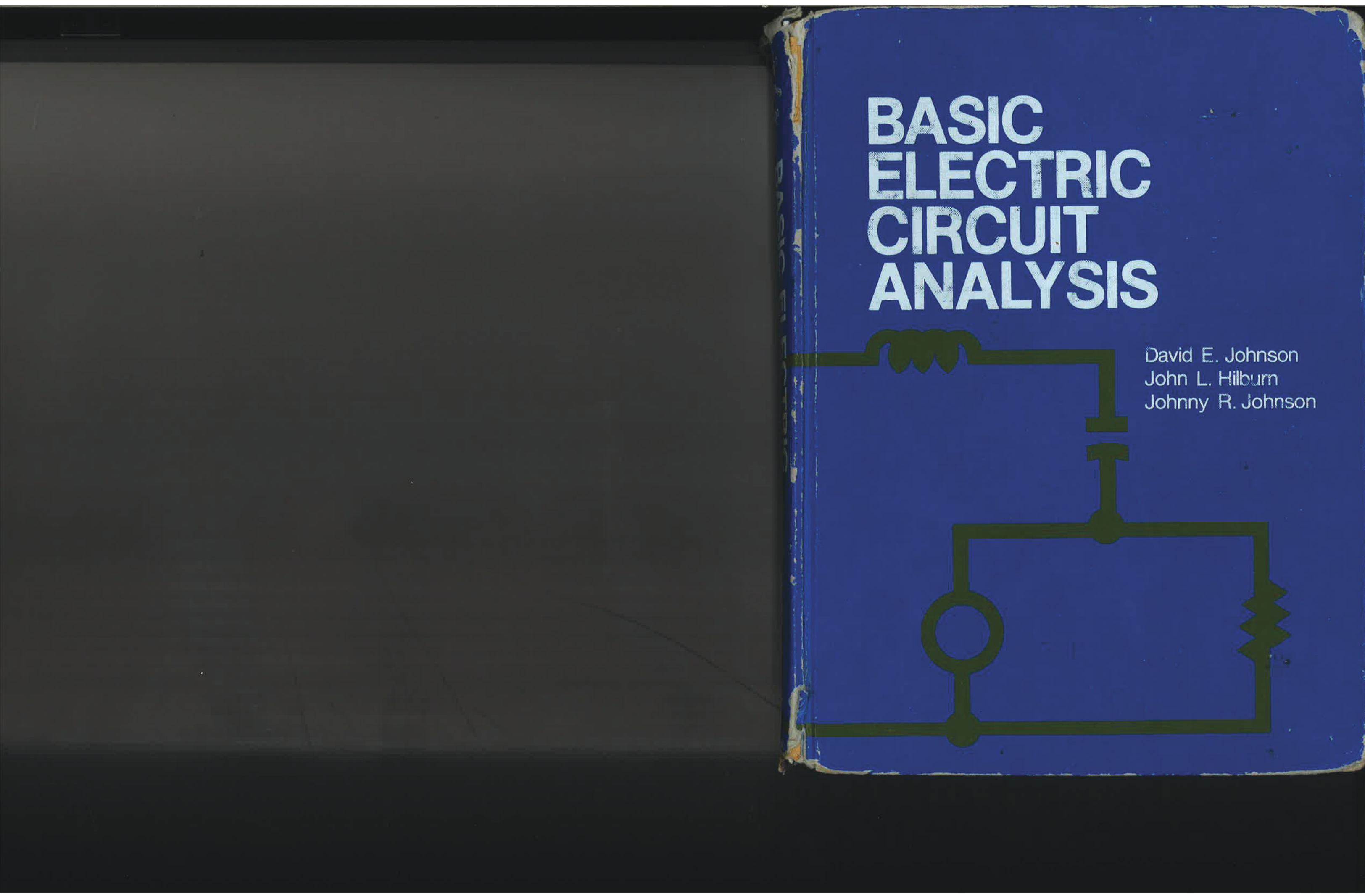
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EXERCISES

10.6.1 Using phasors, find the ac steady-state current i if $v = 12 \cos (1000t + 30°)$ V in (a) Fig. 10.9(a) for $R = 4$ kΩ, (b) Fig. 10.11(a) for $L = 20$ mH, and (c) Fig. 10.13(a) for $C = 1\ \mu$F.

Ans. (a) $3 \cos (1000t + 30°)$ mA, (b) $0.6 \cos (1000t - 60°)$ A, (c) $12 \cos (1000t + 120°)$ mA

10.6.2 In Ex. 10.6.1, find i in each case at $t = 1$ ms.

Ans. (a) 0.142 mA, (b) 0.599 A, (c) −11.987 mA

10.7 IMPEDANCE AND ADMITTANCE

Let us now consider a general phasor circuit with two accessible terminals, as shown in Fig. 10.15. If the time-domain voltage and current at the terminals are given by (10.38), then the phasor quantities at the terminals are

$$\mathbf{V} = V_m\underline{/\theta} \qquad \mathbf{I} = I_m\underline{/\phi} \tag{10.46}$$

We define the ratio of the phasor voltage to the phasor current as the *impedance* of the circuit, which we denote by **Z**. That is,

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \tag{10.47}$$

which by (10.46) is

$$\mathbf{Z} = |\mathbf{Z}|\underline{/\theta_Z} = \frac{V_m}{I_m}\underline{/\theta - \phi} \tag{10.48}$$

where $|\mathbf{Z}|$ is the magnitude and θ_Z the angle of **Z**. Evidently,

$$|\mathbf{Z}| = \frac{V_m}{I_m}, \qquad \theta_Z = \theta - \phi$$

Impedance, as is seen from (10.47), plays the role, in a general circuit, of resistance in resistive circuits. Indeed, (10.47) looks very much like Ohm's law; also like resistance, impedance is measured in ohms, being a ratio of volts to amperes.

It is important to stress that impedance is a complex number, being the ratio of two complex numbers, but it is *not* a phasor. That is, it has no corresponding sinusoidal time-domain function of any physical meaning, as current and voltage phasors have.

The impedance **Z** is written in polar form in (10.48); in rectangular form it is

generally denoted by

$$\mathbf{Z} = R + jX \tag{10.49}$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistive component*, or simply *resistance*, and $X = \text{Im}\,\mathbf{Z}$ is the *reactive component*, or *reactance*. In general, $\mathbf{Z} = \mathbf{Z}(j\omega)$ is a complex function of $j\omega$, but $R = R(\omega)$ and $X = X(\omega)$ are real functions of ω. Both R and X, like **Z**, are measured in ohms. Evidently, comparing (10.48) and (10.49) we may write

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta_Z = \tan^{-1}\frac{X}{R}$$

and

$$R = |\mathbf{Z}| \cos \theta_Z$$

$$X = |\mathbf{Z}| \sin \theta_Z$$

These relations are shown graphically in Fig. 10.16.

As an example, suppose in Fig. 10.15 that $\mathbf{V} = 10\underline{/56.1°}$ V and $\mathbf{I} = 2\underline{/20°}$ A. Then we have

$$\mathbf{Z} = \frac{10\underline{/56.1°}}{2\underline{/20°}} = 5\underline{/36.1°}\ \Omega$$

In rectangular form this is

$$\mathbf{Z} = 5(\cos 36.1° + j \sin 36.1°) = 4 + j3\ \Omega$$

The impedances of resistors, inductors, and capacitors are readily found from their V-I relations of (10.40), (10.43), and (10.45). Distinguishing their impedances with subscripts R, L, and C, respectively, we have, from these equations and (10.47),

$$\mathbf{Z}_R = R$$
$$\mathbf{Z}_L = j\omega L = \omega L\underline{/90°} \tag{10.50}$$
$$\mathbf{Z}_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = \frac{1}{\omega C}\underline{/-90°}$$

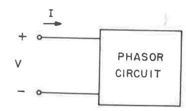

FIGURE 10.15 *General phasor circuit*

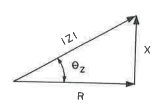

FIGURE 10.16 *Graphical representation of impedance*