Multiuser Spatio-Temporal Coding for Wireless Communications

Jibing Wang and Kung Yao

Department of Electrical Engineering
University of California, Los Angeles

***Abstract*- In this paper, we consider how to design the optimal space-temporal communication systems for the multiple access channels with inter symbol interference (ISI) under the assumption that the transmitters know the channel state information (CSI) perfectly. We optimize the spatio-temporal vector coding (STVC) and the space-frequency multiple input multiple output (MIMO) discrete matrix multitone (DMMT) information structure for the multiple access channels such that the total information capacity is maximized. The optimization problem is a convex programming problem and the iterative waterfilling algorithm [10] is employed to find the optimal solution. The simulation results show that a remarkable capacity improvement is achievable by the joint multiuser spatio-temporal optimization.**

I. INTRODUCTION

Space-time coding has become an active research area recently [1, 2]. Most work, including the space-time trellis coding [3] and space-time block coding [4], assumes the flat fading channels. However in broadband wireless communications, the frequency selective channels are encountered. Possible solutions include combining the space-time coding with multiple input multiple output (MIMO) equalizer [5] or with OFDM [6,7]. On the other hand, in cellular communication systems, the multiple access interference (MAI) is dominant. How to design the space-time codes in multiuser environment is still an open issue.

In [8] the information capacity for wireless MIMO dispersive channels is analyzed under the assumption that the channel side information is available at both the transmitter and the receiver. The asymptotically optimal spatio-temporal vector coding (STVC) and the space-frequency MIMO discrete matrix multitone (DMMT) information structure are proposed to achieve the channel capacity. In this paper, we basically extend the work in [8] to the multiuser scenario. We analyze, from the information capacity point of view, how to design the optimal spatio-temporal communication systems over the multiple access dispersive channels. We assume that the transmitters know the channel state information (CSI) perfectly. The iterative waterfilling algorithm [10] is employed to optimize the STVC and the space-frequency DMMT structures over the multiple access channels such that the total information capacity is maximized. Our simulation results show that the system capacity can be greatly increased by the joint multiuser spatio-temporal optimization.

The rest of the paper is organized as follows. Section II provides the MIMO multiple access ISI channel models. The multiuser spatio-temporal total capacity is maximized by employing the iterative waterfilling algorithm. To reduce the computation complexity, the multiuser DMMT structure is discussed in section III. We prove that the optimal transmitter design can be solved by the iterative waterfilling algorithm. We give some simulation results in section IV. Finally conclusions are drawn in section V.

II. MULTIUSER SPATIO-TEMPORAL VECTOR CODING

A. *Channel Models*

The input-output model for the M-user multiple access ISI channel corrupted by additive noise is given by

$$y_k = \sum_{i=1}^{M} \sum_{m=0}^{v} h_m^i x_{k-m}^i + n_k \, , \tag{1}$$

where $\{x_m^i\}$ and $\{h_m^i\}_{m=0}^{v}$ are the transmitted data and the channel impulse response coefficients for the i-th user, respectively. v is known as the channel memory and n_k is the additive gaussian noise. In the vector coding case each input block of size N is padded with v zeros [8]. Therefore the channel model of (1) can be expressed in the matrix form as

$$\begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+N+v-1} \end{bmatrix} = \sum_{i=1}^{M} \begin{bmatrix} h_0^i & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ h_v^i & & h_0^i & 0 & \vdots \\ 0 & h_v^i & \cdots & h_0^i & \vdots \\ 0 & \cdots & h_v^i & \cdots & h_0^i \\ \vdots & \cdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & h_v^i \end{bmatrix} \begin{bmatrix} x_k^i \\ x_{k+1}^i \\ \vdots \\ x_{k+N-1}^i \end{bmatrix} + \begin{bmatrix} n_k \\ n_{k+1} \\ \vdots \\ n_{k+N+v-1} \end{bmatrix}, \tag{2}$$

or more compactly

$$\mathbf{y} = \sum_{i=1}^{M} \mathbf{H}_i \mathbf{x}_i + \mathbf{n} \, . \tag{3}$$

Now, consider the MIMO multiple access ISI channel models with M_T transmit antennas at each transmitter (mobile) and M_R receive antennas at the receiver (basestation). Assume that the receive vector of each receive antenna is $\mathbf{y}^j, j = 1, \cdots, M_R$, and that for the i-th user, the transmit data vector of each transmit antenna is $\mathbf{x}_i^k, k = 1, \cdots, M_T$.

For the i-th user, the channel matrix from the k-th transmit antenna to the j-th receive antenna is denoted by $\mathbf{H}_i^{j,k}, j = 1, \cdots, M_R, k = 1, \cdots, M_T, i = 1, \cdots, M$.

Denote

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}^1 \\ \vdots \\ \mathbf{y}^{M_R} \end{bmatrix}, \tag{4}$$

$$\mathbf{x}_i = \begin{bmatrix} \mathbf{x}_i^1 \\ \vdots \\ \mathbf{x}_i^{M_T} \end{bmatrix}, i = 1, \cdots, M \ , \tag{5}$$

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{H}_i^{1,1} & \cdots & \mathbf{H}_i^{1,M_T} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_i^{M_R,1} & \cdots & \mathbf{H}_i^{M_R,M_T} \end{bmatrix}, i = 1, \cdots, M \ . \tag{6}$$

Then for this multiuser MIMO multiple access channel, we obtain exactly the same input output relation as (3).

B. Multiuser Spatio-Temporal Total Capacity

The normalized (per transmission) sum capacity for the multiple access channels is given by

$$I_{sum} = \frac{1}{N+v} \log \left| \mathbf{R}_n + \sum_{i=1}^{M} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^* \right| - \frac{1}{N+v} \log |\mathbf{R}_n| \ , \tag{7}$$

where * denotes conjugate transpose, $\mathbf{R}_n = E(\mathbf{nn}^*)$, and $\mathbf{R}_i = E(\mathbf{x}_i \mathbf{x}_i^*)$, for $i = 1, \cdots, M$. Assume the respective input power constraints are

$$trace(\mathbf{R}_i) = E_i \ , \ \ i = 1, \cdots, M \ . \tag{8}$$

Therefore, the sum capacity optimization problem can be stated as

$$\text{maximize} \quad \frac{1}{N+v} \log \left| \mathbf{R}_n + \sum_{i=1}^{M} \mathbf{H}_i \mathbf{R}_i \mathbf{H}_i^* \right| - \frac{1}{N+v} \log |\mathbf{R}_n| \quad ,$$

$$\text{subject to} \quad trace(\mathbf{R}_i) = E_i \ , \text{for} \ \ i = 1, \cdots, M \ ,$$

$$\mathbf{R}_i \geq 0 \ , \ \ \text{for} \ \ i = 1, \cdots, M \ .$$

It is not hard to see that the above problem is a convex programming problem, and the iterative multiuser waterfilling algorithm converges to the optimal solution from any starting point [10]. The optimum set of covariance matrices can be found by iteratively waterfilling each user's covariance matrix regarding the other user's signal as additional noise. More specifically, $\{\mathbf{R}_i\}_{i=1}^M$ are given as following: let the eigenvectors of $\mathbf{R}_i$ match to those of $\mathbf{H}_i^* \left(\mathbf{R}_n + \sum_{j=1, j \neq i}^{M} \mathbf{H}_j \mathbf{R}_j \mathbf{H}_j^* \right)^{-1} \mathbf{H}_i$, while using the waterfilling algorithm [9] to generate the eigenvalues of $\mathbf{R}_i$. All of the input covariance matrices are optimized iteratively in this manner. We can easily see that $\{\mathbf{R}_i\}_{i=1}^M$ obtained by this algorithm are positive semidefinite matrices.

Assume the eigendecomposition of $\mathbf{R}_i = \mathbf{V}_i \mathbf{\Lambda}_i \mathbf{V}_i^*$, then for the *i*-th user, the subchannels that should be used for transmission are determined by the nonzero elements of the diagonal matrix $\mathbf{\Lambda}_i$, and the corresponding columns of $\mathbf{V}_i$ are the optimum transmit filters for those subchannels.

III. Multiuser Discrete Matrix Multitone

In the space-time vector coding, the main disadvantage is the associated computational complexity of the eigendecomposition. Complexity can be greatly reduced by using the discrete matrix multitone (DMMT) [8]. In DMMT systems, each length-*N* input block is extended to the length-*N*+*v* block by adding the cyclic prefix. Denote $\mathbf{R}_i^{j,k} = E\left(\mathbf{x}_i^j \mathbf{x}_i^{k^*}\right) \in C^{N \times N}$, for $i = 1, \cdots, M$ $j = 1, \cdots, M_T, k = 1, \cdots, M_T$. Then we have that the insertion of the cyclic prefix makes the matrices $\mathbf{R}_i^{j,k}$ and $\mathbf{H}_i^{j,k}$ circulant.

Definition [11]: A matrix A that has the following form

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^{1,1} & \cdots & \mathbf{A}^{1,J} \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{K,1} & \cdots & \mathbf{A}^{K,J} \end{bmatrix}$$

is a matrix with circulant block if $\mathbf{A}^{k,j}$ is circulant *N* by *N* matrix for all $k = 1, \ldots, K, j = 1, \cdots, J$.

From this definition, we have that for all $i = 1, \cdots, M$, $\mathbf{R}_i$ and $\mathbf{H}_i$ are matrices with circulant block. Now our optimization problem is to find the set of positive semidefinite matrices $\{\mathbf{R}_i\}_{i=1}^M$ with circulant block under the respective trace constraints such that the sum of the total capacity in (7) is maximized. Suppose that the noise is the additive white gaussian noise (AWGN), and

$$\mathbf{R}_n = N_0 \mathbf{I} \ . \tag{9}$$

We will show that the iterative waterfilling algorithm will converge to the positive semidefinite matrices with circulant block. First we give without proof some basic properties of matrices with circulant block.

Proposition 1. The product and sum of the matrices with circulant block are also matrices with circulant block. This is easy to shown since we know that the product and sum of the circulant matrices are also circulant matrices.

From **Proposition 1** we can see that the sum capacity optimization problem with the additional constraints that $\{\mathbf{R}_i\}_{i=1}^M$ are matrices with circulant block is still a convex programming problem.

Proposition 2 A matrix $\mathbf{A}$ is a Hermitian matrix with circulant block if and only if it has the following decomposition

$$\mathbf{A} = \mathbf{F}^* \mathbf{P} \mathbf{V} \mathbf{\Lambda} \mathbf{V}^* \mathbf{P}^* \mathbf{F} \ , \tag{10}$$

where $\mathbf{F}$ is the block diagonal inverse discrete Fourier transform (IDFT) matrix [8] with each diagonal block the unitary IDFT matrix; $\mathbf{P}$ is the permutation matrix in [8]; $\mathbf{V}$ is block diagonal, with each block containing the unitary matrices; and $\mathbf{\Lambda}$ is a diagonal real matrix.

Proposition 3. If $\mathbf{A}$ is a nonsingular Hermitian matrix with circulant block, $\mathbf{A}^{-1}$ is also a Hermitian matrix with circulant block. This is easy to show from **Proposition 2**.

Lemma $\mathbf{H}_i^*\left(\mathbf{R}_n + \sum_{j=1, j\neq i}^{M} \mathbf{H}_j\mathbf{R}_j\mathbf{H}_j^*\right)^{-1}\mathbf{H}_i$ is a Hermitian matrix with circulant block, if $\mathbf{R}_n$ is a Hermitian matrix with circulant block, and for all j=1,…,M, $\mathbf{H}_j$ are matrices with circulant block, and $\mathbf{R}_j$ are Hermitian matrices with circulant block. The lemma can be easily proved by using **Proposition 1** and **3**.

Now comes the main result.

Theorem The optimal set of Hermitian matrices $\{\mathbf{R}_i\}_{i=1}^{M}$ with circulant block under the respective trace constraints (8) can be found by the iterative waterfilling algorithm such that the sum capacity of (7) is maximized.

Proof: Without loss of generality, we choose the initial value of the iterative algorithm to be diagonal matrices. Then from the above Lemma, at the first step of the iteration, $\mathbf{A}_i = \mathbf{H}_i^*\left(\mathbf{R}_n + \sum_{j=1, j\neq i}^{M} \mathbf{H}_j\mathbf{R}_j\mathbf{H}_j^*\right)^{-1}\mathbf{H}_i$ is a Hermitian matrix with circulant block for all i=1,…,M. Therefore $\mathbf{A}_i$ has the eigendecomposition form (10) as shown in **Proposition 2**. The iterative waterfilling optimizes $\mathbf{R}_i$ such that $\mathbf{R}_i$ has the same eigenvectors as $\mathbf{A}_i$, hence $\mathbf{R}_i = \mathbf{F}^*\mathbf{P}\mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^*\mathbf{P}^*\mathbf{F}$, where $\mathbf{\Lambda}_i$ is a diagonal positive definite real matrix. From **Proposition 2**, $\mathbf{R}_i$ is also a Hermitian matrix with circulant block. By induction, we can see that after each iteration step, $\mathbf{R}_i$ is Hermitian and with circulant block for all i=1,…,M. We also know that the sum capacity increases after each iteration [10]. Since the programming problem is convex, this iterative waterfilling will converge to the global optimal solution. Therefore, the Hermitian matrices with circulant block can be found by the iterative waterfilling algorithm. ■

From this theorem, we know that the optimal transmitter design for the multiuser DMMT system can be solved by the iterative waterfilling algorithm.

IV. SIMULATION RESULTS

We use a 4-path channel model for all of the users with the average power of each path given by {0dB, -0.25dB, -0.6dB, -1dB}, and each path obeys the identical independent Rayleigh fading. The channel parameters of all the users are generated randomly. For the multiuser STVC cases, assume that the energy constraints of all the users obey that $E_i = \frac{S_x(N+v)}{M}$, for $i = 1,\cdots,M$. Due to the extra energy required to transmit the cyclic prefix, the input energy constraint for the multiuser DMMT system is assumed as $E_i = \frac{S_x N}{M}$. Define the *SNR* as $SNR = \frac{S_x}{N_0}$. In the simulation we assume that the block length N is equal to 10, while there are 5 users in the multiple access channel. The capacity versus *SNR* of the multiuser 3×3 STVC system is compared to the capacity of the 2×2 and 1×1 cases in Fig. 1. A similar comparison for the multiuser DMMT system is depicted in Fig. 2. As we can see, remarkable performance gains are achievable by utilizing the space-diversity. We also notice that the multiuser STVC channel capacity is slightly higher than the multiuser DMMT channel capacity. The difference is basically due to the transmitted power penalty associated with the cyclic prefix.

V. CONCLUSIONS

In this paper, the total information capacities of the STVC and DMMT over the multiple access ISI channels are maximized by optimizing the covariance matrices of all the users. The iterative waterfilling algorithm is employed to find the optimal solution. The simulation results show that a significant capacity gain is achieved by the joint multiuser and spatial-temporal optimization. From the optimization we obtain the optimal space-time or space-frequency parallel channels for each user.

ACKNOWLEDGEMENT

This work was partially supported by NASA-Dryden grant NCC-2-374 and a UC CORE grant sponsored by ST Microelectronics, INC.

REFERENCES

[1] A. Naguib and A.R. Calderbank, "Space-time coding and signal processing for high data rate wireless communications," *Wire. Commun. Mob. Comput*, pp.13-34, Jan 2001.

[2] Z. Liu, G. B. Giannakis, S. Zhou, and B. Muquet, "Space-time coding for broadband wireless communications," *Wire. Commun. Mob. Comput*, pp.35-53, Jan 2001.

[3] V. Tarokh, N. Seshadri and A.R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp.744-765, Mar 1998.

[4] V. Tarokh, H. Jafarkhani and A.R. Calderbank**,** **"**Space-time block coding for wireless communications: performance results, " *IEEE JSAC*, vol.17, pp.451-460, Mar 1999.

[5] W. Choi and J. M. Cioffi, "Space-time block codes over frequency selective Rayleigh fading channels," in *Proc. VTC'99*: pp. 2541-2545.

[6] H. Bölcskei and A. J. Paulraj, "Space-frequency coded broadband OFDM systems," in *Proc WCNC'00*, pp.1-6, Sep, 2000.

[7] H. Bölcskei, D. Gesbert, and A. J. Paulraj, "On the capacity of OFDM-based spatial multiplexing systems," *IEEE Trans. Commun.*, accepted for publication.

[8] G.G. Raleigh and J. M. Cioffi, **"**Spatio-temporal coding for wireless communication,**"** *IEEE Trans. Comm*, vol. 46, pp. 357-366, Mar 1998.

[9] N. Al-Dhahir and J. M. Cioffi, "Block transmission over dispersive channels: Transmit filter optimization and realization, and MMSE-DFE receiver performance," *IEEE Trans. Inform. Theory*, vol. 42, pp. 137-160, Jan 1996.

[10] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, “Iterative water-filling for Gaussian vector multiple access channels, ” In *Proc. IEEE ISIT*, Jun 2001.
[11] P. Davis, *Circulant matrices*. John Wiley & Sons, 1979.

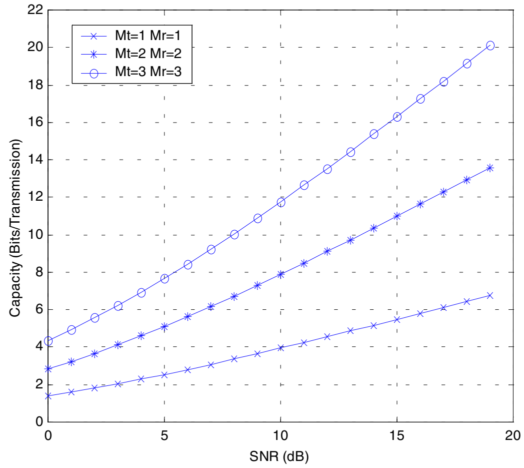

Fig.1. Capacity versus *SNR* for the multiuser STVC system: block length *N*=10, M=5

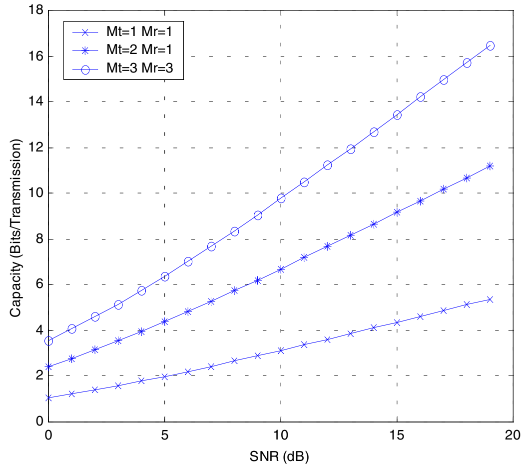

Fig.2. Capacity versus *SNR* for the multiuser DMMT system: block length *N*=10, M=5