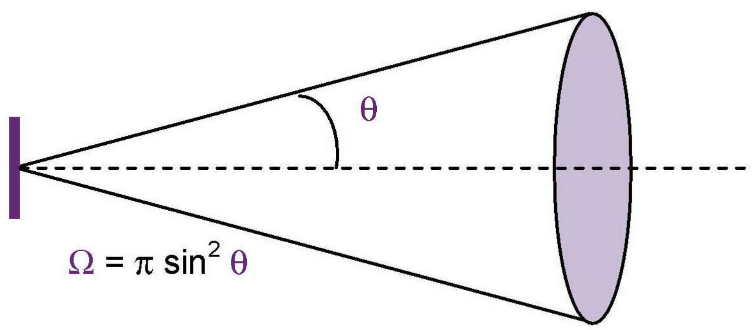
Solid Angle and Projected Solid Angle

The relationship between solid angle and projected solid angle can be confusing. Projected solid angle has meaning primarily for a small Lambertian source, which has intensity that varies as the cosine of the angle with the surface normal. The **projected solid angle**, Ω, is the **solid angle**, ω, weighted by the cosine of the angle with the surface normal.



When the solid angle is large enough so that the angle with the surface normal is not the same over the entire solid angle, the total projected solid angle must be computed by integrating the incremental projected solid angles. See the reference by Bartell for a more detailed explanation.

For some special cases, the integration results in simple expressions, such as for a large circular cone that is normal to a surface and subtends a half angle, θ.



A hemisphere has 2π steradians (solid angle) but π projected steradians (projected solid angle).

Spectroradiometric and Radiometric Quantities

In the spectral dimension of illumination, the most general view looks at the spectral density—the amount of radiation per unit wavelength interval. In terms of the four spatial quantities already considered, the spectral quantities are **spectral flux**, Φ_λ; **spectral irradiance**, E_λ; **spectral intensity**, I_λ; and **spectral radiance**, L_λ. These quantities, usually written with a subscript to indicate that they are integrable, must be integrated to determine the amount of radiation in a particular spectral band. For example, the **total radiant flux**, Φ (in units of watts), in the band between wavelength $\lambda 1$ and wavelength $\lambda 2$ is

$$\Phi\,(\lambda 1, \lambda 2) = \int_{\lambda 1}^{\lambda 2} \Phi_\lambda(\lambda)\cdot d\lambda.$$

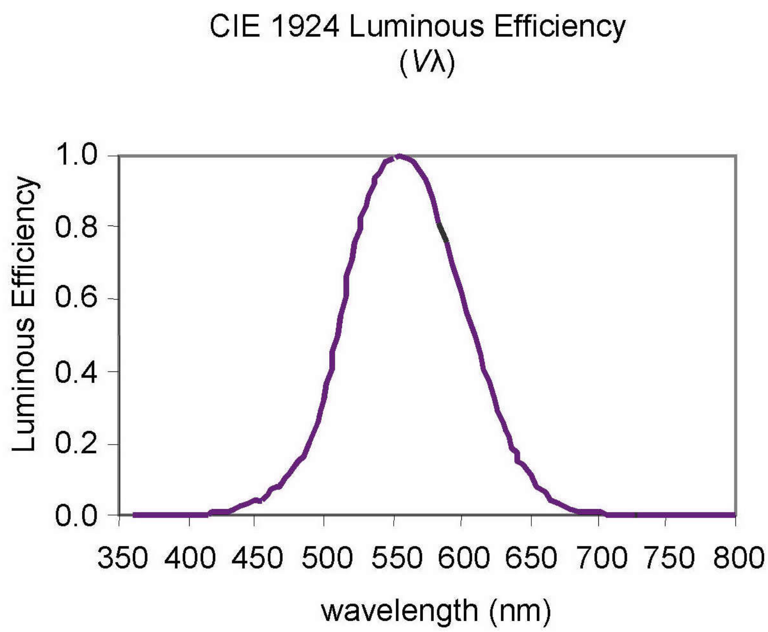
Similar expressions can be written for the **total irradiance**, E (watts/m^2); **total radiant intensity**, I (watts/ sr); and **total radiance**, L (watts/m^2·sr).

Photometry measures the response of the human eye to light. Although not everyone has exactly the same response, the standardized **CIE 1924 luminous efficiency function** works very well for most people. (The CIE is the International Commission on Illumination.) This function, shown on the following page, is designated $V(\lambda)$. The values for this function, in 5-nm increments, are given in the Appendix. Not coincidentally, this function is identical to the **CIE color matching function**, $\bar{y}$. The unit of **luminous (photopic) flux** is the **lumen**. The luminous flux is found from the spectral flux and the $V(\lambda)$ function from the following relationship:

$$\text{luminous flux} = 683\int \Phi_\lambda(\lambda)\cdot V(\lambda)\cdot d\lambda.$$

The factor of 683 in this equation comes directly from the definition of the fundamental unit of luminous intensity, the **candela**.

Photometric Quantities



Notes on notation:

- The photopic quantities of flux, irradiance, intensity, and radiance are called **luminous flux**, **illuminance**, **luminous intensity**, and **luminance**, respectively.
- These quantities are sometimes notated with a subscript "*v*" (for visual), as Φ_v, E_v, I_v, and L_v. But often the subscript is omitted since the meaning is usually clear from the context, and it could be confused with the subscript notation often reserved for integrable quantities.
- The designations Φ, E, I, and L are common but not universally standard. Another set of symbols sometimes used is P, H, J, and N, respectively, for radiometric quantities; P_λ, H_λ, J_λ, and N_λ for spectral quantities; and F, E, I, and B for the corresponding photometric quantities.
- Solid angle and projected solid angle are not always distinguished by ω and Ω, respectively.

Matrix of Basic Quantities

		SPECTRAL		
		Radio-metric	**Spectral**	**Photopic**
SPATIAL	**Flux**	Power watts (W)	Power/ wavelength interval watts/nm	Luminous flux lumens (lm)
	Flux/ area	Irradiance W/m^2	Spectral irradiance $W/m^2 \cdot nm$	Illuminance lm/m^2 or lux
	Flux/ solid angle	(Radiant) intensity W/sr	Spectral intensity $W/sr \cdot nm$	(Luminous) intensity lm/sr or candela (cd)
	Flux/ area· solid angle	Radiance $W/m^2 \cdot sr$	Spectral radiance $W/m^2 \cdot sr \cdot nm$	Luminance $lm/m^2 \cdot sr$ or cd/m^2 or nit

The table above shows the four spatial quantities and the three spectral categories that are discussed in the preceding pages. These create 12 distinct cells that cover the vast majority of specifications for illumination systems.

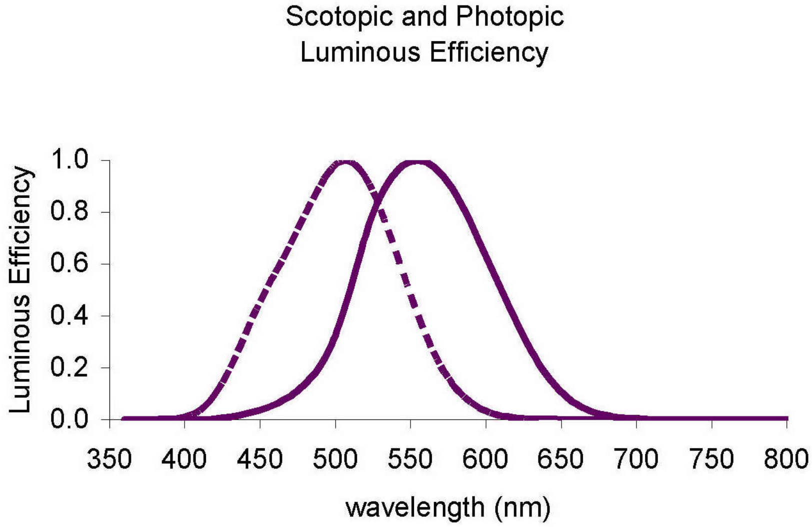
With two exceptions, both used mainly in the United States, work in illumination is almost always done in SI units. The two exceptions (both deprecated) are:

Illuminance
1 footcandle (lm/ft^2) = 10.764 lux (lm/m^2)

Luminance
1 footlambert ($candela/\pi ft^2$) = 3.426 nit ($candela/m^2$)

Photopic and Scotopic Vision

The human visual system responds to light over a wide dynamic range, in excess of 6 orders of magnitude. To achieve this dynamic range, the mechanisms for high-light-level vision and low-light-level vision are different. The high-level region, called the **photopic region**, is active at luminance levels above about 3 cd/m^2. The low-level region, called the **scotopic region**, is active below approximately 0.01 cd/m^2. The region between pure photopic and pure scotopic is called the **mesopic region**, where the visual response is a mixture of the two. The **photopic efficiency**, usually designated $V(\lambda)$, peaks at 555 nm, while the **scotopic efficiency**, usually designated $V'(\lambda)$, peaks at 507 nm.

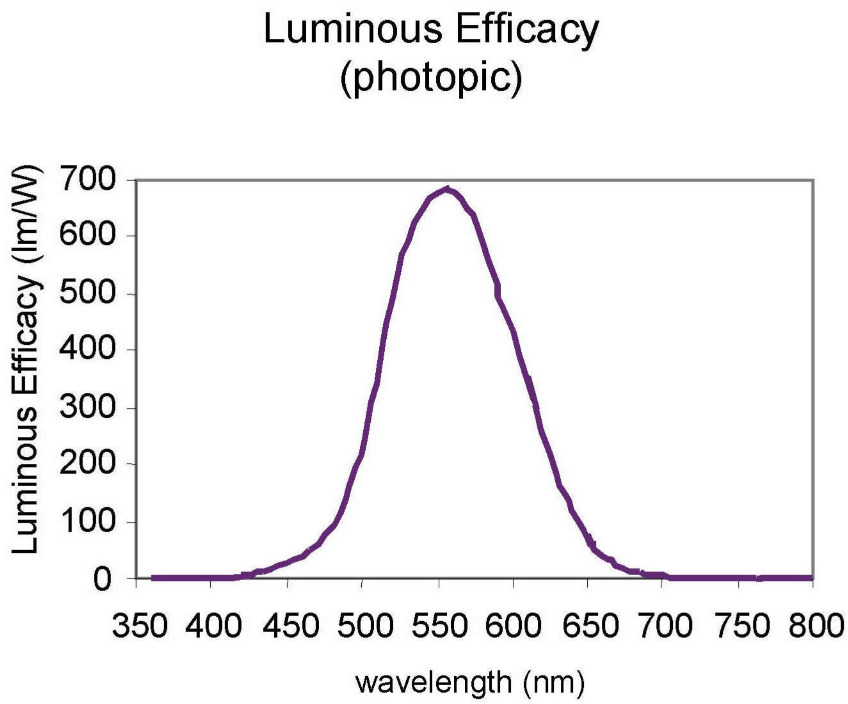


The values for photopic efficiency and scotopic efficiency, both in 5-nm increments, are given in the Appendix.

Luminous Efficacy

Luminous efficacy, quantified in lumens per watt, is a measure of the ability of a light source to produce a visual response from its power. In the photopic region, luminous efficacy peaks at 683 lumens per watt at 555 nm. In fact, the lumen is defined in terms of the power at 555 nm (frequency of 540×10^{12} Hz). Specifically, the definition (adopted in 1979) is in terms of the candela (lumen per steradian).

> The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation at a frequency of 540×10^{12} Hz and that has a radiant intensity in that direction of 1/683 watt per steradian.



It is usually clear from the context whether the power is the radiated power (as in the discussion above) or, often for lamps, the "wall-plug" power.

Typical Values of Illumination Quantities

Irradiance and Illuminance	
Direct sunlight	1000 W/m^2 (250–2500 nm)
Direct sunlight	100,000 lux
Shade	10,000 lux
Overcast day	1,000 lux
Office space	300–600 lux
Full moon	0.2 lux
Quarter moon	0.01 lux
Moonless clear night	0.001 lux
Luminous Intensity	
Automobile headlight	5,000–20,000 cd
Household flashlight	100–1,000 cd
100-W tungsten lightbulb	100 cd
LED traffic signal	250–700 cd
Single LED	1 mcd–25 cd
Radiance and Luminance	
Sun	2×10^7 W/m^2·sr (250–2500nm)
Sun	2×10^9 nit
Frosted lightbulb	100,000 nit
Fluorescent lamp	5,000 nit
Computer screen	100 nit

Wavelength Ranges for Illumination	
UV-C*	250 to 280 nm
UV-B	280 to 315 nm
UV-A	315 to 400 nm
Visible	~360–400 to ~760–800 nm
Near-infrared (NIR)†	760 nm to 1.1 µm

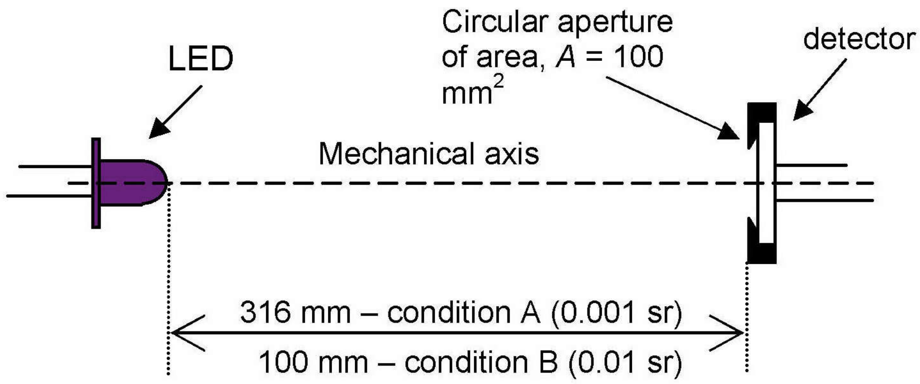
* Actual definition of UV-C is 100 to 280 nm. However, the range from 100 to 250 nm is not of interest for illumination systems.

† Actual definition of NIR is to 1.4 µm. However, 1.1 µm is the upper limit for silicon-based detectors.

Averaged LED Intensity

In 1997 the CIE established a special quantity for light-emitting diodes (LEDs) called the **averaged LED intensity**. This was introduced because, as stated in CIE Publication 127:2007, *Measurement of LEDs*, "There are significant differences between LEDs and other light sources which made it necessary for the CIE to introduce a new quantity for their characterization with precisely defined measurement conditions."

To obtain **averaged LED intensity**, the LED is measured on its mechanical axis (in line with the package) by a circular detector of area 100 mm^2 at a prescribed distance from the front tip of the LED package. Two distances are used: 316 mm **(condition A)** and 100 mm **(condition B)**, with the solid angles defined as 0.001 sr and 0.01 sr, respectively. The measurements made are notated as $I_{\mathrm{LED\ A}}$ and $I_{\mathrm{LED\ B}}$ in units of intensity (candela or W/sr). Since the entire measurement geometry is completely defined, the measurements should be repeatable.



Light Source Color

The perceived color of a light source is quantified by its **chromaticity**. Chromaticity is calculated from the spectral density of the light source, S_λ, and the **CIE color matching functions**, $\bar{x}, \bar{y}$, and $\bar{z}$ as follows:

$$X = \int S_\lambda(\lambda) \cdot \bar{x}(\lambda) \cdot d\lambda, \qquad x = \frac{X}{X+Y+Z},$$

$$Y = \int S_\lambda(\lambda) \cdot \bar{y}(\lambda) \cdot d\lambda,$$

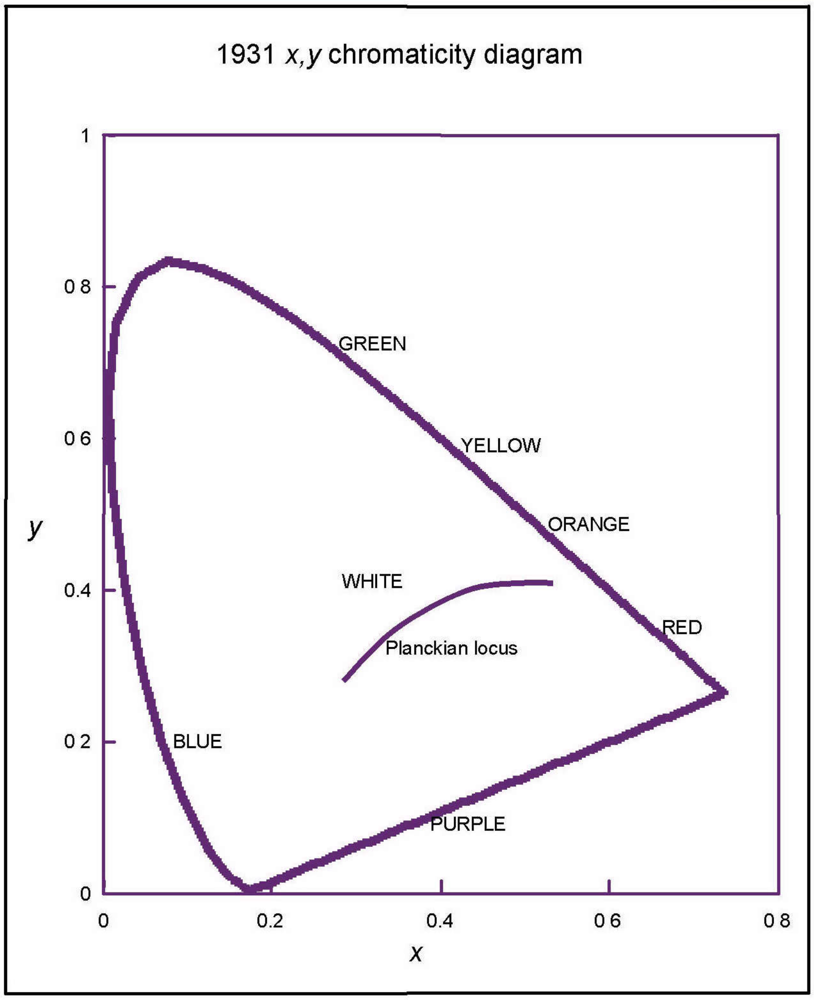
$$Z = \int S_\lambda(\lambda) \cdot \bar{z}(\lambda) \cdot d\lambda, \qquad y = \frac{Y}{X+Y+Z},$$

where X, Y, and Z are called **tristimulus values**, and x and y are the **CIE 1931 chromaticity coordinates**. The integrals above are usually calculated as block summations from 360 to 830 nm, generally at 1-nm or 5-nm intervals. A table of $\bar{x}, \bar{y}$, and $\bar{z}$, in 5-nm intervals can be found in the Appendix.

The 1931 chromaticity coordinates (x, y) are common coordinates for light-source colors and are represented graphically by the familiar "horseshoe" graph. All of the possible colors of light are contained inside the horseshoe shape, with the pure monochromatic spectral colors around the curved perimeter, the purples along the straight line at the bottom, and less-saturated colors in the interior. The various shades of white, which are of the most interest in illumination systems, occupy the central region.

Those white lights that have near-blackbody spectra (such as tungsten incandescent lamps) lie along the **Planckian locus**. The lower blackbody temperatures lie toward the red, and the higher temperatures toward the blue.

Chromaticity Diagram



Two other coordinate systems are used to describe the chromaticity of light sources: the **CIE 1960 UCS coordinate system** (u, v), and the **CIE 1976 UCS coordinate system** (u', v'). Both attempt to portray equal perceived color differences by equal distances.

$$u = 4X/(X + 15Y + 3Z) = 4x/(-2x + 12y + 3),$$
$$v = 6Y/(X + 15Y + 3Z) = 6y/(-2x + 12y + 3),$$
$$u' = 4X/(X + 15Y + 3Z) = 4x/(-2x + 12y + 3),$$
$$v' = 9Y/(X + 15Y + 3Z) = 9y/(-2x + 12y + 3).$$

The CIE 1960 UCS coordinate system is obsolete except for calculating **correlated color temperature**.

Color Temperature and CCT

Any light source whose chromaticity coordinates fall directly on the Planckian locus has a **color temperature** equal to the blackbody temperature of the Planckian radiator with those coordinates. Color temperature is usually expressed in Kelvins (K). The concept of color temperature is especially useful for incandescent lamps, which very closely approximate a blackbody spectrum throughout the visible region. For these lamps, the color temperature also defines the spectrum in this region.

For white lights that don't have chromaticity coordinates that fall exactly on the Planckian locus but do lie near it, the **correlated color temperature (CCT)** is used. The CCT of a light source, also expressed in Kelvins, is defined as the temperature of the blackbody source that is closest to the chromaticity of the source in the CIE 1960 UCS (*u*, *v*) system. CCT is an essential metric in the general lighting industry to specify the perceived color of fluorescent lights and other nonincandescent white-light sources such as LEDs and high intensity discharge HID lamps.

The difference in perceived color is closely related to the reciprocal of CCT. The reciprocal is expressed in reciprocal megakelvin $(\mathrm{MK})^{-1}$, with one $(\mathrm{MK})^{-1}$ approximately equal to a just-noticeable color difference:

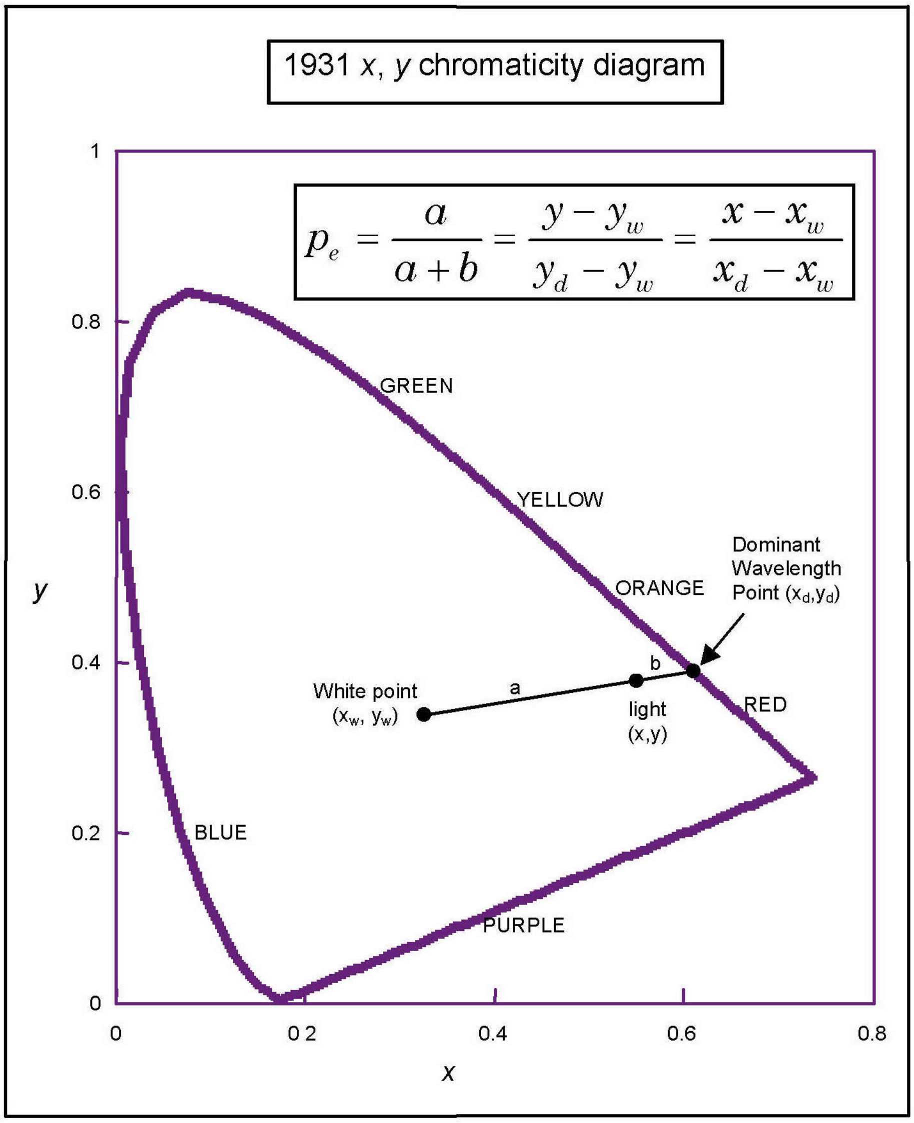
$$(MK)^{-1} = 10^6 / CCT.$$

There are limitless different spectra, all with the same CCT, that may have little or no resemblance to the blackbody curve for that temperature or to each other.

There is no approved method for computing CCT nor is there a simple and accurate closed-form expression. One simple and accurate method is to use a program such as Excel with *solver* to find the blackbody temperature that minimizes the distance between its (*u*, *v*) coordinates and those of the light in question.

Dominant Wavelength and Purity

Colored light sources can be modeled as a mixture of a monochromatic source and a white light. The wavelength of this theoretical monochromatic source is called the **dominant wavelength**, λ_d, and is the perceived color of the light. The percent of the total power provided by the monochromatic source is the **purity**, p_e.



The choice of the "white point" is arbitrary. Very often, the default choice is an "equal energy white" ($x = 0.3333$, $y = 0.3333$).

Surface Color

The color of a surface, like that of a light source, can be quantified. The **reflectance factor**, $R(\lambda)$, of the surface is combined with the spectral density of the illumination, S_λ, and the CIE color matching functions, $\bar{x}$, $\bar{y}$, and $\bar{z}$, in the calculation of tristimulus values:

$$X = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{x}(\lambda)\cdot d\lambda,$$

$$Y = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{y}(\lambda)\cdot d\lambda,$$

$$Z = k\int S_\lambda(\lambda)\cdot R(\lambda)\cdot \bar{z}(\lambda)\cdot d\lambda,$$

$$k = \frac{100}{\int S_\lambda(\lambda)\cdot \bar{y}(\lambda)\cdot d\lambda}.$$

Or, in summation form:

$$Y = k\sum_{\lambda=380}^{780} S_\lambda(\lambda)\ R(\lambda)\ \bar{y}(\lambda), \qquad X = k\sum_{\lambda=380}^{780} S_\lambda(\lambda)\ R(\lambda)\ \bar{x}(\lambda),$$

$$Z = k\sum_{\lambda=380}^{780} S_\lambda(\lambda)\ R(\lambda)\ \bar{z}(\lambda)\ , \qquad k = 100\Big/\sum_{\lambda=380}^{780} S_\lambda(\lambda)\ \bar{y}(\lambda).$$

The chromaticity of light sources is 2D, with the photometric value of luminous flux, illuminance, luminous intensity, or luminance playing the role of the third "dimension." However, the chromaticity of surfaces is 3D, with a "lightness" dimension included. Common 3D surface-color spaces are derived from the tristimulus values.

x, *y*, *Y* – CIE 1931 chromaticity plus *Y* tristimulus value

L*,u*,v* – CIE 1976 (L*u*v*) color space; CIELUV

L*,a*,b* – CIE 1976 (L*a*b*) color space; CIELAB

W*U*V*– CIE 1964 uniform space coordinates (obsolete except for the calculation of color rendering index)

Color of Fluorescent Surfaces

The phenomenon of **fluorescence** is characterized by the absorption of light at one wavelength and the nearly instantaneous emission at a longer wavelength. Various surfaces of interest in illumination systems, such as road signs designed to convert the blue-rich skylight at twilight to more visible yellow, also exhibit intentional fluorescent properties to make them brighter or more detectable.

The calculation of the chromaticity of fluorescent surfaces has a degree of complexity that is not present for nonfluorescent surfaces. For fluorescent surfaces, the reflectance factor $R(\lambda)$ is replaced by the **Donaldson matrix**, $D(\mu,\lambda)$, where μ is the absorbed or excitation wavelength, and λ is the emission wavelength:

$$X = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{x}(\lambda),$$

$$Y = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{y}(\lambda),$$

$$Z = k \sum_{\lambda=380}^{780} \sum_{\mu=300}^{780} S_{\mu}(\mu)\, D(\mu,\lambda)\, \bar{z}(\lambda).$$

Note that for nonfluorescent surfaces, the Donaldson matrix collapses to the reflectance factor for the diagonal elements ($\mu = \lambda$), and is zero everywhere else. In fact, the reflectance factor can be thought of as a special case of the Donaldson matrix.

> The normalization constant, k, is the same as for non-fluorescent surfaces.

The details of computing the chromaticity of fluorescent surfaces can be found in ASTM Standard E2152-01, *Standard Practice for Computing the Colors of Fluorescent Objects from Bispectral Photometric Data.*

Color Rendering and CRI

The color of an object depends on both the object and the illuminating source. Some sources are better than others at rendering the colors of objects. This ability is quantified in a figure of merit called the **color rendering index (CRI)** for the source. The CRI is calculated by comparing the chromaticity coordinates of eight prescribed nonfluorescent **test-color samples** (specified by their spectral radiance factors) under the source being evaluated with the chromaticity coordinates of the same eight samples under a **reference illuminant**. The reference illuminant is a blackbody radiator (with the same CCT as the source) for sources with a CCT of less than 5000 K, and a phase of daylight (with the same CCT as the source) for sources with a CCT of 5000 K or higher. The distance between the two chromaticity coordinates for a particular test-color sample represents the color difference between the sample illuminated by the source being evaluated and the same sample illuminated by the reference illuminant. The **general color rendering index**, designated R_a, is the average distance (in W^*, U^*, V^* space) between the eight pairs of chromaticity coordinates (a pair for each test-color sample). It is normalized so a source that is identical to its reference illuminant has a CRI of 100, and a "warm white" fluorescent lamp has a CRI of about 50. Here are examples of typical CRI values for several illuminating sources:

Sunlight (CIE D65)	100
Tungsten lamp (CIE A)	100
Xenon	97
White light LED (blue + YAG)	83
Compact fluorescent lamp	80
Daylight fluorescent	75
Metal halide lamp	61
Warm white fluorescent	52
High-pressure sodium lamp	20
White light from RGB LED combination	20 to 65
White light from four LED combination	up to 90

Calculating CRI and Problems with CRI

Complete details for calculating **CRI** are beyond the scope of this guide. The information is available in two CIE publications (both are necessary):

- CIE 13.3-1995, *Method of Measuring and Specifying Color Rendering Properties of Light Sources*
- CIE Publication 15:2004, *Colorimetry, 3rd Edition*

With these publications comes a software disk for calculating CRI from a light source spectral density. The publications and disk can be purchased from the CIE at www.cie.co.at/cie/index.html.

Work on the CRI began in the 1940s when widespread use of fluorescent tubes began for general lighting. The figure of merit has come under criticism lately for several reasons. Among them:

- The eight test-color samples are all moderate in saturation. CRI does not produce numbers that correspond well with observations on highly saturated colors.
- The CRI is considered to be less accurate when the test illuminant and the reference illuminant differ by more than 0.0054 (in u, v space). Many real illuminants are farther away than this from their respective reference illuminants.
- Many different illuminants, such as sunlight and tungsten lamps, have “perfect” CRIs near 100, yet they render colors quite differently.
- New white light sources, such as combinations of LEDs, seem to perceptually render colors far better than is predicted by their rather low CRIs.

The CIE is expected to issue a new recommendation for quantifying color rendering within the next several years.

Typical Source Parameters

Spectral densities for several common light sources are displayed on the following pages. The table below lists some of the relevant parameters for these sources.

Typical values for common illumination sources

Source	Approx. CCT (K)	CRI (R_a)	Luminous Efficacy (lm/W)
Tungsten†	2800 to 3200	100	12 to 20
Sunlight	6500	100	100‡
Daylight fluorescent	6360	75	55
Warm white fluorescent	3000	52	60
High-pressure sodium	2030	20	115
Metal halide	4020	61	90
Xenon	5000 to 6000	97	10 to 20

† Tungsten lamps allow a great deal of control over these parameters. Guidelines for controlling tungsten lamps are detailed on the next page.

‡ Lumens per radiated watt. All others are "wall-plug."

LEDs are becoming common illumination sources. They produce white light by using a blue LED and one or more phosphors. Depending on the phosphor used, they can have CCTs from less than 3000 K to in excess of 9000 K. The luminous efficacies also vary over a wide range, from 40 or 50 lm/W for high-flux devices and up to and above 100 lm/W for low-flux devices, with CRIs up to about 80 (R_a). White light is also produced from LEDs by mixing the light from several single-color LEDs. LED lighting technology is changing extremely rapidly as newer devices are developed.

Tungsten Lamps

Tungsten filament incandescent lamps, particularly tungsten halogen lamps, are often used in illumination systems. Paired with stable, current-controlled power supplies, they provide extremely stable sources. If the current is altered, the filament temperature changes, with a resulting change in filament resistance. Therefore, the voltage change is not linearly related to the current change. Similar nonlinear relationships hold for other characteristics of the lamp, such as power consumed, luminous flux output, efficacy (lumens/Watt), color temperature, and lifetime.

Tungsten halogen lamps should be operated between 95% and 105% of their **rated current**. Within this operating envelope, the relationships between the operating parameters are approximately exponential. Modifying the current to a value different from the rated current changes the other parameters according to the exponents shown in the table below:

Parameter	Exponent
Color temperature (K)	0.80
Lifetime (hours)	–25
Luminous efficacy (lumens/Watt)	3.6
Luminous flux (lumens)	6.5
Power (Watts)	2.9
Voltage (volts)	1.9

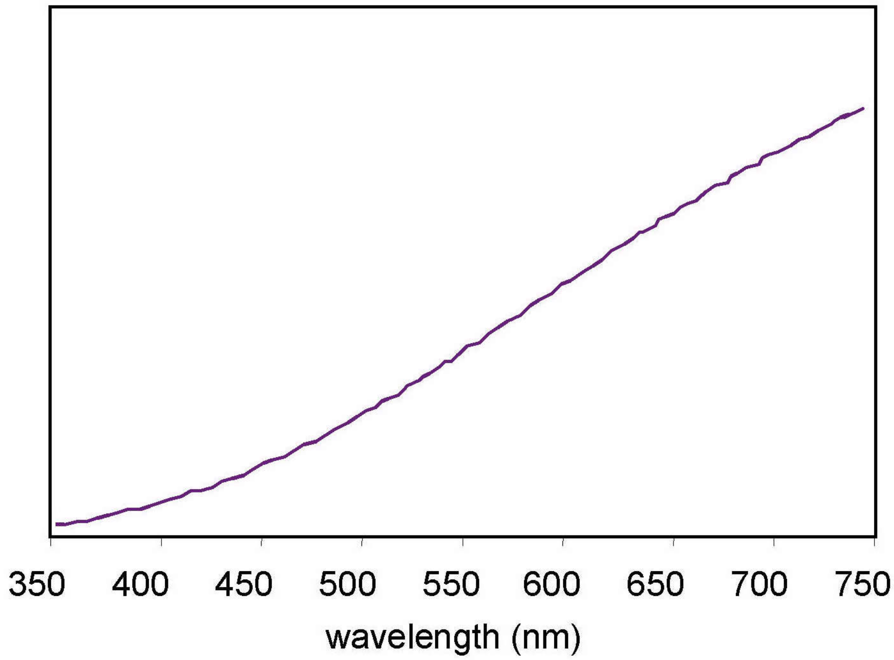
An example of a lamp operated at 5% over its rated current is shown below:

Parameter	Rated	Operated
Current	6.02 A	$6.02 \times 1.05 = 6.32$
Color temperature	2900 K	$2900 \times 1.05^{0\ 8} = 3015$
Lifetime	1000 hrs	$1000 \times 1.05^{-25} = 295$
Luminous efficacy	16 lm/W	$16 \times 1.05^{3\ 6} = 19$
Luminous flux	1040 lm	$1040 \times 1.05^{6\ 5} = 1428$
Power	65 W	$65 \times 1.05^{2\ 9} = 75$
Voltage	10.8 V	$10.8 \times 1.05^{1\ 9} = 11.8$

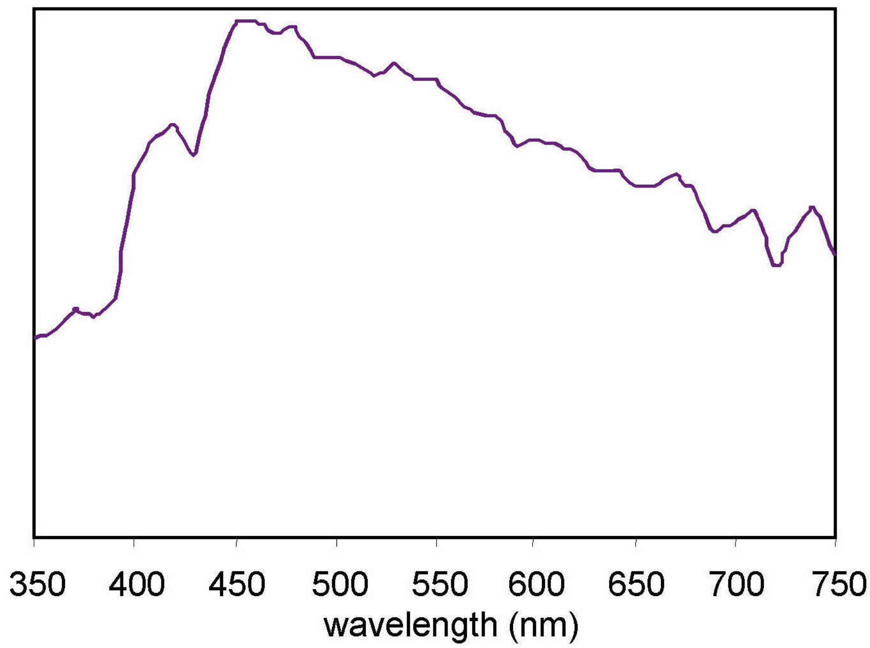
Tungsten and Sunlight

This page and those that follow show typical spectra of several common illumination sources.

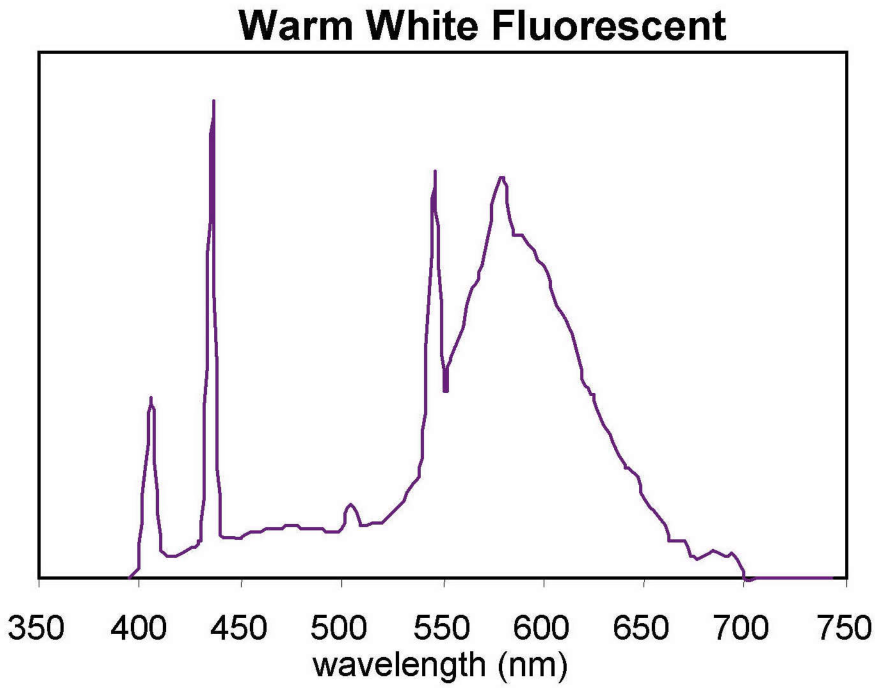
Tungsten Lamp (CIE A)

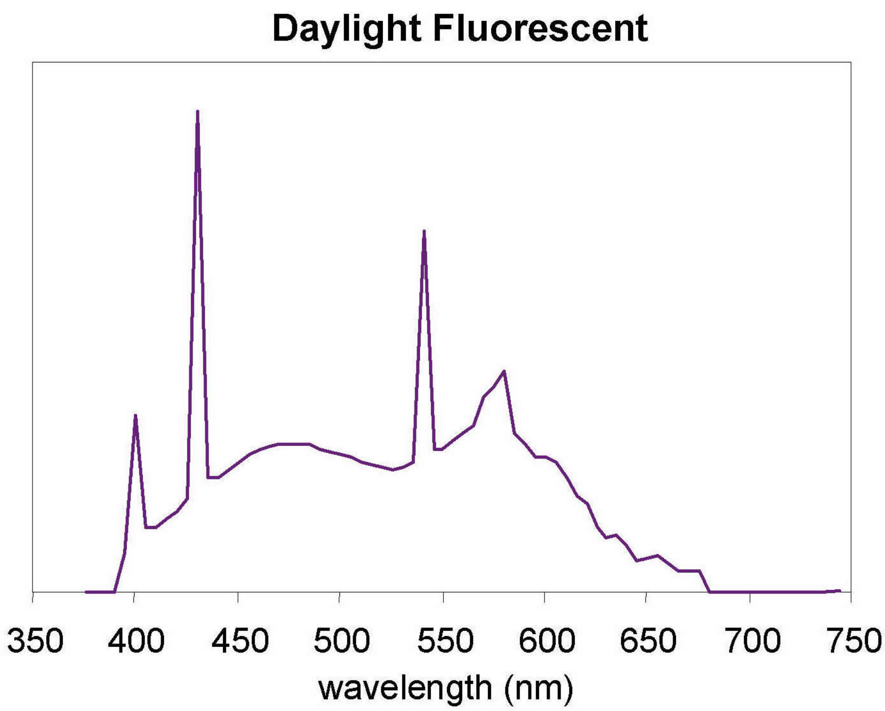


Sunlight (CIE D65)

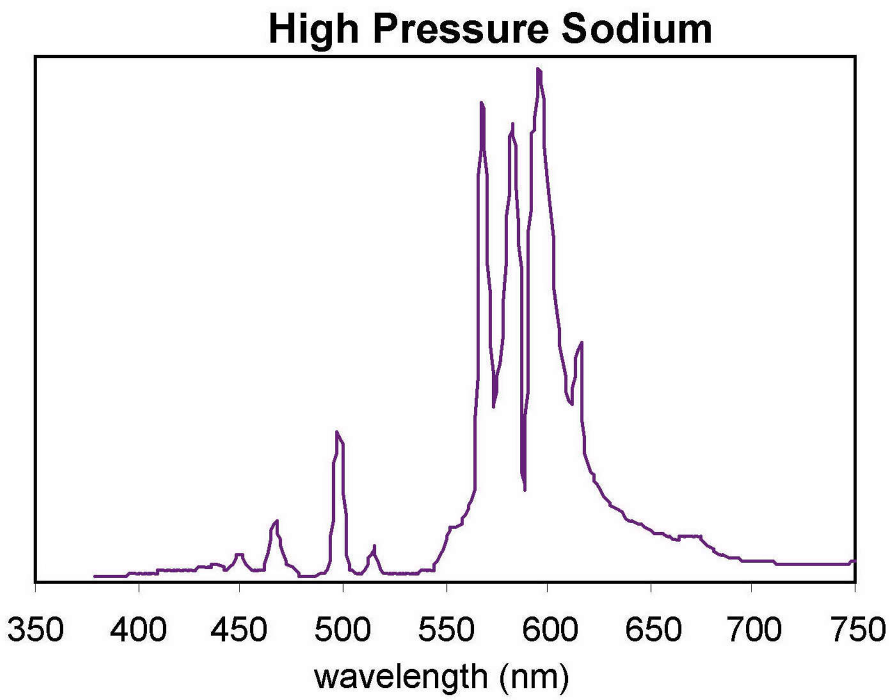


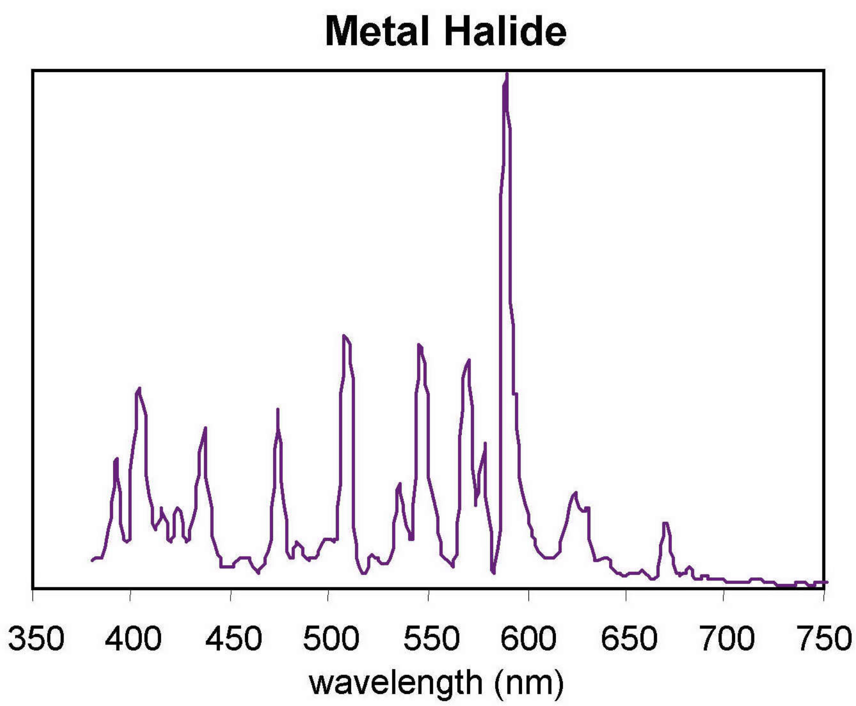
Fluorescent Lamps



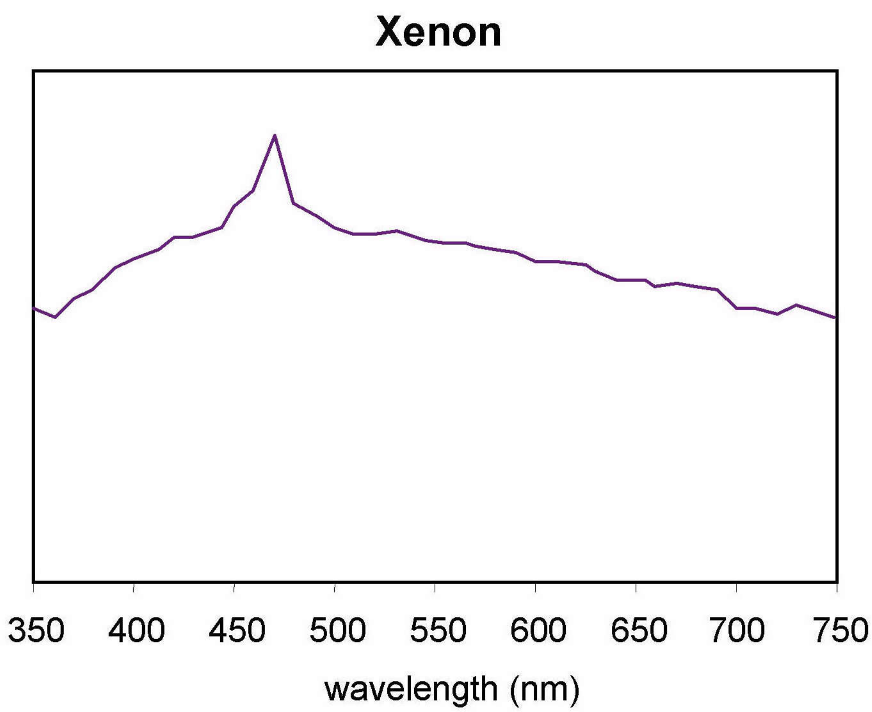


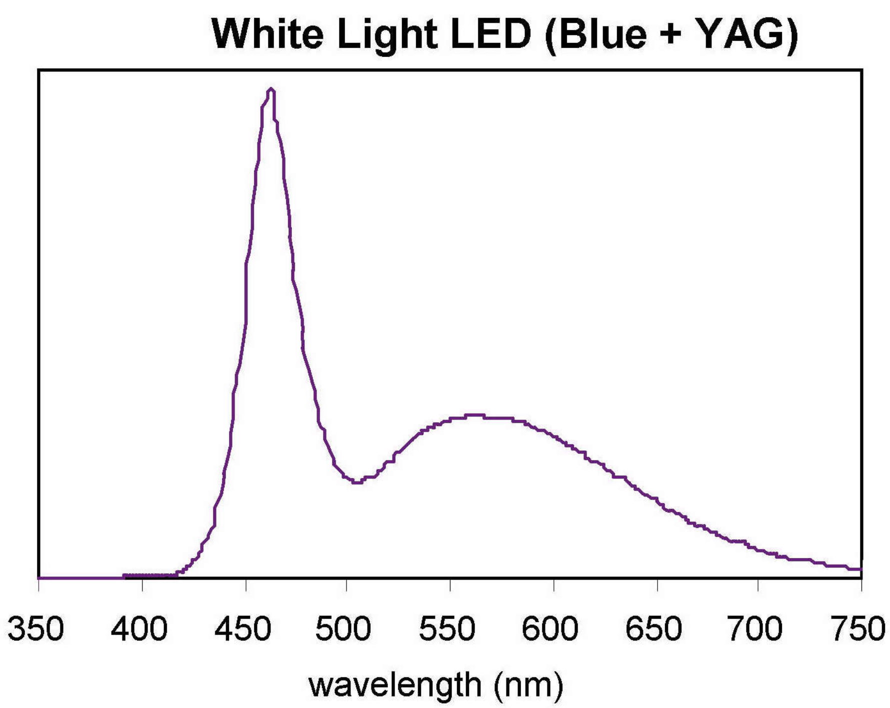
H.P. Sodium and Metal Halide





Xenon and White LEDs

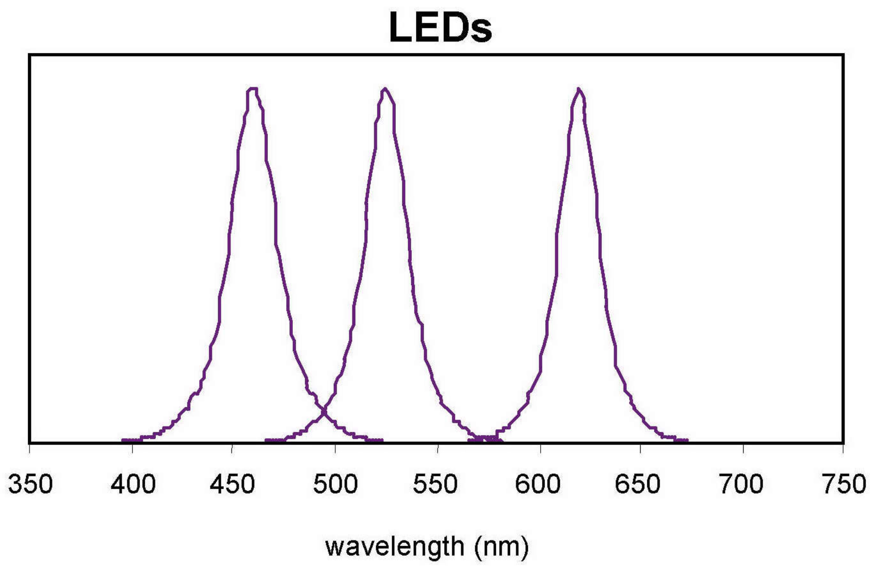




Light Emitting Diodes (LEDs)

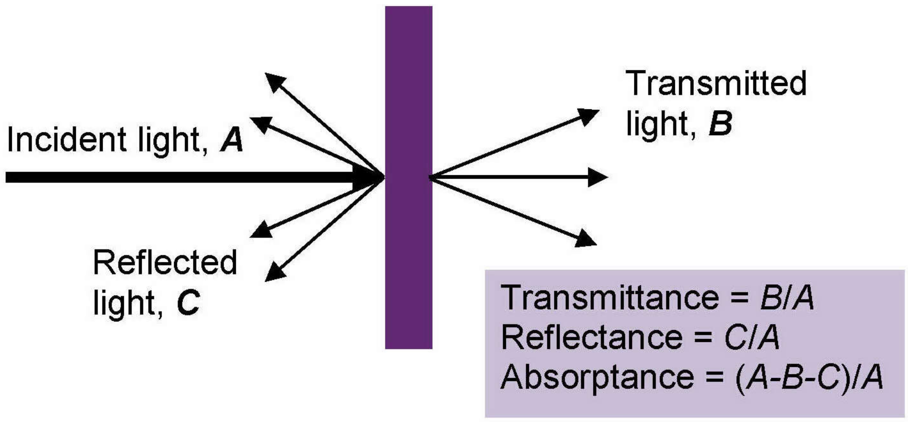
LEDs are moderately narrowband emitters with an approximately Gaussian spectral shape. The spectrum of an LED is often expressed by a single wavelength, with four different single-wavelength descriptions in general use. The most common spectrum-based description is the **peak wavelength**, λ_p, which is the wavelength of the peak of the spectral density curve. Less common is the **center wavelength**, $\lambda_{0.5m}$, which is the wavelength halfway between the two points with a spectral density of 50% of the peak. For a symmetrical spectrum, the peak and center wavelengths are identical. However, many LEDs have slightly asymmetrical spectra. Least common is the **centroid wavelength**, λ_c, which is the mean wavelength. The peak, center, and centroid wavelengths are all derived from a plot of $S_\lambda(\lambda)$ versus λ. The fourth description, the **dominant wavelength**, λ_d, is a colorimetric quantity that is described in the section on **color**. It is the most important description in visual illumination systems because it describes the perceived color of the LED.

Spatially, LEDs, especially those in lens-end packages, are often described by their **viewing angle**, which is the full angle between points at 50% of the peak intensity.



Transmittance, Reflectance, and Absorptance

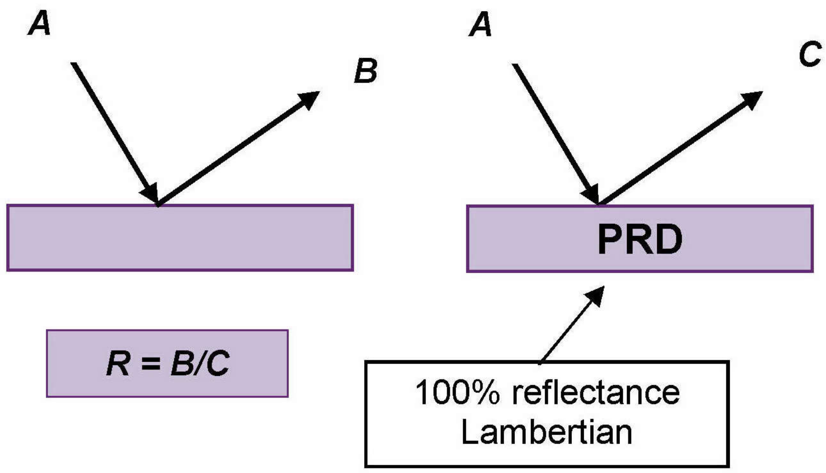
Several alternative methods describe the response of materials to illumination. One common approach is the ratio of the light that is transmitted, reflected, or absorbed to the incident light. This method describes a material by its **transmittance**, τ, its **reflectance**, ρ, or its **absorptance**, α. Do not confuse absorptance with **absorbance**, A, which is equivalent to **optical density (OD)** and is a conversion of transmittance or reflectance to a log scale. For example, 10% transmittance can be described as 1A, 1% as 2A, etc.



A material that produces intensity proportional to the cosine of the angle with the surface normal is called Lambertian. The radiance of a **Lambertian surface** is constant with viewing direction (since the projected area of a viewed surface is also proportional to the cosine of the angle with the surface normal). Furthermore, the directional distribution of scattered light is independent of the directional distribution of the incident illumination. It is impossible to tell, by looking at a Lambertian surface, where the incident light comes from. Perfectly Lambertian surfaces don't really exist, but many materials, such as matte paper, flat paint, and sandblasted metal (in reflection), as well as opal glass and sandblasted quartz (in transmission), are good Lambertian approximations over a wide range of incidence and view angles.

Reflectance Factor and BRDF

A quantity sometimes confused with reflectance is the **reflectance factor**, *R*. The reflectance factor is defined in terms of a hypothetical **perfectly reflecting diffuser (PRD)**, a surface that is perfectly Lambertian and has a 100% reflectance. The reflectance factor is the ratio of the amount of light reflected from the material to the amount of light that would be reflected from a PRD if similarly illuminated and similarly viewed.



Notes on reflectance (ρ) and the reflectance factor (R):

- For a **Lambertian surface**, ρ and R are identical.
- **Reflectance** must be between 0 and 1. The reflectance factor is not similarly bound. A highly polished mirror, for example, has near-zero R for any nonspecular incident and viewing angles, and a very high R (>1.0) for any specular incident and viewing angles.
- The reflectance factor is more closely related to the **bidirectional reflectance distribution function (BRDF)** than to reflectance. The BRDF is defined as the radiance of a surface divided by its irradiance:

$$\text{BRDF} = L/E.$$

- The reflectance factor measures per hemisphere (there are π projected steradians in a hemisphere) what the BRDF measures per projected steradian:

$$R = \text{BRDF}\cdot\pi.$$

Harvey / *ABg* Method

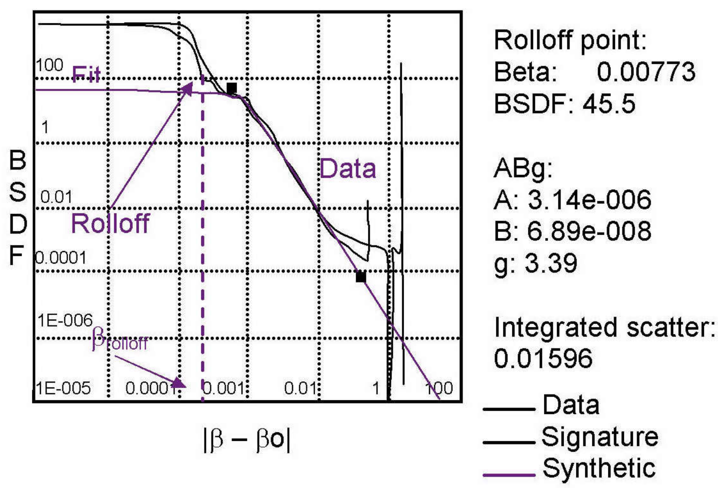
The **Harvey** or ***ABg* method** is used to parameterize scatter from a weakly scattering surface, which is typical for optical surfaces such as lenses and mirrors. It also can be used to model Lambertian surfaces and anisotropic (i.e., asymmetric) scatter. An example for a three-axis polished surface is provided here, which has a **total integrated scatter (TIS** or **TS)** of about 1.6%. The vertical scale represents the **BSDF**, for which an *R* (reflection) or *T* (transmission) can be substituted for the *S* (surface). The horizontal scale represents the absolute difference between $\beta_0 = \sin\theta_0$, or the specular direction, and $\beta = \sin\theta$, or any direction away from specular. Note that both axes are plotted in log space such that the roll-off slope is linear. The *ABg* parameters are:

- *g* is the slope of the roll-off as shown in the figure whose value of 0 defines a **Lambertian surface**.
- *B* is the roll-off parameter defined as

$$B = \left|\beta_{\text{rolloff}}\right|^{g}.$$

- *A* is the amplitude factor and can be found from

$$BSDF = \frac{A}{B + \left|\beta - \beta_0\right|^{g}}.$$



Directional Properties of Materials

The reflectance of a material can depend on the direction of the incident light. This dependence is often indicated by a number or letter.

- ρ(0°): reflectance for normal incidence.
- ρ(45°): reflectance for a 45-deg oblique incidence.
- ρ(*d*) or ρ(*h*): reflectance for diffuse illumination.

The **reflectance factor** of a material can depend on both the direction of illumination and the viewing geometry. This is usually indicated by two letters or numbers, the first indicating the incident geometry and the second the viewing geometry.

- R(0°/45°): the reflectance factor for normal incidence and a 45-deg oblique viewing (a common geometry for measuring the color of a surface).
- R(0°/*d*): the reflectance factor for normal incidence and diffuse (everything except the specular) viewing only.
- R(8°/*h*): the reflectance factor for near-normal incidence and hemispherical (everything, including the specular) viewing.
- R(45°/*h*): the reflectance factor for hemispherical illumination and a 45-deg oblique viewing.

The same notation used for reflecting materials can be applied to transmitting materials, where transmittance τ can be dependent on incident geometry, and the transmittance factor, *T*, on both the incident and transmitting geometries. The use of the transmittance factor is not as common as transmittance, reflectance, and the reflectance factor.

Some materials have reflecting properties that are not the same for every azimuthal angle, even for the same elevation angle, e.g., the specular geometry of mirrorlike surfaces has vastly different reflecting properties than any geometry with the same incident and reflecting elevation angles that are not both in the same plane with the surface normal.

Retroreflectors—Geometry

Retroreflectors reflect incident light back toward the direction of the light source, operating over a wide range of angles of incidence. Typically they are constructed in one of two different forms, 90-deg corner cubes or high index-of-refraction transparent spheres with a reflective backing. Retroreflectors are used in transportation systems as unlighted night-time roadway and waterway markers, as well as in numerous optical systems, including lunar ranging. Some are made of relatively inexpensive plastic pieces or flexible plastic sheeting, and some are made of high-priced precision optics.

The performance of retroreflectors is characterized within a geometrical coordinate system, usually with three angles for the incident and viewing geometries and a fourth orientation angle for prismatic designs like corner cubes, which are not rotationally isotropic in their performance. All the geometric variations are described in detail in ASTM E808-01, *Standard Practice for Describing Retroreflection*, along with expressions for converting from one geometric system to another.

Two angles commonly used to specify the performance of retroreflectors are the **entrance angle**, β, and the **observation angle**, α. The entrance angle is the angle between the illumination direction and the normal to the retroreflector surface. High-quality retroreflectors work over fairly wide entrance angles, up to 45-deg or more (up to 90 deg for pavement marking). The observation angle, the angle between the illumination direction and the viewing direction, is generally very small, often one degree or less.

Another useful angle for interpreting the performance of retroreflectors is the **viewing angle**,υ, the angle between the viewing direction and the normal to the retroreflector surface.

Retroreflectors—Radiometry

The performance of retroreflectors is quantified by several coefficients. These are the most common:

R_I, **coefficient of retroreflected luminous intensity**,

$$R_I = \frac{I}{E_\perp},$$

where $E_\perp$ is the illuminance on a plane normal to the direction of illumination, and I is the intensity of the illuminated retroreflector.

R_A, **coefficient of retroreflection**,

$$R_A = \frac{R_I}{A} = \frac{I/A}{E_\perp},$$

where A is the area of the retroreflector.

R_L, **coefficient of retroreflected luminance**,

$$R_L = \frac{R_A}{\cos\upsilon} = \frac{L}{E_\perp}$$

is the ratio of the luminance in the direction of observation to $E_\perp$.

R_Φ, **coefficient of retroreflected luminous flux**:

$$R_\Phi = \frac{R_A}{\cos\beta}.$$

R_F, **retroreflectance factor**

$$R_F = \frac{\pi \cdot R_I}{A \cdot \cos\beta \cdot \cos\upsilon} = \frac{\pi \cdot R_A}{\cos\beta \cdot \cos\upsilon} = \frac{\pi \cdot R_L}{\cos\beta}.$$

It is the retroreflectance factor, R_F that is numerically equivalent to the reflectance factor, R.

Retroreflectors are often specified by the coefficient of retroreflection, R_A, for various observation angles and entrance angles.

Values for R_A of several hundred (cd/m^2)/lux are not uncommon, corresponding to reflectance factors up to and over 1000.

Lambertian and Isotropic Models

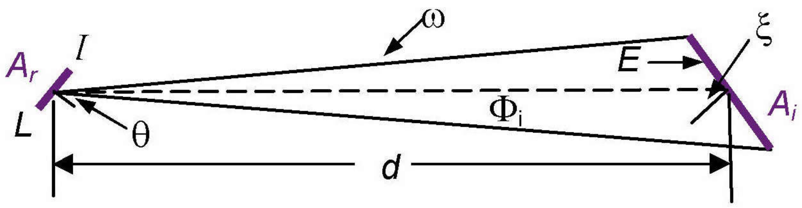
There are no direct "conversion factors" between the four basic quantities in illumination: **flux**, Φ; **irradiance**, E; **intensity**, I; and **radiance**, L. But for many situations, knowledge of one factor allows the calculation of the others. Making this calculation usually requires knowledge of the directional properties of the illuminating source, or at least a fair model of these directional properties. The two most common models are isotropic and Lambertian.

An **isotropic source** is defined here as having intensity independent of direction. For a **Lambertian source**, the radiance is independent of direction and the intensity is therefore proportional to the cosine of the angle with the surface normal. A few nearly isotropic sources exist, such as a round, frosted light bulb, a frosted ball-end on a fiber, and a line filament (in one plane, anyway). However, most flat radiators, diffusely reflecting surfaces, and exit pupils of illuminating optical systems are more nearly Lambertian than isotropic. Reasonable predictions can be made by modeling them as Lambertian.

The model of directional illumination properties need only apply, of course, over the range of angles applicable to your particular situation. In many cases, the mutually contradictory models of an isotropic and a Lambertian source are used simultaneously. This is valid over small angular ranges where the cosine of the angle with the surface normal doesn't change much. This assumption is not all that restricting. For example, for a small Lambertian source illuminating an on-axis circular area, the error in flux caused by using an isotropic model is less than 1% for a subtended full angle of 22 deg [NA = 0.19, *f*/2.6], less than 5% for a full angle of 50 deg [NA = 0.42, *f*/1.2], and less than 10% for 70 deg [NA = 0.57, f/0.9]. However, for a full angle of 180 deg (a full hemisphere), the error is 100%!

Known Intensity

Consider a small source at a distance. For a **known intensity** that is essentially constant over all relevant directions, i.e., toward the illuminated area:



where I is the intensity of the radiating area in the direction of the illuminated area;

A_r is the radiating area;

θ is the angle between the normal to the radiating area and the direction of illumination;

$A_r \cos\theta$ is the projected radiating area as viewed from the illuminated area;

A_i is the illuminated area;

ξ (*xi*) is the angle between the normal to the illuminated area and the direction of illumination (assumed constant over this small angular range);

d is the distance between the two areas (assumed to be constant);

ω is the solid angle formed by the illuminated area when viewed from the radiating area (assumed to be small);

$\Omega = \omega \cos\theta$ is the corresponding projected solid angle (for small solid angles);

E is the irradiance at the illuminated area;

Φ_i is the total flux irradiating the illuminated area; and

L is the radiance of the radiating area.

$$E = \frac{I \cos\xi}{d^2}, \qquad \Phi_i = I\omega, \qquad L = \frac{I}{A_r \cos\theta}.$$

Known Flux and Known Radiance

If, in the same situation, the **flux** within the **solid angle** is known, then the intensity is

$$I = \Phi_i / \omega ,$$

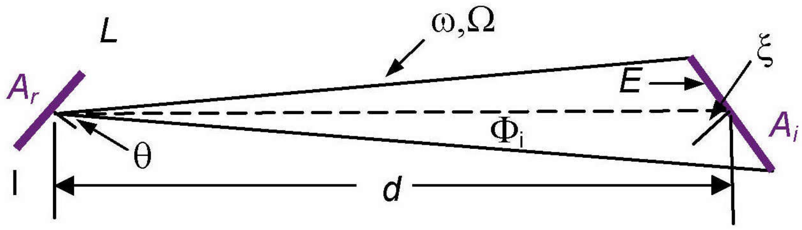
the irradiance is

$$E = \Phi_i / A_i ,$$

and the radiance is

$$L = \frac{\Phi_i}{\omega \, A_r \cos\theta} = \frac{\Phi_i}{\Omega \, A_r} .$$

Consider the same situation, but not necessarily with a small radiating area or small illuminated area:



If the radiance is known and the radiating area is small, then

$$I = L \, A_r \cos\theta .$$

If cosθ is essentially constant from all points on the radiating area to all points on the illuminated area, then

$$\Phi_i = L \, A_r \, \omega \cos\theta = L \, A_r \, \Omega .$$

If cosθ varies substantially over the illuminated area, then the second form of this equation, using the projected solid angle, should be used.

Since there are π projected steradians in a hemisphere, the total flux radiated (for a Lambertian radiator) is

$$\Phi_r = L \, A_r \, \pi .$$

The irradiance at the illuminated area (E) is

$$E = \frac{\Phi_i}{A_i} = \frac{L \, A_r \, \Omega}{A_i} = \frac{L \, A_r \cos\theta \cos\xi}{d^2} = L \;\; \Omega_i ,$$

where Ω_i is the projected solid angle of the radiating area when viewed from the illuminated spot.