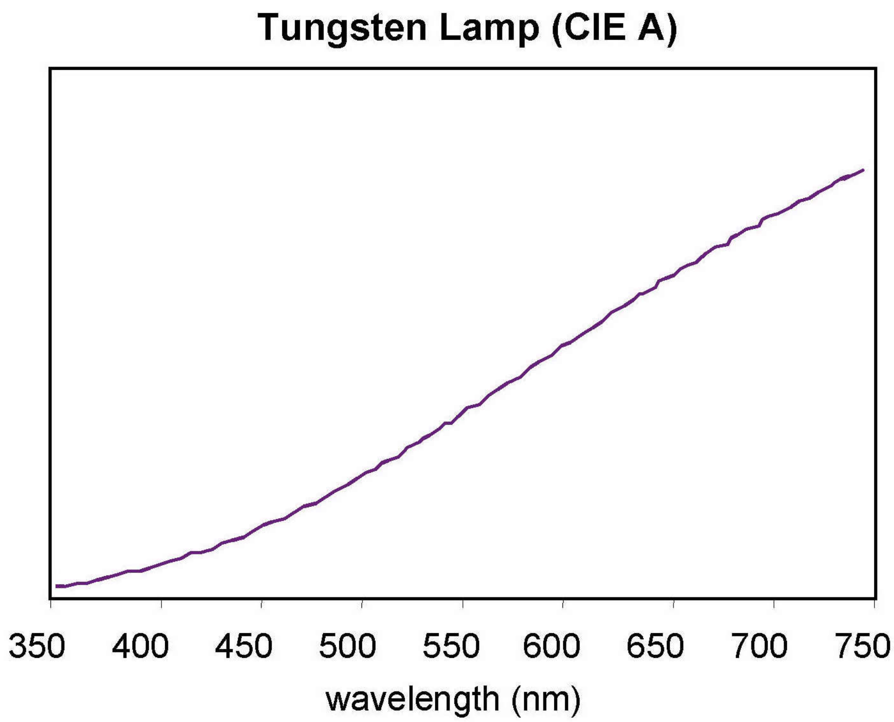
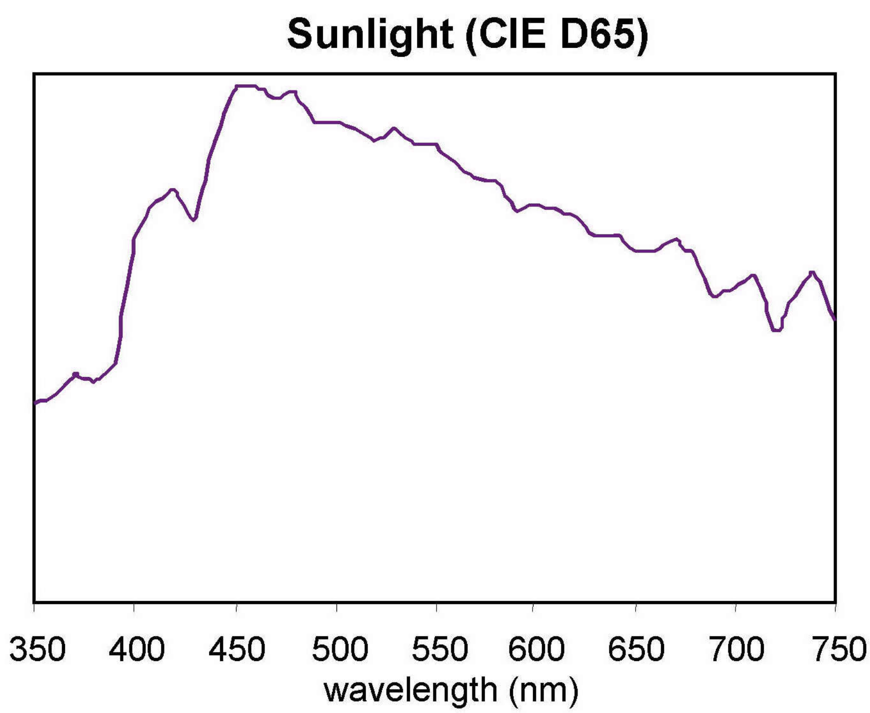
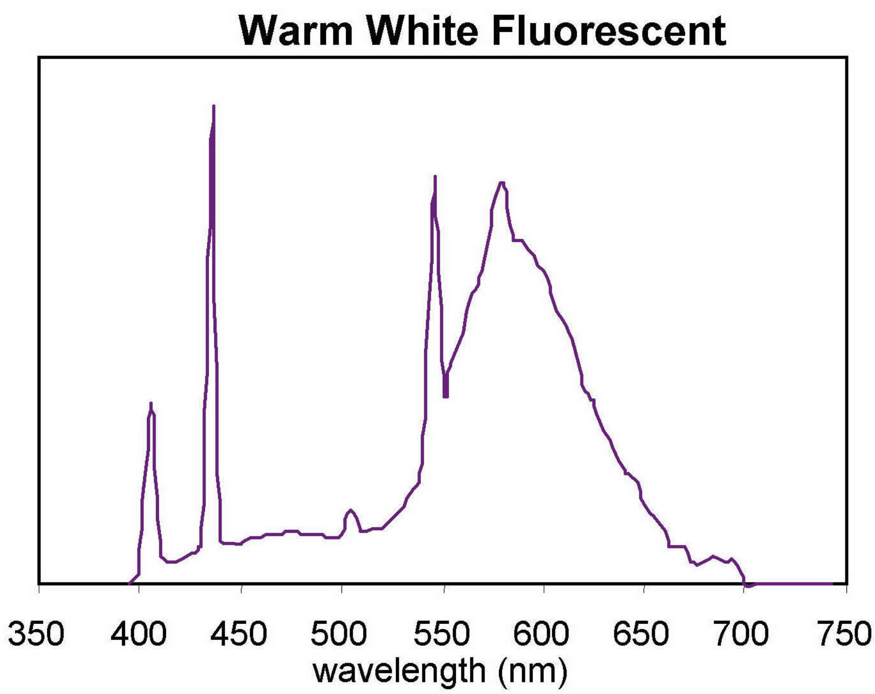
Tungsten and Sunlight

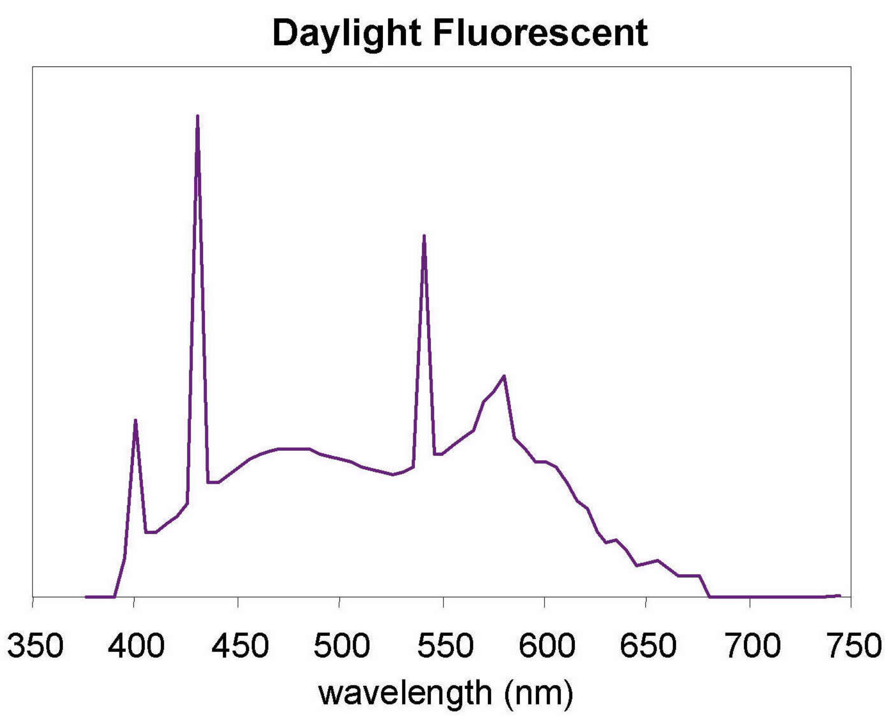
This page and those that follow show typical spectra of several common illumination sources.



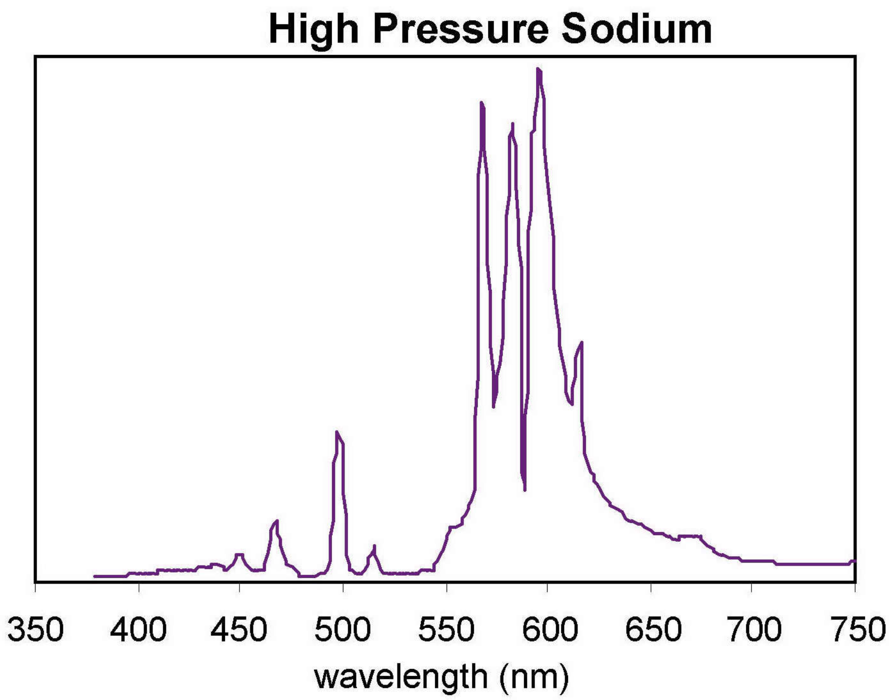


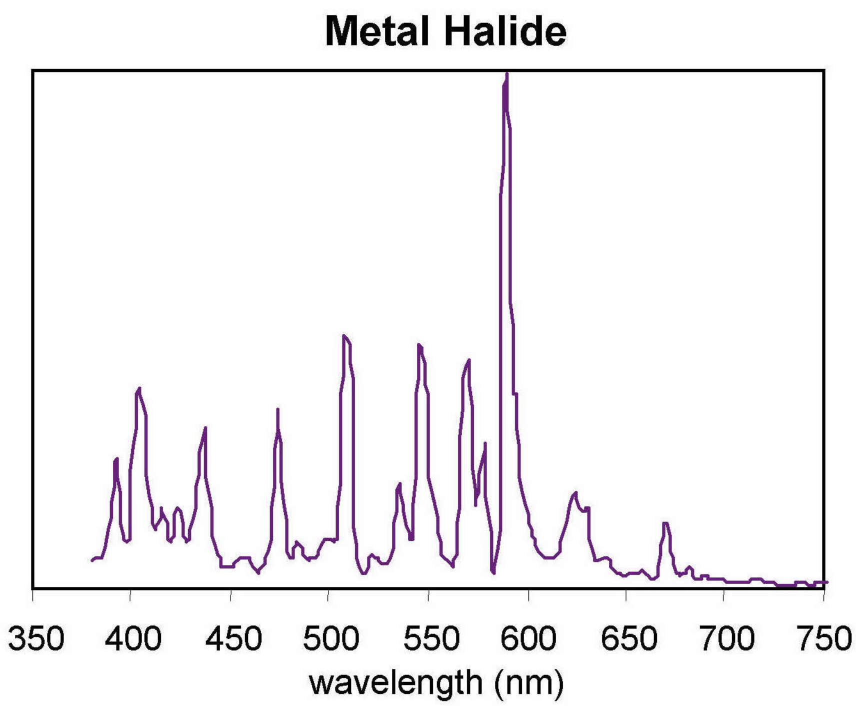
Fluorescent Lamps



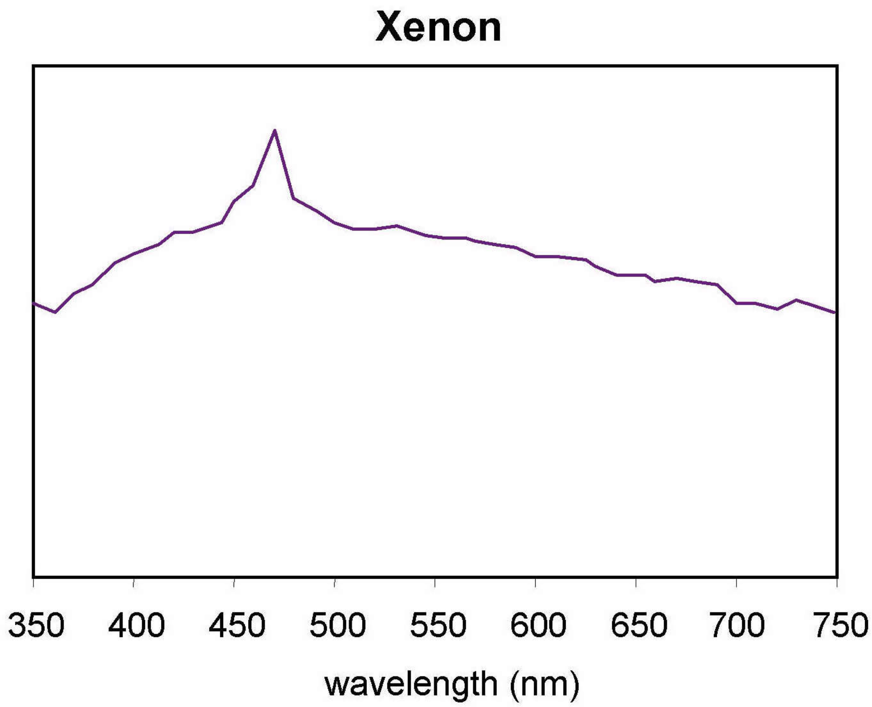


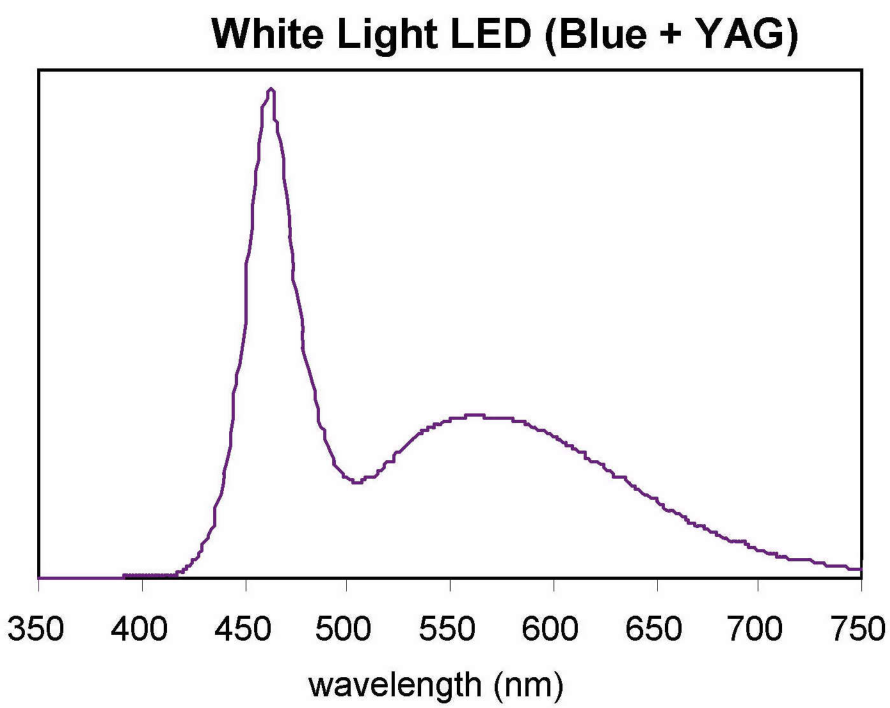
H.P. Sodium and Metal Halide





Xenon and White LEDs

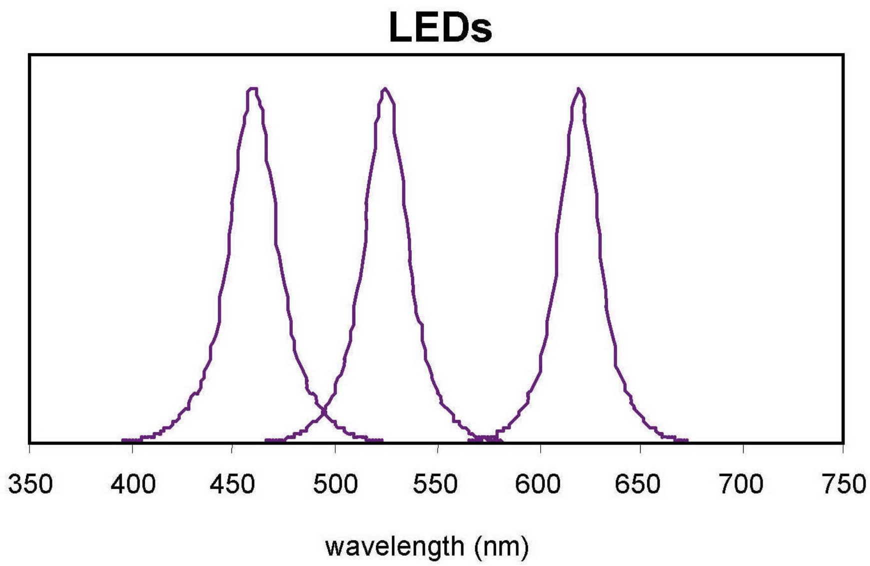




Light Emitting Diodes (LEDs)

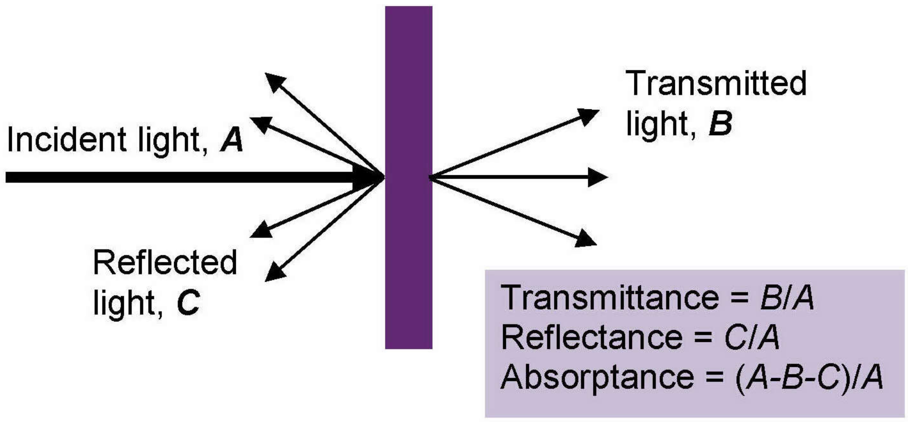
LEDs are moderately narrowband emitters with an approximately Gaussian spectral shape. The spectrum of an LED is often expressed by a single wavelength, with four different single-wavelength descriptions in general use. The most common spectrum-based description is the **peak wavelength**, λ_p, which is the wavelength of the peak of the spectral density curve. Less common is the **center wavelength**, $\lambda_{0.5m}$, which is the wavelength halfway between the two points with a spectral density of 50% of the peak. For a symmetrical spectrum, the peak and center wavelengths are identical. However, many LEDs have slightly asymmetrical spectra. Least common is the **centroid wavelength**, λ_c, which is the mean wavelength. The peak, center, and centroid wavelengths are all derived from a plot of $S_\lambda(\lambda)$ versus λ. The fourth description, the **dominant wavelength**, λ_d, is a colorimetric quantity that is described in the section on **color**. It is the most important description in visual illumination systems because it describes the perceived color of the LED.

Spatially, LEDs, especially those in lens-end packages, are often described by their **viewing angle**, which is the full angle between points at 50% of the peak intensity.



Transmittance, Reflectance, and Absorptance

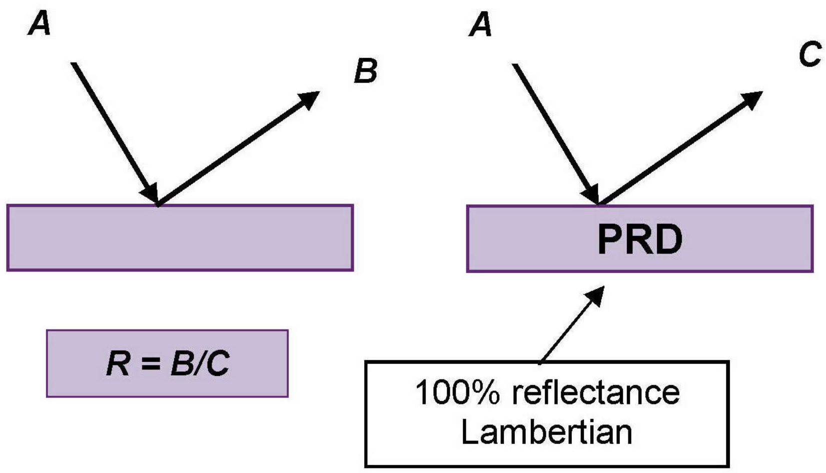
Several alternative methods describe the response of materials to illumination. One common approach is the ratio of the light that is transmitted, reflected, or absorbed to the incident light. This method describes a material by its **transmittance**, τ, its **reflectance**, ρ, or its **absorptance**, α. Do not confuse absorptance with **absorbance**, A, which is equivalent to **optical density (OD)** and is a conversion of transmittance or reflectance to a log scale. For example, 10% transmittance can be described as 1A, 1% as 2A, etc.



A material that produces intensity proportional to the cosine of the angle with the surface normal is called Lambertian. The radiance of a **Lambertian surface** is constant with viewing direction (since the projected area of a viewed surface is also proportional to the cosine of the angle with the surface normal). Furthermore, the directional distribution of scattered light is independent of the directional distribution of the incident illumination. It is impossible to tell, by looking at a Lambertian surface, where the incident light comes from. Perfectly Lambertian surfaces don't really exist, but many materials, such as matte paper, flat paint, and sandblasted metal (in reflection), as well as opal glass and sandblasted quartz (in transmission), are good Lambertian approximations over a wide range of incidence and view angles.

Reflectance Factor and BRDF

A quantity sometimes confused with reflectance is the **reflectance factor**, R. The reflectance factor is defined in terms of a hypothetical **perfectly reflecting diffuser (PRD)**, a surface that is perfectly Lambertian and has a 100% reflectance. The reflectance factor is the ratio of the amount of light reflected from the material to the amount of light that would be reflected from a PRD if similarly illuminated and similarly viewed.



Notes on reflectance (ρ) and the reflectance factor (R):

- For a **Lambertian surface**, ρ and R are identical.
- **Reflectance** must be between 0 and 1. The reflectance factor is not similarly bound. A highly polished mirror, for example, has near-zero R for any nonspecular incident and viewing angles, and a very high R (>1.0) for any specular incident and viewing angles.
- The reflectance factor is more closely related to the **bidirectional reflectance distribution function (BRDF)** than to reflectance. The BRDF is defined as the radiance of a surface divided by its irradiance:

$$\mathrm{BRDF} = L/E.$$

- The reflectance factor measures per hemisphere (there are π projected steradians in a hemisphere) what the BRDF measures per projected steradian:

$$R = \mathrm{BRDF}\cdot\pi.$$

Harvey / *ABg* Method

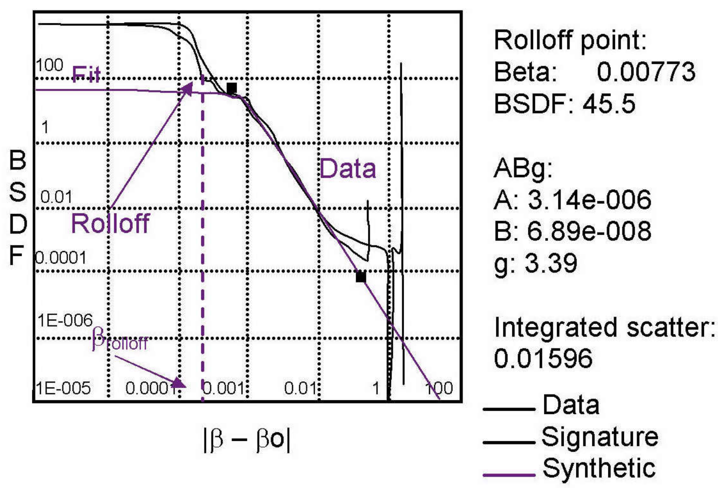
The **Harvey** or ***ABg* method** is used to parameterize scatter from a weakly scattering surface, which is typical for optical surfaces such as lenses and mirrors. It also can be used to model Lambertian surfaces and anisotropic (i.e., asymmetric) scatter. An example for a three-axis polished surface is provided here, which has a **total integrated scatter (TIS** or **TS)** of about 1.6%. The vertical scale represents the **BSDF**, for which an *R* (reflection) or *T* (transmission) can be substituted for the *S* (surface). The horizontal scale represents the absolute difference between $\beta_0 = \sin\theta_0$, or the specular direction, and $\beta = \sin\theta$, or any direction away from specular. Note that both axes are plotted in log space such that the roll-off slope is linear. The *ABg* parameters are:

- *g* is the slope of the roll-off as shown in the figure whose value of 0 defines a **Lambertian surface**.
- *B* is the roll-off parameter defined as

$$B = \left|\beta_{\text{rolloff}}\right|^{g}.$$

- *A* is the amplitude factor and can be found from

$$BSDF = \frac{A}{B + \left|\beta - \beta_0\right|^{g}}.$$



Directional Properties of Materials

The reflectance of a material can depend on the direction of the incident light. This dependence is often indicated by a number or letter.

- $\rho(0°)$: reflectance for normal incidence.
- $\rho(45°)$: reflectance for a 45-deg oblique incidence.
- $\rho(d)$ or $\rho(h)$: reflectance for diffuse illumination.

The **reflectance factor** of a material can depend on both the direction of illumination and the viewing geometry. This is usually indicated by two letters or numbers, the first indicating the incident geometry and the second the viewing geometry.

- R(0°/45°): the reflectance factor for normal incidence and a 45-deg oblique viewing (a common geometry for measuring the color of a surface).
- R(0°/*d*): the reflectance factor for normal incidence and diffuse (everything except the specular) viewing only.
- R(8°/*h*): the reflectance factor for near-normal incidence and hemispherical (everything, including the specular) viewing.
- R(45°/*h*): the reflectance factor for hemispherical illumination and a 45-deg oblique viewing.

> The same notation used for reflecting materials can be applied to transmitting materials, where transmittance τ can be dependent on incident geometry, and the transmittance factor, T, on both the incident and transmitting geometries. The use of the transmittance factor is not as common as transmittance, reflectance, and the reflectance factor.

Some materials have reflecting properties that are not the same for every azimuthal angle, even for the same elevation angle, e.g., the specular geometry of mirrorlike surfaces has vastly different reflecting properties than any geometry with the same incident and reflecting elevation angles that are not both in the same plane with the surface normal.

Retroreflectors—Geometry

Retroreflectors reflect incident light back toward the direction of the light source, operating over a wide range of angles of incidence. Typically they are constructed in one of two different forms, 90-deg corner cubes or high index-of-refraction transparent spheres with a reflective backing. Retroreflectors are used in transportation systems as unlighted night-time roadway and waterway markers, as well as in numerous optical systems, including lunar ranging. Some are made of relatively inexpensive plastic pieces or flexible plastic sheeting, and some are made of high-priced precision optics.

The performance of retroreflectors is characterized within a geometrical coordinate system, usually with three angles for the incident and viewing geometries and a fourth orientation angle for prismatic designs like corner cubes, which are not rotationally isotropic in their performance. All the geometric variations are described in detail in ASTM E808-01, *Standard Practice for Describing Retroreflection*, along with expressions for converting from one geometric system to another.

Two angles commonly used to specify the performance of retroreflectors are the **entrance angle**, β, and the **observation angle**, α. The entrance angle is the angle between the illumination direction and the normal to the retroreflector surface. High-quality retroreflectors work over fairly wide entrance angles, up to 45-deg or more (up to 90 deg for pavement marking). The observation angle, the angle between the illumination direction and the viewing direction, is generally very small, often one degree or less.

Another useful angle for interpreting the performance of retroreflectors is the **viewing angle**,υ, the angle between the viewing direction and the normal to the retroreflector surface.

Retroreflectors—Radiometry

The performance of retroreflectors is quantified by several coefficients. These are the most common:

R_I, **coefficient of retroreflected luminous intensity**,

$$R_I = \frac{I}{E_\perp},$$

where $E_\perp$ is the illuminance on a plane normal to the direction of illumination, and I is the intensity of the illuminated retroreflector.

R_A, **coefficient of retroreflection**,

$$R_A = \frac{R_I}{A} = \frac{I/A}{E_\perp},$$

where A is the area of the retroreflector.

R_L, **coefficient of retroreflected luminance**,

$$R_L = \frac{R_A}{\cos\upsilon} = \frac{L}{E_\perp}$$

is the ratio of the luminance in the direction of observation to $E_\perp$.

R_Φ, **coefficient of retroreflected luminous flux**:

$$R_\Phi = \frac{R_A}{\cos\beta}.$$

R_F, **retroreflectance factor**

$$R_F = \frac{\pi \cdot R_I}{A \cdot \cos\beta \cdot \cos\upsilon} = \frac{\pi \cdot R_A}{\cos\beta \cdot \cos\upsilon} = \frac{\pi \cdot R_L}{\cos\beta}.$$

It is the retroreflectance factor, R_F that is numerically equivalent to the reflectance factor, R.

Retroreflectors are often specified by the coefficient of retroreflection, R_A, for various observation angles and entrance angles.

Values for R_A of several hundred (cd/m^2)/lux are not uncommon, corresponding to reflectance factors up to and over 1000.

Lambertian and Isotropic Models

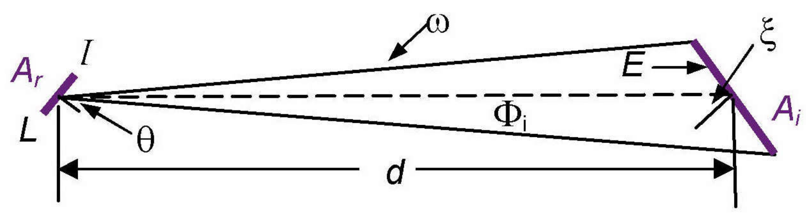
There are no direct "conversion factors" between the four basic quantities in illumination: **flux**, Φ; **irradiance**, E; **intensity**, I; and **radiance**, L. But for many situations, knowledge of one factor allows the calculation of the others. Making this calculation usually requires knowledge of the directional properties of the illuminating source, or at least a fair model of these directional properties. The two most common models are isotropic and Lambertian.

An **isotropic source** is defined here as having intensity independent of direction. For a **Lambertian source**, the radiance is independent of direction and the intensity is therefore proportional to the cosine of the angle with the surface normal. A few nearly isotropic sources exist, such as a round, frosted light bulb, a frosted ball-end on a fiber, and a line filament (in one plane, anyway). However, most flat radiators, diffusely reflecting surfaces, and exit pupils of illuminating optical systems are more nearly Lambertian than isotropic. Reasonable predictions can be made by modeling them as Lambertian.

The model of directional illumination properties need only apply, of course, over the range of angles applicable to your particular situation. In many cases, the mutually contradictory models of an isotropic and a Lambertian source are used simultaneously. This is valid over small angular ranges where the cosine of the angle with the surface normal doesn't change much. This assumption is not all that restricting. For example, for a small Lambertian source illuminating an on-axis circular area, the error in flux caused by using an isotropic model is less than 1% for a subtended full angle of 22 deg [NA = 0.19, *f*/2.6], less than 5% for a full angle of 50 deg [NA = 0.42, *f*/1.2], and less than 10% for 70 deg [NA = 0.57, f/0.9]. However, for a full angle of 180 deg (a full hemisphere), the error is 100%!

Known Intensity

Consider a small source at a distance. For a **known intensity** that is essentially constant over all relevant directions, i.e., toward the illuminated area:



where I is the intensity of the radiating area in the direction of the illuminated area;

A_r is the radiating area;

θ is the angle between the normal to the radiating area and the direction of illumination;

$A_r \cos\theta$ is the projected radiating area as viewed from the illuminated area;

A_i is the illuminated area;

ξ (*xi*) is the angle between the normal to the illuminated area and the direction of illumination (assumed constant over this small angular range);

d is the distance between the two areas (assumed to be constant);

ω is the solid angle formed by the illuminated area when viewed from the radiating area (assumed to be small);

$\Omega = \omega \cos\theta$ is the corresponding projected solid angle (for small solid angles);

E is the irradiance at the illuminated area;

Φ_i is the total flux irradiating the illuminated area; and

L is the radiance of the radiating area.

$$E = \frac{I \cos\xi}{d^2}, \qquad \Phi_i = I\omega, \qquad L = \frac{I}{A_r \cos\theta}.$$

Known Flux and Known Radiance

If, in the same situation, the **flux** within the **solid angle** is known, then the intensity is

$$I = \Phi_i / \omega \,,$$

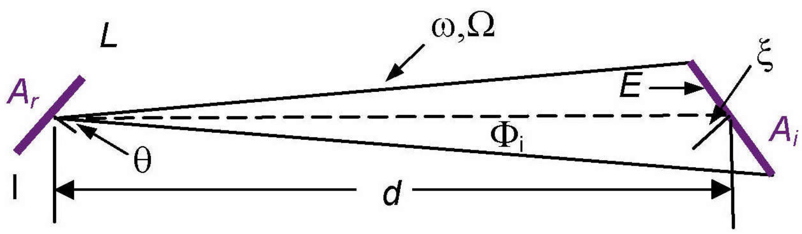
the irradiance is

$$E = \Phi_i / A_i \,,$$

and the radiance is

$$L = \frac{\Phi_i}{\omega \, A_r \, \cos\theta} = \frac{\Phi_i}{\Omega \, A_r} \,.$$

Consider the same situation, but not necessarily with a small radiating area or small illuminated area:



If the radiance is known and the radiating area is small, then

$$I = L \, A_r \, \cos\theta \,.$$

If cosθ is essentially constant from all points on the radiating area to all points on the illuminated area, then

$$\Phi_i = L \, A_r \, \omega \cos\theta = L \, A_r \, \Omega \,.$$

If cosθ varies substantially over the illuminated area, then the second form of this equation, using the projected solid angle, should be used.

Since there are π projected steradians in a hemisphere, the total flux radiated (for a Lambertian radiator) is

$$\Phi_r = L \, A_r \, \pi \,.$$

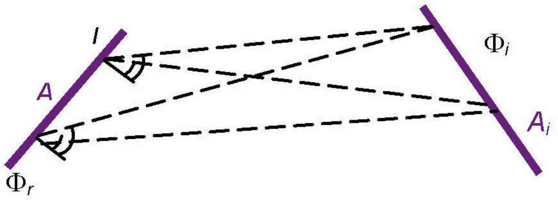
The irradiance at the illuminated area (*E*) is

$$E = \frac{\Phi_i}{A_i} = \frac{L \, A_r \, \Omega}{A_i} = \frac{L \, A_r \, \cos\theta \cos\xi}{d^2} = L \;\; \Omega_i \,,$$

where Ω_i is the projected solid angle of the radiating area when viewed from the illuminated spot.

Form Factor and Average Projected Solid Angle

Here the approximations of constant cosines cannot be used.



The angles between the normal to the radiating surface and the directions to points on the illuminating surface vary not only with the locations of the points on the illuminated surface, but also with the locations of points on the radiating surface. The concept of projected solid angle takes the former into account, but not the latter. What is needed is an **average projected solid angle**, $\bar{\Omega}_{r \text{ to } i}$, which is the projected solid angle subtended by the illuminated area and averaged over all points on the radiating area. Then the illuminating flux, Φ_i, from a **Lambertian radiator** is

$$\Phi_i = L\, A_r\, \bar{\Omega}_{r\, to\, i} = \frac{\Phi_r}{\pi}\, \bar{\Omega}_{r\, to\, i}.$$

In practice, the average projected solid angle is not used. However, its geometrical equivalent, called the **form factor**, $F_{a \text{ to } b}$, is used. The only difference between the form factor and the average projected solid angle is a multiplier of π:

$$F_{a\, to\, b} = \bar{\Omega}_{a\, to\, b} / \pi.$$

The form factor measures in hemispheres what the average projected solid angle measures in projected steradians. The form factor also can be interpreted as the portion of the flux leaving a Lambertian radiator, a, that illuminates a surface, b:

$$\Phi_i = \Phi_r\, F_{r\, to\, i}.$$

Note that the form factor is directional, as are the solid and the projected solid angles. $F_{a \text{ to } b}$ is not in general equal to $F_{b \text{ to } a}$. However, the product of the area and the form factor is constant:

$$A_a\, F_{a\, to\, b} = A_b\, F_{b\, to\, a}.$$

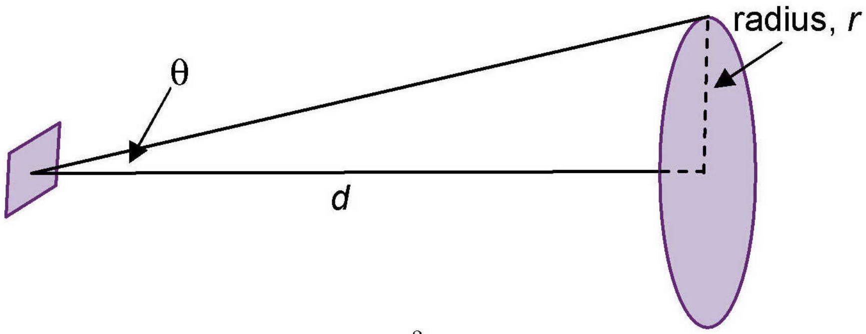
Configuration Factor

The **form factor** and the **average projected solid angle** both link two extended areas. The form factor measures in hemispheres what the average projected solid angle measures in projected steradians. Another term, the **configuration factor**, C, is similarly related to the **projected solid angle**, linking a small area with an extended area. Like the form factor, the configuration factor measures in hemispheres what the projected solid angle measures in projected steradians:

$$C = \Omega/\pi.$$

Tables of configuration factors and form factors for myriad geometries can be found in handbooks on illumination, in books on radiative heat transfer (where the issues are identical to illumination by Lambertian radiators), and on the Internet. Three cases with applicability to many optical situations are listed here:

Case 1: Small area to an extended circular area; both areas parallel and with axial symmetry.



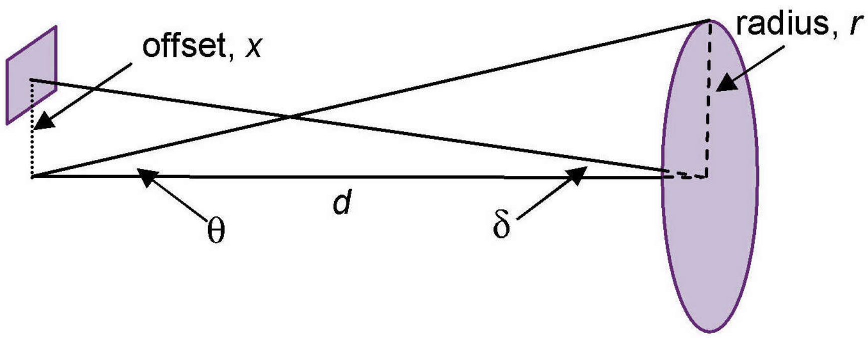
$$C = \frac{r^2}{r^2 + d^2} = \sin^2\theta$$

and

$$\Omega = \frac{\pi r^2}{r^2 + d^2} = \pi \sin^2\theta.$$

Useful Configuration Factor

Case 2: Small area to an extended circular area; both areas parallel, but without axial symmetry.



$$C = \frac{1}{2}\left(1 - \frac{1 + \left(d/x\right)^2 - \left(r/x\right)^2}{\left[\left\{1 + \left(d/x\right)^2 + \left(r/x\right)^2\right\}^2 - 4\left(r/x\right)^2\right]^{1/2}}\right)$$

or, equivalently:

$$C = \frac{1}{2}\left(1 - \frac{1 + \tan^2\delta - \tan^2\theta}{\left[\tan^4\delta + \left(2\tan^2\delta\right)\left(1 - \tan^2\theta\right) + \sec^4\theta\right]^{1/2}}\right)$$

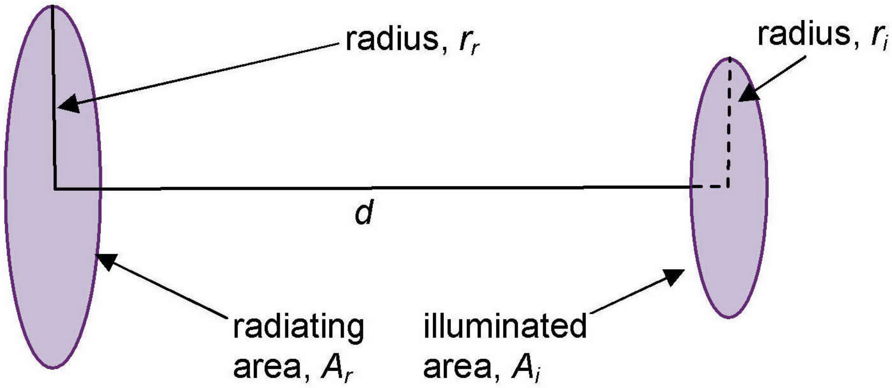
and

$$\Omega = \pi \cdot C.$$

These expressions degenerate to the expressions for case 1 above when x, or equivalently, δ, is equal to zero.

Useful Form Factor

Case 3: An extended circular area illuminating another extended circular area; both areas parallel and centered on the same axis.



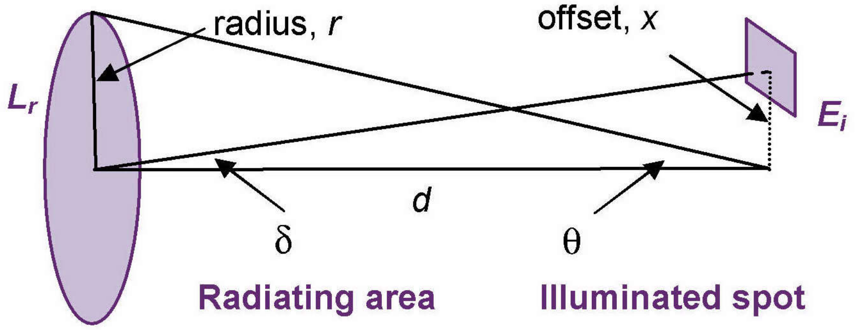
$$F_{r\,to\,i} = \frac{1}{2}\left\{1 + \frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2} - \left[\left(1 + \frac{1+\left(\frac{r_i}{d}\right)^2}{\left(\frac{r_r}{d}\right)^2}\right)^2 - 4\left(\frac{r_i}{r_r}\right)^2\right]^{\frac{1}{2}}\right\}.$$

Some numerical values of $F_{r\ to\ i}$ for this case are shown in the table below for several sizes of radiating and illuminated disks (each expressed as a multiple of the distance between the two parallel circular areas that are centered on the same axis).

Form Factor, $F_{r\ to\ i}$							
		r_i/d					
		0.03	**0.10**	**0.30**	**1.00**	**3.00**	**10.0**
r_r/d	**0.03**	.001	.010	.083	.500	.900	.990
	0.10	.001	.010	.082	.499	.900	.990
	0.30	.001	.009	.077	.489	.899	.990
	1.00	.000	.005	.044	.382	.890	.990
	3.00	.000	.001	.009	.099	.718	.989
	10.0	.000	.000	.001	.010	.089	.905

Irradiance from a Uniform Lambertian Disk

Many illumination situations can be modeled as illumination by a **uniform circular Lambertian disk**, with the illuminated area parallel to the disk and at some distance from it.



The irradiance at the illuminated spot is equal to the radiance of the radiating area times the projected solid angle of the radiating area when viewed from the illuminated spot:

$$E_i = L_r \ \Omega_i .$$

If the illuminated spot is on axis ($x = 0$, $\delta = 0$), then

$$E_i = \pi L_r \sin^2\theta = \pi L_r \frac{r^2}{r^2 + d^2} .$$

If the spot is offset from the axis, it is necessary to use the projected solid angle or the configuration factor discussed previously for case 2:

$$\Omega_i = \frac{\pi}{2}\left\{1 - \frac{1+\tan^2\delta - \tan^2\theta}{\left[\tan^4\delta + \left(2\tan^2\delta\right)\left(1-\tan^2\theta\right) + \sec^4\theta\right]^{1/2}}\right\}.$$

Note: The configuration factor, form factor, and projected solid angle are useful mainly when the radiation pattern is Lambertian or nearly Lambertian.

Cosine Fourth and Increase Factor

Consider the previous case of illumination by a uniform circular Lambertian disk, with the illuminated area parallel to the disk and at some distance from it. For many values of aperture size (θ) and field angle (δ), the irradiance falls off very nearly at $\cos^4\delta$, a phenomenon often referred to as the **cosine-fourth law**.

> Two of the cosine terms in the $\cos^4$ law are due to the fact that, off axis, the distance increases with the cosine of δ and the inverse square law applies. The third cosine factor comes from the Lambertian source, and the fourth from the fact that the illuminated surface is inclined to the direction of propagation.

In reality, the $\cos^4$ "law" is not exactly true, and is far from true for large values of θ and δ. The table below displays values of the **increase factor**, *F*′, which is the multiplier that must be applied to the irradiance calculated by using the axial irradiance and $\cos^4$ falloff. *F*′ compensates for the inaccuracy in the "cosine-fourth" assumption:

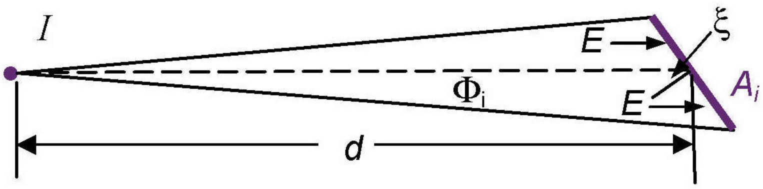
$$E_i = \pi L_r \sin^2\theta \cdot \cos^4\delta \cdot F'$$

Increase Factor, *F*′

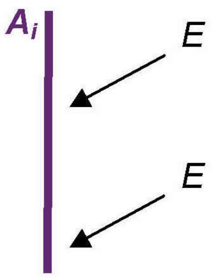
		θ					
	(deg)	**1.8**	**3.6**	**7.2**	**14.5**	**30.0**	**45.0**
	NA	**0.03**	**0.06**	**0.13**	**0.25**	**0.50**	**0.71**
	***f*/#**	**16**	**8**	**4**	**2**	**1**	**0.71**
δ(deg)	**0**	1.00	1.00	1.00	1.00	1.00	1.00
	10	1.00	1.00	1.00	1.01	1.03	1.05
	20	1.00	1.00	1.01	1.03	1.11	1.20
	30	1.00	1.00	1.01	1.05	1.23	1.49
	40	1.00	1.00	1.02	1.08	1.37	1.94
	60	1.00	1.01	1.02	1.09	1.48	2.69

The $\cos^4$ approximation is valid within a few percent up to very large apertures and field angles.

Known Irradiance



If a surface is illuminated by a source of uniform intensity at a distance *d* and the irradiance on the surface is known, then the intensity of the source is



$$I = \frac{E \cdot d^2}{\cos \xi}.$$

For any surface that is illuminated by uniform irradiance, the total flux illuminating the surface is

$$\Phi = E \cdot A_i.$$

The radiance of the surface, caused by the light reflecting from the surface, depends on the reflecting properties of the surface.

If the surface is Lambertian over all angles of reflection (for this incident geometry), then

$$L = \frac{\rho \cdot E}{\pi},$$

where ρ is the reflectance of the surface for the relevant incident geometry.

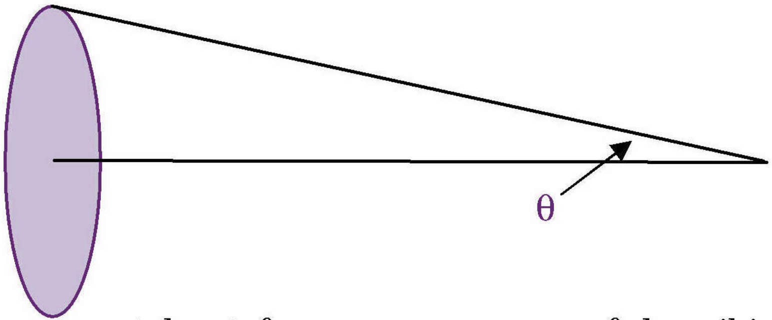
If the surface is not Lambertian over all angles but is Lambertian over the direction of concern, then

$$L = \frac{R \cdot E}{\pi},$$

where *R* is the reflectance factor of the surface for the relevant incident geometry and for the direction of concern.

ω, Ω, NA, and *f*/# for a Circular Cone

The case of a circular disk subtending a known half-angle, θ, shows up often in illumination situations.



There are at least four common ways of describing the cone: **solid angle** (ω), **projected solid angle** (Ω), **numerical aperture (NA)**, and ***f*-number** (*f*/#):

$$\omega = 2\pi(1-\cos\theta) \qquad \Omega = \pi \sin^2\theta$$
$$NA = n \cdot \sin\theta \qquad f/\# = 1/2\sin\theta,$$

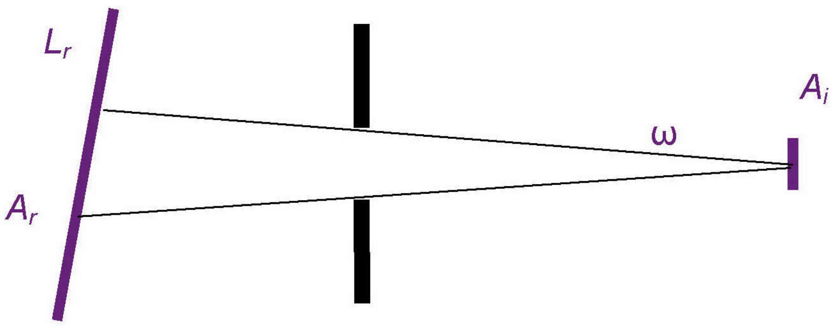
where n is the index of refraction.

Cone subtended by a circular disk				
θ(deg)	ω	Ω	NA/n	f/#
1.8	0.003	0.003	0.03	16.00
3.6	0.012	0.012	0.06	8.00
7.2	0.049	0.049	0.13	4.00
12.7	0.154	0.152	0.22	2.27
14.5	0.200	0.196	0.25	2.00
20.0	0.379	0.367	0.34	1.46
25.0	0.589	0.561	0.42	1.18
30.0	0.842	0.785	0.50	1.00
35.0	1.14	1.03	0.57	0.87
40.0	1.47	1.30	0.64	0.78
45.0	1.84	1.57	0.71	0.71
50.0	2.24	1.84	0.77	0.65
60.0	3.14	2.36	0.87	0.58
70.0	4.13	2.77	0.94	0.53
80.0	5.19	3.05	0.98	0.51
90.0	6.28	3.14	1.00	0.50

Invariance of Radiance

Unlike intensity, which is associated with a specific point, and irradiance, which is associated with a specific surface, radiance is associated with the propagating light rays themselves. This distinction is not trivial and implies that the radiance of a surface can be considered separate from the actual physical emitter or reflector that produces the radiance.

Consider a uniform Lambertian radiating source, A_r, with radiance, L_r, illuminating an area, A_i, through a limiting aperture that limits the solid angle of the source to ω:



The physical location of the radiating source is irrelevant. Only the solid angle matters. In fact, the physical location (and shape) can be assumed to be anywhere (and any shape) as long as the solid angle is the same. All of the following descriptions of the radiating area, A_1, A_2, and A_3, are equivalent to A_r from an illumination point of view:

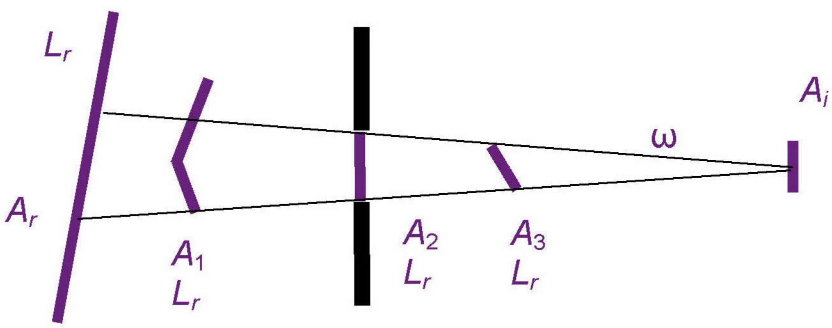
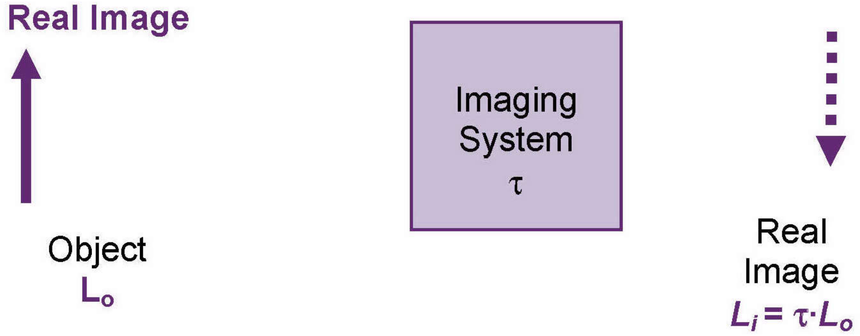


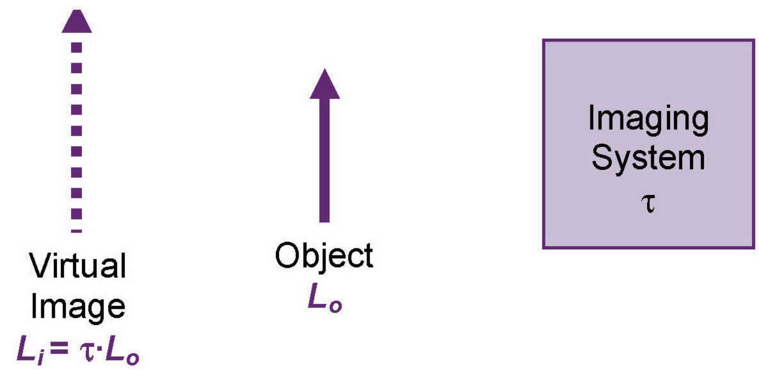
Image Radiance

In an imaging system with no vignetting or significant aberrations, for Lambertian objects, point-by-point, the radiance of an image is equal to the radiance of the object except for losses due to reflection, absorption, and scattering. These losses are usually combined into a single value of **transmittance**, τ. This equivalence of radiance is true for virtual as well as real images, and for reflective or refractive imaging systems.

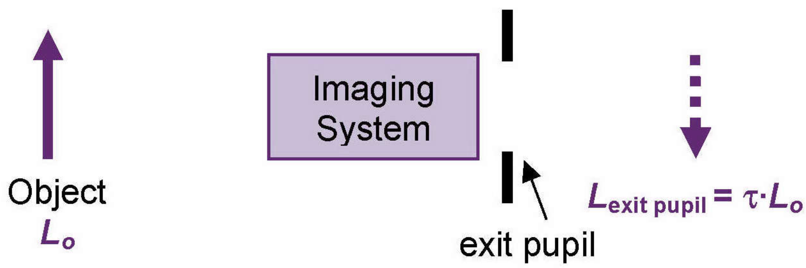
Real Image



Virtual Image

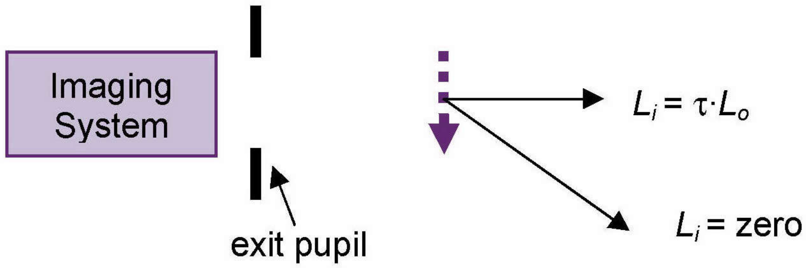


Viewed from any point on a real image, the entire exit pupil of the optical system is also the radiance of the corresponding object point but reduced by τ.



Limitations on Equivalent Radiance

In all cases, the image radiance only exists when the image is viewed through the exit pupil of the imaging system. When viewed in a direction that doesn't include the pupil, the radiance is zero.



If the object is not Lambertian, then the angular distribution of radiance of the image is also not Lambertian. The relationship between the angular distributions of object and image radiances is not straightforward and must be determined by ray tracing on the specific system. However, in many practical cases, the entrance pupil of the imaging system subtends a small angle from the object, and the source is essentially Lambertian over this small angle.

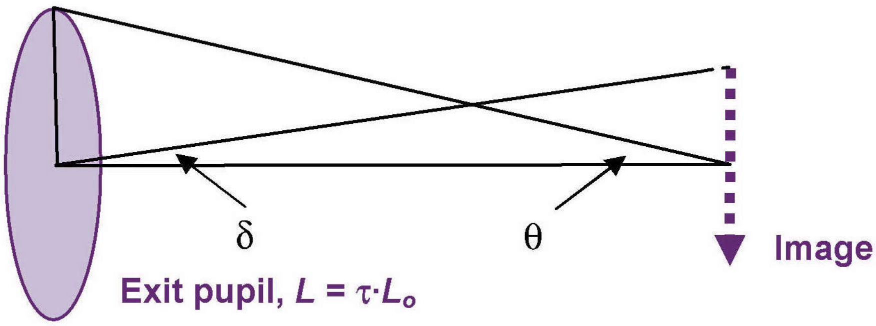
If the object and the image are in media of different refractive indices, n_o for the object and n_i for the image, then the expression for equivalent radiance is

$$\frac{L_i}{n_i^2} = \tau \cdot \frac{L_o}{n_o^2}.$$

The point-by-point equivalence of radiance from object to image is only valid for well-corrected optical systems. For systems that suffer from aberrations or are not in focus, each small point in the object is mapped to a "blur spot" in the image. Thus, the radiance of any small spot in the image is related to the average of the radiances of the corresponding spot in the object and its surrounding area.

Image Irradiance

Since the exit pupil, when viewed from the image, has the radiance of the object, then the irradiance at the image is the same as the irradiance from a source of the same size as the exit pupil and the same radiance as the object (reduced by τ). In most imaging systems, the exit pupil is round and the irradiance is the same as the irradiance from a uniform **Lambertian disk**:



$$E_i = \pi\, \tau L_o \sin^2\theta \cdot \cos^4\delta \cdot F'.$$

A table of values for the **increase factor**, F, is presented in the section on illumination transfer. F is very close to 1.0 except for a combination of large field angle (δ) and large aperture (θ), which is not a common combination in imaging systems.

The $\cos^4\delta$ term contributes to substantial field darkening in wide-angle imaging systems—for example, $\cos^4 45$ deg = 0.25.

If the physical aperture stop is not the limiting aperture for all the rays converging to an off-axis image point, the light is vignetted. The irradiance at image points where there is **vignetting** will be lower than predicted.

On axis, $\cos^4\delta = 1.0$ and $F = 1.0$. The image irradiance on axis, E_{i0}, is

$$E_{i0} = \pi\, \tau L_o \sin^2\theta .$$

f/#, Working *f/#*, *T/#*, NA, Ω

For a camera working at infinite conjugates (distant object, magnification, |m|<<1), the image irradiance can be expressed in terms of the lens' ***f*-number**, *f/#*:

$$E_{i0} = \frac{\pi \, \tau \, L_o}{4\,(f/\#)^2}.$$

This *f/#*, usually associated with a lens, is an "infinite conjugates" quantity. When a lens is used at finite conjugates, the **working *f*-number**, ***f/#*$_w$**, describes the cone angle illuminating the image:

$$f/\#_w = (f/\#)\cdot(1-m),$$

where m is the lateral magnification of the image (negative for real images), and the axial image irradiance is:

$$E_{i0} = \frac{\pi \, \tau \, L_o}{4\,(f/\#_w)^2}.$$

Note that ***f/#*$_w$** degenerates to the conventional "infinite conjugates" ***f/#*** when the lens is used at infinite conjugates.

Occasionally, a lens will be designated with a ***T*-number**, T/#, which combines the *f/#* and the transmittance into a single quantity,

$$T/\# = \frac{f/\#}{\sqrt{\tau}} \text{ with axial image irradiance: } E_{i0} = \frac{\pi \, L_o}{4\,(T/\#)^2}.$$

Another descriptor of the image illumination cone angle is the **numerical aperture**, NA,

$$NA = \sin\theta \text{ with axial image irradiance: } E_{i0} = \pi \, \tau L_o \, NA^2.$$

In all cases, even without circular symmetry, on or off axis, the cone illuminating the image can be described by its **projected solid angle**, Ω, with image irradiance:

$$E_i = \tau L_o \, \Omega.$$

Flux and Étendue

The total **flux** reaching the image is the product of the image irradiance and the area of the image. The **image irradiance** is proportional to the projected **solid angle** of the exit pupil when viewed from the image:

$$\Phi_i = \tau L_o \, a_i \Omega_i ,$$

where Ω_i is the projected solid angle of the exit pupil viewed from the image, a_i is the area of the image, L_o is the [assumed uniform] radiance of the object, and τL_o is the radiance of the exit pupil.

The flux reaching the image also can be expressed in terms of the radiance of the exit pupil, τL_o, the area of the exit pupil, a_p, and the projected solid angle of the image when viewed from the exit pupil, Ω_p:

$$\Phi_i = \tau L_o \, a_p \Omega_p .$$

The quantity $a\Omega$, representing the area of a plane in the optical system times the projected solid angle of another plane when viewed from it, appears equivalently in both expressions. This **area-solid-angle-product** is a fundamental property of the optical system that determines the amount of light that can get through the system. It is called the **throughput** or **étendue**.

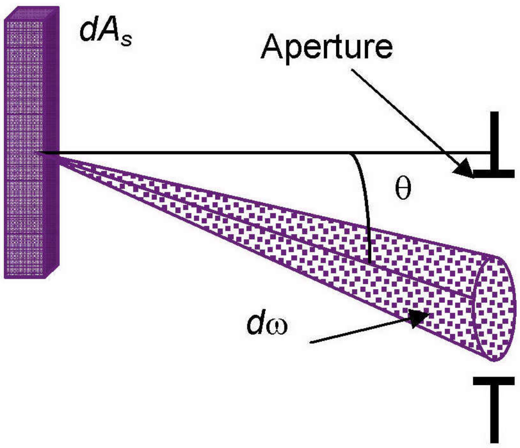
> The radiance of an object is invariant and cannot be increased by an optical system, and the étendue is a fundamental property of an optical system. These two concepts mean that, for a source of given radiance and a given optical system, the maximum flux that can be transmitted through the system is predetermined.

And, without "throwing away" light, the étendue cannot be decreased, but area and solid angle can be traded off.

Generalized Étendue

The terminology for illumination in nonimaging systems is the same as that for imaging systems; however, the range of validity is extended to include all angular space, while that of imaging systems is limited to paraxial systems. With this taken into account, étendue is often called **generalized étendue**. In this domain the étendue cannot be regarded as the simple product of the area and solid angle; it must be integrated per the following equation and figure:

$$\mathcal{E} = n^2 \iint_{\text{aperture}} \cos\theta \, dA_s \, d\omega,$$



where n is the refractive index, θ is the angle from the normal, dA_s is the differential source area, and $d\omega$ is the differential solid angle.

The total flux through the aperture is found by integrating the radiance over the aperture:

$$\Phi = \iint_{\text{aperture}} L(\mathbf{r}, \hat{\mathbf{a}}) \cos\theta \, dA_s \, d\omega,$$

where $\mathbf{r}$ and $\hat{\mathbf{a}}$ denote the positional and directional aspects of source emission. Assuming that the source is Lambertian so radiance is independent of angle, then

$$\Phi = L_s \iint_{\text{aperture}} \cos\theta \, dA_s \, d\omega = \frac{L_s \mathcal{E}}{n^2}.$$

Note that total flux is the product of the radiance and the geometrical étendue factor. This also shows the **conservation of étendue** that follows from the conservations of radiance and energy.

Concentration

Concentration (*C*) is a term associated with the generalized étendue. It represents the ability to transfer more light into a desired area by using the conservation of étendue to alter the angle at the output of an optical system. It is defined as the ratio of the input area (A) to the output aperture area (A') that transmits the prescribed flux from area A. For this reason it is called the **concentration ratio**:

$$C = A/A' .$$

This expression, a limit factor of the laws of thermodynamics, is a forbearer of the invariance of radiance and étendue.

In a 2D system, which is analogous to an extruded trough, and a 3D system, which is analogous to a well, we find that the respective concentrations are given by

$$C_{2D} = \frac{a}{a'} = \frac{n' \sin \theta'}{n \sin \theta} \quad \text{and} \quad C_{3D} = \frac{A}{A'} = \left(\frac{n' \sin \theta'}{n \sin \theta} \right)^2 ,$$

where a and a' are the aperture widths, A and A' are the aperture areas, n is the input index, n' is the output index, θ' is the output angle, and θ is the input angle. **Optimal concentration** is realized when the output angle is $\pi/2$, giving

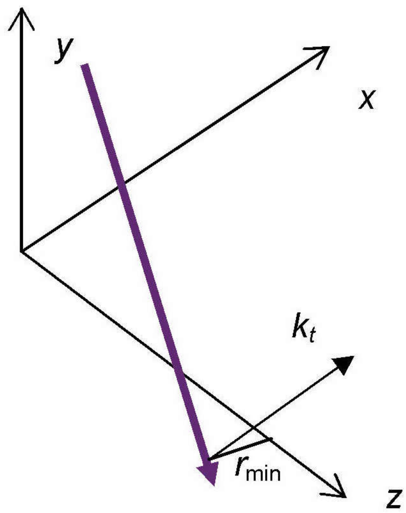
$$C_{2D,opt} = \frac{n'}{n \sin \theta_a} \quad \text{and} \quad C_{3D,opt} = \left(\frac{n'}{n \sin \theta_a} \right)^2 ,$$

where θ_a, the **acceptance angle**, is the prescribed upper input angle over which conservation of étendue is maintained.

Skew Invariant

The **skew invariant** is another limiting factor in nonimaging system design. Its definition is rather esoteric:

$$f_{\text{skew}}(s) = \frac{d\mathcal{E}(s)}{ds},$$



where $s = r_{\min}k_t$, and $r_{\min}$ is the ray's closest approach to the optical axis (z, as shown), and k_t is the tangential component of the ray's propagation direction.

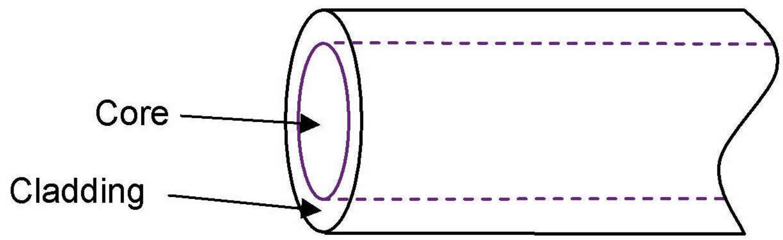
A simpler way to think about the skew invariant is to recognize that in a rotationally symmetric system (e.g., a lens), loss is introduced from the input to the output if the two spatial distributions are not the same shape. For example, if the object shape is a uniform square but a uniform round output is desired, then transfer losses will be produced.

To maximize transfer efficiency with different distributions, the symmetry of the optical system must be broken; or, in other words, there must be a "twist" in the optical components to force rays out of their respective sagittal planes. Many nonimaging optical systems take advantage of this property by including faceted reflectors (e.g., segmented headlights), segmented lenses (e.g., pillow optics for projection displays), or 3D edge-ray concentrators that employ V-wedges near the source (i.e., solar concentrators).

For a rotationally symmetric system, the rotational skewness of each ray is conserved or invariant. This skew invariant is given by the first derivative of the étendue.

Fibers—Basic Description

Optical fibers, **lightpipes**, and **lightguides** are all variations on the same theme. They each contain a central transparent core, usually circular in cross-section, surrounded by an annular cladding. The cladding has a lower index of refraction than the core.



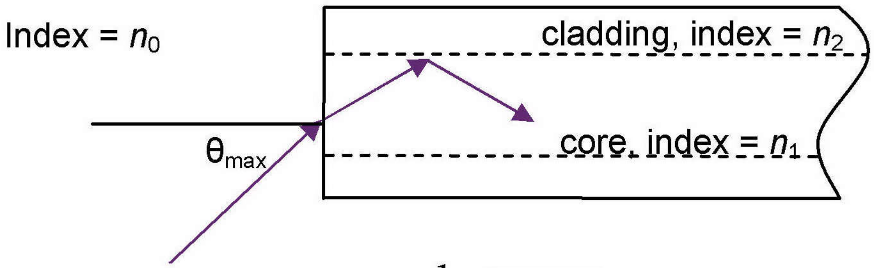
The core can transmit light for long distances with low loss because of total internal reflection at the interface between the core and the cladding. The primary purpose of the cladding is to maintain the integrity of this interface. Without it, total internal reflection would occur at a core-air interface, but dust, nicks, abrasions, oils, and other contamination on the interface would reduce the transmission to unacceptably low levels.

> Sometimes layers of buffering and/or jacketing are placed outside the cladding for additional protection.

The core diameter can range from very small, on the order of the wavelength of light, to a centimeter or more. The very thin cores are essentially waveguides and not used for illumination. Flexible glass and quartz fibers have core diameters ranging from approximately 50 microns to about 1 millimeter. If they are thicker than that, they are rigid and called rods or light pipes. Plastic fibers are flexible at thicker core diameters. Sometimes liquid cores and plastic cladding are used to make flexible, high-transmittance lightguides that are over a centimeter in core diameter.

Numerical Aperture and Étendue

The maximum angle that a fiber can accept and transmit depends on the indices of refraction of the core and cladding (as well as the index of the surrounding medium, usually air, $n_0 = 1$).



$$\sin\theta_{max} = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2}$$

and the NA is

$$NA = n_0 \sin\theta_{max} = \sqrt{n_1^2 - n_2^2}.$$

The fiber has a maximum acceptance projected **solid angle**, $\Omega = \pi\sin^2\theta_{max}$, and an acceptance area, the cross-sectional area of the core. Together, they define a throughput or étendue for the fiber in air:

$$\text{Étendue} = \frac{\pi^2}{4} d^2 NA^2,$$

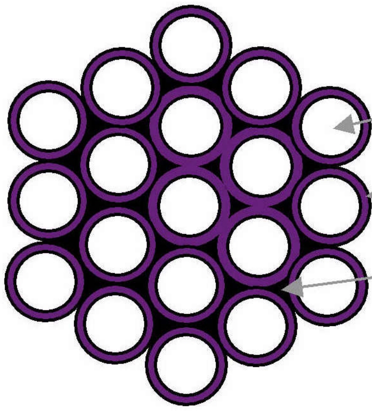
where d is the core diameter.

This étendue defines the maximum flux-carrying capability of the fiber when presented with a source of radiance.

Note: A fiber illuminated at less than its maximum acceptance angle will, theoretically, preserve the maximum illumination angle at its output. However, bending and scattering at the core-cladding interface broadens this angle toward the maximum allowable. This effect is not important in illumination systems in which it is desirable to utilize the maximum étendue of low-throughput components such as fibers and fill the full input NA.

Fiber Bundles

To achieve high throughput with flexible glass or quartz fibers, multiple fibers are often arranged in a bundle, such as the 19-fiber tightly packed bundle shown below:



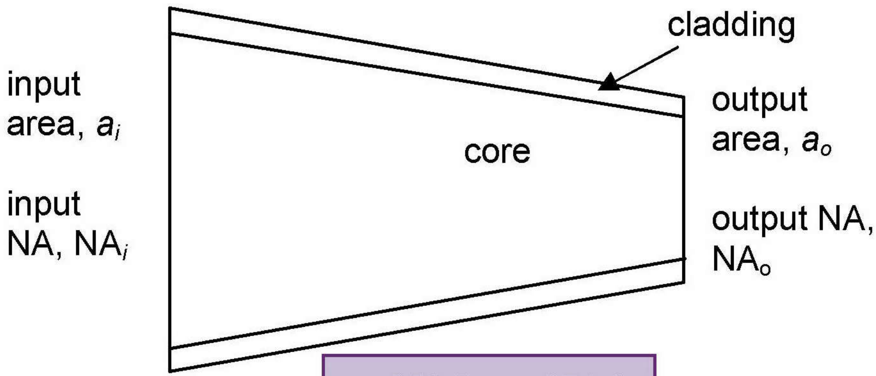
The ratio of the light-carrying core area to the area of the entire bundle is called the **packing fraction (pf)**, and can be as high as 85%. This packing fraction reduces the effective area of the bundle and, correspondingly, its étendue.

In addition to flexibility, fiber bundles have other possible advantages in illumination systems:

- **Shape Conversion:** In some situations, such as when illuminating a spectrometer, it can be useful to convert a circular cross-section of fibers to a line cross-section to align with, or actually become, the entrance slit to the spectrometer.
- **Splitting the Bundle:** By feeding a large fiber bundle with a single light source and splitting the bundle into two or more branches, it is possible to illuminate multiple locations, from multiple angles, with one source.
- **Mixed Bundle:** When illuminating with light over a wide spectral band, such as the full solar spectrum (~250 to 2500 nm), a mixed bundle of high OH silica fibers for good UV transmission and low OH silica fibers for good IR transmission can compensate for the lack of an adequate single-fiber type.

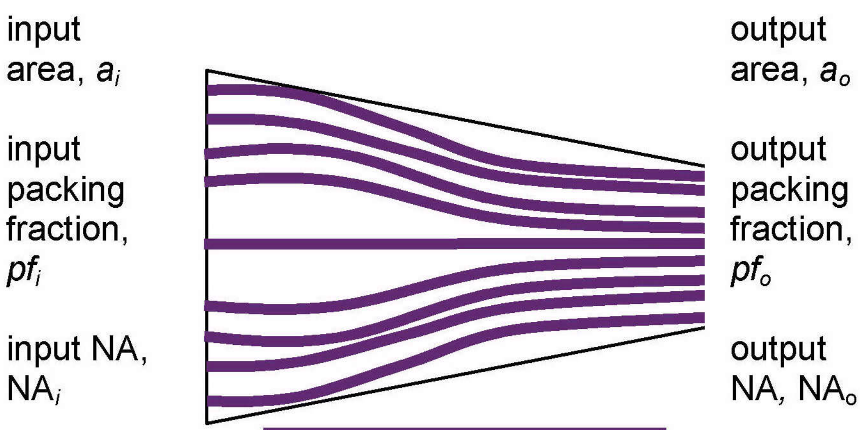
Tapered Fibers and Bundles

By tapering a single fiber, it is possible to trade off between area and solid angle while keeping the product (étendue) approximately constant.



$$a_i \cdot NA_i^2 \approx a_o \cdot NA_o^2$$

On the other hand, when a bundle of straight fibers is tapered, the tradeoff is between the area and packing fraction.



$$NA_o = NA_i = NA_{fiber}$$

$$a_o \cdot pf_o = a_i \cdot pf_i$$