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Addressing the gap in scheduling research: a review of optimization and heuristic methods in production scheduling

B. L. MACCARTHY† and JIYIN LIU

This paper considers the gap between scheduling theory and scheduling practice. The development and the main results of classical scheduling theory are reviewed and presented in an easily accessible way. Recent trends in scheduling research which attempt to make it more relevant and applicable are described. The nature of the gap between theory and practice is discussed. The failure of classical scheduling theory to address the total environment within which the scheduling function operates is noted. However, scheduling research in operations management and manufacturing systems tends to ignore the rich vein of methods, techniques and results in the classical theory. The need for an integrated scheduling research effort, containing elements of both approaches, is stressed.

1. Introduction

Scheduling may be defined as the allocation of resources over time to perform tasks. The importance of good scheduling strategies in production environments in today's competitive markets cannot be overstressed. The need to respond to market demands quickly and to run plants efficiently gives rise to complex scheduling problems in all but the simplest production environments.

The theory of scheduling has received a lot of attention from OR practitioners, management scientists, production and operations research workers and mathematicians since the early 1950s. A number of books have been published on the subject, e.g. Muth and Thompson (eds) 1963, Conway *et al.* (1967), Elmaghraby (ed) 1973, Baker (1974), Rinnooy Kan (1976), French (1982), Bellman *et al.* (1982). Review articles of varying breadths and depths which survey the development of scheduling theory include Mellor (1966), Lenstra *et al.* (1977), Graham *et al.* (1979), Graves (1981), Frost (1984), Blazewicz *et al.* (1988), Rodammer and White (1988), Buxey (1989), Kovalev *et al.* (1989), White (1990).

The utilization of classical scheduling theory in most production environments is minimal. In many production environments scheduling and plant loading is frequently carried out by first line management. In some sectors it may be delegated to shift leaders, foremen, or chargehands. In many cases there is no appreciation that a body of theory exists which may relate to some or perhaps all of the scheduling problems. More importantly the consequences of poor scheduling strategies on overall company performance is generally not appreciated.

This paper is aimed at researching the gap between scheduling theory and scheduling practice. Firstly, the state-of-the-art in classical scheduling theory is

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† To whom correspondence must be addressed.

Department of Manufacturing Engineering and Operations Management, University of Nottingham, University Park, Nottingham NG7 2RD, UK.

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reviewed in sections 2 and 3. The emphasis is on clarity and simplicity in outlining the main developments, solution approaches and the most significant results. This review emphasizes major trends and does not attempt fully comprehensive coverage. Secondly, research which attempts to make classical scheduling theory more useful in practice is discussed in section 4. In section 5 the need for an integrated approach to production scheduling research is discussed.

2. Classical scheduling theory and its development

Although the definition of scheduling given at the beginning of the paper is widely applicable it is conventional (and appropriate in this case) to denote the tasks as jobs and the resources as machines. The definitions and assumptions in classical scheduling theory are now outlined. The limitations of these definitions and assumptions are discussed in sections 4 and 5.

2.1. *Definitions and assumptions*

A general scheduling problem may be stated thus:

n jobs $\{J_1, J_2, \ldots, J_n\}$ have to be processed. m machines $\{M_1, M_2, \ldots, M_m\}$ are available. A subset of these machines is required to complete the processing of each job. The flow pattern or order of machines for any job may or may not be fixed for some or all jobs. The processing of job J_j on machine M_i is called an operation, denoted by O_{ij}. For each operation O_{ij}, there is an associated processing time t_{ij}. In addition, there may be a ready time (or release date) r_j associated with each job, at which time J_j is available for processing, and/or a due date d_j, by which time J_j should be completed. A schedule in this context is an assignment of jobs over time onto machines. The scheduling problem is to find a schedule which optimizes some performance measure.

The stated scheduling problem may be generalized further by replacing machines by processing stages which may contain several machines.

The following assumptions appear frequently in scheduling theory literature:

(1) Machines are always available and never break down.
(2) Each machine can process at most one job at any time.
(3) Any job can be processed on at most one machine at any time.
(4) Ready times of all jobs are zero, i.e. all jobs are available at the commencement of processing.
(5) No pre-emption is allowed—once an operation is started it is continued until complete.
(6) Setup times are independent of the schedules and are included in processing times.
(7) Processing times and technological constraints are deterministic and known in advance and similarly for due dates, where appropriate.

In classical scheduling theory the planning framework or time horizon in which a scheduling problem may arise or a schedule be applicable is generally not considered. The implicit assumption is not only that decision-making is short term in a static, deterministic environment but also the fact that researchers have realized that problem complexity increases further if a dynamic environment is considered.

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2.2.1. *Classification of scheduling problems*

In defining a scheduling problem both the technological constraints on jobs and the scheduling objectives must be specified.

Technological constraints are determined principally by the flow pattern of the jobs on machines. In this context the following definitions are useful:

(1) *Job shop*: each job has its own individual flow pattern or specific route through the machines which must be adhered to.
(2) *Flow shop*: each job has an identical flow pattern.
(3) *Open shop*: there is no specified flow pattern for any jobs.
(4) *Permutation flow shop*: a flow shop in which the order of processing of jobs on all machines is constrained to be the same.
(5) *Single machine shop*: only one machine is available.

In cases (1), (2) and (3) the schedule may produce a different order of jobs on machines in the shop. When processing stages are considered rather than machines the following definitions are useful.

(6) *Parallel machines*: k identical machines in a single processing stage. Each job needs one and only one of these machines.
(7) *Job shop with duplicate machines*: A job shop in which there are k_i identical machines in each stage ($i = 1, \ldots, m$) and any job requiring that stage needs to be processed on one and only one of these machines.

The diagram in Fig. 1 illustrates schematically the relationship between the different machine environments. At the expense of clarity it could be extended further, e.g. a flow shop with duplicate machines.

Within any of these environments scheduling may be attempted with respect to various objectives. Mellor (1966) lists 27 different objectives. A useful classification for single objective problems was given by Baker (1974). For the jth job define the following measures:

Completion time C_j

Flow time $F_j = C_j - r_j$†

Waiting time $W_j = C_j - r_j - \sum_{i=1}^{m} t_{ij}$

Lateness $L_j = C_j - d_j$

Tardiness $T_j = \max\{0, L_j\}$

Baker (1974) noted three types of decision-making goals prevalent in scheduling and indicated commonly used measures of schedule performance which are associated with them:

- *Efficient utilization of resources*: Maximum completion time (or makespan) $C_{\max}$.
- *Rapid response to demands*: Mean completion time $\bar{C}$, mean flow time $\bar{F}$, or mean waiting time $\bar{W}$.

† Clearly completion times and flow times are equivalent when ready times are all zero.

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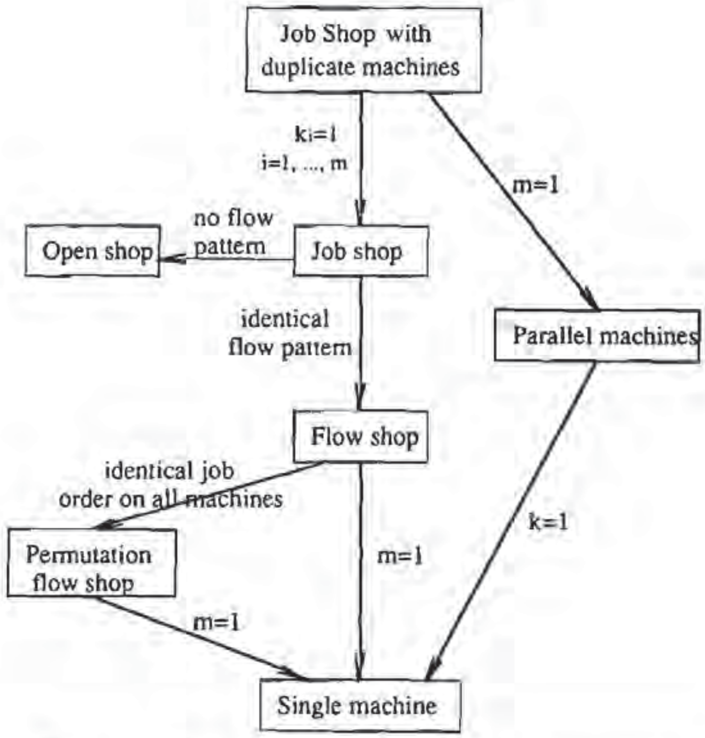

Figure 1. Relationships between machine shop environments.

- *Close conformance to prescribed deadlines*: Mean tardiness $\bar{T}$, maximum tardiness T_{max}, and the number of tardy jobs N_T.

Most other commonly used measures may also be viewed within this framework. Other approaches to measure schedule performance are considered in section 4.

2.2.2. *Notation for scheduling problems*

Conway *et al.* (1967) give a classification scheme for scheduling problems based on four descriptors A/B/C/D which has since been followed by a number of researchers. It is used here in an extended form to include a larger problem set.

Definition	Possible value
A—number of jobs.	any positive integer, usually n.
B—number of machines.	any positive integer, frequently m.

Note: When parallel machines are considered the value of this descriptor is the number of processing stages and the number of machines at each stage is included in descriptor C.

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C—flow pattern and further technological and management constraints. Possible values are:

|| : single machine
J : job shop
F : flow shop
O : open shop
F,perm : permutation flow shop
k-parallel : k-machines in parallel
J,k-parallel : job shop with k parallel machines at each stage

The following abbreviations have been used to represent additional constraints which may occur in more complex scheduling environments:

r_j : jobs with different ready times
str : string jobs
prec : precedence constraints
prmt : pre-emption is allowed
unit : unit processing times
eq : equal processing time for all jobs
depend : dependent jobs
setup : sequence-dependent setup times

D—criteria to be optimized

Usually minimization of schedule performance measures noted in section 2.2.1, e.g. C_{max}, $\bar{F}$, etc.

For convenience here those problems for which the assumptions in 2.1 hold are referred to as basic problems. These can all be described easily in the above notation. For example, $n/m/J/C_{max}$ refers to the job shop scheduling problem with n jobs and m machines which attempts to minimize makespan. Non-basic problems are ones where some of the assumptions are not valid and/or extra conditions apply, e.g. $r_j > 0$ for some j. In presenting these problems, descriptor C may become long. Graham *et al.* (1979) introduced another notation based on three descriptors $\alpha/\beta/\gamma$. The first descriptor α defines the flow pattern together with the number of machines. The second descriptor β represents other constraints on jobs. The third descriptor γ defines the scheduling criterion. Although this notation can represent non-basic problems easily and has been used by some authors (e.g. Lawler *et al.* 1989), we use Conway's notation with the refinements defined above in this paper. This notation has been used widely for a long time and is familiar to most manufacturing and scheduling researchers. Converting between the two notations is simple.

2.3. *Historical development of classical scheduling theory*

Like many OR application areas the study of scheduling theory began in the early 1950s. Johnson's article (Johnson 1954) is acknowledged as a pioneering work. It presented an efficient optimal algorithm for $n/2/F/C_{max}$ and generalized the method for some special cases of $n/3/F/C_{max}$. Jackson (1955) and Smith (1956) gave various optimal rules for single-machine problems. These early works formed the basis for much of the development of classical scheduling theory.

Later several kinds of general optimization procedures were applied to scheduling problems. These included mixed and pure integer programming formulations (Wagner 1959, Bowman 1959), dynamic programming (Held and Karp 1962), and branch

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