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Electrical Engineering Reference Manual

for the Electrical and Computer PE Exam

Sixth Edition

John A. Camara, PE

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Preface to the Sixth Edition

The sixth edition of the *Electrical Engineering Reference Manual* is completely new, yet it owes much to its predecessors and to the outstanding guidance of Professional Publications, Inc.

This text is written as a broad review of electrical engineering design, analysis, and operational fundamentals. In choosing the subjects and preparing the contents, I was directed by the need for a comprehensive study guide for the newly developed Principles and Practice of Engineering (PE) examination in electrical and computer engineering, as well as an all-inclusive reference book for both the practicing engineer and the undergraduate electrical engineering student.

This book strives to meet the needs of three groups. For PE candidates, it is an efficient resource for exploring the exam topics systematically and exhaustively. For the practicing electrical or electronics engineer, it functions as a comprehensive reference, discussing all aspects of these fields in a realistic manner and incorporating the most common formulas and data. Finally, for the engineering student, it presents a thorough review of the fundamentals of electrical engineering.

The *Reference Manual* covers all the topics on the electrical and computer engineering PE exam, providing sufficient background information to preclude the necessity of referring to other texts. Additional subjects not specifically covered on the exam but essential for full understanding of exam topics are explained as well. The mathematical, theoretical, and practical applications of each topic are explored, so the reader may focus on any or all of these areas.

A number of features enhance the usefulness of the text itself. The introduction gives details on how to use this book efficiently. Lists of codes, handbooks, and references recommended for additional study are provided. The means for accessing online updates (and errata) is given. Appendices are provided to exhibit mathematical, basic theoretical, and practical data. A glossary of common electrical terms is included. A comprehensive index is provided to aid in the search for specific information.

The sources used in assembling the content of this work include: (1) information on the electrical and computer PE exam made public by the National Council of Examiners for Engineering and Surveying (NCEES); (2) PE exam review course material published by the National Society of Professional Engineers; (3) electrical engineering curricula at leading colleges and universities; (4) current literature in the field of electrical engineering; (5) information on the numerous electrical engineering websites on the Internet; and (6) survey comments from those who have recently taken the PE exam and/or have purchased previous editions of this text.

Changes from the previous edition are numerous. All are meant to enhance the enduring usefulness of the book. Here are some of the major differences between the fifth and sixth editions.

- Mathematics chapters are added for those who need an extra review before moving on to the electrical engineering information.
- Theory and Fields topics are added, both to improve readers' understanding of principles and to prepare them for potential changes to the PE exam structure.
- Information from the fifth edition is completely rewritten to reflect the mindset of a practicing engineer, and it is separated into generally smaller, more logical divisions.
- The Power topic is expanded to include Generation Systems; Transmission and Distribution Systems; and Lightning Protection and Grounding.
- The Electronics topic is updated to reflect the latest advances.
- The Computers topic is added. The Digital Systems information is expanded to include the basics of interfaces, protocols, and standards.
- The Communications information is gathered into a single topic and expanded to reflect recent progress in the field.
- A chapter on Biomedical Engineering is incorporated.
- Sections on Electrical Materials, Law, Ethics, and Electrical Engineering Frontiers are added for completeness.
- The National Electrical Code is addressed as a separate topic, and there is complete coverage of the 1999 NEC.
- SI units are used, except where common practice dictates customary U.S. units (as in the National Electrical Code topic).

- Nomenclature used is consistent from chapter to chapter and with current SI system usage. Varying symbology is mentioned to minimize confusion when using other texts.
- Topics and individual chapters are arranged in a logical, progressive manner appropriate to the new organization of the PE exam.

The change in the electrical and computer engineering PE exam to a breadth-and-depth format is mirrored in this book. The *Reference Manual* is meant as a compendium of the *breadth* of electrical engineering, providing the *depth* required in individual sections to enable engineers to gain a solid understanding of theory and practical applications.

The scope of this book is beyond that of the PE exam, as it is intended to be. When the exam is complete, you will still need a resource for electrical engineering information, either in your work or simply to satisfy the curiosity that makes us thinking humans. In both the exam and your career, I hope this book serves you well.

Should you find an error in this book, know that it is mine, and that I regret it. Beyond that, I hope two things happen. First, please let me know about it, either by using the "Errata" section on the PPI website at **www.ppi2pass.com** or by filling out the errata card found in this book. Second, I hope you learn something from the error—I know I will! I would appreciate constructive comments and suggestions for improvement, additional questions, and recommendations for expansion so that new editions or similar texts will more nearly meet the needs of future examinees.

John A. Camara, PE

26 DC Circuit Fundamentals

Nomenclature

A	area	m^2
d	diameter	m
E	electromotive force	V
G	conductance	S
i, I	current	A
l	length	m
N	number	–
P	power	W
Q	charge	C
R	resistance	Ω
S	surface area	m^2
t	time	s
T	temperature	°C
v, V	voltage	V
W	work	J

Symbols

α	thermal coefficient of resistivity	1/°C
ρ	resistivity	Ω·cm or Ω·m
σ	conductivity	S/m

Subscripts

0	initial
cmils	circular mils
Cu	copper
e	equivalent
fl	full load
nl	no load
N	Norton
oc	open circuit
s	shunt or source
sc	short circuit
Th	Thevenin

1. VOLTAGE

Voltage is the energy, or work, per unit charge exerted in moving a charge from one position to another. One of the positions is a reference position and is given an arbitrary value of zero. Voltage is also referred to as the *potential difference* to distinguish it from the unit of potential difference, the volt. One volt, V, is the potential difference if one joule of energy moves one coulomb of charge from the reference position to another position. Thus 1 V = 1 J/C. Additional equivalent units are W/A, C/F, A/S, and Wb/s.

In practical terms, a potential of one volt is defined as the potential existing between two points of a conducting wire carrying a constant current, I, of one ampere when the power dissipated between these two points is one watt. When the voltage refers to a source of electrical energy, it is termed the *electromotive force* (emf) and sometimes given the symbol E. In a *direct-current* (DC) *circuit*, the voltage, v, may vary in amplitude but not polarity. In many applications, the voltage magnitude, V, is constant as well. That is, it does not vary with time.

2. CURRENT

Current is the amount of charge transported past a given point per unit time. When current is constant or time-invariant, it is given the symbol I.[1] The unit of

[1] The symbol for current is derived from the French word intensité.

Circuit Theory

measure for current is the ampere (A), which is equivalent to coulombs per second (C/s). That is, one ampere is equal to the flow of one coulomb of charge past a plane surface, S, in one second. Though current is now known to be electron movement, it was originally viewed as flowing positive charges. This frame-of-reference current is called *conventional current* and is used in this and most other texts. When current refers to the actual flow of electrons, it is termed *electron current*. These concepts are illustrated in Fig. 26.1.

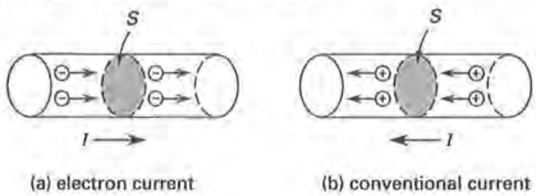


Figure 26.1 *Current Flow*

Current does not "flow," though this term is often used in conjunction with current. Additionally, due to the conservation of charge, all circuit elements are electrically neutral, meaning that no net positive or negative charge accumulates on any circuit element.

Example 26.1

A conductor has a constant current of 10 A. How many electrons pass a fixed plane in the conductor in 1 min?

Solution

First, using amperes, determine the charge flowing per minute.

$$10\ \text{A} = \left(10\ \frac{\text{C}}{\text{s}}\right)\left(60\ \frac{\text{s}}{\text{min}}\right) = 600\ \text{C/min}$$

Using the charge of an electron, compute the number of electrons flowing.

$$\frac{600\ \dfrac{\text{C}}{\text{min}}}{1.602\times 10^{-19}\ \dfrac{\text{C}}{\text{electron}}} = 3.75\times 10^{21}\ \text{electrons/min}$$

3. RESISTANCE

Resistance, R, is the property of a circuit element that impedes current flow.[2] It is measured in ohms, Ω. A circuit with zero resistance is a *short circuit*, while a circuit with infinite resistance is an *open circuit*.

Resistors are often constructed from carbon compounds, ceramics, oxides, or coiled wire. Adjustable resistors are known as *potentiometers* and *rheostats*.

The resistance of any substance depends on its *resistivity* (ρ), its length, and its cross-sectional area. Assuming resistivity and cross-sectional area are constant, the resistance of a substance is

$$R = \frac{\rho l}{A} \qquad 26.1$$

Though SI system units are the standard, the units of area, A, are often in the English Engineering System, especially in many tabulated values in the National Electrical Code. The area is given and tabulated for various conductors in *circular mils*, abbreviated cmil. One cmil is the area of a 0.001 in diameter circle. The concept of area in circular mils is represented by Eqs. 26.2, 26.3, and 26.4.

$$A_{\text{cmil}} = \left(\frac{d_{\text{inches}}}{0.001}\right)^2 \qquad 26.2$$

$$A_{\text{in}^2} = 7.854\times 10^{-7}\times A_{\text{cmil}} \qquad 26.3$$

$$A_{\text{cm}^2} = 5.067\times 10^{-6}\times A_{\text{cmil}} \qquad 26.4$$

Resistivity is dependent on temperature. In most conductors, it increases with temperature, since at higher temperatures electron movement through the lattice structure becomes increasingly difficult. The variation of resistivity with temperature is specified by a *thermal coefficient of resistivity*, α, with typical units of 1/°C. The actual resistance or resistivity for a given temperature is calculated with Eqs. 26.5 and 26.6.

$$R = R_0(1+\alpha\Delta T) \qquad 26.5$$

$$\rho = \rho_0(1+\alpha\Delta T) \qquad 26.6$$

The thermal coefficients for several common conducting materials are given in Table 26.1.

Table 26.1 *Approximate Temperature Coefficients of Resistance[a] and Percent Conductivities*

material	α, 1/°C	% conductivity
aluminum, 99.5% pure	0.00423	63.0
aluminum, 97.5% pure	0.00435	59.8
constantan[b]	0.00001	3.1
copper, IACS (annealed)	0.00402	100.0
copper, pure annealed	0.00428	102.1
copper, hard drawn	0.00402	97.8
gold, 99.9% pure	0.00377	72.6
iron, pure	0.00625	17.5
iron wire, EBB[c]	0.00463	16.2
iron wire, BB[d]	0.00463	13.5
manganin[e]	0.00000	3.41
nickel	0.00622	12.9
platinum, pure	0.00367	14.6
silver, pure annealed	0.00400	108.8
steel wire	0.00463	11.6
tin, pure	0.00440	12.2
zinc, very pure	0.00406	27.7

(Multiply 1/°C by 0.5556 to obtain 1/°F.)

[a]between 0°C and 100°C
[b]58% Cu, 41% Ni, 1% Mn
[c]Extra Best Best grade
[d]Best Best grade
[e]84% Cu, 4% Ni, 12% Mn

[2]Resistance is the real part of *impedance*, which opposes changes in current flow.

Example 26.2

What is the resistance of 305 m of 10 AWG (with an area of approximately 10,000 cmil) copper conductor with a resistivity of 2.82×10^{-6} Ω·cm?

Solution

From Eq. 26.1, and using the conversion of Eq. 26.4, the resistance is

$$\begin{aligned} R &= \frac{\rho l}{A} \\ &= \frac{(2.82 \times 10^{-6}\ \Omega\text{·cm})(305\ \text{m})\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)}{(10{,}000\ \text{cmil})\left(\frac{5.067 \times 10^{-6}\ \text{cm}^2}{1\ \text{cmil}}\right)} \\ &= 1.70\ \Omega \end{aligned}$$

4. CONDUCTANCE

The reciprocal of resistivity is *conductivity*, σ, measured in siemens per meter (S/m). The reciprocal of resistance is *conductance*, G, measured in siemens (S). Siemens is the metric unit for the older unit known as the *mho* (1/Ω).

$$\sigma = \frac{1}{\rho} \qquad 26.7$$

$$G = \frac{1}{R} \qquad 26.8$$

The *percent conductivity* is the ratio of a given substance's conductivity to the conductivity of standard IACS (International Annealed Copper Standard) copper, simply called *standard copper*. Alternatively, the percent conductivity is the ratio of standard copper's resistivity to the substance's resistivity.

$$\begin{aligned} \%\ \text{conductivity} &= \frac{\sigma}{\sigma_{\text{Cu}}} \times 100\% \\ &= \frac{\rho_{\text{Cu}}}{\rho} \times 100\% \end{aligned} \qquad 26.9$$

The standard resistivity of copper at 20°C is approximately

$$\begin{aligned} \rho_{\text{Cu,20°C}} &= 1.7241 \times 10^{-6}\ \Omega\text{·cm} \\ &= 0.3403\ \Omega\text{·cmil/cm} \end{aligned} \qquad 26.10$$

The percent conductivities for various substances are given in Table 26.1.

5. OHM'S LAW

Voltage, current, and resistance are related by Ohm's law.[3] The numerical result of Ohm's law is called the *voltage drop* or the *IR* drop.[4]

$$V = IR \qquad 26.11$$

Ohm's law presupposes a *linear circuit*, that is, a circuit consisting of linear elements and linear sources. In mathematical terms, this means that voltage plotted against current will result in a straight line with the slope represented by resistance. A *linear element* is a passive element, such as a resistor, whose performance can be portrayed by a linear voltage-current relationship. A *linear source* is one whose output is proportional to the first power of the voltage or current in the circuit. Many elements are linear or can be represented by equivalent linear circuits over some portion of their operation. Most sources, though not linear, can be represented as ideal sources with resistors in series or parallel to account for the nonlinearity.

6. POWER

If a steady current and voltage produce work, W, in time interval t, the electric power (energy conversion rate) is

$$P = \frac{W}{t} = \frac{VIt}{t} = VI = \frac{V^2}{R} \qquad 26.12$$

If the voltage and current vary with time, Eq. 26.12 still applies with these terms expressed as v and i. Equation 26.12 then represents the instantaneous power. The same is true for Eq. 26.13.

Equation 26.13 represents a form of power sometimes referred to as *I squared R* (I^2R) *losses*. Equation 26.13 is also the mathematical statement of *Joule's law of heating effect*.

$$P = I^2R \qquad 26.13$$

7. DECIBELS

Power changes in many circuits range over decades, that is, over several orders of magnitude. As a result, decibels are adopted to express such changes. The use of

[3]This book uses the convention where uppercase letters represent fixed, maximum, effective values or direct current (DC) values, and lowercase letters represent values that change with time, such as alternating current (AC) values. Direct current (DC) values can change in amplitude over time (but not polarity or direction), and for this reason are sometimes shown as lowercase letters.

[4]It is sometimes helpful to consider electrical problems and theories in terms of their mechanical analogs. The mechanical analogy to Ohm's law in terms of fluid flow is as follows: Voltage is the pressure, current is the flow, and resistance is the head loss caused by friction and restrictions within the system.

decibels also has an advantage in that the power gain (or loss) of cascaded stages in series is the sum of the individual stage decibel gains (or losses), making the mathematics easier. Strictly speaking, decibels refer to a power ratio with the denominator arbitrarily chosen as a particular value, P_0. The reference value changes, depending upon the specific area of electrical engineering usage. For example, communications and acoustics use different references.[5] Decibels have come into common usage for referring to voltage and current ratios as well, though an accurate comparison between circuits requires equivalent resistances between the two terminals being compared.

$$\begin{aligned} \text{ratio (in dB)} &= 10\log_{10}\left(\frac{P}{P_0}\right) \\ &\approx 20\log_{10}\left(\frac{V_2}{V_1}\right) \\ &= 20\log_{10}\left(\frac{I_2}{I_1}\right) \end{aligned} \qquad 26.14$$

Time-varying quantities can also be in the ratio. Care must be taken to compare only instantaneous values or effective (rms) or time-averaged values to one another.

8. ENERGY SOURCES

Sources of electrical energy include friction between dissimilar substances, contact between dissimilar substances, thermoelectric action (for example, the Thomson, Peltier, and Seebeck effects), the Hall effect, electromagnetic induction, the photoelectric effect, and chemical action. These sources of energy manifest themselves by the potential, V, they generate. When sources of energy are processed through certain electronic circuits, a current source can be created.

An *ideal voltage source* supplies power at a constant voltage, regardless of the current drawn. An *ideal current source* supplies power in terms of a constant current, regardless of the voltage between its terminals. However, real sources have internal resistances that, at higher currents, reduce the available voltage. Consequently, a *real voltage source* cannot maintain a constant voltage when the currents are large. A *real current source* cannot maintain a constant current completely independent of the voltage at its terminals. Real and ideal voltage and current sources are shown in electrical schematic form in Fig. 26.2.

[5] Communications uses many such reference values. One references all power changes to 1 kW. In audio acoustics, the reference is the power necessary to cause the minimum sound pressure level audible to the human ear at 2000 Hz, that is, a pressure of 20 μPa.

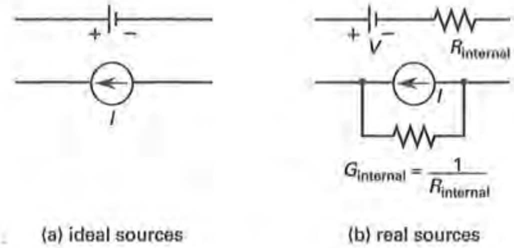


Figure 26.2 *Ideal and Real Energy Sources*

The change in the output of a voltage source is measured as its *regulation*. The formula is given by

$$\text{regulation} = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\% \qquad 26.15$$

Independent sources deliver voltage and current at their rated values regardless of circuit parameters. *Dependent sources* deliver voltage and current at levels determined by a voltage or current somewhere else in the circuit.

9. VOLTAGE SOURCES IN SERIES AND PARALLEL

Voltage sources connected in series, as in Fig. 26.3(a), can be reduced to an equivalent circuit with a single voltage source and a single resistance, as in Fig. 26.3(b). The equivalent voltage and resistance are calculated per Eqs. 26.16 and 26.17.

$$V_e = \sum V_i \qquad 26.16$$

$$R_e = \sum R_i \qquad 26.17$$

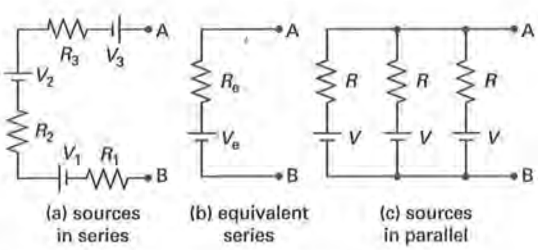


Figure 26.3 *Voltage Sources in Series and Parallel*

Millman's theorem states that N identical voltage sources of voltage V and resistance R connected in parallel, as in Fig. 26.3(c), can be reduced to an equivalent circuit with a single voltage source and a single resistance, as in Fig. 26.3(b). The equivalent voltage and resistance are calculated per Eqs. 26.18 and 28.19.

$$V_e = V \qquad 26.18$$

$$R_e = \frac{R}{N} \qquad 26.19$$

Nonidentical sources can be connected in parallel, but the lower-voltage sources may be "charged" by the higher-voltage sources. That is, the current may enter the positive terminal of the source. A loop current analysis is needed to determine the current direction and magnitude through the voltage sources.

10. CURRENT SOURCES IN SERIES AND PARALLEL

Currents sources may be placed in parallel. The circuit is then analyzed using any applicable method to determine the power delivered to the various elements. Current sources of differing magnitude cannot be placed in series without damaging one of them.

11. SOURCE TRANSFORMATIONS

A voltage source of V_s volts with an internal series resistance of R_s ohms can be represented by an equivalent circuit with a current source of I_s amps with an internal parallel resistance of the same R_s, and vice versa. Consequently, if Eq. 26.20 is valid, the circuits in Fig. 26.4 are equivalent.

$$V_s = I_s R_s \qquad 26.20$$

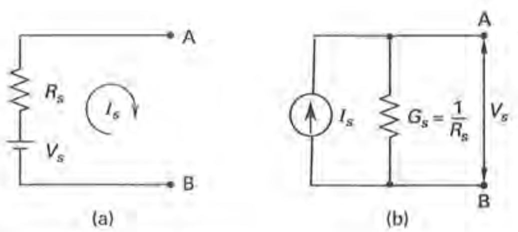


Figure 26.4 Equivalent Sources

12. MAXIMUM ENERGY TRANSFER

The maximum energy transfer, that is, the maximum power transfer, from a voltage source is attained when the series source resistance, R_s, of Fig. 26.4(a) is reduced to the minimum possible value, with zero being the ideal case. This assumes the load resistance is fixed and the source resistance can be changed. Though this is the ideal situation, it is not often the case. Where the load resistance varies and the source resistance is fixed, maximum power transfer occurs when the load and source resistances are equal.

Similarly, the maximum power transfer from a current source occurs when the parallel source resistance, R_s, of Fig. 26.4(b) is increased to the maximum possible value. This assumes the load resistance is fixed and the source resistance can be changed. In the more common case, where the load resistance varies and the source resistance is fixed, the maximum power transfer occurs when the load and source resistances are equal.

The maximum power transfer occurs in any circuit when the load resistance equals the Norton or Thevenin equivalent resistance.

13. VOLTAGE AND CURRENT DIVIDERS

At times, a source voltage will not be at the required value for the operation of a given circuit. For example, the standard automobile battery voltage is 12 V, while some electronic circuitry uses 8 V for DC biasing. One method of obtaining the required voltage is by use of a circuit referred to as a *voltage divider*. A voltage divider is illustrated in Fig. 26.5(a). The voltage across resistor 2 is

$$V_2 = V_s\left(\frac{R_2}{R_1 + R_2}\right) \qquad 26.21$$

If the fraction of the source voltage, V_s, is found as a function of V_2, the result is known as the *gain* or the *voltage-ratio transfer function*. Rearranging Eq. 26.21 to show the specific relationship between R_1 and R_2 gives

$$\frac{V_2}{V_s} = \frac{1}{1 + \frac{R_1}{R_2}} \qquad 26.22$$

An analogous circuit called a *current divider* can be used to produce a specific current. A current divider circuit is shown in Fig. 26.5(b). The current through resistor 2 is

$$I_2 = I_s\left(\frac{R_1}{R_1 + R_2}\right) = I_s\left(\frac{G_2}{G_1 + G_2}\right) \qquad 26.23$$

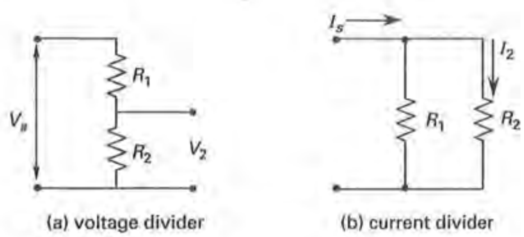


Figure 26.5 Divider Circuits

14. KIRCHHOFF'S LAWS

A fundamental principle in the design and analysis of electrical circuits is that the dimensions of the circuit are small. That is, the circuit's dimensions are small when compared to the wavelengths of the electromagnetic quantities that pass within them. Thus, only time variations need be considered, not spatial variations.

Consequently, Maxwell's equations, which are partial integrodifferential equations, become ordinary integrodifferential equations that vary only with time.

The net result is that Gauss's law, in integral form, reduces to Kirchhoff's current law. This occurs because the net charge enclosed in any particular volume of the circuit is zero and thus the sum of the currents at any point must be zero.

Another result is that Faraday's law of electromagnetic induction reduces to Kirchhoff's voltage law. This is because, with a "small" circuit, the surface integral of magnetic flux density is zero; thus the sum of the voltages around any closed loop must be zero.

15. KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltages around any closed path within a circuit or network is zero. Some of the voltages may be sources, while others will be voltages due to current in passive elements. Voltages through the passive elements are often called *voltage drops*. A *voltage rise* may also appear to occur across passive elements in the initial stages of the application of Kirchhoff's voltage law. For resistors, this is only because the direction of the current is arbitrarily chosen. For inductors and capacitors, a voltage rise indicates the release of magnetic or electrical energy. Stated in terms of the voltage rises and drops, KVL is

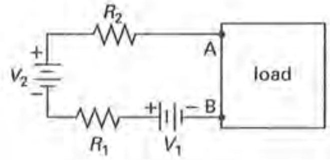
$$\sum_{\text{loop}} \text{voltage rises} = \sum_{\text{loop}} \text{voltage drops} \qquad 26.24$$

Kirchhoff's voltage law can also be stated as follows: The sum of the voltages around a closed loop must equal zero. The reference directions for voltage rises and drops used in this book are shown in Fig. 26.6.[6]

Figure 26.6 Voltage Reference Directions

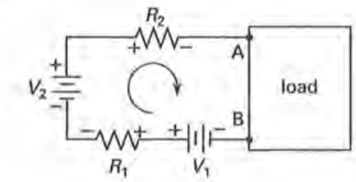
Example 26.3

Consider the following circuit. Determine the expression for V_{AB} across the load in the circuit in the figure.



Solution

For consistency in all problems, apply KVL in a clockwise direction as shown. The unknown voltage is V_{AB}, which indicates that the voltage is arbitrarily referenced from A to B. That is, terminal A is assumed to be the high potential (+). Assign polarities to the resistors based on the assumed direction.



The KVL expression for the loop, starting at terminal B, is

$$V_1 - V_{R_1} + V_2 - V_{R_2} - V_{AB} = 0$$

Rearrange and solve for V_{AB}.

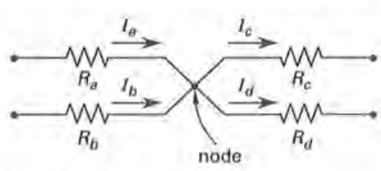
$$V_{AB} = V_1 - V_{R_1} + V_2 - V_{R_2}$$

16. KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law (KCL) states that the algebraic sum of the currents at a node is zero. A *node* is a connection of two or more circuit elements. When the connection is between two elements, the node is a *simple node*. When the connection is between three or more elements, the node is referred to as a *principal node*. Stated in terms of the currents directed into and out of the node, KCL is

$$\sum_{\text{node}} \text{currents in} = \sum_{\text{node}} \text{currents out} \qquad 26.25$$

Kirchhoff's current law can also be stated as follows: The sum of the currents flowing out of a node must equal zero.[7] The reference directions for positive currents (currents "out") and negative currents (currents "in") used in this book are illustrated in Fig. 26.7.



(a) I_a, I_b: negative currents or currents in
(b) I_c, I_d: positive currents or currents out

Figure 26.7 Current Reference Directions

[6]This alternative statement of Kirchhoff's voltage law and the arbitrary directions chosen are sometimes useful in simplifying the writing of the mathematical equations used in applying KVL.

[7]This alternative statement of Kirchhoff's current law and the arbitrary directions chosen are sometimes useful in simplifying the writing of the mathematical equations used in applying KCL.

17. SERIES CIRCUITS

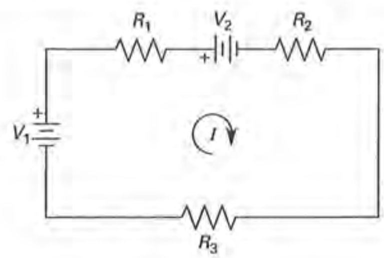


Figure 26.8 Series Circuit

A simple series circuit, as shown in Fig. 26.8, has the following properties (generalized to any number of voltage sources and series resistances).

- Current is the same through all the circuit elements.

$$I = I_{R_1} = I_{R_2} = I_{R_3} \cdots = I_{R_N} \quad 26.26$$

- The equivalent resistance is the sum of the individual resistances.

$$R_e = R_1 + R_2 + R_3 \cdots + R_N \quad 26.27$$

- The equivalent applied voltage is the sum of all the voltage sources, with the polarity considered.

$$V_e = \pm V_1 \pm V_2 \cdots \pm V_N \quad 26.28$$

- The sum of the voltage drops across all circuit elements is equal to the equivalent applied voltage (KVL).

$$V_e = IR_e \quad 26.29$$

18. PARALLEL CIRCUITS

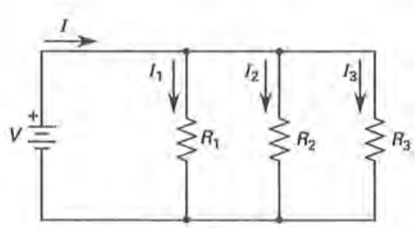


Figure 26.9 Parallel Circuit

A simple parallel circuit with only one active source, as shown in Fig. 26.9, has the following properties (generalized to any number of resistors).

- The voltage across all legs of the circuit is the same.

$$\begin{aligned} V &= V_1 = V_2 = V_3 \cdots = V_N \\ &= I_1R_1 = I_2R_2 = I_3R_3 \cdots = V_N \end{aligned} \quad 26.30$$

- The reciprocal equivalent resistance is the sum of the reciprocals of the individual resistances. The equivalent conductance is the sum of the individual conductances.

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \cdots + \frac{1}{R_N} \quad 26.31(a)$$

$$G_e = G_1 + G_2 + G_3 \cdots + G_N \quad 26.31(b)$$

- The total current is the sum of the currents in the individual legs of the circuit (KCL).

$$\begin{aligned} I &= I_1 + I_2 + I_3 \cdots + I_N \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \cdots + \frac{V}{R_N} \\ &= V(G_1 + G_2 + G_3 \cdots + G_N) \end{aligned} \quad 26.32$$

19. ANALYSIS OF COMPLICATED RESISTIVE NETWORKS

The following is a general method that can be used to determine the current flow and voltage drops within a complex resistive circuit.

step 1: If the circuit is three-dimensional, draw a two-dimensional representation.

step 2: Combine series voltage sources.

step 3: Combine parallel current sources.

step 4: Combine series resistances.

step 5: Combine parallel resistances.

step 6: Repeat steps 2–5 as needed to obtain the current and/or voltage at the desired point, junction, or branch of the circuit. (Do not simplify the circuit beyond what is required to determine the desired quantities.)

step 7: If applicable, utilize the delta-wye transformation of Sec. 26-20.

Circuit Theory

20. DELTA-WYE TRANSFORMATIONS

Electrical resistances arranged in the shape of the Greek letter delta or the English letter Y (wye) are known as *delta-wye configurations*. They are also called *pi-T configurations*. Two such circuits are shown in Fig. 26.10.

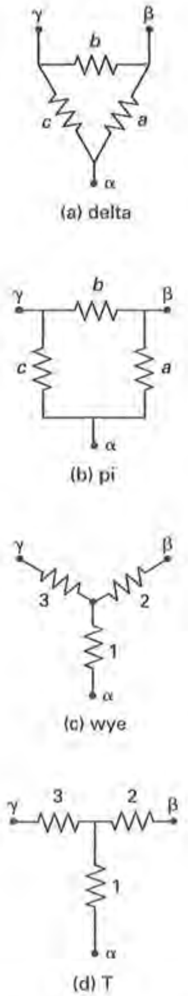


Figure 26.10 *Delta (Pi)-Wye (T) Configurations*

The equivalent resistances, which allow transformation between configurations, are

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3} \qquad 26.33$$

$$R_b = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} \qquad 26.34$$

$$R_c = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} \qquad 26.35$$

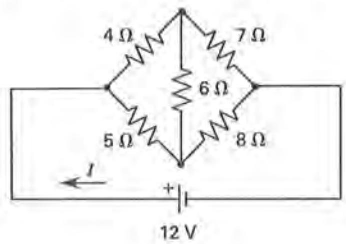
$$R_1 = \frac{R_aR_c}{R_a + R_b + R_c} \qquad 26.36$$

$$R_2 = \frac{R_aR_b}{R_a + R_b + R_c} \qquad 26.37$$

$$R_3 = \frac{R_bR_c}{R_a + R_b + R_c} \qquad 26.38$$

Example 26.4

Simplify the circuit and determine the total current.



Solution

Convert the 4 Ω-5 Ω-6 Ω delta connection to wye form.

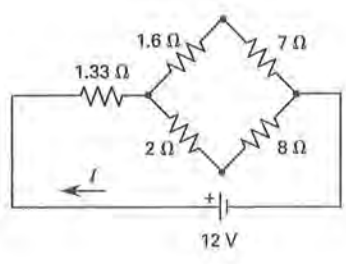
$$R_1 = \frac{R_aR_c}{R_a + R_b + R_c} = \frac{(5\ \Omega)(4\ \Omega)}{5\ \Omega + 6\ \Omega + 4\ \Omega}$$

$$= \frac{20}{15} = 1.33\ \Omega$$

$$R_2 = \frac{(5)(6)}{15\ \Omega} = 2\ \Omega$$

$$R_3 = \frac{(6)(4)}{15} = 1.6\ \Omega$$

The transformed circuit is



The total equivalent resistance is

$$R_e = 1.33 + \frac{1}{\frac{1}{1.6+7} + \frac{1}{2+8}} = 5.95\ \Omega$$

The current is

$$I = \frac{V}{R_e} = \frac{12\ \text{V}}{5.95\ \Omega} = 2.02\ \text{A}$$

21. SUBSTITUTION THEOREM

The *substitution theorem*, also known as the *compensation theorem*, states that any branch in a circuit can be replaced by a substitute branch, as long as the branch voltage and current remain the same, without affecting voltages and currents in any other portion of the circuit.

22. RECIPROCITY THEOREM

In any linear, time-independent circuit with independent current and voltage sources, the ratio of the current in a short circuit in one part of the network to the output of a voltage source in another part is constant—even when the positions of the voltage source and the short-circuit positions are interchanged. This principle of *reciprocity* is also applicable to the ratio of the current from a current source and the voltage across an open circuit. Reciprocity between a voltage source and short circuit is illustrated in Fig. 26.11.

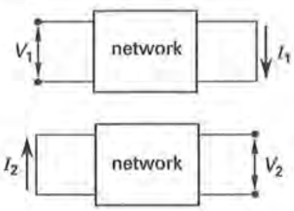


Figure 26.11 *Reciprocal Measurements*

Alternatively, if the network is comprised of linear resistors, the ratio of the applied voltage to the current measured at any point is constant—even when the positions of the source and meter are interchanged. The ratio in both cases is

$$R_{\text{transfer}} = \frac{V_1}{I_1} = \frac{V_2}{I_2} \qquad 26.39$$

The ratio is called the *transfer resistance*. The term *transfer* is used when a given response is determined at a point in a network other than where the driving force is applied. In this case, the voltage is the driving force on one side of the circuit and the current is measured on the other side.

23. SUPERPOSITION THEOREM

The principle of *superposition* is that the response (that is, the voltage across or current through) of a linear circuit element in a network with multiple independent sources is equal to the response obtained if each source is considered individually and the results are summed. The steps involved in determining the desired quantity are as follows.

step 1: Replace all sources except one with their internal resistances.[8] Replace ideal current sources with open circuits. Replace ideal voltage sources with short circuits.

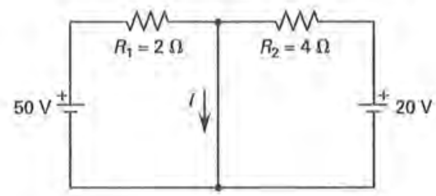
step 2: Compute the desired quantity, either voltage or current, for the element in question attributable to the single source.

step 3: Repeat steps 1 and 2 for each of the sources in turn.

step 4: Sum the calculated values obtained for the current or voltage obtained in step 2. The result is the actual value of the current or voltage in the element for the complete circuit.

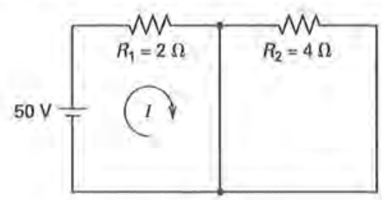
Example 26.5

Determine the current through the center leg.



Solution

First, work with the left (50 V) battery. Short out the right (20 V) battery.

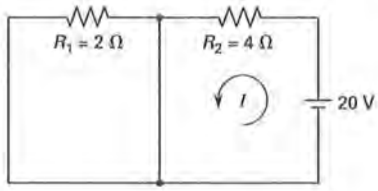


No current will flow through R_2. The equivalent resistance and current are

$$R_e = 2\ \Omega$$
$$I = \frac{V}{R_e} = \frac{50\ \text{V}}{2\ \Omega} = 25\ \text{A}$$

Next, work with the right (20 V) battery. Short out the left (50 V) battery.

[8] As in all the DC theorems presented in this chapter, this principle also applies to AC circuits or networks. In the case of AC circuits or networks, the term *impedance* would be applicable instead of *resistance*.



No current will flow through R_1. The equivalent resistance and current are

$$R_e = 4\ \Omega$$
$$I = \frac{V}{R_e} = \frac{20\ \text{V}}{4} = 5\ \text{A}$$

Considering both batteries, the total current flowing is 25 A + 5 A = 30 A.

24. THEVENIN'S THEOREM

Thevenin's theorem states that, insofar as the behavior of a linear circuit at its terminals is concerned, any such circuit can be replaced by a single voltage source, V_{Th}, in series with a single resistance, R_{Th}. The method for determining and utilizing the *Thevenin equivalent circuit*, with designations referring to Fig. 26.12, follows.

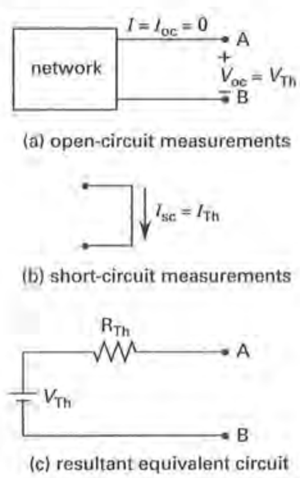


Figure 26.12 Thevenin Equivalent Circuit

step 1: Separate the network that is to be changed into a Thevenin equivalent circuit from its load at two terminals, say A and B.

step 2: Determine the open-circuit voltage, V_{oc}, at terminals A and B.

step 3: Short circuit terminals A and B and determine the current, I_{sc}.

step 4: Calculate the Thevenin equivalent voltage and resistance from the following equations.

$$V_{\text{Th}} = V_{\text{oc}} \qquad 26.40$$

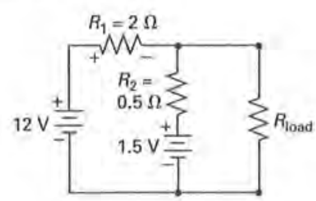
$$R_{\text{Th}} = \frac{V_{\text{oc}}}{I_{\text{sc}}} \qquad 26.41$$

step 5: Using the values calculated in Eqs. 26.40 and 26.41, replace the network with the Thevenin equivalent. Reconnect the load at terminals A and B. Determine the desired electrical parameters in the load.

Steps 3 and 4 can be altered by using the following shortcut. Determine the Thevenin equivalent resistance by looking into terminals A and B toward the network with all the power sources altered. Specifically, change independent voltage sources into short circuits and independent current sources into open circuits, then calculate the resistance of the altered network. The resulting resistance is the Thevenin equivalent resistance.

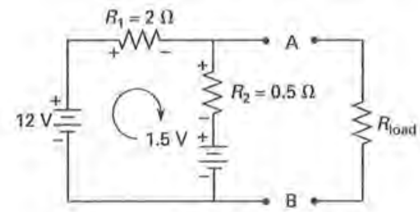
Example 26.6

Determine the Thevenin equivalent circuit that the load resistor, R_{load}, sees.



Solution

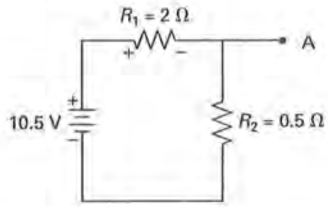
step 1: Separate the load resistor from the portion of the network to be changed.



step 2: Apply Kirchhoff's voltage law to find the open-circuit voltage. Starting at terminal A, the equation is

$$12\ \text{V} - V_{R_1} - V_{R_2} - 1.5\ \text{V} = 0$$
$$V_{R_1} + V_{R_2} = 10.5\ \text{V}$$

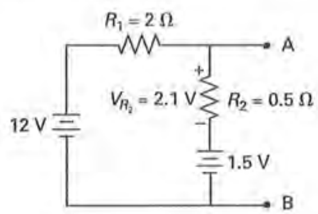
For clarification, the circuit is redrawn using the result calculated.



The voltage across R_2 is found using the voltage-divider concept.

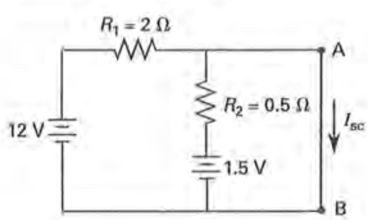
$$V_{R_2} = (10.5\ \text{V})\left(\frac{R_2}{R_1 + R_2}\right)$$
$$= (10.5\ \text{V})\left(\frac{0.5\ \Omega}{2\ \Omega + 0.5\ \Omega}\right) = 2.1\ \text{V}$$

The circuit branch from A to B can now be illustrated as follows.



The open-circuit voltage is the voltage across R_2 and the 1.5 V source, that is $V_{oc} = 3.6$ V. (This can be found by using KVL around the loop containing the terminals A and B.)

step 3: Short terminals A and B to determine the short-circuit current.



Using either Ohm's law or KVL around the outer loop, determine the short-circuit current contribution from the 12 V source.

$$I_{sc} = \frac{12\ \text{V}}{2\ \Omega} = 6\ \text{A}$$

Using either Ohm's law or KVL around the loop containing the 1.5 V source, R_2, and terminals A and B, determine the short-circuit current contribution from the 1.5 V source.

$$I_{sc} = \frac{1.5\ \text{V}}{0.5\ \Omega} = 3\ \text{A}$$

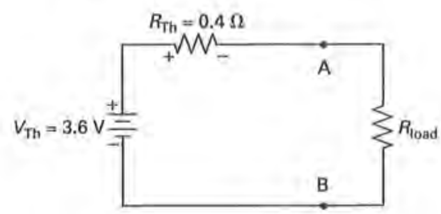
The total short-circuit current, using the principle of superposition, is 6 A+3 A = 9 A.

step 4: Using Eqs. 26.40 and 26.41, calculate the Thevenin equivalent voltage and resistance.

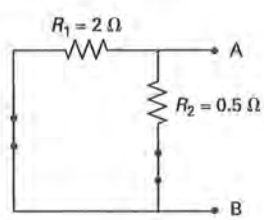
$$V_{Th} = V_{oc} = 3.6\ \text{V}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{3.6\ \text{V}}{9\ \text{A}} = 0.4\ \Omega$$

step 5: Replace the network with the Thevenin equivalent.



The method outlined is systematic. The shortcut for finding the Thevenin equivalent circuit mentioned earlier in this section would have simplified obtaining the solution. For example, short the voltage sources and determine the Thevenin equivalent resistance.



$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(2\ \Omega)(0.5\ \Omega)}{2\ \Omega + 0.5\ \Omega} = 0.4\ \Omega$$

The short-circuit current, found using superposition, is 9 A. The Thevenin equivalent voltage can then be found by combining Eqs. 26.29 and 26.41.

$$V_{Th} = I_{Th} R_{Th}$$

Circuit Theory

Substituting the short-circuit current for I_{sc} and the calculated value for R_{Th} results in an identical answer, as required by Thevenin's theorem.

25. NORTON'S THEOREM

Norton's theorem states that, insofar as the behavior of a linear circuit at its terminals is concerned, any such circuit can be replaced by a single current source, I_N, in parallel with a single resistance, R_N. The method for determining and utilizing the *Norton equivalent circuit*, with designations referring to Fig. 26.13, follows.

step 1: Separate the network that is to be changed into a Norton equivalent circuit from its load at two terminals, say A and B.

step 2: Determine the open-circuit voltage, V_{oc}, at terminals A and B.

step 3: Short circuit terminals A and B and determine the current, I_{sc}.

step 4: Calculate the Norton equivalent current and resistance from the following equations.

$$I_N = I_{sc} \quad 26.42$$

$$R_N = \frac{V_{oc}}{I_{sc}} \quad 26.43$$

step 5: Using the values calculated in Eqs. 26.42 and 26.43, replace the network with the Norton equivalent. Reconnect the load at terminals A and B. Determine the desired electrical parameters in the load.

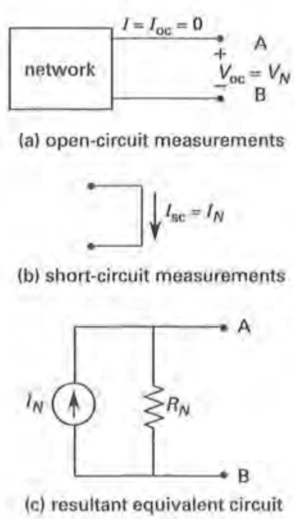


Figure 26.13 *Norton Equivalent Circuit*

Steps 3 and 4 can be altered by using the following shortcut. Determine the Norton equivalent resistance by looking into terminals A and B toward the network with all the power sources altered. Specifically, change independent voltage sources into short circuits and independent current sources into open circuits, then calculate the resistance of the altered network. The resulting resistance is the Norton equivalent resistance.

The Norton equivalent resistance equals the Thevenin equivalent resistance for identical networks.

26. LOOP-CURRENT METHOD

The *loop-current method* is a systematic network-analysis procedure that uses currents as the unknowns. It is also called *mesh analysis* or the *Maxwell loop-current method*. The method uses Kirchhoff's voltage law and is performed on planar networks. It requires $n-1$ simultaneous equations for an n-loop system. The method's steps are as follows.

step 1: Select $n-1$ loops, that is, one loop less than the total number of possible loops.

step 2: Assume current directions for the selected loops. (While this is arbitrary, clockwise directions will always be chosen in this text for consistency. Any incorrectly chosen current direction will result in a negative result when the simultaneous equations are solved in step 4.) Show the direction of the current with an arrow.

step 3: Write Kirchhoff's voltage law for each of the selected loops. Assign polarities based on the direction of the loop current. The voltage of a source is positive when the current flows out of the positive terminal, that is, from the negative terminal to the positive terminal inside the source. The selected direction of the loop current always results in a voltage drop in the resistors of the loop (see Fig. 26.6(c)). Where two loop currents flow through an element, they are summed to determine the voltage drop in that element, using the direction of the current in the loop for which the equation is being written as the positive (i.e., correct) direction. (Any incorrect direction for the loop current will be indicated by a negative sign in the solution in step 4.)

step 4: Solve the $n-1$ equations (from step 3) for the unknown currents.

step 5: If required, determine the actual current in an element by summing the loop currents flowing through the element. (Sum the absolute values to obtain the correct magnitude. The correct direction is given by the loop current with the positive value.)

27. NODE-VOLTAGE METHOD

The *node-voltage method* is a systematic network-analysis procedure that uses voltages as the unknowns. The method uses Kirchhoff's current law. It requires $n-1$ equations for an n-principal node system (equations are not necessary at simple nodes, that is, nodes connecting only two circuit elements). The method's steps are as follows.

step 1: Simplify the circuit, if possible, by combining resistors in series or parallel or by combining current sources in parallel. Identify all nodes. (The minimum total number of equations required will be $n-1$, where n represents the number of principal nodes.)

step 2: Choose one node as the reference node, that is, the node that will be assumed to have ground potential (0 V). (To minimize the number of terms in the equations, pick the node with the largest number of circuit elements as the reference node.)

step 3: Write Kirchhoff's current law for each principal node except the reference node, which is assumed to have a zero potential.

step 4: Solve the $n-1$ equations (from step 3) to determine the unknown voltages.

step 5: If required, use the calculated node voltages to determine any branch current desired.

28. DETERMINATION OF METHOD

When analyzing electrical networks, the method used depends on the circuit elements and their configurations. The loop-current method using Kirchhoff's voltage law is used in circuits without current sources. The node-voltage method using Kirchhoff's current law is used in circuits without voltage sources. When both types of sources are present, one of the following two methods may be used.

method 1: Use each of Kirchhoff's laws, assigning voltages and currents as needed, and substitute any known quantity into the equations as written.

method 2: Use source transformation or source shifting to change the appearance of the circuit so that it contains only the desired sources, that is, voltage sources when using KVL and current sources when using KCL. (Source shifting is manipulating the circuit so that each voltage source has a resistor in series and each current source has a resistor in parallel.)

Use the method that results in the least number of equations. The loop-current method produces $n-1$ equations, where n is the total number of loops. The node-voltage method produces $n-1$ equations, where n is the number of principal nodes. Count the number of loops and the number of principal nodes prior to writing the equations; whichever is least determines the method used.

Additional methods exist, some of which require less work. The advantage of the loop-current and node-voltage methods is that they are systematic and thus guarantee a solution.

29. PRACTICAL APPLICATION: BATTERIES

In general, a battery is defined as a direct-current voltage source made up of one or more units that convert chemical, thermal, nuclear, or solar energy into electrical energy. The most widely used battery type is one that converts the chemical energy contained in its active materials directly into electrical energy by means of an oxidation-reduction reaction.[9] A battery consists of two dissimilar metals, an anode and a cathode, immersed in an electrolyte. The anode is the component that gives up electrons, that is, it is oxidized during the reaction. The anode is labeled as the positive terminal since by definition this is the terminal through which current enters the battery. (The current is conventional current flow. Therefore, positive "charges" enter the anode and it is thus the source of the electrons to the external circuit.) The cathode is reduced during the reaction. The transfer of charge is completed within the electrolyte by the flow of ions. This is shown for a single cell in Fig. 26.14.

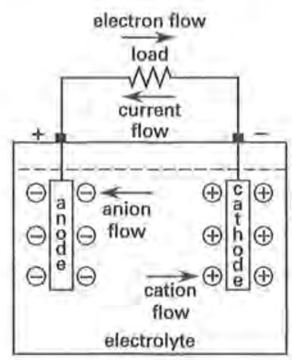


Figure 26.14 *Electrochemical Battery*

The battery terminals have no absolute voltage value. They have a value relative only to each other, and thus the terminals and the battery are said to *float*. Usually, one of the terminals is assigned as the reference and given the value of 0 V. This terminal is then the *reference* or *datum* and is said to be *grounded*. If the battery is allowed to float and the circuit is grounded elsewhere, all voltages are assumed to be measured with respect to this ground. The earth is generally regarded as being

[9]In *nonelectrochemical reactions*, the transfer of electrons takes place directly and only heat is involved.

at 0 V and any circuit tied to earth by an electrical wire is grounded. Various symbols for grounds are shown in Fig. 26.15.

Figure 26.15 *Ground Symbols*

The voltage of a battery is determined by the number of cells used. The voltage of the cell is determined by the materials used, as this determines the half-cell oxidation potentials. The capacity of the battery is determined by the amount of materials used and is measured in ampere-hours (A·h). One gram-equivalent weight of material supplies approximately 96,480 C or 26.805 A·h of electric charge.

A *primary battery* is one that uses an electrochemical reaction that is not efficiently reversible. An example is the common flashlight battery. This type of battery is also called a *dry cell*, as the electrolyte is a moist paste instead of a liquid solution. A *secondary battery* is rechargeable and has a much higher energy density. An example is a lead-acid storage battery used in automobiles. Such batteries are treated as ideal voltage sources with a series resistance representing internal resistance. Using the specified voltage and resistance, any network containing a battery is analyzed using the techniques described in this chapter.

27 AC Circuit Fundamentals

Nomenclature

B	susceptance	S, Ω^{-1}, or mho
C	capacitance	F
f	function	–
f	frequency	Hz, s^{-1}, or cycles/s
G	conductance	S, Ω^{-1}, or mho
i	instantaneous current	A
I	effective or DC current	A
Im	imaginary	–
L	inductance	H
p	instantaneous power	W
pf	power factor	–
P	power	W
Q	reactive power	var (volt-amps reactive or VAR)
R	resistance	Ω
Re	real portion[1]	–
S	apparent power	voltampere (volt-amps or VA)
t	time	s
T	period	s
v, V	voltage	V
X	reactance	Ω
Y	admittance	S, Ω^{-1}, or mho
Z	complex number	–
Z	impedance	Ω

Symbols

θ	phase angle	rad
ψ	phase difference angle	rad
ϕ	impedance angle	rad
φ	current angle $(\theta \pm \phi)$	rad
Ψ	power factor angle	rad
ω	angular frequency	rad/s

Subscripts

0	initial
ave	average
C	capacitor
e	equivalent
eff	effective
i	imaginary
I	current
L	inductor
m	maximum
p	peak
r	real
rms	root-mean-square
R	resistor
s	source
thr	threshold
V	voltage
Z	impedance

1. FUNDAMENTALS

Alternating waveforms have currents and voltages that vary with time in a symmetrical manner. Possible waveforms include square, sawtooth, and triangular along with many variations on these themes. However, for the most part, the variations are sinusoidal in time for many applications in electrical engineering. In this book, unless otherwise specified, currents and voltages referred to are sinusoidal.[2] When sinusoidal, the waveform is nearly always referred to as AC, that is, *alternating current*, indicating that the current is produced by the application of a sinusoidal voltage. This means that the flow of electrons changes directions, unlike DC circuits, where the flow of electrons is unidirectional (though the

[1]Do not confuse this symbology, Re, for that of the Reynolds number.

[2]Nearly all periodic functions can be represented as sinusoidal functions. The superposition theorem allows the effects of individual sinusoids to be summed to obtain an overall effect, which simplifies the mathematics necessary to obtain a result.

Circuit Theory

magnitude can change in time). A circuit is said to be in a *steady-state* condition if the current and voltage time variation is purely constant (DC) or purely sinusoidal (AC).[3] In this book, unless otherwise specified, the circuits referred to are in a steady-state condition. Importantly, all the methods, basic definitions, and equations presented in Ch. 26 involving DC circuits are applicable to AC circuits. AC electrical parameters have both magnitudes and angles. Nevertheless, following common practice, phasor notation will be used only in cases where not doing so would cause confusion.

2. VOLTAGE

Sinusoidal variables can be expressed in terms of sines or cosines without any loss of generality.[4] A sine waveform is often the standard. If this is the case, Eq. 27.1 gives the value of the instantaneous voltage as a function of time.

$$v(t) = V_m \sin(\omega t + \theta) \qquad 27.1$$

The *maximum value* of the sinusoid is given the symbol V_m and is also known as the *amplitude*. If $v(t)$ is not zero at $t = 0$, the sinusoid is *shifted* and a *phase angle*, θ, must be used, as shown in Fig. 27.1.[5] Also shown in Fig. 27.1 is the *period*, T, which is the time that elapses in one cycle of the sinusoid. The *cycle* is the smallest portion of the sinusoid that repeats.

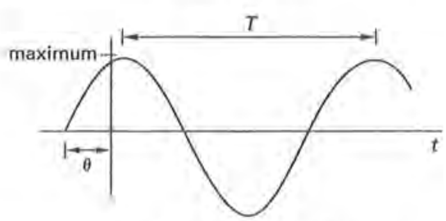


Figure 27.1 *Sinusoidal Waveform with Phase Angle*

Since the horizontal axis of the voltage in Fig. 27.1 corresponds to time, not distance, the waveform does not have a wavelength. The frequency, f, of the sinusoid is the reciprocal of the period in hertz (Hz). The angular frequency, ω, in rad/s can also be used.

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad 27.2$$

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad 27.3$$

[3]Steady-state AC may have a *DC offset*.

[4]The point at which time begins, that is, where $t = 0$, is of no consequence in steady-state AC circuit problems, as the signal repeats itself every cycle. Therefore, it makes no difference whether a sine or cosine waveform is used (though, if one exists, care must be taken to keep the phase angle correct).

[5]The term *phase* is not the same as *phase difference*, which is the difference between corresponding points on two sinusoids of the same frequency.

An AC voltage waveform without a phase angle is plotted as a function of differing variables in Fig. 27.2.

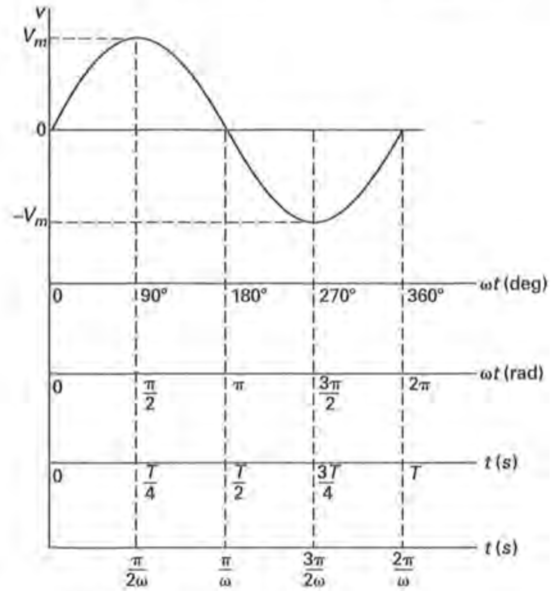


Figure 27.2 *Sine Wave Plots*

Exponentials, $e^{j\theta}$ and $e^{-j\theta}$, can be combined to produce $\sin\theta$ and $\cos\theta$ terms. As a result, sinusoids can be represented in the following equivalent forms.

- trigonometric: $V_m \sin(\omega t + \theta)$
- exponential: $V_m e^{j\theta}$
- polar or phasor: $V_m \angle\theta$
- rectangular: $V_r + jV_i$

The use of complex exponentials and phasor analysis allows sinusoidal functions to be more easily manipulated mathematically, especially when dealing with derivatives. Exponents are manipulated algebraically, and the resulting sinusoid is then recovered using Euler's relation. This avoids complicated trigonometric mathematics. The angles used in exponentials and with angular frequency, ω, must be in radians. From Euler's relation,

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad 27.4$$

$$j = \sqrt{-1} \qquad 27.5$$

Therefore, to regain the sinusoid from the exponential, use Euler's equation, keeping in mind two factors. First, either the real or the imaginary part contains the sinusoid desired, not both.[6] Second, the exponential should

[6]The cosine function could be used as the reference sinusoid, and is in some texts. In this text, unless otherwise specified, assume the sine function is the desired form. The desired form is determined by the representation of the voltage source waveform as either a cosine or a sine function (see Eq. 27.1). Either way, only the phase angle changes.

not be factored out of any equation until all the derivatives, or integrals, have been taken.

3. CURRENT

Current is the net transfer of electric charge per unit time. When a circuit's driving force, the voltage, is sinusoidal, the resulting current is also (though it may differ by an amount called the *phase angle difference*). The equations and representations in Sec. 27-2, as well as any other representations of sinusoidal waveforms provided in this chapter, apply to current as well as to voltage.

4. IMPEDANCE

Electrical impedance, also known as *complex impedance*, is the total opposition a circuit presents to alternating current. It is equal to the ratio of the complex voltage to the complex current. Impedance, then, is a ratio of phasor quantities and is not itself a function of time. Relating voltage and current in this manner is analogous to Ohm's law, which for AC analysis is referred to as *extended Ohm's law*. Impedance is given the symbol $\mathbf{Z}$ and is measured in ohms. The three passive circuit elements (resistors, capacitors, and inductors), when used in an AC circuit, are assigned an angle, θ, known as the *impedance angle*. This angle corresponds to the phase difference angle produced when a sinusoidal voltage is applied across that element alone.

Impedance is a complex quantity with a magnitude and associated angle. It can be written in *phasor form*—also known as *polar form*—for example, $\mathbf{Z}\angle\theta$, or in *rectangular form* as the complex sum of its resistive (R) and reactive (X) components.

$$\mathbf{Z} \equiv R + jX \qquad 27.6$$

$$R = Z\cos\phi \quad \begin{bmatrix}\text{resistive or}\\ \text{real part}\end{bmatrix} \qquad 27.7$$

$$X = Z\sin\phi \quad \begin{bmatrix}\text{reactive or}\\ \text{imaginary part}\end{bmatrix} \qquad 27.8$$

The resistive and reactive components can be combined to form an *impedance triangle*. Such a triangle is shown in Fig. 27.3.

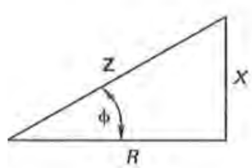


Figure 27.3 *Impedance Triangle*

The characteristics of the passive elements in AC circuits, including impedance, are given in Table 27.1.

Table 27.1 *Characteristics of Resistors, Capacitors, and Inductors*

	resistor	capacitor	inductor
value	R (Ω)	C (F)	L (H)
reactance, X	0	$\frac{-1}{\omega C}$	ωL
rectangular impedance, $\mathbf{Z}$	$R + j0$	$0 - \frac{j}{\omega C}$	$0 + j\omega L$
phasor impedance, $\mathbf{Z}$	$R\angle 0°$	$\frac{1}{\omega C}\angle -90°$	$\omega L\angle 90°$
phase	in-phase	leading	lagging
rectangular admittance, $\mathbf{Y}$	$\frac{1}{R} + j0$	$0 + j\omega C$	$0 - \frac{j}{\omega L}$
phasor admittance, $\mathbf{Y}$	$\frac{1}{R}\angle 0°$	$\omega C\angle 90°$	$\frac{1}{\omega L}\angle -90°$

5. ADMITTANCE

The reciprocal of impedance is called *admittance*, $\mathbf{Y}$. Admittance is useful in analyzing parallel circuits, as admittances can be added directly. The defining equation is

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{Z}\angle -\phi \qquad 27.9$$

The reciprocal of the resistive portion of an impedance is known as *conductance*, G. The reciprocal of the reactive part is termed the *susceptance*, B.

$$G = \frac{1}{R} \qquad 27.10$$

$$B = \frac{1}{X} \qquad 27.11$$

Using these definitions and multiplying by a complex conjugate, admittance can be written in terms of resistance and reactance. Using the same method, impedance can be written in terms of conductance and susceptance. Equations 27.12 and 27.13 show this and can be used for conversion between admittance and impedance and vice versa.

$$\mathbf{Y} = G + jB = \frac{R}{R^2 + X^2} - j\left(\frac{X}{R^2 + X^2}\right) \qquad 27.12$$

$$\mathbf{Z} = R + jX = \frac{G}{G^2 + B^2} - j\left(\frac{B}{G^2 + B^2}\right) \qquad 27.13$$

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6. VOLTAGE SOURCES

The energy for AC voltage sources comes primarily from electromagnetic induction. The concepts of ideal and real sources, as well as regulation, apply to AC sources (see Sec. 26-8). Independent sources deliver voltage and current at their rated values regardless of circuit parameters. Dependent sources, often termed *controlled sources*, deliver voltage and current at levels determined by a voltage or current somewhere else in the circuit. These types of sources occur in electronic circuitry and are also used to model electronic elements, such as transistors. The symbols used for AC voltage sources are shown in Fig. 27.4.

Figure 27.4 *AC Voltage Sources Symbology*

7. AVERAGE VALUE

In purely mathematical terms, the average value of a periodic waveform is the first term of a Fourier series representing the function, that is, it is the zero frequency or DC value. If a function, $f(t)$, repeats itself in a time period T, then the average value of the function is given by Eq. 27.14. In Eq. 27.14, t_1 is any convenient time for evaluating the integral, that is, the time that simplifies the integration. The integral itself is computed over the period.

$$f_{\text{ave}} = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)dt \qquad 27.14$$

Integration can be interpreted as the area under the curve of the function. Equation 27.14 divides the net area of the waveform by the period T. This concept is illustrated in Fig. 27.5 and stated in mathematical terms by Eq. 27.15.

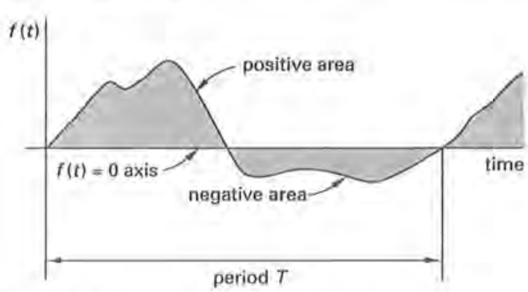


Figure 27.5 *Average Value Areas Defined*

$$f_{\text{ave}} = \frac{\text{positive area} - \text{negative area}}{T} \qquad 27.15$$

For the function shown in Fig. 27.5, the area above the axis is called the positive area and the area below is called the negative area. The average value is the net area remaining after the negative area is subtracted from the positive area and the result is divided by the period.

For any periodic voltage, Eq. 27.16 calculates the average value.

$$V_{\text{ave}} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)d\theta = \frac{1}{T}\int_0^{T} v(t)dt \qquad 27.16$$

Any waveform that is symmetrical with respect to the horizontal axis will result in a value of zero for Eq. 27.16. While mathematically correct, the electrical effects of such a voltage occur on both the positive and negative half-cycles. Therefore, the average is instead taken over only half a cycle. This is equivalent to determining the average of a *rectified waveform*, that is, the absolute value of the waveform. The average voltage for a rectified sinusoid is

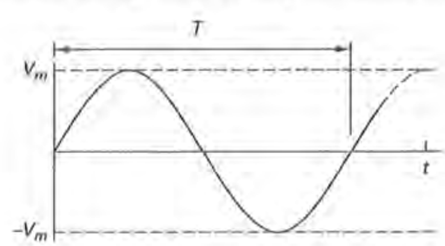
$$V_{\text{ave}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)d\theta$$

$$= \frac{2V_m}{\pi} \quad \text{[rectified sinusoid]} \qquad 27.17$$

A DC current equal in value to the average value of a rectified AC current produces the same electrolytic effects, such as capacitor charging, plating operations, and ion formation. Nevertheless, not all AC effects can be accounted for using the average value. For instance, in a typical DC meter, the average DC current determines the response of the needle. An AC current sinusoid of equal average magnitude will not result in the same effect since the torque on each half-cycle is in opposite directions, resulting in a net zero effect. Unless the AC signal is rectified, the reading will be zero.

Example 27.1

What is the average value of the pure sinusoid shown?



Solution

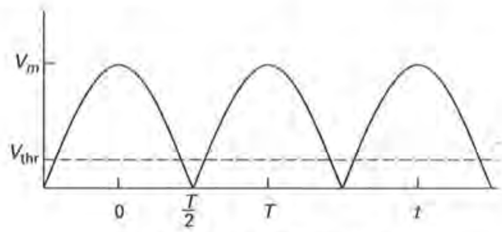
The sinusoid shown is a sine wave. Substituting into Eq. 27.14 with $t_1 = 0$ gives

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{T}\int_{t_1}^{t_1+T} f(t)dt = \frac{1}{T}\int_0^T V_m \sin t dt \\ &= \left(\frac{V_m}{T}\right)\left(\cos t\,\big|_0^T\right) = \left(\frac{V_m}{T}\right)\left(\cos T - \cos(0)\right) \\ &= \left(\frac{V_m}{T}\right)(1-1) = 0 \end{aligned}$$

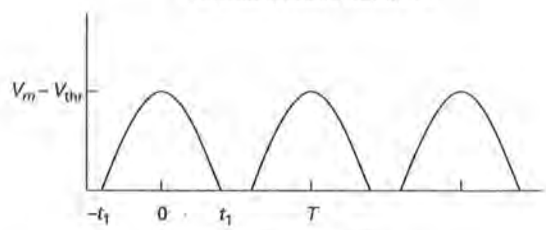
This result is as expected, since a pure sinusoid has equal positive and negative areas.

Example 27.2

Diodes are used to rectify AC waveforms. Real diodes will not pass current until a threshold voltage is reached. As a result, the output is the difference between the sinusoid and the threshold value. This is illustrated in the following figures.



(a) ideal rectified waveform



(b) net voltage

What is the expression for the average voltage on the output of a real rectifier diode with a sinusoidal voltage input?

Solution

Because of the symmetry, the average value can be found using half the average determined from Eq. 27.16 by integrating from zero to $T/2$, simplifying the calculation. The waveform is represented by

$$v(t) = \begin{cases} V_m \cos\left(\left(\frac{\pi}{T}\right)t\right) - V_{\text{thr}} & [\text{for } 0 < t < t_1] \\ 0 & [\text{for } t_1 < t < {}^{T}\!/_{2}] \end{cases}$$

At $t = t_1$, the following condition exists.

$$V_m \cos\left(\left(\frac{\pi}{T}\right)t_1\right) = V_{\text{thr}}$$

The equation for one-half the average is

$$\begin{aligned} \tfrac{1}{2}v_{\text{ave}} &= \frac{1}{T}\int_0^{t_1}\left(V_m\cos\left(\left(\frac{\pi}{T}\right)t\right) - V_{\text{thr}}\right)dt \\ &= \left(\frac{1}{T}\right)\left(\frac{T}{\pi}\right)V_m \sin\left(\left(\frac{\pi}{T}\right)t_1\right) - V_{\text{thr}}\left(\frac{t_1}{T}\right) \end{aligned}$$

Using the equation for the condition at $t = t_1$ and the trigonometric identity $\sin^2 x + \cos^2 x = 1$ results in the following equations.

$$\sin\left(\left(\frac{\pi}{T}\right)t_1\right) = \sqrt{1 - \left(\frac{V_{\text{thr}}}{V_m}\right)^2}$$

$$\frac{t_1}{T} = \frac{1}{\pi}\arccos\left(\frac{V_{\text{thr}}}{V_m}\right)$$

Substituting into the equation for one-half the average and rearranging gives the following final result.

$$\begin{aligned} v_{\text{ave}} &= \left(\frac{2}{\pi}\right)V_m\sqrt{1 - \left(\frac{V_{\text{thr}}}{V_m}\right)^2} \\ &\quad - \left(\frac{2}{\pi}\right)V_{\text{thr}}\arccos\left(\frac{V_{\text{thr}}}{V_m}\right) \end{aligned}$$

The arccos must be expressed in radians. The conclusion drawn is that real diode average values are more complex than values for ideal diode full-wave rectification. However, if ideal diodes are assumed, that is, if the threshold voltage is considered negligible, the following errors are generated.

- 1% error for $V_{\text{thr}}/V_m = 0.0064$
- 2% error for $V_{\text{thr}}/V_m = 0.0128$
- 5% error for $V_{\text{thr}}/V_m = 0.0321$
- 10% error for $V_{\text{thr}}/V_m = 0.0650$

For most practical applications, the ideal diode assumption results in an error of less than 10%.

8. ROOT-MEAN-SQUARE VALUE

In purely mathematical terms, the *effective value* (also known as the *root-mean-square* (rms) *value*) of a periodic waveform represented by a function, $f(t)$, which repeats itself in a time period is given by Eq. 27.18. In Eq. 27.18, t_1 is any convenient time for evaluating the integral, that is, the time that simplifies the integration. The integral itself is computed over the period.

$$f_{\text{rms}}^2 = \frac{1}{T}\int_{t_1}^{t_1+T} f^2(t)dt \qquad 27.18$$

For any periodic voltage, Eq. 27.19 calculates the effective or rms value.

$$V = V_{\text{eff}} = V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)d\theta}$$

$$= \sqrt{\frac{1}{T}\int_0^{T} v^2(t)dt} \qquad \textit{27.19}$$

Normally when a voltage or current variable, V or I, is left unsubscripted, it represents the effective (rms) value.[7] Rectification of the waveform is not necessary to calculate the effective value. (The squaring of the waveform ensures the net result of integration is something other than zero for a sinusoid.) For a sinusoidal waveform, $V = V_m/\sqrt{2} \approx 0.707V_m$. A DC current of magnitude I produces the same heating effect as an AC current of magnitude I_{eff}.

Table 27.2 gives the characteristics of various commonly encountered alternating waveforms. In this table, two additional terms are introduced. The *form factor* is given by

$$\text{FF} = \frac{V_{\text{eff}}}{V_{\text{ave}}} \qquad \textit{27.20}$$

The *crest factor*, CF, also known as the *peak factor* or *amplitude factor*, is

$$\text{CF} = \frac{V_m}{V_{\text{eff}}} \qquad \textit{27.21}$$

Nearly all periodic functions can be represented by a Fourier series. A Fourier series can be written as

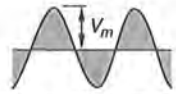
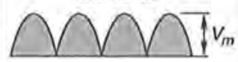
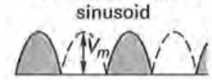
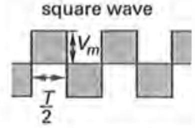
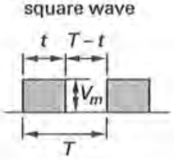
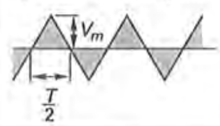
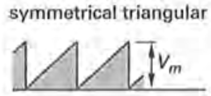
$$f(t) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\left(\frac{2\pi n}{T}\right)t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\left(\frac{2\pi n}{T}\right)t\right) \qquad \textit{27.22}$$

The average value is the first term, as mentioned in Sec. 27-7. The rms value is

$$f_{\text{rms}} = \sqrt{\left(\tfrac{1}{2}a_0\right)^2 + \tfrac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)} \qquad \textit{27.23}$$

[7]The value of the standard voltage in the United States, reported as 115–120 V, is an effective value.

Table 27.2 *Characteristics of Alternating Waveforms*

waveform	$\frac{V_{\text{ave}}}{V_m}$	$\frac{V_{\text{rms}}}{V_m}$	FF	CF
sinusoid 	0	$\frac{1}{\sqrt{2}}$	–	$\sqrt{2}$
full-wave rectified sinusoid 	$\frac{2}{\pi}$	$\frac{1}{\sqrt{2}}$	$\frac{\pi}{2\sqrt{2}}$	$\sqrt{2}$
half-wave rectified sinusoid 	$\frac{1}{\pi}$	$\frac{1}{2}$	$\frac{\pi}{2}$	2
symmetrical square wave 	0	1	–	1
unsymmetrical square wave 	$\frac{t}{T}$	$\sqrt{\frac{t}{T}}$	$\sqrt{\frac{T}{t}}$	$\sqrt{\frac{T}{t}}$
sawtooth and symmetrical triangular 	0	$\frac{1}{\sqrt{3}}$	–	$\sqrt{3}$
sawtooth and symmetrical triangular 	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$

Example 27.3

A peak sinusoidal voltage, V_p, of 170 V is connected across a 240 Ω resistor in a light bulb. What is the power dissipated by the bulb?

Solution

From Table 27.2, the effective voltage is

$$V = \frac{V_p}{\sqrt{2}} = \frac{170\ \text{V}}{\sqrt{2}} = 120.21\ \text{V}$$

The power dissipated is

$$P = \frac{V^2}{R} = \frac{(120.21\ \text{V})^2}{240\ \Omega} = 60.21\ \text{W}$$

Example 27.4

What is the rms value of a constant 3 V signal?

Solution

Using Eq. 27.18, the rms value is

$$v_{\text{rms}}^2 = \frac{1}{T}\int_0^T (3)^2 dt = \left(\frac{1}{T}\right)(9)(T-0)$$

$$v_{\text{rms}} = \sqrt{9} = 3\ \text{V}$$

9. PHASE ANGLES

AC circuit elements inductors and capacitors have the ability to store energy in magnetic and electric fields, respectively. Consequently, the voltage and current waveforms, while the same shape, differ by an amount called the *phase angle difference*, ϕ. Ordinarily, the voltage and current sinusoids do not peak at the same time. In a *leading circuit*, the phase angle difference is positive and the current peaks before the voltage. A leading circuit is termed a *capacitive circuit*. In a *lagging circuit*, the phase angle difference is negative and the current peaks after the voltage. A lagging circuit is termed an *inductive circuit*. These cases are represented mathematically as

$$v(t) = V_m \sin(\omega t + \theta) \quad \text{[reference]} \qquad 27.24$$

$$i(t) = I_m \sin(\omega t + \theta + \phi) \quad \text{[leading]} \qquad 27.25$$

$$i(t) = I_m \sin(\omega t + \theta - \phi) \quad \text{[lagging]} \qquad 27.26$$

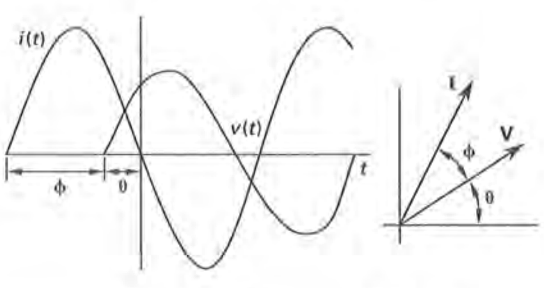


Figure 27.6 Leading Phase Angle Difference

10. SINUSOID

The most important waveform in electrical engineering is the sine function, or a waveform that is sinusoidal in value with respect to time. A sinusoidal waveform has an amplitude, also known as a magnitude or peak value, that remains constant, such as V_m. The waveform, however, repeats or goes through cycles. From Eq. 27.1, the sinusoid can be characterized by three quantities: magnitude, frequency, and phase angle.[8] The major properties of the sinusoid are covered in Sec. 27-2.

A circuit processes or changes waveforms. This is called *signal processing* or *waveform processing*. *Signal analysis* is the determination of these waveforms. The sinusoid can be represented as a phasor and used to analyze electrical circuits only if the circuit is in a steady-state condition. The terms "AC" and "DC" imply steady-state conditions. For AC circuits, "sinusoidal steady-state" indicates that all voltages and currents within the circuit are sinusoids of the same frequency as the excitation, that is, the driving voltage.

Example 27.5

During an experiment, a reference voltage of $v(t) = 170 \sin \omega t$ is used. A measurement of a second voltage, $v_2(t)$ occurs. The second voltage reaches its peak 2.5 ms before the reference voltage and has the same peak value 20 ms later. The peak value is 1.8 times the reference peak. What is the expression for $v_2(t)$?

Solution

Any sinusoidal voltage can be represented as in Eq. 27.1.

$$v(t) = V_m \sin(\omega t + \theta)$$

The peak value (equivalent to V_m) is 1.8 times the reference, or $1.8 \times 170\ \text{V} = 306\ \text{V}$. Calculate the angular frequency using Eq. 27.3 and the given period of 20 ms.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{20\ \text{ms}} = 314\ \text{rad/s}$$

The phase angle is determined by changing the time, 2.5 ms, into its corresponding angle, θ, as follows.

$$\theta = \omega t = \left(314\ \frac{\text{rad}}{\text{s}}\right)(2.5 \times 10^{-3}\ \text{s}) = 0.79\ \text{rad}$$

Converting 0.79 rad gives a result of 45°. Substituting the calculated values gives the result.

$$v_2(t) = V_m \sin(\omega t + \theta) = 306 \sin(314t + 45^\circ)$$

Note that the result mixes the use of radians (314) and degrees (45). This is often done for clarity. Ensure one or the other is converted prior to calculations. It is often best to deal with all angles in radians.

[8]Note that a phase angle of ±90° changes the sine function into a cosine function. Also, frequency refers in this case to the angular frequency, ω, in radians per second. The term frequency also refers to the term f, measured in cycles per second or hertz (Hz).

Circuit Theory

11. PHASORS

The addition of voltages and currents of the same frequency is simplified mathematically by treating them as phasors.[9] Using a method called the *Steinmetz algorithm*, the following two quantities, introduced in Sec. 27-2, are considered analogs.

$$v(t) = V_m \sin(\omega t + \theta) \text{ and } V = V_m \angle \theta \qquad 27.27$$

Both forms show the magnitude and the phase, but the *phasor*, $V_m\angle\theta$, does not show frequency. In the phasor form, the frequency is implied. The phasor form of Eq. 27.27 is shown in Fig. 27.7.

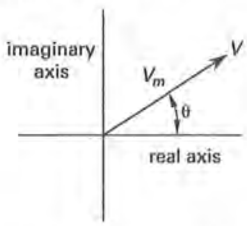


Figure 27.7 *Phasor of Magnitude V_m and Angle θ*

The phasor is actually a point, but is represented by an arrow of magnitude V_m at an angle θ with respect to a reference, normally taken to be $\theta = 0$. A *reference phasor* is one of known value, usually the driving voltage of a circuit (i.e., the voltage phasor). The reference phasor would be shown in the position of the real axis with θ equal to zero. Phasors are summed using phasor addition, commonly referred to as *vector addition*.[10] The Steinmetz algorithm is illustrated in Fig. 27.8.

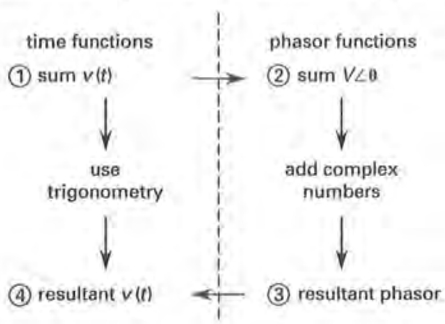


Figure 27.8 *Steinmetz Algorithm Steps*

In electric circuits with sinusoidal waveforms, an alternate phasor representation is often used. The alternate representation is called the *effective value phasor notation*. In this notation, the rms values of the voltage and current are used instead of the peak values. The angles are also given in degrees. The phasor notation of Eq. 27.27 is then modified to

$$V = V_{\text{rms}}\angle\theta = \frac{V_m}{\sqrt{2}}\angle\theta \qquad 27.28$$

Ensure that angles substituted into exponential forms are in radians or errors in mathematical calculations will result.

12. COMPLEX REPRESENTATION

Phasors are plotted in the complex plane. Figure 27.9 shows the relationships among the complex quantities introduced.

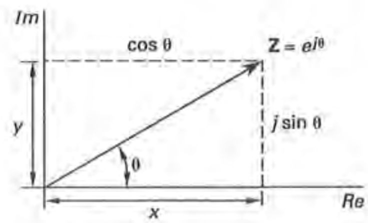


Figure 27.9 *Complex Quantities*

In Fig. 27.9, **Z** represents a complex number, not the impedance. Equation 27.1 is repeated here for convenience.

$$v(t) = V_m \sin(\omega t + \theta) \qquad 27.29$$

Using Euler's relation, Eq. 27.4, the voltage $v(t)$ can be represented as

$$v(t) = V_m e^{j(\omega t + \theta)} \qquad 27.30$$

Consequently, $v(t)$ is the imaginary part of Eq. 27.30. If the cosine function is used, $v(t)$ is the real part of Eq. 27.30. The fixed portion of Eq. 27.30 ($e^{j\theta}$) can be separated from the time-variable portion ($e^{j\omega t}$) of the function. This is shown in Fig. 27.10(a) and (b). The magnitude of $e^{j\omega t}$ remains equal to one, but the angle increases (rotates counterclockwise) linearly with time. All functions with an $e^{j\omega t}$ term are assumed to rotate counterclockwise with an angular velocity of ω in the complex plane. The voltage can thus be represented as in Fig. 27.10(c) and

$$v(t) = V_m e^{j\omega t} e^{j\theta} \qquad 27.31$$

If the rotating portion of Eq. 27.31 is assumed to exist, the voltage can be written

$$v(\theta) = V_m e^{j\theta} \qquad 27.32$$

Changing Eq. 27.32 to the phasor form yields

$$\mathbf{V} = V_m \angle\theta \qquad 27.33$$

[9]For all circuit elements at the same frequency, the circuit must be in a steady-state condition.
[10]The term "vector addition" is not technically correct since phasors, with the exception of impedance and admittance phasors, rotate with time. The methods, however, are identical.

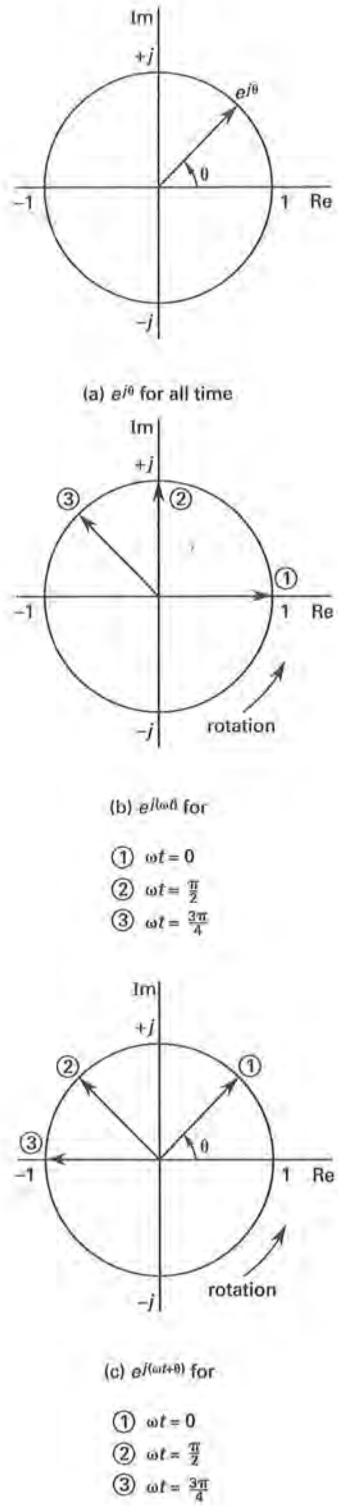


Figure 27.10 Phasor Rotation in the Complex Plane

The properties of complex numbers, which can represent voltage, current, or impedance, are summarized in Table 27.3. The designations used in the table are illustrated in Fig. 27.9.

13. RESISTORS

Resistors oppose the movement of electrons. In an *ideal* or *pure resistor*, no inductance or capacitance exists. The magnitude of the impedance is the resistance, R, with units of ohms and an impedance angle of zero. Therefore, voltage and current are in phase in a purely resistive circuit.

$$\mathbf{Z}_R = R\angle 0 = R + j0 \qquad 27.34$$

14. CAPACITORS

Capacitors oppose the movement of electrons by storing energy in an electric field and using this energy to resist changes in voltage over time. Unlike a resistor, an *ideal* or *perfect capacitor* consumes no energy. Equation 27.35 gives the impedance of an ideal capacitor with capacitance C. The magnitude of the impedance is termed the *capacitive reactance*, X_C, with units of ohms and an impedance angle of $-\pi/2(-90°)$. Consequently, the current leads the voltage by 90° in a purely capacitive circuit.[11]

$$\mathbf{Z}_C = X_C\angle{-90°} = 0 + jX_C \qquad 27.35$$

$$X_C = \frac{-1}{\omega C} = \frac{-1}{2\pi f C} \qquad 27.36$$

15. INDUCTORS

Inductors oppose the movement of electrons by storing energy in a magnetic field and using this energy to resist changes in current over time. Unlike a resistor, an *ideal* or *perfect inductor* consumes no energy. Equation 27.37 gives the impedance of an ideal inductor with inductance L. The magnitude of the impedance is termed the *inductive reactance*, X_L, with units of ohms and an impedance angle of $+\pi/2$ (+90°). Consequently, the current lags the voltage by 90° in a purely inductive circuit.[12]

$$\mathbf{Z}_L = X_L\angle{+90°} = 0 + jX_L \qquad 27.37$$

$$X_L = \omega L = 2\pi f L \qquad 27.38$$

[11] The impedance angle for a capacitor is negative, but the current phase angle difference is positive—hence the term "leading." This occurs mathematically since the current is obtained by dividing the voltage by the impedance: $\mathbf{I} = \mathbf{V}/\mathbf{Z}$.

[12] The impedance angle for an inductor is positive, but the current phase angle difference is negative—hence the term "lagging." This occurs mathematically since the current is obtained by dividing the voltage by the impedance: $\mathbf{I} = \mathbf{V}/\mathbf{Z}$.

Table 27.3 Properties of Complex Numbers

	rectangular form	polar/exponential form
	$\mathbf{Z} = x + jy$	$\mathbf{Z} = \lvert\mathbf{Z}\rvert\angle\theta$ $\mathbf{Z} = \lvert\mathbf{Z}\rvert e^{j\theta} = \lvert\mathbf{Z}\rvert\cos\theta + j\lvert\mathbf{Z}\rvert\sin\theta$
relationship between forms	$x = \lvert\mathbf{Z}\rvert\cos\theta$ $y = \lvert\mathbf{Z}\rvert\sin\theta$	$\lvert\mathbf{Z}\rvert = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
complex conjugate	$\mathbf{Z}^* = x - jy$ $\mathbf{ZZ}^* = (x^2 + y^2) = \lvert z\rvert^2$	$\mathbf{Z}^* = \lvert\mathbf{Z}\rvert e^{-j\theta} = \lvert\mathbf{Z}\rvert\angle{-\theta}$ $\mathbf{ZZ}^* = (\lvert\mathbf{Z}\rvert e^{j\theta})(\lvert\mathbf{Z}\rvert e^{-j\theta}) = \lvert\mathbf{Z}\rvert^2$
addition	$\mathbf{Z}_1 + \mathbf{Z}_2 = (x_1 + x_2) + j(y_1 + y_2)$	$\mathbf{Z}_1 + \mathbf{Z}_2 = (\lvert\mathbf{Z}_1\rvert\cos\theta_1 + \lvert\mathbf{Z}_2\rvert\cos\theta_2) + j(\lvert\mathbf{Z}_1\rvert\sin\theta_1 + \lvert\mathbf{Z}_2\rvert\sin\theta_2)$
multiplication	$\mathbf{Z}_1\mathbf{Z}_2 = (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1)$	$\mathbf{Z}_1\mathbf{Z}_2 = \lvert\mathbf{Z}_1\rvert\lvert\mathbf{Z}_2\rvert\angle\theta_1 + \theta_2$
division	$\frac{\mathbf{Z}_1}{\mathbf{Z}_2} = \frac{(x_1x_2 + y_1y_2) + j(x_2y_1 - x_1y_2)}{\lvert\mathbf{Z}_2\rvert^2}$	$\frac{z_1}{z_2} = \frac{\lvert\mathbf{Z}_1\rvert}{\lvert\mathbf{Z}_2\rvert}\angle\theta_1 - \theta_2$

16. COMBINING IMPEDANCES

Impedances in combination are like resistors: Impedances in series are added, while the reciprocals of impedances in parallel are added. For series circuits, the resistive and reactive parts of each impedance element are calculated separately and summed. For parallel circuits, the conductance and susceptance of each element are summed. The total impedance is found by a complex addition of the resistive (conductive) and reactive (susceptive) parts. It is convenient to perform the addition in rectangular form.

$$Z_e = \sum Z = \sqrt{\left(\sum R\right)^2 + \left(\sum X_L - \sum X_C\right)^2} \quad \text{[series]} \qquad 27.39$$

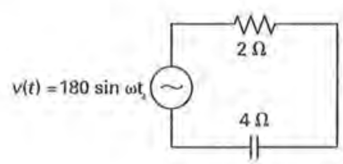
$$\frac{1}{Z_e} = \sum\frac{1}{Z} = Y_e = \sqrt{\left(\sum G\right)^2 + \left(\sum B_L - \sum B_C\right)^2} \quad \text{[parallel]} \qquad 27.40$$

The impedance of various series-connected circuit elements is given in App. 27.A. The impedance of various parallel-connected circuit elements is given in App. 27.B.

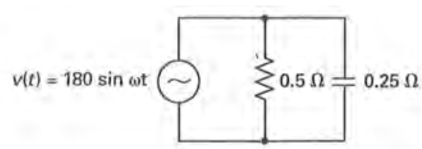
Example 27.6

Determine the impedance and admittance of the following circuits.

(a)



(b)



Solution

(a) From Eq. 27.39,

$$Z = \sqrt{R^2 + Z_C^2} = \sqrt{(2\ \Omega)^2 + (4\ \Omega)^2} = 4.47\ \Omega$$

$$\phi = \arctan\left(\frac{X_C}{R}\right) = \arctan\left(\frac{-4}{2}\right) = -63.4^\circ$$

$$\mathbf{Z} = 4.47\angle{-63.4^\circ}\Omega$$

From Eq. 27.9,

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{4.47\angle{-63.4^\circ}\Omega} = 0.224\angle 63.4^\circ \text{ S}$$

(b) Since this is a parallel circuit, work with the admittances.

$$G = \frac{1}{R} = \frac{1}{0.5\ \Omega} = 2 \text{ S}$$

$$B_C = \frac{1}{X_C} = \frac{1}{0.25\ \Omega} = 4 \text{ S}$$

$$Y = \sqrt{G^2 + B_C^2} = \sqrt{(2\text{ S})^2 + (4\text{ S})^2} = 4.47 \text{ S}$$

$$\phi = \arctan\left(\frac{B_C}{G}\right) = \arctan\left(\frac{4}{2}\right) = 63.4^\circ$$

$$\mathbf{Y} = 4.47\angle 63.4^\circ \text{ S}$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{4.47\angle 63.4^\circ \text{ S}} = 0.224\angle{-63.4^\circ}\ \Omega$$

17. OHM'S LAW

Ohm's law for AC circuits with linear circuit elements is similar to Ohm's law for DC circuits.[13] The difference is that all the terms are represented as phasors.[14]

$$\mathbf{V} = \mathbf{IZ} \qquad 27.41$$

$$V\angle\theta_V = (I\angle\theta_I)(Z\angle\phi_Z) \qquad 27.42$$

18. POWER

The instantaneous power, p, in a purely resistive circuit is given by

$$\begin{aligned} p_R(t) &= i(t)v(t) = (I_m \sin\omega t)(V_m \sin\omega t) \\ &= I_m V_m \sin^2 \omega t \\ &= \tfrac{1}{2} I_m V_m - \tfrac{1}{2} I_m V_m \cos 2\omega t \end{aligned} \qquad 27.43$$

The second term of Eq. 27.43 integrates to zero over the period. Therefore, the *average power* dissipated is

$$P_R = \tfrac{1}{2} I_m V_m = \frac{V_m^2}{2R} = I_{\text{rms}} V_{\text{rms}} = IV \qquad 27.44$$

[13]Ohm's law can be used on *nonlinear devices* (NLD) if the region analyzed is restricted to be approximately linear. When this condition is applied, the analysis is termed *small signal analysis*.

[14]In general, phasors are considered to rotate with time. A secondary definition of a phasor is any quantity that is a complex number. As a result, the impedance is also considered a phasor even though it does not change with time.

Equation 27.28 was used to change the maximum, or peak, values into the more usable rms, or effective, values.

In a purely capacitive circuit, the current leads the voltage by 90°. This allows the current term of Eq. 27.43 to be replaced by a cosine (since the sine and cosine differ by a 90° phase). The instantaneous power in a purely capacitive circuit is

$$\begin{aligned} p_C(t) &= i(t)v(t) \\ &= (I_m \cos\omega t)(V_m \sin\omega t) \\ &= I_m V_m \sin\omega t \cos\omega t \\ &= \tfrac{1}{2} I_m V_m \sin 2\omega t \end{aligned} \qquad 27.45$$

Since the $\sin 2\omega t$ term integrates to zero over the period, the average power is zero. Nevertheless, power is stored during the charging process and returned to the circuit during the discharging process.

$$P_C = 0 \qquad 27.46$$

Similarly, the instantaneous power in an inductor is

$$p_L(t) = -\tfrac{1}{2} I_m V_m \sin 2\omega t \qquad 27.47$$

Again, the $\sin 2\omega t$ integrates to zero over the period, and the average power is zero. Nevertheless, power is stored during the expansion of the magnetic field and returned to the circuit during the contraction of the magnetic field.

$$P_L = 0 \qquad 27.48$$

19. REAL POWER AND THE POWER FACTOR

In a circuit that contains all three circuit elements (resistors, capacitors, and inductors) or the effects of all three, the average power is calculated from Eq. 27.14 as

$$P_{\text{ave}} = \frac{1}{T}\int_0^T i(t)v(t)dt \qquad 27.49$$

Let the generic voltage and current waveforms be represented by Eqs. 27.50 and 27.51. The current angle φ is equal to $\theta \pm \phi$.[15]

$$i(t) = I_m \sin(\omega t + \varphi) \qquad 27.50$$

$$v(t) = V_m \sin(\omega t + \theta) \qquad 27.51$$

[15]Using a current angle simplifies the derivational mathematics (not shown) and clarifies the definition of the power factor angle given in this section.

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Substituting Eqs. 27.50 and 27.51 into Eq. 27.49 gives

$$P_{\text{ave}} = \left(\frac{I_m V_m}{2}\right)\cos\Psi = I_{\text{rms}}V_{\text{rms}}\cos\Psi \qquad 27.52$$

The angle $\Psi = \theta - \varphi$ is the *power factor angle*. It represents the difference between the voltage and current angles. This difference is the impedance angle, $\pm\phi$. Since the $\cos(-\phi) = \cos(+\phi)$, only the absolute value of the impedance angle is used. Because the absolute value is used in the equation, the terms "leading" (for a capacitive circuit) and "lagging" (for an inductive circuit) must be used when describing the power factor. The power factor of a purely resistive circuit equals one; the power factor of a purely reactive circuit equals zero.

Equation 27.52 determines the magnitude of the product of the current and voltage in phase with one another, that is, of a resistive nature. Consequently, Eq. 27.52 determines the power consumed by the resistive elements of a circuit. This average power is called the *real power* or *true power*, and sometimes the *active power*. The term $\cos\Psi$, or $\cos|\phi|$, is called the *power factor* or *phase factor* and given the symbol pf. The power factor is also equal to the ratio of the real power to the apparent power (see Sec. 27-22). Often, no subscript is used on P when it represents the real power, nor are subscripts used on rms or effective values. The real power is then given by

$$P = IV\cos\Psi = IV\text{pf} \qquad 27.53$$

It is important to realize that the real power cannot be obtained by multiplying phasors. Doing so results in the addition of the voltage and current angles when the difference is required.

$$P \neq I\angle\varphi V\angle\theta = VI\angle\theta + \varphi \qquad 27.54$$

If phasor power is used, the difficulty can be alleviated by using Eq. 27.55.

$$P = \text{Re}\{\mathbf{VI}^*\} \qquad 27.55$$

Equation 27.55 is equivalent to Eq. 27.53.

20. REACTIVE POWER

The *reactive power*, Q, measured in units of VAR, is given by

$$Q = IV\sin\Psi \qquad 27.56$$

The reactive power, sometimes called the *wattless power*, is the product of the rms values of the current and voltage multiplied by the *quadrature* of the current. $\sin\Psi$ is called the *reactive factor*.[16] The reactive power represents the energy stored in the inductive and capacitive elements of a circuit.

21. APPARENT POWER

The apparent power, S, measured in voltamperes, is given by

$$S = IV \qquad 27.57$$

The apparent power is the product of the rms values of the voltage and current without regard to the angular relationship between them. The apparent power is representative of the combination of real and reactive power (see Sec. 22). As such, not all of the apparent power is dissipated or consumed. Nevertheless, electrical engineers must design systems to adequately handle this power as it exists in the system.

22. COMPLEX POWER: THE POWER TRIANGLE

The real, reactive, and apparent powers can be related to one another as vectors. The *complex power vector*, $\mathbf{S}$, is the vector sum of the *reactive power vector*, $\mathbf{Q}$, and the *real power vector*, $\mathbf{P}$. The magnitude of $\mathbf{S}$ is given by Eq. 27.57. The magnitude of $\mathbf{Q}$ is given by Eq. 27.56. The magnitude of $\mathbf{P}$ is given by Eq. 27.53. In general, $\mathbf{S} = \mathbf{I}^*\mathbf{V}$ where $\mathbf{I}^*$ is the complex conjugate of the current, that is, the current with the phase difference angle reversed. This convention is arbitrary, but emphasizes that the *power angle*, ϕ, associated with $\mathbf{S}$ is the same as the overall impedance angle, ϕ, whose magnitude equals the power factor angle, Ψ.

The relationship between the magnitudes of these powers is

$$S^2 = P^2 + Q^2 \qquad 27.58$$

A drawing of the power vectors in a complex plane is shown in Fig. 27.11 for leading and lagging conditions. The following relationships are determined from the drawing.

$$P = S\cos\phi \qquad 27.59$$

$$Q = S\sin\phi \qquad 27.60$$

[16]The power factor angle, Ψ, is used instead of the absolute value of the impedance angle (as in the power factor) since the sine function results in positive and negative values depending upon the difference between the voltage and current angles. If Ψ were not used, $\pm\phi$ would have to be used.

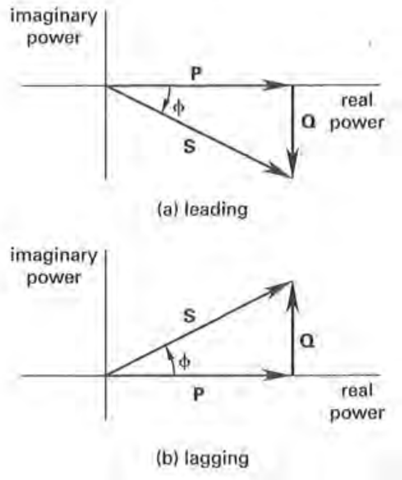
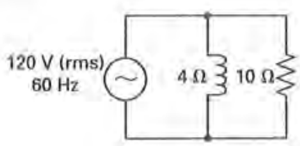


Figure 27.11 *Power Triangle*

Example 27.7

For the circuit shown, find the (a) apparent power, (b) real power, and (c) reactive power; and (d) draw the power triangle.



Solution

The equivalent impedance is

$$\frac{1}{Z} = \frac{1}{j4} + \frac{1}{10} = -j0.25 + 0.10 \text{ S}$$

$$Z = 3.714 \ \Omega$$

$$\phi_Z = \arctan\left(\frac{0.25}{0.10}\right) = 68.2^\circ$$

The total current is

$$I = \frac{V}{Z} = \frac{120\angle 0^\circ \text{ V}}{3.714\angle 68.2^\circ \ \Omega} = 32.31\angle -68.2^\circ \text{ A}$$

(a) The apparent power is

$$S = I^*V = (32.31\angle 68.2^\circ \text{ A})(120\angle 0^\circ \text{ V})$$
$$= 3877\angle 68.2^\circ \text{ VA}$$

(The angle of apparent power is usually not reported.)

(b) The real power is

$$P = \frac{V_R^2}{R} = \frac{(120\angle 0^\circ \text{ V})^2}{10 \ \Omega} = 1440 \text{ W}$$

Alternatively, the real power can be calculated from Eq. 27.59.

$$P = S\cos\phi = (3877)\cos(68.2^\circ) = 1440 \text{ W}$$

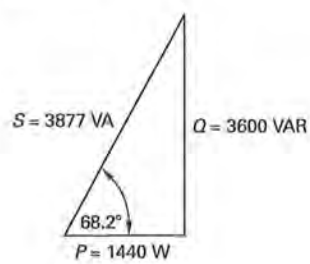
(c) The reactive power is

$$Q = \frac{V_L^2}{X_L} = \frac{(120 \text{ V})^2}{4 \ \Omega} = 3600 \text{ VAR}$$

Alternatively, the reactive power can be calculated from Eq. 27.60.

$$Q = S\sin\phi = (3877)(\sin 68.2^\circ) = 3600 \text{ VAR}$$

(d) The real power is represented by the vector (in rectangular form) $1440 + j0$. The reactive power is represented by the vector $0 + j3600$. The apparent power is represented (in phasor form) by the vector $3877\angle 68.2^\circ$. The power triangle is



23. MAXIMUM POWER TRANSFER

Assuming a fixed primary impedance, the maximum power condition in an AC circuit is similar to that in a DC circuit. That is, the maximum power is transferred from the source to the load when the impedances match. This occurs when the circuit is in resonance. Resonance is discussed more fully in Ch. 29. The conditions for maximum power transfer, and thus resonance, are given by

$$R_{\text{load}} = R_s \qquad 27.61$$

$$X_{\text{load}} = -X_s \qquad 27.62$$

24. AC CIRCUIT ANALYSIS

All of the methods and equations presented in Ch. 26, such as Ohm's and Kirchhoff's laws and loop-current and node-voltage methods can be used to analyze AC circuits as long as complex arithmetic is utilized.

29 Linear Circuit Analysis

Nomenclature

A	area	m^2
b	y-intercept	–
C	arbitrary constant	–
C	capacitance	F
f	frequency	Hz, s^{-1}, or cycles/s
i	instantaneous current	A
I	effective or DC current	A
l	length	m
L	self-inductance	H
m	slope	–
M	mutual inductance	H
n	integer	–
N	number of turns	–
Q	charge	C
r	distance	m
R	resistance	Ω
t	time	s
T	temperature	°C
U	energy	J
v	instantaneous voltage	V
V	effective or DC voltage	V
Z	impedance	Ω

Symbols

α	thermal coefficient of resistance	1/°C
ε	permittivity	F/m
ε_0	free-space permittivity	8.854×10^{-12} F/m
ε_r	relative permittivity	–
κ	arbitrary constant	–
μ	permeability	H/m
μ_0	free-space permeability	1.2566×10^{-6} H/m
ρ	resistivity	Ω·cm
Ψ	magnetic flux	Wb
ω	angular frequency	rad/s
℧	conductance	S

Subscripts

0	initial or free space (vacuum)
C	capacitor
e	equivalent
fl	full load
int	internal or source
l	load
L	inductor
m	magnetizing
nl	no load
N	Norton
oc	open circuit
s	source
sc	short circuit
Th	Thevenin

1. FUNDAMENTALS

Circuit analysis is fundamental to the practice of electrical engineering. Such analysis is based on Kirchhoff's two circuit laws and the values, as well as the fluctuations, of the *circuit variables*, that is, the voltages between terminals and the currents within the network. Circuit theory is nominally divided into determination of *equivalent circuits* and *mathematical analysis* of those equivalent circuits.[1] Equivalent circuits are composed of *lumped elements*. A lumped element is a

[1]In digital systems, the final equivalent circuit is a logic diagram.

Circuit Theory

single element representing a particular electrical property in the entire circuit, or in some portion of the circuit.[2] Equivalent circuit theory is sometimes called *network theory*. The term "network," while often used synonymously with "circuit," is usually reserved for more complicated arrangements of elements. In fact, "circuit" often designates a single loop of a network. Networks are often modeled using ideal elements.

2. IDEAL INDEPENDENT VOLTAGE SOURCES

An ideal independent voltage source maintains the voltage at its terminals regardless of the current flowing through the terminals. The symbology for an ideal independent voltage source and its characteristics is shown in Fig. 29.1. The subscript s generally indicates an independent source. The source value can vary with time, but its effective value does not change. That is, the magnitude of its variations in time is without regard to the current. The polarity assigned is arbitrary, but it is often assigned in the direction of positive power flow out of the source. This type of notation is called *source notation*.

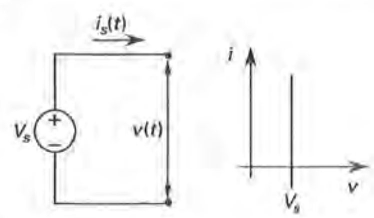


Figure 29.1 *Ideal Voltage Source*

A *real voltage source* cannot maintain the voltage at its terminals when the current becomes too large (in either the positive or the negative direction). The equivalent circuit of a real voltage source is shown in Fig. 29.2. The decrease in voltage as the current increases is measured by the *voltage regulation* given in Eq. 29.1.

$$\text{regulation} = \frac{V_{\text{nl}} - V_{\text{fl}}}{V_{\text{fl}}} \times 100\% \qquad 29.1$$

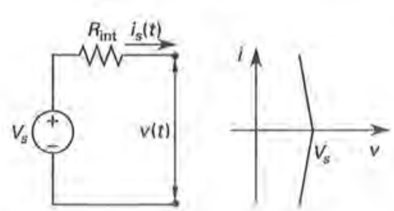


Figure 29.2 *Real Voltage Source*

[2]Electrical properties are distributed throughout elements, though they are concentrated in certain areas. Analyzing such *distributed elements* is difficult and they are thus represented by lumped parameters. This method is important in the analysis of electrical energy transmission.

3. IDEAL INDEPENDENT CURRENT SOURCES

An *ideal independent current source* maintains the current regardless of the voltage at its terminals. The symbology for an ideal independent current source and its characteristics is shown in Fig. 29.3, the subscript s indicating the independent source. The source value can vary with time, but its effective value does not change. That is, the magnitude of its variations in time is without regard to the voltage. The polarity is again arbitrary but is assigned using the source notation.

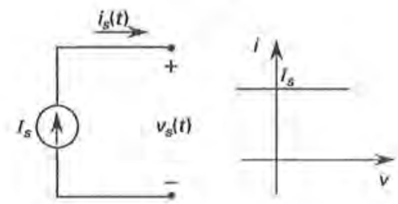


Figure 29.3 *Ideal Current Source*

A *real current source* cannot maintain the current at its terminals when the voltage becomes too large (in either the positive or the negative direction). The equivalent circuit of a real current source is shown in Fig. 29.4.

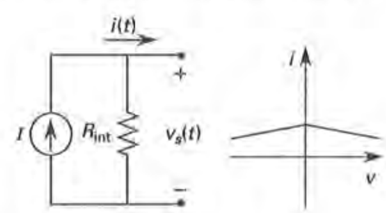


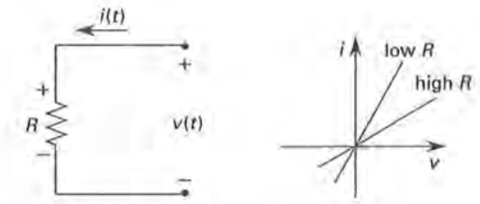
Figure 29.4 *Real Current Source*

4. IDEAL RESISTORS

An *ideal resistor* maintains the same resistance at its terminals regardless of the power consumed. That is, it is a linear element whose voltage/current relationship is given by Ohm's law.

$$v = iR \qquad 29.2$$

The symbology for an ideal resistor is identical to that for a real resistor and is shown in Fig. 29.5, along with the characteristics of the ideal resistor. The polarity is assigned so as to make Eq. 29.2, Ohm's law, valid as written. This is sometimes called the *sink notation*, as it corresponds to positive power flow into the resistor.



(a) ideal and real (b) ideal

Figure 29.5 Resistor

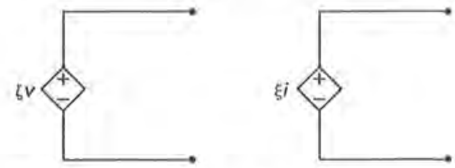
A real resistor undergoes changes in resistance as the temperature changes. The resistance of metals and most alloys increases with temperature while that of carbon and electrolytes decreases. The resistance change is accounted for in circuit design by ensuring the design allows for proper circuit functioning over the expected temperature range (see Sec. 26-3).

5. DEPENDENT SOURCES

Any circuit that is to deliver more power to the load than is available at the signal frequency must use an *active device*. An active device is defined as a component capable of amplifying the current or voltage, such as a transistor. An active device uses a source of power other than the input signal in order to accomplish the modification. In order to analyze circuits containing active devices, the devices are replaced by an equivalent circuit of *passive linear elements*, that is, elements that are not sources of energy (such as resistors, inductors, and capacitors), and one or more dependent sources. Such sources may be treated as standard sources using all the linear circuit analysis techniques, with the exception of superposition. Superposition can only be used if the device is limited to operation over a linear region.

6. DEPENDENT VOLTAGE SOURCES

A *dependent voltage source* is one in which the output is controlled by a variable elsewhere in the circuit. For this reason, dependent voltage sources are sometimes called *controlled voltage sources*. The controlling variable can be another voltage, a current, or any other physical quantity, such as light intensity or temperature. The symbology for dependent voltage sources is shown in Fig. 29.6. A *voltage-controlled voltage source* is controlled by a voltage, v, present somewhere else in the circuit. A *current-controlled voltage source* is controlled by a current, i, present somewhere else in the circuit. The terms ζ and ξ are constants.

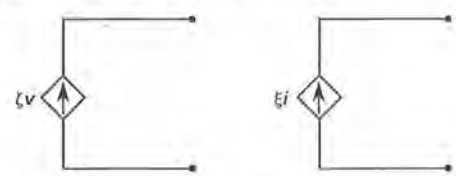


(a) voltage-controlled voltage source (b) current-controlled voltage source

Figure 29.6 Dependent Voltage Sources

7. DEPENDENT CURRENT SOURCES

A *dependent current source* is one in which the output is controlled by a variable elsewhere in the circuit. For this reason, dependent current sources are sometimes called *controlled current sources*. The controlling variable can be another voltage, a current, or any other physical quantity, such as light intensity or temperature. The symbology for dependent current sources is shown in Fig. 29.7. A *voltage-controlled current source* is controlled by a voltage, v, present somewhere else in the circuit. A *current-controlled current source* is controlled by a current, i, present somewhere else in the circuit. The terms ζ and ξ are constants.



(a) voltage-controlled current source (b) current-controlled current source

Figure 29.7 Dependent Current Sources

8. LINEAR CIRCUIT ELEMENTS

The basic circuit elements are resistance, capacitance, and inductance. Related to the basic element of inductance are mutual inductance and ideal (linear) transformers. A *linear network* is one in which the resistance, capacitance, and inductance are constant with respect to current and voltage, and in which the current or voltage of sources is independent or directly proportional to the other currents and voltages, or their derivatives, in the network. That is, a linear network is one that consists of linear elements and linear sources.[3] Circuit elements or sources are linear if the superposition principle is valid for them, and vice versa. A summary of linear circuit element parameters is given in Table 29.1. The time domain and frequency domain behavior of linear circuit elements, along with their defining equations, are shown in Table 29.2.

[3]A linear source is one in which the output is proportional to the first power of a voltage or current in the circuit.

9. RESISTANCE

Resistance is the opposition to current flow in a conductor. Technically, it is the opposition that a device or material offers to the flow of direct current. In AC circuits, it is the real part of the complex impedance. The resistance of a conductor depends on the resistivity, ρ, of the conducting material. The resistivity is a function of temperature that varies approximately linearly in accordance with Eq. 29.3. A slightly different version of this equation was given in Sec. 26-3. Here, the resistivity is referenced to 20°C and is written ρ_{20}. For copper, ρ_{20} is approximately 1.8×10^{-8} Ω·m. The thermal coefficient of resistivity, also called the *temperature coefficient of resistivity*, is referenced to 20°C and written α_{20}. For copper, α_{20} is approximately 3.9×10^{-3} C^{-1}. The equation for resistivity at any temperature, T (in °C), then becomes[4]

$$\rho = \rho_{20}\left(1 + \alpha_{20}\left(T - 20\right)\right) \qquad 29.3$$

The resistance of a conductor of length l and cross-sectional area A is

$$R = \frac{\rho l}{A} \qquad 29.4$$

The defining equation for resistance is Ohm's law, or the extended Ohm's law for AC circuits, which relates the linear relationship between the voltage and the current.

$$V = IR \quad \text{or} \quad \mathbf{V} = \mathbf{IZ}_R \qquad 29.5$$

The power dissipated by the resistor is[5]

$$P = IV = I^2R = \frac{V^2}{R} \qquad 29.6$$

The equivalent resistance of resistors in series is

$$R_e = R_1 + R_2 + \ldots R_n \qquad 29.7$$

The equivalent resistance of resistors in parallel is

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n} \qquad 29.8$$

10. CAPACITANCE

Capacitance is a measure of the ability to store charge. Technically, it is the opposition a device or material offers to changing voltage. In a DC circuit, that is, a circuit in which the voltage is constant, the capacitor appears as an open circuit (after any transient condition has subsided). In a DC circuit, if the voltage is zero, the capacitor appears as a short circuit. In AC circuits, it is the constant of proportionality between the changing voltage and current.

$$i = C\left(\frac{dv}{dt}\right) \qquad 29.9$$

Capacitance is a function of the insulating material's *permittivity*, ε, between the conducting portions of the capacitor. The permittivity is the product of the *free-space permittivity*, ε_0, and the *relative permittivity* or *dielectric constant*, ε_r. For free space, that is, a vacuum, the permittivity is 8.854×10^{-12} F/m. The dielectric constant varies, and for many linear materials it has values between 1 and 10.

$$\varepsilon = \varepsilon_r\varepsilon_0 \qquad 29.10$$

The capacitance of a simple parallel plate capacitor is given by Eq. 29.11. The term r is the distance between the two parallel plates of equal area, A, and ε is the permittivity of the material between the plates.

$$C = \frac{\varepsilon A}{r} \qquad 29.11$$

The defining equation for capacitance, which relates the amount of charge stored to the voltage across the capacitor terminals, is

$$C = \frac{Q}{V} \qquad 29.12$$

The (average) power dissipated by a capacitor is zero. The (average) energy stored in the electric field of a capacitor is

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \tfrac{1}{2}\left(\frac{Q^2}{C}\right) \qquad 29.13$$

The equivalent capacitance of capacitors in series is

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} \qquad 29.14$$

The equivalent capacitance of capacitors in parallel is

$$C_e = C_1 + C_2 + \ldots + C_n \qquad 29.15$$

From Eq. 29.9, the voltage cannot change instantaneously, as this would make the current and the power infinite, which clearly cannot be the case. Using this fact, the transient behavior of capacitors can be analyzed. Using this principle is the equivalent of treating a capacitor's arbitrary constant κ in Table 29.1 as the initial voltage, V_0.

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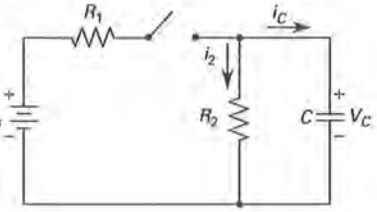
[4] Copper with a conductivity of 100% will experience a resistance change of approximately 10% for a 25°C (77°F) temperature change.

[5] When the term "power" is used, the average power is the referenced quantity. The instantaneous power depends on the time within the given cycle and is written p.

Example 29.1

In the circuit shown, the switch has been open for an "extended time," which in such circuits is considered to be a minimum of five time constants. The capacitor has no charge on it at $t = 0$ when the switch is closed. What is the current through the capacitor at the instant the switch is closed?



Solution

Since the voltage across the capacitor cannot change instantaneously, $v(0^-) = v(0^+)$, and the capacitor remains uncharged. The capacitor thus initially acts as a short circuit. All the current flows through the capacitor, bypassing R_2. The current is given by

$$i_c(0^+) = \frac{V_s}{R_1}$$

The initial voltage across the capacitor, $V_c(0^+)$, is zero.

Example 29.2

Using the circuit given in Ex. 29.1, what is the steady-state voltage across the capacitor?

Solution

The term "steady-state" is another way of saying "an extended period of time," which in such circuits is considered to be a minimum of five time constants. The capacitor will be fully charged and will act as an open circuit. Consequently, $dv/dt = 0$ and $i_c = 0$. All the current flows through R_2 and is given by

$$I_{R_2} = \frac{V_s}{R_1 + R_2}$$

Since they are in parallel, the voltage across the capacitor is the same as the voltage across R_2. Thus,

$$\begin{aligned} V_c(\infty) &= V_{R_2} = I_2 R_2 \\ &= \left(\frac{V_s}{R_1 + R_2}\right) R_2 = V_s \left(\frac{R_2}{R_1 + R_2}\right) \end{aligned}$$

The steady-state current through the capacitor, $I_c(\infty)$, is zero.

11. INDUCTANCE

Inductance is a measure of the ability to store magnetic energy. Technically, it is the opposition a device or material offers to changing current. In a DC circuit, that is, a circuit in which the current is constant, the inductor appears as a short circuit (after any transient condition has subsided). In a DC circuit, if the current is zero, the inductor appears as an open circuit. In AC circuits, it is the constant of proportionality between the changing current and voltage.

$$v = L\left(\frac{di}{dt}\right) \qquad 29.16$$

Inductance is a function of a medium's *permeability*, μ, between the magnetically linked portions of the inductor. The permeability is the product of the *free-space permeability*, μ_0, and the *relative permeability* or *permeability ratio*, μ_r. For free space, that is, a vacuum, the permeability is 1.2566×10^{-6} H/m. The permeability ratio varies. Depending on the material and the flux density, it can have values in the hundreds and sometimes thousands.

$$\mu = \mu_r \mu_0 \qquad 29.17$$

The inductance of a simple toroid shown in Fig. 29.8 is given by Eq. 29.18. The term l is the average flux path distance around the toroid of equal area, A, and μ is the permeability of the medium between the coil turns, N.

$$L = \frac{\mu N^2 A}{l} \qquad 29.18$$

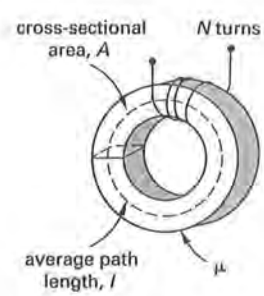


Figure 29.8 *Toroid Inductance Terms*

The defining equation for inductance, which relates the amount of flux present to the current flowing through the inductor terminals, is

$$L = \frac{\Psi}{I} \qquad 29.19$$

The (average) power dissipated by an inductor is zero. The (average) energy stored in the magnetic field of an inductor is given by Eq. 29.20. Positive energy indicates energy stored in the inductor. Negative energy indicates energy supplied to the circuit by the inductor.

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\Psi I = \tfrac{1}{2}\left(\frac{\Psi^2}{L}\right) \qquad 29.20$$

The equivalent inductance of inductors in series is

$$L_e = L_1 + L_2 + \ldots + L_n \qquad 29.21$$

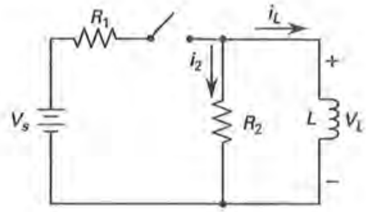
The equivalent inductance of inductors in parallel is

$$\frac{1}{L_e} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n} \qquad 29.22$$

From Eq. 29.16, the current cannot change instantaneously, as this would make the voltage and the power infinite, which clearly cannot be the case. Using this fact, the transient behavior of inductors can be analyzed. Using this principle is the equivalent of treating an inductor's arbitrary constant κ in Table 29.1 as the initial current, I_0.

Example 29.3

In the circuit shown, the switch has been open for an extended time, that is, greater than five time constants. The inductor has no associated magnetic field on it at $t = 0$ when the switch is closed. What is the current through the inductor at the instant the switch is closed?



Solution

Since the current across the inductor cannot change instantaneously, $i(0^-) = i(0^+)$ and the inductor remains without a magnetic field. The inductor thus initially acts as an open circuit. All the current flows through R_2.

$$i_L(0^+) = 0$$

Example 29.4

Using the circuit given in Ex. 29.3, what is the initial voltage across the inductor?

Solution

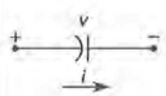
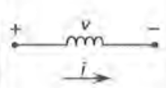
All the current flows through R_2 and is given by

$$I_{R_2} = \frac{V_s}{R_1 + R_2}$$

Since they are in parallel, the voltage across the inductor is the same as the voltage across R_2. Thus,

$$\begin{aligned} V_L(0^+) &= V_{R_2} = I_2R_2 \\ &= \left(\frac{V_s}{R_1 + R_2}\right)R_2 = V_s\left(\frac{R_2}{R_1 + R_2}\right) \end{aligned}$$

Table 29.1 *Linear Circuit Element Parameters*

circuit element	voltage	current	instantaneous power	average power	average energy stored
	$v = iR$	$i = \frac{v}{R}$	$p = iv = i^2R = \frac{v^2}{R}$	$P = IV = I^2R = \frac{V^2}{R}$	–
	$v = \frac{1}{C}\int i\,dt + \kappa$	$i = C\left(\frac{dv}{dt}\right)$	$p = iv = Cv\left(\frac{dv}{dt}\right)$	0	$U = \frac{1}{2}CV^2$
	$v = L\left(\frac{di}{dt}\right)$	$i = \frac{1}{L}\int v\,dt + \kappa$	$p = iv = Li\left(\frac{di}{dt}\right)$	0	$U = \frac{1}{2}LI^2$

Example 29.5

Using the circuit given in Ex. 29.3, what is the steady-state value of the current through the inductor?

Solution

The term "steady-state" is another way of saying "an extended period of time," which in such circuits is considered to be a minimum of five time constants. The inductor's magnetic field will be at a maximum and the inductor acts as a short circuit. Consequently, $di/dt = 0$, and $v_L = 0$. All the current thus flows through the inductor, bypassing R_2, and, using Ohm's law, is

$$i_L(\infty) = \frac{V_s}{R_1}$$

12. MUTUAL INDUCTANCE

Mutual inductance is the ratio of the electromotive force induced in one circuit to the rate of change of current in the other circuit. Applicable equations are given in Table 29.2. The concept is explained in Ch. 28. The total energy stored in a system involving mutual inductance is

$$U = \tfrac{1}{2}L_1I_1^2 + \tfrac{1}{2}L_2I_2^2 + MI_1I_2 \qquad 29.23$$

13. LINEAR SOURCE MODELS

Electric power sources tend to be nonlinear. In order to analyze such sources using circuit analysis, they are modeled as linear sources in combination with linear impedance. Such a source is classified as an *independent source*.

Dependent sources, whose output depends on a parameter elsewhere in the circuit, are associated with active devices, such as a transistor. These are not actually sources of power but can also be modeled as linear sources in combination with an impedance. The range of operation over which the model is valid must be selected to be approximately linear. Circuit analysis of this type is called *small-signal analysis*.

A third source type is the *transducer* that produces a current or voltage output from a nonelectric input. *Thermocouples* are transducers that produce voltages as functions of temperature. *Photodiodes* are transducers that produce currents as functions of incident illumination level. *Microphones* are transducers that produce voltages as functions of incident sound intensity.

All these sources can be modeled by linear approximation to measured values. Specific models for the electronic components are given in Ch. 43.

Circuit Theory

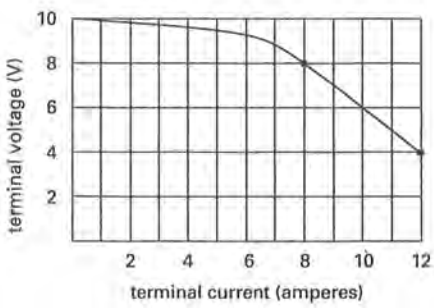
Table 29.2 *Linear Circuit Parameters, Time and Frequency Domain Representation*

parameter	defining equation	time domain	frequency domain[a]
resistance	$R = \frac{v}{i}$	$v = iR$	$V = IR$
capacitance	$C = \frac{Q}{V}$	$i = C\left(\frac{dv}{dt}\right)$	$I = j\omega CV$
self-inductance	$L = \frac{\Psi}{I}$	$v = L\left(\frac{di}{dt}\right)$	$V = j\omega LI$
mutual inductance[b]	$M_{12} = M_{21} = M = \frac{\Psi_{12}}{I_2}$ $= \frac{\Psi_{21}}{I_1}$	$v_1 = L_1\left(\frac{di_1}{dt}\right) + M\left(\frac{di_2}{dt}\right)$ $v_2 = L_2\left(\frac{di_2}{dt}\right) + M\left(\frac{di_1}{dt}\right)$	$V_1 = j\omega(L_1I_1 + MI_2)$ $V_2 = j\omega(L_2I_2 + MI_1)$

[a]The voltages and currents in the frequency domain column are not shown as vectors; for example, **I** or **V**. They are, however, shown in phasor form. Across any single circuit element (or parameter), the phase angle is determined by the impedance and embodied by the j. If the $j\omega$ (C or L or M) were not shown and instead given as **Z**, the current and voltage would be shown as **I** and **V**, respectively.
[b]Currents I_1 and I_2 (in the mutual inductance row) are in phase. The angle between either I_1 or I_2 and V_1 or V_2 is determined by the impedance angle embodied by the j. Even in a real transformer, this relationship holds, since the equivalent circuit accounts for any phase difference with a magnetizing current, I_m (see Sec. 28-5).

Example 29.6

The accompanying figure shows a representative plot of battery voltage versus load current. (a) What is the linear expression of the battery for the range of operation from 8 to 12 A? (b) Show the associated model.



Solution

(a) Using the *two-point method*, equivalent to determining the parameters of a straight line in the formula $y = mx + b$, for the points (8,8) and (4,12) gives

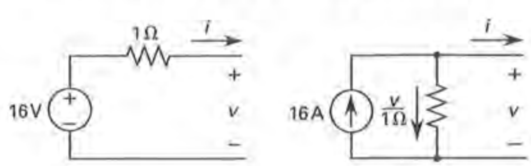
$$\frac{I-8}{V-8} = \frac{12-8}{4-8} = -1$$

(b) Either the voltage or the current may be selected as the dependent variable.

$$V = -I + 16 \quad [\text{I}]$$

$$I = -V + 16 \quad [\text{II}]$$

Equation I is a loop voltage equation with the terminal voltage determined by two sources, one independent (the 16 V) and the other dependent on the current (the $-1\ \Omega$). Equation II is a node current equation with the terminal current determined by two sources, one independent (the 16 A) and the other dependent on the voltage (the $-1\ \Omega^{-1}$). The resulting models are



[I] [II]

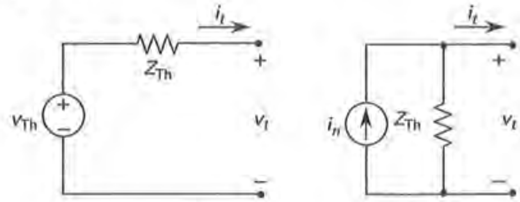
14. SOURCE TRANSFORMATIONS

An electrical source can be modeled as either a voltage source in series with an impedance or a current source in parallel with an impedance. The two models are equivalent in that they have the same terminal voltage/current relationships. Changing from one model to the other is called *source transformation* and is a *circuit reduction* technique. The models' parameters are determined by

$$v_{\text{Th}} = i_N Z_{\text{Th}} \qquad 29.24$$

$$i_N = \frac{v_{\text{Th}}}{Z_{\text{Th}}} \qquad 29.25$$

Using Eqs. 29.24 and 29.25, it is possible to convert from one model to the other. The models are shown in Fig. 29.9.



(a) equivalent voltage source (b) equivalent current source

Figure 29.9 *Source Transformation Models*

The source transformation is not defined for $Z = 0$ or $Z = \infty$. Also, both the open-circuit voltage and the short-circuit current direction must be the same in each model (see Sec. 26-11).

15. SERIES AND PARALLEL SOURCE COMBINATION RULES

One aspect of circuit reduction is the removal of sources that are not required. Section 29-14 referred to ideal current and voltage sources. These can be manipulated, combined, or eliminated as indicated by the following rules.

- An ideal voltage source cannot be placed in parallel with a second ideal voltage source, since the resulting voltage is indeterminate.
- An ideal current source cannot be placed in series with a second ideal current source, since the resulting current is indeterminate.
- An ideal voltage source in parallel with an ideal current source makes the current source redundant. The voltage is set by the voltage source, which absorbs all the current from the current source.
- An ideal current source in series with an ideal voltage source makes the voltage source redundant. The current is set by the current source, which is able to overcome any voltage of the voltage source.
- Ideal voltage sources in series can be combined algebraically using Kirchhoff's voltage law.
- Ideal current sources in parallel can be combined algebraically using Kirchhoff's current law.

The rules above can be used in an iterative manner to reduce a network to a single equivalent source and impedance. This technique is useful during phasor analysis or where a capacitance or inductance is present in a complicated network and the transient current or voltage is of interest. The iterative technique is also useful in the analysis of networks containing nonlinear or active devices.

16. REDUNDANT IMPEDANCES

Circuit reduction occurs when unnecessary impedances are removed according to the following rules.

- Impedance in parallel with an ideal voltage source may be removed. Theoretically, the ideal voltage supplies whatever energy is required to maintain the voltage, so the current flowing through the parallel impedance has no impact on the remainder of the circuit. Care must be exercised when calculating the source current to take into account the current through the parallel impedance.
- Impedance in series with an ideal current source may be removed. Theoretically, the ideal current source supplies whatever energy is required to maintain the current, so the energy loss in the series impedance has no impact on the remainder of the circuit. The voltage across the current source or current source/series impedance combination is determined by the remainder of the circuit.

17. DELTA-WYE TRANSFORMATIONS

Electrical impedances arranged in the shape of the Greek letter delta (Δ) or the English letter *Y* (wye) are known as *delta-wye configurations*, or *pi-T configurations*. A delta-wye configuration can be useful in reducing circuits in power networks.

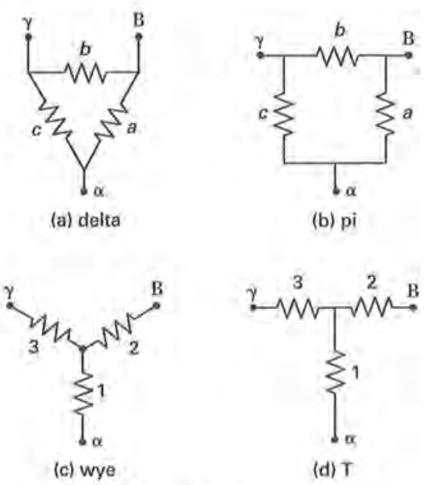


Figure 29.10 *Delta (Pi)-Wye (T) Configurations*

The equivalent impedances, which allow transformation between configurations, are

$$Z_a = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3} \qquad 29.26$$

$$Z_b = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1} \qquad 29.27$$

$$Z_c = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2} \qquad 29.28$$

$$Z_1 = \frac{Z_aZ_c}{Z_a + Z_b + Z_c} \qquad 29.29$$

$$Z_2 = \frac{Z_aZ_b}{Z_a + Z_b + Z_c} \qquad 29.30$$

$$Z_3 = \frac{Z_bZ_c}{Z_a + Z_b + Z_c} \qquad 29.31$$

18. THEVENIN'S THEOREM

Thevenin's theorem states that, insofar as the behavior of a linear circuit at its terminals is concerned, any such circuit can be replaced by a single voltage source, $\mathbf{V}_{\text{Th}}$, in series with a single impedance, $\mathbf{Z}_{\text{Th}}$. The method for determining and utilizing the *Thevenin equivalent circuit*, with designations referring to Fig. 29.11, follows.

step 1: Separate the network that is to be changed into a Thevenin equivalent circuit from its load at two terminals, say A and B.

step 2: Determine the open-circuit voltage, $\mathbf{V}_{\text{oc}}$, at terminals A and B.

step 3: Short-circuit terminals A and B and determine the current, $\mathbf{I}_{\text{sc}}$.

step 4: Calculate the Thevenin equivalent voltage and resistance from the following equations.

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{oc}} \qquad 29.32$$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{oc}}}{\mathbf{I}_{\text{sc}}} \qquad 29.33$$

step 5: Using the values calculated in Eqs. 29.32 and 29.33, replace the network with the Thevenin equivalent. Reconnect the load at terminals A and B. Determine the desired electrical parameters in the load.

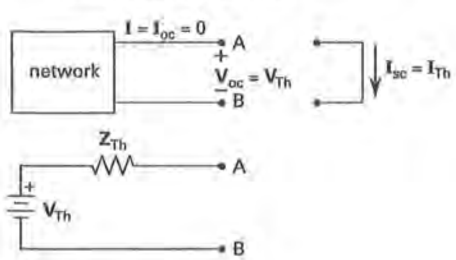


Figure 29.11 *Thevenin Equivalent Circuit*

Circuit Theory

Steps 3 and 4 can be altered by using the following shortcut. Determine the Thevenin equivalent impedance by looking into terminals A and B toward the network with all the power sources altered. Specifically, change independent voltage sources into short circuits and independent current sources into open circuits. Then calculate the resistance of the altered network. The resulting resistance is the Thevenin equivalent impedance.

19. NORTON'S THEOREM

Norton's theorem states that, insofar as the behavior of a linear circuit at its terminals is concerned, any such circuit can be replaced by a single current source, $\mathbf{I}_N$, in parallel with a single impedance, $\mathbf{Z}_N$. The method for determining and utilizing the *Norton equivalent circuit*, with designations referring to Fig. 29.12, follows.

step 1: Separate the network that is to be changed into a Norton equivalent circuit from its load at two terminals, say A and B.

step 2: Determine the open-circuit voltage, $\mathbf{V}_{oc}$, at terminals A and B.

step 3: Short-circuit terminals A and B and determine the current, $\mathbf{I}_{sc}$.

step 4: Calculate the Norton equivalent current and resistance from the following equations.

$$\mathbf{I}_N = \mathbf{I}_{sc} \qquad 29.34$$

$$\mathbf{Z}_N = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} \qquad 29.35$$

step 5: Using the values calculated in Eqs. 29.34 and 29.35, replace the network with the Norton equivalent. Reconnect the load at terminals A and B. Determine the desired electrical parameters in the load.

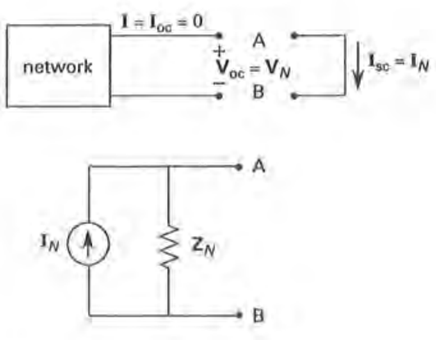


Figure 29.12 *Norton Equivalent Circuit*

Steps 2 and 4 can be altered by using the following shortcut. Determine the Norton equivalent impedance by looking into terminals A and B toward the network with all the power sources altered. Specifically, change independent voltage sources into short circuits and independent current sources into open circuits. Then calculate the resistance of the altered network. The resulting impedance is the Norton equivalent impedance.

The Norton equivalent impedance equals the Thevenin equivalent impedance for identical networks.

20. MAXIMUM POWER TRANSFER THEOREM

The maximum energy, that is, maximum power transfer from a voltage source, occurs when the series source impedance, $\mathbf{Z}_s$, is reduced to the minimum possible value, with zero being the ideal case. This assumes the load impedance is fixed and the source impedance can be changed. Though this is the ideal situation, it is not often the case. Where the load impedance varies and the source impedance is fixed, maximum power transfer occurs when the load and source impedances are complex conjugates. That is, $\mathbf{Z}_l = \mathbf{Z}_s^*$ or $R_{\text{load}} + jX_{\text{load}} = R_s - jX_s$.

Similarly, the maximum power transfer from a current source occurs when the parallel source impedance, $\mathbf{Z}_s$, is increased to the maximum possible value. This assumes the load impedance is fixed and the source impedance can be changed. In the more common case where the load impedance varies and the source impedance is fixed, the maximum power transfer occurs when the load and source impedances are complex conjugates.

The maximum power transfer in any circuit occurs when the load impedance equals the complex conjugate of the Norton or Thevenin equivalent impedance.

21. SUPERPOSITION THEOREM

The principle of *superposition* is that the response of (that is, the voltage across or current through) a linear circuit element in a network with multiple independent sources is equal to the response obtained if each source is considered individually and the results summed. The steps involved in determining the desired quantity follow.

step 1: Replace all sources except one by their internal resistances. Ideal current sources are replaced by open circuits. Ideal voltage sources are replaced by short circuits.

step 2: Compute the desired quantity, either voltage or current, for the element in question due to the single source.

step 3: Repeat steps 1 and 2 for each of the sources in turn.

step 4: Sum the calculated values obtained for the current or voltage obtained in step 2. The result is the actual value of the current or voltage in the element for the complete circuit.

Superposition is not valid for circuits in which the following conditions exist.

- The capacitors have an initial charge (i.e., an initial voltage) not equal to zero. This principle can be used if the charge is treated as a separate voltage source and the equivalent circuit analyzed.
- The inductors have an initial magnetic field (i.e., an initial current) not equal to zero. This principle can be used if the energy in the magnetic field is treated as a separate current source and this equivalent circuit analyzed.
- Dependent sources are used.

22. MILLER'S THEOREM

A standard two-port network is shown in Fig. 29.13(a). *Miller's theorem* states that if an admittance, Y, is connected between the input and output terminals of a two-port network, as shown in Fig. 29.13(b), the output voltage is linearly dependent on the input voltage. Further, the circuit can be transformed as shown in Fig. 29.13(c). The transformation equations, with A as a constant that must be determined by independent means, are

$$Y_1 = Y(1 - A) \quad 29.36$$

$$Y_2 = Y\left(1 - \frac{1}{A}\right) \quad 29.37$$

Similar formulas can be derived from Eqs. 29.36 and 29.37 for impedances Z_1 and Z_2, though the theorem is not often used in such a manner. Miller's theorem is useful in transistor high-frequency amplifiers, among other applications.

23. KIRCHHOFF'S LAWS

Once an equivalent circuit is determined and reduced to its simplest form, Kirchhoff's voltage and current laws are used to determine the circuit behavior. If the KVL and KCL equations are written in terms of instantaneous quantities, $v(t)$ and $i(t)$, a differential equation results with an order equal to the number of independent energy-storing devices in the circuit. The solution of this equation results in a steady-state component and a transient component, which decays with time. If the equation is written in terms of phasor quantities, $\mathbf{V}$ and $\mathbf{I}$, an algebraic equation results. The solution of this equation alone results in a steady-state component. For any equation with an order higher than two, the most efficient solution method becomes a computer-aided design software package.

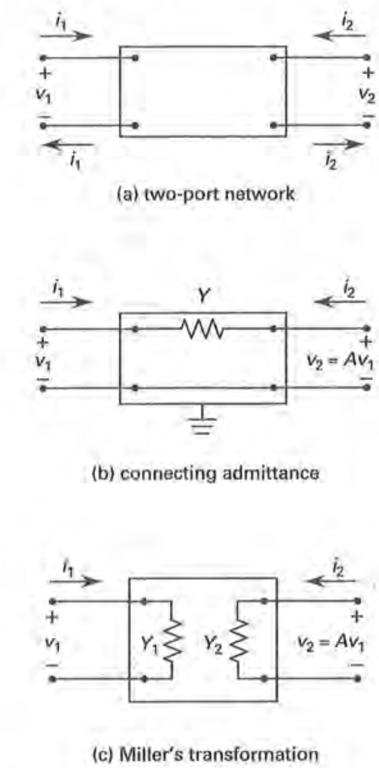


Figure 29.13 *Miller's Theorem*

24. KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law (KVL) states that the algebraic sum of the phasor voltages around any closed path within a circuit or network is zero. Stated in terms of the voltage rises and drops, the sum of the phasor voltage rises equals the sum of the phasor voltage drops around any closed path within a circuit or network. The method for applying KVL follows.

step 1: Identify the loop.

step 2: Pick a loop direction.[6]

step 3: Assign the loop current in the direction picked in step 2.

step 4: Assign voltage polarities consistent with the loop current direction in step 3.

step 5: Apply KVL to the loop using Ohm's law to express the voltages across each circuit element.

step 6: Solve the equation for the desired quantity.

[6]Clockwise is the direction chosen as the standard in this text. Any direction is allowed. The resulting mathematics determines the correct direction of current flow and voltage polarities. Consistency is the key to preventing calculation errors.

25. KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law (KCL) states that the algebraic sum of the phasor currents at any node is zero. Stated in terms of the currents out of and into a node, the sum of the currents directed out of a node must equal the sum of the currents directed into the same node. The method for applying KCL follows.

step 1: Identify the nodes and pick a reference or datum node.

step 2: Label the node-to-datum voltage for each unknown node.

step 3: Pick a current direction for each path at every node.[7]

step 4: Apply KCL to the nodes using Ohm's law to express the currents through each circuit branch.

step 5: Solve the equations for the desired quantity.

26. LOOP ANALYSIS

The *loop-current* method is a systematic network-analysis procedure that uses currents as the unknowns. It is also called *mesh analysis* or the *Maxwell loop-current* method. The method uses Kirchhoff's voltage law and is performed on planar networks. It requires $n-1$ simultaneous equations for an n-loop system. The method's steps are as follows.

step 1: Select $n-1$ loops, that is, one loop less than the total number of possible loops.

step 2: Assign current directions for the selected loops. (While this is arbitrary, clockwise directions will always be chosen in this text for consistency.) Any incorrectly chosen current direction will cause a negative result when the simultaneous equations are solved in step 4. Show the direction of the current with an arrow.

step 3: Write Kirchhoff's voltage law for each of the selected loops. Assign polarities based on the direction of the loop current. The voltage of a source is positive when the current flows out of the positive terminal, that is, from the negative terminal to the positive terminal inside the source. The selected direction of the loop current always results in a voltage drop in the resistors of the loop (see Fig. 26.6(c)). Where two loop currents flow through an element, they are summed to determine the voltage drop in that element, using the direction of the current in the loop for which the equation is being written as the positive (i.e., correct) direction. Any incorrect direction for the loop current will be indicated by a negative sign in the solution in step 4.

step 4: Solve the $n-1$ equations from step 3 for the unknown currents.

step 5: If required, determine the actual current in an element by summing the loop currents flowing through the element. Sum the absolute values to obtain the correct magnitude. The correct direction is given by the loop current with the positive value.

27. NODE ANALYSIS

The *node-voltage* method is a systematic network-analysis procedure that uses voltages as the unknowns. The method uses Kirchhoff's current law. It requires $n-1$ equations for an n-principal node system (equations are not necessary at simple nodes, that is, nodes connecting only two circuit elements). The method's steps are as follows.

step 1: Simplify the circuit, if possible, by combining resistors in series or parallel or by combining current sources in parallel. Identify all nodes. The minimum number of equations required will be $n-1$ where n represents the number of principal nodes.

step 2: Choose one node as the reference node, that is, the node that will be assumed to have ground potential (0 V). To minimize the number of terms in the equations, select the node with the largest number of circuit elements to serve as the reference node.

step 3: Write Kirchhoff's current law for each principal node except the reference node, which is assumed to have a zero potential.

step 4: Solve the $n-1$ equations from step 3 to determine the unknown voltages.

step 5: If required, use the calculated node voltages to determine any branch current desired.

28. DETERMINATION OF METHOD

The method used to analyze an electrical network depends on the circuit elements and their configurations. The loop-current method employing Kirchhoff's voltage law is used in circuits without current sources. The node-voltage method employing Kirchhoff's current law is used in circuits without voltage sources. When both types of sources are present, one of two methods may be used.

method 1: Use each of Kirchhoff's laws, assigning voltages and currents as needed, and substitute any known quantity into the equations as written.

[7]Current flow out of the node is the direction chosen as the standard for positive current in this text. Any direction is allowed. The resulting mathematics determines the correct direction of current flow and voltage polarities. Consistency is the key to preventing calculation errors.

method 2: Use source transformation or source shifting to change the appearance of the circuit so that it contains only the desired sources, that is, voltage sources when using KVL and current sources when using KCL. *Source shifting* is manipulating the circuit so that each voltage source has a resistor in series and each current source has a resistor in parallel.

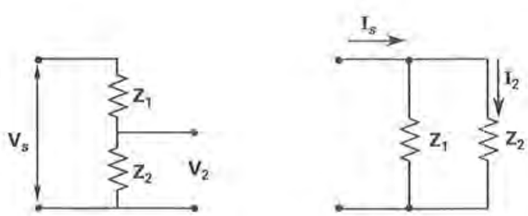
Use the method that results in the least number of equations. The loop-current method produces $n-1$ equations where n is the total number of loops. The node-voltage method produces $n-1$ equations where n is the number of principal nodes. Count the number of loops and the number of principal nodes prior to writing the equations. Whichever is least determines the method used.

Additional methods exist, some of which require less work. The advantage of the loop-current and node-voltage methods is that they are systematic and thus guarantee a solution.

29. VOLTAGE AND CURRENT DIVIDERS

At times, a source voltage will not be at the required value for the operation of a given circuit. For example, the household voltage of 120 V is too high to properly bias electronic circuitry. One method of obtaining the required voltage without using a transformer is to use a *voltage divider*. A voltage divider is illustrated in Fig. 29.14(a). The voltage across impedance 2 is

$$\mathbf{V}_2 = \mathbf{V}_s \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \right) \qquad 29.38$$



(a) voltage divider (b) current divider

Figure 29.14 Divider Circuits

If the fraction of the source voltage, $\mathbf{V}_s$, is found as a function of $\mathbf{V}_2$, the result is known as the *gain* or the *voltage-ratio transfer function*. Rearranging Eq. 29.38 to show the specific relationship between $\mathbf{Z}_1$ and $\mathbf{Z}_2$ gives

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{1}{1 + \frac{\mathbf{Z}_1}{\mathbf{Z}_2}} \qquad 29.39$$

A specific current can be obtained through an analogous circuit called a *current divider*, shown in Fig. 29.14(b). The current through impedance 2 is

$$\mathbf{I}_2 = \mathbf{I}_s \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} \right) = \mathbf{I}_s \left(\frac{\mathbf{G}_2}{\mathbf{G}_1 + \mathbf{G}_2} \right) \qquad 29.40$$

30. STEADY-STATE AND TRANSIENT IMPEDANCE ANALYSIS

When the electrical parameters of a circuit do not change with time, the circuit is said to be in a *steady-state* condition. In DC circuits, this condition exists when the magnitude of the parameter is constant. In AC circuits, this condition exists when the frequency is constant. A *transient* is a temporary phenomenon occurring prior to a network reaching steady state.

DC steady-state impedance analysis is based on constant voltages and currents and, therefore, time derivatives of zero. The DC impedances are

- Resistance:

$$Z_R|_{\text{DC}} = R \qquad 29.41$$

- Inductance:

$$v = L\left(\frac{di}{dt}\right) = L(0) = 0 \qquad 29.42$$

$$Z = \frac{v}{i} = \frac{0}{i} = 0 \qquad 29.43$$

$$Z_L|_{\text{DC}} = 0 \quad \text{[short circuit]} \qquad 29.44$$

- Capacitance:

$$i = C\left(\frac{dv}{dt}\right) = C(0) = 0 \qquad 29.45$$

$$Z = \frac{v}{i} = \frac{v}{0} \rightarrow \infty \qquad 29.46$$

$$Z_C|_{\text{DC}} = \infty \quad \text{[open circuit]} \qquad 29.47$$

AC steady-state impedance analysis is based on the phasor form where $df(t)/dt = j\omega f(t)$. The AC steady-state impedances are

- Resistance:

$$Z_R|_{\text{AC}} = R \qquad 29.48$$

- Inductance:

$$v = L\left(\frac{di}{dt}\right) = Lj\omega i \qquad 29.49$$

$$Z = \left(\frac{v}{i}\right) = \frac{Lj\omega i}{i} = j\omega L \qquad 29.50$$

$$Z_L|_{\text{AC}} = j\omega L \qquad 29.51$$

* Capacitance:

$$i = C\left(\frac{dv}{dt}\right) = Cj\omega v \qquad 29.52$$

$$Z = \frac{v}{i} = \frac{v}{Cj\omega v} = \frac{1}{j\omega C} \qquad 29.53$$

$$Z_C|_{\text{AC}} = \frac{1}{j\omega C} \qquad 29.54$$

Transient impedance analysis is based on the phasor form with the complex variable s substituted for $j\omega$. The variable $s = \sigma + j\omega$ and is the same as the Laplace transform variable. The derivative is $df(t)/dt = sf(t)$. The transient impedances are

* Resistance:

$$Z_R = R \qquad 29.55$$

* Inductance:

$$Z_L = sL \qquad 29.56$$

* Capacitance:

$$Z_C = \frac{1}{sC} \qquad 29.57$$

Transient impedances are useful in the analysis of stability as well as during transients.

31. TWO-PORT NETWORKS

An electric circuit or network is often used to connect a source to a load, modifying the source energy or information in a given manner as required or desired by the load. If the circuit is such that the current flow into one terminal is equal to the current flow out of a second terminal, the terminal pair is called a *port*. Any number of ports is possible for a given network, the most common being the *two-port network* shown in Fig. 29.15. Four variables exist in this representation: v_1, v_2, i_1, and i_2. The port using the subscript 1 is the *input port*, and the port using the subscript 2 is the *output port*.

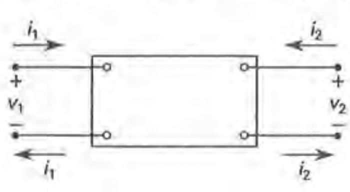


Figure 29.15 *Two-Port Network*

Two-port networks can represent either three- or four-terminal devices, such as transistors or transformers (see Fig. 29.16). The electrical properties of the device within the network are described by a set of parameters identified by double subscripts, the first representing the row and the second the column of a matrix. The *parameter type* is determined by the selection of independent variables for which to solve.

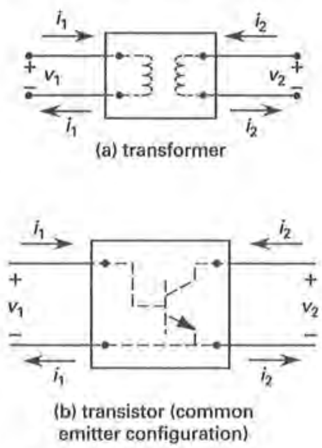


Figure 29.16 *Transformer and Transistor Two-Port Networks*

Open-circuit impedance parameters, or z parameters, occur when the two currents are selected as the independent variables. The z parameter model and *deriving equations* for the individual parameters are shown in Fig. 29.17. The applicable matrix and resulting equations follow.

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad 29.58$$

$$v_1 = z_{11}i_1 + z_{12}i_2 \qquad 29.59$$

$$v_2 = z_{21}i_1 + z_{22}i_2 \qquad 29.60$$

Short-circuit admittance parameters, or y parameters, occur when the two voltages are selected as the independent variables. The y parameter model and *deriving equations* for the individual parameters are shown in Fig. 29.18. The applicable matrix and resulting equations follow.

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad 29.61$$

$$i_1 = y_{11}v_1 + y_{12}v_2 \qquad 29.62$$

$$i_2 = y_{21}v_1 + y_{22}v_2 \qquad 29.63$$

Hybrid parameters, or h parameters, occur when one voltage and one current are picked as the independent variables. The h parameter model and *deriving equations* for the individual parameters are shown in

Fig. 29.19. The applicable matrix and resulting equations follow.

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \qquad 29.64$$

$$v_1 = h_{11}i_1 + h_{12}v_2 \qquad 29.65$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \qquad 29.66$$

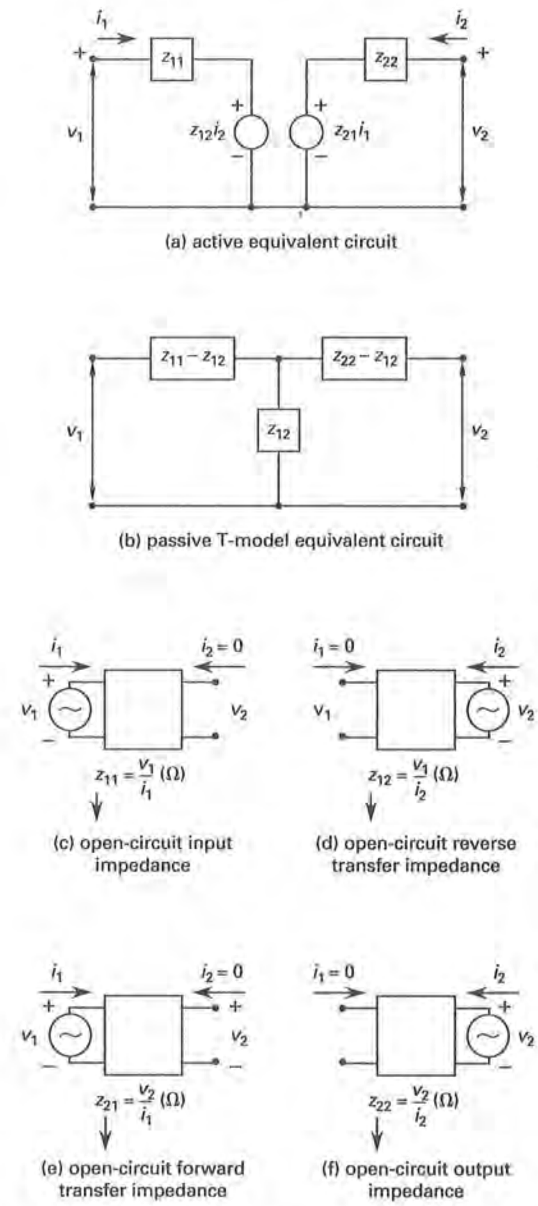


Figure 29.17 *Impedance Model Parameters*

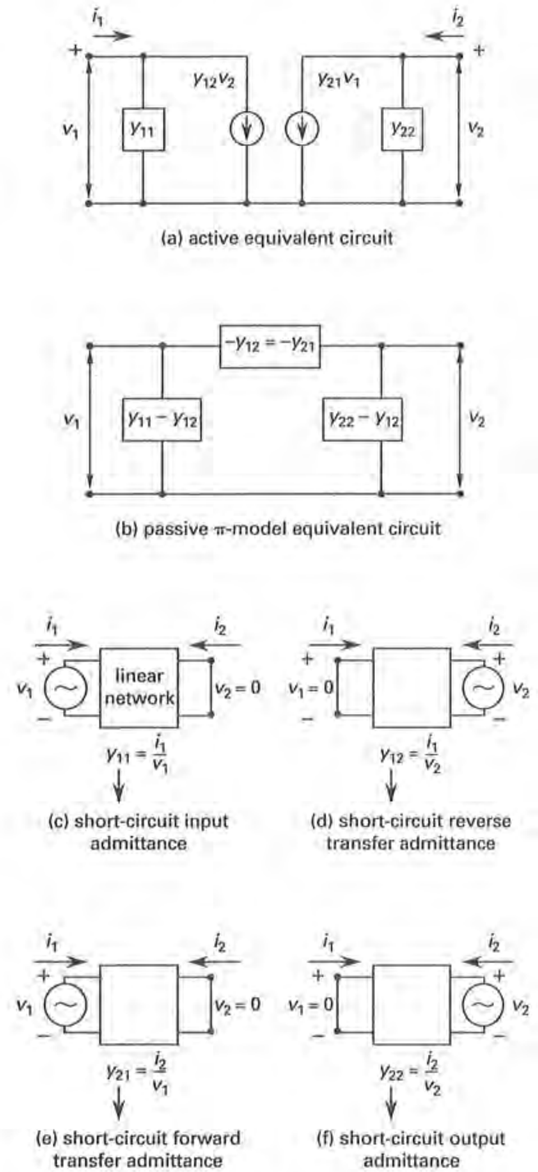


Figure 29.18 *Admittance Model Parameters*

Inverse hybrid parameters, or g parameters, also occur when one voltage and one current are picked as the independent variables. The applicable matrix and resulting equations follow.

$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \qquad 29.67$$

$$i_1 = g_{11}v_1 + g_{12}i_2 \qquad 29.68$$

$$v_2 = g_{21}v_1 + g_{22}i_2 \qquad 29.69$$

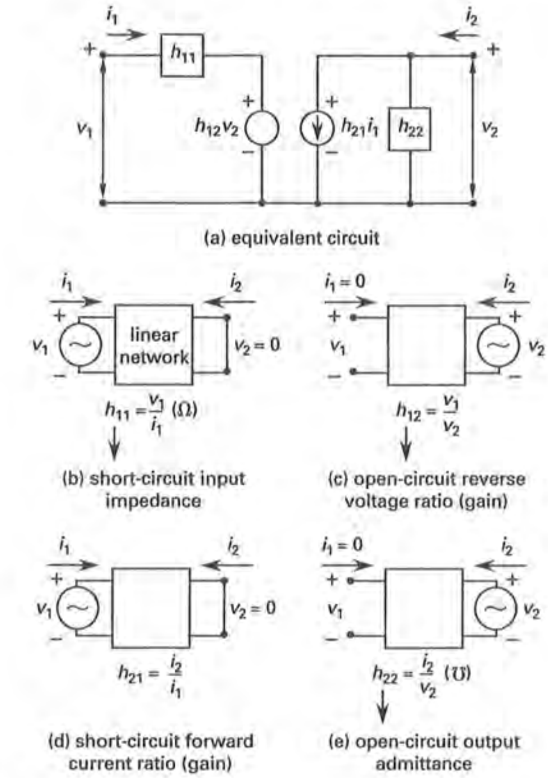


Figure 29.19 *Hybrid Model Parameters*

Other parameter representations are possible. Some examples include the *transmission line* or *chain parameters*, and the *inverse transmission line parameters.* The choice of parameter type depends on numerous factors, including whether or not all the parameters exist or are defined for a given network, the mathematical convenience of using a certain set of parameters, and the accuracy of sensitivity of the parameters when considered as part of the overall circuit to which it is connected. The parameters and their deriving equations are summarized in Table 29.3. The formulas for conversion between the parameter types are given in App. 29.A.

Table 29.3 *Two-Port Network Parameters*

representation	deriving equations			
impedance $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$z_{11} = \frac{V_1}{I_1}$ $I_2 = 0$	$z_{12} = \frac{V_1}{I_2}$ $I_1 = 0$	$z_{21} = \frac{V_2}{I_1}$ $I_2 = 0$	$z_{22} = \frac{V_2}{I_2}$ $I_1 = 0$
admittance $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$y_{11} = \frac{I_1}{V_1}$ $V_2 = 0$	$y_{12} = \frac{I_1}{V_2}$ $V_1 = 0$	$y_{21} = \frac{I_2}{V_1}$ $V_2 = 0$	$y_{22} = \frac{I_2}{V_2}$ $V_1 = 0$
hybrid $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$h_{11} = \frac{V_1}{I_1}$ $V_2 = 0$	$h_{12} = \frac{V_1}{V_2}$ $I_1 = 0$	$h_{21} = \frac{I_2}{I_1}$ $V_2 = 0$	$h_{22} = \frac{I_2}{V_2}$ $I_1 = 0$
inverse hybrid $\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$	$g_{11} = \frac{I_1}{V_1}$ $I_2 = 0$	$g_{12} = \frac{I_1}{I_2}$ $V_1 = 0$	$g_{21} = \frac{V_2}{V_1}$ $I_2 = 0$	$g_{22} = \frac{V_2}{I_2}$ $V_1 = 0$
transmission or chain $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	$A = \frac{V_1}{V_2}$ $I_2 = 0$	$B = \frac{-V_1}{I_2}$ $V_2 = 0$	$C = \frac{I_1}{V_2}$ $I_2 = 0$	$D = \frac{-I_1}{I_2}$ $V_2 = 0$
inverse transmission $\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$	$\alpha = \frac{v_2}{v_1}$ $i_1 = 0$	$\beta = \frac{v_2}{i_1}$ $v_1 = 0$	$\gamma = \frac{-i_2}{v_1}$ $i_1 = 0$	$\delta = \frac{-i_2}{i_1}$ $v_i = 0$

30 Transient Analysis

Nomenclature

a	constant	–
A	natural response coefficient	–
b	constant	–
c	constant	–
C	capacitance	F
f	function	–
G	conductance	S
i, I	current	A
L	self-inductance	H
pf	power factor	–
P	power	W
q	instantaneous charge	C
Q	quality factor	–
R	resistance	Ω
s	Laplace transform variable	–
t	time	s
v, V	voltage	V
x	variable	–
X	reactance	Ω
y	variable	–
Y	admittance	S

Symbols

κ	integration constant	–
σ	damping, real part of s	–
τ	time constant	s
ϕ	angle	rad
ϕ	phase difference angle	rad
ω_0	zero resistance frequency	rad/s

Subscripts

0	initial
bat	battery
C	capacitor
d	delay
L	inductor
m	maximum
R	resistor
s	source
ss	steady-state
tr	transient

1. FUNDAMENTALS

Whenever a network or circuit undergoes a change, the currents and voltages experience a transitional period during which their properties shift from their former values to their new steady-state values. This time period is called a *transient*. The determination of a circuit's behavior during this period is *transient analysis*. There are three classes of transient problems, all based on the nature of the energy source. These are *DC switching transients*, *AC switching transients*, and *pulse transients*. *Switching transients* occur as a result of a change in topology of the circuit, that is, the physical elements within the circuit change (become connected or disconnected) as a result of the operation of a switch. The switch can be a physical switch or an electrical one, such as a transistor operating in the switching mode. A circuit's response to a switching transient is known as a *step response*.

Pulse transients involve a change in the current or voltage waveform, not in the topology of a circuit. A circuit's response to a pulse transient is known as an *impulse response*.

The energy storage elements in electric circuits are the capacitor and the inductor. When one of these is present, the mathematical representation of the circuit is a first-order differential equation of the form

$$f(t) = b\left(\frac{dx}{dt}\right) + cx \qquad 30.1$$

Either the current or the voltage can be represented as the dependent variable, that is, $f(t)$, in Eq. 30.1. For capacitors, the voltage is used because the energy storage is a function of voltage and a true differential equation results (the capacitor current is an integral

Circuit Theory

equation). For inductors, similar reasoning holds and the current is considered the dependent variable, $f(t)$. The solution to Eq. 30.1 is of the general form

$$x(t) = \kappa + Ae^{-\frac{t}{\tau}} \qquad 30.2$$

See Table 30.1 for the applicable equations for both the inductor and capacitor.

The term A in Eq. 30.2 is the natural response coefficient and is set to match the conditions at some known time in the transient, typically at the onset. The term κ is value-dependent on the forcing function, $f(t)$. The term τ is called the *time constant* and is equal to

$$\tau = \frac{b}{c} \qquad 30.3$$

The time constant for an RC circuit is

$$\tau = RC \qquad 30.4$$

The time constant for an RL circuit is

$$\tau = \frac{L}{R} \qquad 30.5$$

There are two parts to Eq. 30.2, or any solution to a first-order differential equation: the homogeneous (or complementary) solution and the particular solution. The homogeneous solution, that is, the solution where $f(t) = 0$, is called the *natural response* of the circuit and is represented by the exponential term in Eq. 30.2. The decay behavior of a general exponential is shown in Fig. 30.1. The particular solution is called the *forced response*, since it depends on the forcing function, $f(t)$, and is represented by κ in Eq. 30.2.

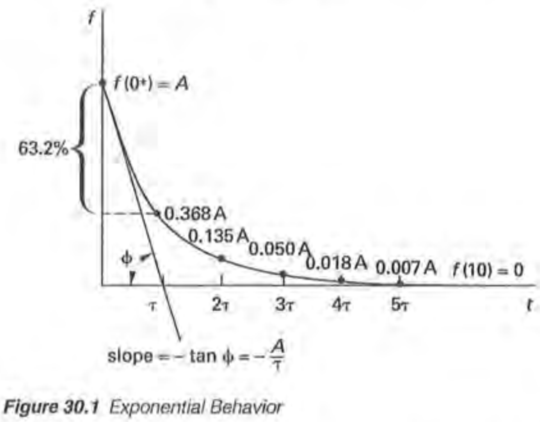


Figure 30.1 *Exponential Behavior*

When two independent energy storage elements are present, the mathematical representation of the circuit is a second-order differential equation of the form

$$f(t) = a\left(\frac{d^2x}{dt^2}\right) + b\left(\frac{dx}{dt}\right) + cx \qquad 30.6$$

There are three forms of the solution, depending on the magnitudes of the constants a, b, and c (see Ch. 31).

2. RESISTOR-CAPACITOR CIRCUITS: NATURAL RESPONSE

Consider the generic resistor-capacitor *source-free circuit* shown in Fig. 30.2. When the charged capacitor, C, is connected to the resistor, R, via a complete electrical path, the capacitor will discharge in an attempt to resist the change in voltage. In the process, the capacitor's stored electrical energy is dissipated in the resistor until no further energy remains and no current flows. This gradual decrease is the transient. The voltage follows the form of Fig. 30.1.

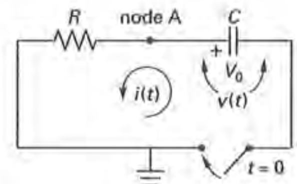


Figure 30.2 *Resistor-Capacitor Circuit: Natural Response*

Example 30.1

Determine the formula for the voltage in the circuit of Fig. 30.2 from $t = 0$ onward.

Solution

Write Kirchhoff's current law (KCL) for the simple node A between the resistor and the capacitor. The reference node is located at the switch, opposite node A. In keeping with the convention of this book, both currents are assumed to flow out of the node, giving

$$i_C + i_R = 0$$

Substitute the expression for current flow through a capacitor and the Ohm's law expression for current flow in a resistor.

$$C\left(\frac{dv}{dt}\right) + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \left(\frac{1}{RC}\right)v = 0$$

This is a homogeneous first-order linear differential equation. The solution could be written directly from the information in the mathematics chapters or by knowing that only an exponential form of v can be linearly combined with its derivative. In order to show the process, separate the variables and integrate both sides.

$$\frac{dv}{v} = -\left(\frac{1}{RC}\right) dt$$

$$\int \frac{1}{v} dv = \int -\left(\frac{1}{RC}\right) dt$$

$$\ln v = -\frac{1}{RC} t + \kappa \quad [\kappa \equiv \text{constant}]$$

$$v(t) = e^{-\left(\frac{1}{RC}\right)t+\kappa} = Ae^{-\frac{t}{RC}} = Ae^{-\frac{t}{\tau}}$$

This is the natural response of the system. To determine the constant A, the value of $v(t)$ at some time must be known. Since the initial voltage is V_0, the value of the constant A is

$$v(0) = V_0 = Ae^{-\frac{0}{\tau}} = A$$

The final solution is

$$v(t) = V_0 e^{-\frac{t}{RC}}$$

3. RESISTOR-CAPACITOR CIRCUITS: FORCED RESPONSE

Consider the generic resistor-capacitor circuit shown in Fig. 30.3. When the charged capacitor, C, is connected to the resistor, R, via a complete electrical path, the capacitor will discharge or charge, depending on the magnitude of V_{bat}, in an attempt to resist the change in voltage. In the process, the capacitor's stored electrical energy is either dissipated or enhanced until no further energy change occurs and no current flows. This gradual decrease or increase is the transient. After the passage of time equal to five time constants, 5τ, the final value is within less than 1% of its steady-state value and the transient is considered complete. The voltage follows the form of Fig. 30.1, or its inverse, with the exception that the final steady-state voltage is the voltage of the driving force.

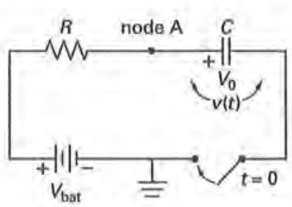


Figure 30.3 *Resistor-Capacitor Circuit: Forced Response*

Example 30.2

Determine the formula for the voltage in the circuit of Fig. 30.3 from $t = 0$ onward.

Solution

Write KCL for the simple node A between the resistor and the capacitor. The node between the negative terminal of the battery and the capacitor, opposite node A, is the reference node. Both currents are assumed to flow out of the node in keeping with the convention of this text, giving

$$i_C + i_R = 0$$

Substitute the expression for current flow through a capacitor and the Ohm's law expression for current flow in a resistor.

$$C\left(\frac{dv}{dt}\right) + \frac{v - V_{\text{bat}}}{R} = 0$$

$$\frac{dv}{dt} + \left(\frac{1}{RC}\right)(v - V_{\text{bat}}) = 0$$

$$\frac{dv}{dt} + \left(\frac{1}{RC}\right)v = \left(\frac{1}{RC}\right)V_{\text{bat}}$$

This is a nonhomogeneous first-order linear differential equation. The known constant to the right of the equal sign is called the *forcing* or *driving function*. Such an equation is solved in two steps. The first step is to determine the homogeneous, or complementary, solution, that is, the solution with the forcing function equal to zero. In Ex. 30.1 this was determined to be

$$v_{\text{tr}}(t) = Ae^{-\frac{t}{RC}}$$

This is the natural response of the system and represents the transient portion of the solution. The next step is to determine the particular solution, or forced response, of the system. Since the particular solution is the steady-state solution, the expectation is that the solution would be of the same form as the forcing function. Thus, assuming a constant, κ, as the particular solution and substituting this into the KCL expression for v gives

$$v_{\text{ss}} = \kappa$$

$$\frac{dv}{dt} + \left(\frac{1}{RC}\right)v = \left(\frac{1}{RC}\right)V_{\text{bat}}$$

$$\frac{dv_{\text{ss}}}{dt} + \left(\frac{1}{RC}\right)v_{\text{ss}} = \left(\frac{1}{RC}\right)V_{\text{bat}}$$

$$\frac{d\kappa}{dt} + \left(\frac{1}{RC}\right)\kappa = \left(\frac{1}{RC}\right)V_{\text{bat}}$$

$$0 + \left(\frac{1}{RC}\right)\kappa = \left(\frac{1}{RC}\right)V_{\text{bat}}$$

$$\kappa = V_{\text{bat}}$$

The particular solution is

$$v(\infty) = v_{\text{ss}} = V_{\text{bat}}$$

Circuit Theory

The total solution is the combination of the complementary (homogeneous) and particular (nonhomogeneous) solutions, that is, the combination of the transient and steady-state solutions. The total solution is

$$v(t) = V_{\text{bat}} + Ae^{-\frac{t}{RC}}$$

The order of the transient and steady-state solutions is unimportant. The exponential term (the transient term) is often listed to the right in an equation.

The natural response coefficient A is again determined by the initial condition $v(0) = V_0$. Thus,

$$v(0) = V_0 = V_{\text{bat}} + Ae^{-\frac{0}{RC}} = V_{\text{bat}} + A$$

$$A = V_0 - V_{\text{bat}}$$

The final solution is

$$v(t) = V_{\text{bat}} + (V_0 - V_{\text{bat}})\, e^{-\frac{t}{RC}}$$

The voltage $v(t)$ is the voltage at node A, that is, the voltage across the capacitor.

4. RESISTOR-INDUCTOR CIRCUITS: NATURAL RESPONSE

Consider the generic resistor-inductor source-free circuit shown in Fig. 30.4. When the inductor, L, whose magnetic field is at a maximum due to initial current, I_0, is connected to the resistor, R, via a complete electrical path, the inductor will discharge in an attempt to resist the change in current. In the process, the inductor's stored magnetic energy is dissipated in the resistor until no further energy remains and no current flows. This gradual decrease is the transient. After the passage of time equal to five time constants, 5τ, the final value is within less than 1% of its steady-state value and the transient is considered complete. The current follows the form of Fig. 30.1.

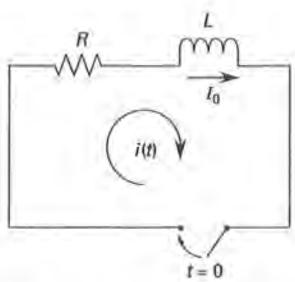


Figure 30.4 *Resistor-Inductor Circuit: Natural Response*

Example 30.3

Determine the formula for the current in the circuit of Fig. 30.4 from $t = 0$ onward.

Solution

Write Kirchhoff's voltage law (KVL) for the loop containing the resistor and the inductor. In keeping with the convention of this book, the reference direction is clockwise, giving

$$v_L + v_R = 0$$

Substitute the expression for voltage across an inductor and the Ohm's law expression for voltage drop in a resistor.

$$L\left(\frac{di}{dt}\right) + iR = 0$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = 0$$

This is a homogeneous first-order linear differential equation. The solution could be written directly from the information in the mathematics chapters or by knowing that only an exponential form of i can be linearly combined with its derivative. The process is similar to that in Ex. 30.1.

$$\frac{di}{i} = -\left(\frac{R}{L}\right)dt$$

$$\int\left(\frac{1}{i}\right)di = \int -\left(\frac{R}{L}\right)dt$$

$$\ln i = -\left(\frac{R}{L}\right)t + \kappa \quad [\kappa \equiv \text{constant}]$$

$$i(t) = e^{-\left(\frac{R}{L}\right)t+\kappa} = Ae^{-\frac{t}{L/R}} = Ae^{-\frac{t}{\tau}}$$

This is the natural response of the system. To determine the constant A, the value of $i(t)$ at some time must be known. Since the initial current is I_0, the value of the constant A is

$$i(0) = I_0 = Ae^{-\frac{0}{\tau}} = A$$

The final solution is

$$i(t) = I_0 e^{-\frac{t}{L/R}}$$

5. RESISTOR-INDUCTOR CIRCUITS: FORCED RESPONSE

Consider the generic resistor-inductor circuit shown in Fig. 30.5. When the inductor, L, whose magnetic field is at a maximum due to initial current, I_0, is connected to the resistor, R, via a complete electrical path, the inductor's magnetic field will collapse or expand in an attempt to resist the change in current. In the process, the inductor's stored magnetic energy is either dissipated or enhanced until no further energy change occurs and no current flows. This gradual decrease or increase is the transient. After the passage of time equal to five time constants, 5τ, the final value is within less than 1% of its steady-state value and the transient is considered complete. The voltage follows the form of Fig. 30.1 or its inverse, with the exception that the final steady-state current is the current resulting from the driving force and is determined by the remainder of the circuit.

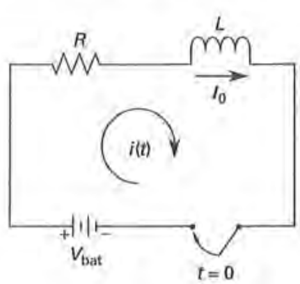


Figure 30.5 Resistor-Inductor Circuit: Forced Response

Example 30.4

Determine the formula for the current in the circuit of Fig. 30.5 from $t = 0$ onward.

Solution

Write KVL for the loop containing the resistor and the inductor. In keeping with the convention of this book, the reference direction is clockwise, giving

$$v_L + v_R - V_{bat} = 0$$

Substitute the expression for voltage across an inductor and the Ohm's law expression for voltage drop in a resistor.

$$L\left(\frac{di}{dt}\right) + iR - V_{bat} = 0$$

$$\frac{di}{dt} + \left(\frac{R}{L}\right)i = \left(\frac{1}{L}\right)V_{bat}$$

$$\frac{di}{dt} + \left(\frac{1}{\frac{L}{R}}\right)i = \left(\frac{1}{L}\right)V_{bat}$$

This is a nonhomogeneous first-order linear differential equation. The known constant to the right of the equal sign is called the *forcing* or *driving function*. Such an equation is solved in two steps. The first step is to determine the particular solution, or forced response, of the system. Since the particular solution is the steady-state solution, the expectation is that the solution would be of the same form as the forcing function. Thus, assuming a constant, κ, as the particular solution and substituting this into the KVL expression for i gives

$$i_{ss} = \kappa$$

$$\frac{di}{dt} + \left(\frac{1}{\frac{L}{R}}\right)i = \left(\frac{1}{L}\right)V_{bat}$$

$$\frac{di_{ss}}{dt} + \left(\frac{1}{\frac{L}{R}}\right)i_{ss} = \left(\frac{1}{L}\right)V_{bat}$$

$$\frac{d\kappa}{dt} + \left(\frac{1}{\frac{L}{R}}\right)\kappa = \left(\frac{1}{L}\right)V_{bat}$$

$$0 + \left(\frac{1}{\frac{L}{R}}\right)\kappa = \left(\frac{1}{L}\right)V_{bat}$$

$$\kappa = \left(\frac{1}{R}\right)V_{bat}$$

The particular solution is

$$i(\infty) = i_{ss} = \left(\frac{1}{R}\right)V_{bat}$$

The next step is to determine the homogeneous, or complementary, solution, that is, the solution with the forcing function equal to zero. In Ex. 30.3 this was determined to be

$$i_{tr}(t) = Ae^{-\frac{t}{L/R}}$$

This is the natural response of the system and represents the transient portion of the solution. The total solution is the combination of the particular (nonhomogeneous) and the complementary (homogeneous) solution, that is, the combination of the steady-state and transient solutions. The total solution is

$$i(t) = \left(\frac{1}{R}\right)V_{bat} + Ae^{-\frac{t}{L/R}}$$

The natural response coefficient A is again determined by the initial condition $i(0) = I_0$. Thus,

$$i(0) = I_0 = \left(\frac{1}{R}\right)V_{bat} + Ae^{-\frac{0}{L/R}} = \left(\frac{1}{R}\right)V_{bat} + A$$

$$A = I_0 - \left(\frac{1}{R}\right)V_{bat}$$

The final solution is

$$i(t) = \left(\frac{1}{R}\right)V_{bat} + \left(I_0 - \left(\frac{1}{R}\right)V_{bat}\right)e^{-\frac{t}{L/R}}$$

The current $i(t)$ is the current through the inductor.

Circuit Theory

6. RC AND RL CIRCUITS: SOLUTION METHOD

The method presented in Secs. 30-2 through 30-5 is valid for any circuit that can be represented by a single equivalent capacitor or inductor and a single equivalent resistor. The solutions were obtained for DC transients. Table 30.1 is a summary of DC transient responses.[1] AC transients are handled in the same manner except that the forcing function, that is, the source, is represented as a sinusoid. Phasor methods, or sinusoidal methods, are then used to solve the resulting differential equation. If the switch closure were instead treated as a pulse from the source and represented by the unit step function, $u(t)$, the voltage applied at $t = 0^+$ would have been $V_{\text{bat}}u(t)$ and the equations handled in the same manner. The steps for solving single resistor and single capacitor/inductor circuits follow.

step 1: Find the steady-state response from the particular solution of the nonhomogeneous equation. The solution will be of the same form as the forcing function.[2] Regardless of transient type, the result will be a steady-state value, κ.

step 2: Determine the transient response from the homogeneous equation or from the known parameters of the time constant. The transient response will be of the form $Ae^{-\frac{t}{\tau}}$.

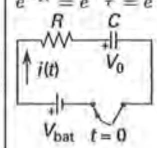
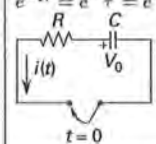
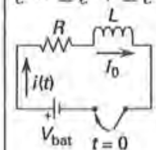
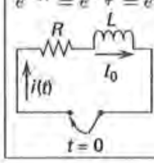
step 3: Sum the steady-state and transient solutions, that is, the particular and complementary solutions, to obtain the total solution. This will be of the form $\kappa + Ae^{-\frac{t}{\tau}}$.

step 4: Determine the initial value from the circuit initial conditions. Use this information to calculate the constant A.

step 5: Write the final solution.

This method is used for resistor-capacitor circuits which are primarily used in electronics. Inductors tend to be large and change value with both temperature and time. They do not lend themselves well to miniaturization. Capacitors are more easily manufactured on integrated circuits. Inductors are important in power circuits that handle large amounts of current or high voltages and in transformers.

Table 30.1 Transient Response

type of circuit	response
series RC, charging	
$\tau = RC$ $e^{-N} = e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$ 	$V_{\text{bat}} = v_R(t) + v_C(t)$ $i(t) = \left(\frac{V_{\text{bat}} - V_0}{R}\right)e^{-N}$ $v_R(t) = i(t)R$ $= (V_{\text{bat}} - V_0)e^{-N}$ $v_C(t) = V_0 + (V_{\text{bat}} - V_0)$ $\times (1 - e^{-N})$ $Q_C(t) = C\big(V_0 + (V_{\text{bat}} - V_0)$ $\times (1 - e^{-N})\big)$
series RC, discharging	
$\tau = RC$ $e^{-N} = e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$ 	$0 = v_R(t) + v_C(t)$ $i(t) = \left(\frac{V_0}{R}\right)e^{-N}$ $v_R(t) = -V_0e^{-N}$ $v_C(t) = V_0e^{-N}$ $Q_C(t) = CV_0e^{-N}$
series RL, charging	
$\tau = \frac{L}{R}$ $e^{-N} = e^{-\frac{t}{\tau}} = e^{-\frac{t}{L/R}}$ 	$V_{\text{bat}} = v_R(t) + v_L(t)$ $i(t) = I_0e^{-N}$ $+ \left(\frac{V_{\text{bat}}}{R}\right)(1 - e^{-N})$ $v_R(t) = i(t)R$ $= I_0Re^{-N}$ $+ V_{\text{bat}}(1 - e^{-N})$ $v_L(t) = (V_{\text{bat}} - I_0R)e^{-N}$
series RL, discharging	
$\tau = \frac{L}{R}$ $e^{-N} = e^{-\frac{t}{\tau}} = e^{-\frac{t}{L/R}}$ 	$0 = v_R(t) + v_L(t)$ $i(t) = I_0e^{-N}$ $v_R(t) = I_0Re^{-N}$ $v_L(t) = -I_0Re^{-N}$

7. RISE TIME

When the input of a capacitive or inductive circuit like the one shown in Fig. 30.6(a) undergoes a step change, the circuit response is called a *step response*. The step response is illustrated in Fig. 30.6(b). The speed of the response may be quantified in several ways. The time constant, τ, is one possibility. The shorter the time constant, the less time it takes for the circuit to reach a specified value. The value of the time constant is dependent on the circuit. A second possibility is the *rise*

[1]The final expression of the response given in the table appears to differ from that given in step 3 of the solution method. The table uses the common form $1 - e^{-t/\tau}$. The forms are equivalent and can be interchanged. Both contain a steady-state and transient component. The steady-state term κ can be found in the $1 - e^{-t/\tau}$ form by letting $t \to \infty$.

[2]If the excitation is constant, the form of the particular solution is a constant and can be determined by simply open-circuiting the terminals of the capacitor and determining the voltage present. This voltage will be the steady-state voltage to which the capacitor will be driven.

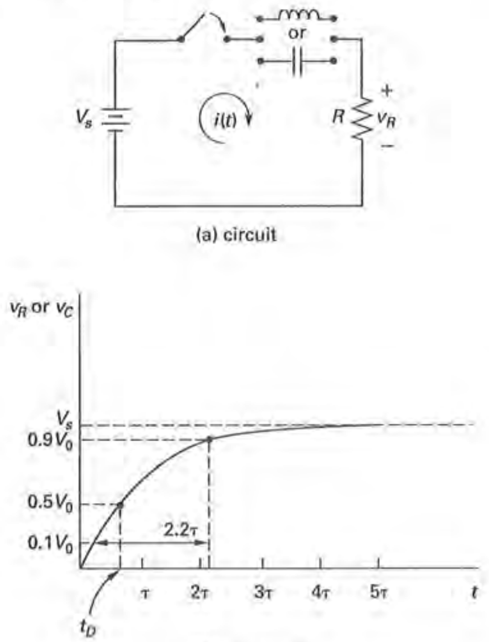
time, which is a *defined quantity* applicable to responses in general regardless of the circuit. Let t_{10} be the time a response has reached 10% of its final steady-state value. Let t_{90} be the time a response has reached 90% of its final steady-state value. The rise time, t_r, is then defined as

$$t_r = t_{90} - t_{10} \qquad 30.7$$

The rise time is related to the time constant by Eq. 30.8.

$$t_r = 2.2\tau \qquad 30.8$$

Another defined quantity is the *time delay*, t_d, which is defined as the time for a response to reach 50% of its final steady-state value. The rise and delay times and their relationship to the time constant are shown graphically in Fig. 30.6(b).



(b) response

Figure 30.6 *Rise Time*

8. DAMPED OSCILLATIONS: RINGING

Some circuits when energized undergo voltage or current oscillations that decrease in magnitude over time. Such oscillation is called *ringing*. An example of such a circuit and its response to a switch closure at $t = 0$ is shown in Fig. 30.7. The capacitor is initially charged to V_0 and the inductor has no initial magnetic field. Writing KVL for the circuit yields

$$\frac{1}{C}\int_0^t i\,dt + iR + L\left(\frac{di}{dt}\right) = V_0 \qquad 30.9$$

Note that the current is the rate of change of the charge on the capacitor, that is, $i = dq/dt$. Rearranging mathematically gives

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\right)\left(\frac{dq}{dt}\right) + \left(\frac{1}{LC}\right)q = \frac{q_0}{LC} \qquad 30.10$$

Representing this nonhomogeneous second-order differential equation in the s domain, appropriate for Laplace transform analysis, results in the following characteristic equation.[3]

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0 \qquad 30.11$$

The roots of Eq. 30.11, found from the quadratic equation, take one of three possible forms: (1) real and unequal, (2) real and equal, or (3) complex conjugates.

Ringing occurs for case 3 with roots given by

$$s_1, s_2 = -\sigma_1 \pm j\omega_1 \qquad 30.12$$

The solution to Eq. 30.10 for the ringing circuit is

$$v_C(t) = V_0 e^{-\sigma_1 t}\cos\omega_1 t \qquad 30.13$$

The term σ_1 is called the *damping*. The exponential, $e^{-\sigma_1 t}$, is termed the *envelope* of the waveform. The frequency of the oscillation is given by ω_1. These quantities are illustrated in Fig. 30.7(b). The damping and frequency are found in terms of the circuit parameters from Eqs. 30.14 and 30.15.

$$\sigma_1 = \frac{R}{2L} \qquad 30.14$$

$$\omega_1 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \qquad 30.15$$

The damping and frequency are also defined in terms of the *quality factor*, Q, of the inductor coil. If ω_0 is the frequency of the circuit when $R = 0$, the quality factor is

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{X_L}{R} = \frac{X_C}{R} \qquad 30.16$$

The damping and frequency can be defined as

$$\sigma_1 = \frac{\omega_0}{2Q} \qquad 30.17$$

$$\omega_1 = \omega_0\sqrt{1 - \left(\frac{1}{2Q}\right)^2} \qquad 30.18$$

[3]A second-order equation should be expected since the circuit contains two independent energy storage devices.

Circuit Theory

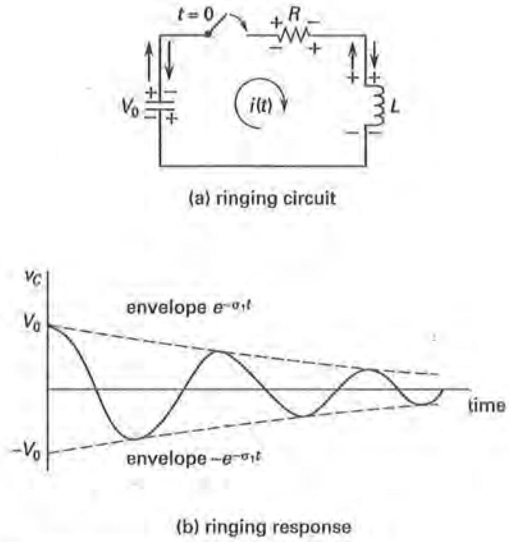


Figure 30.7 Ringing Circuit

9. SUSTAINED OSCILLATIONS: RESONANCE

Capacitors and inductors within the same circuit exchange energy. In the ideal, that is, lossless, circuit shown in Fig. 30.8, this exchange can continue indefinitely and is known as a *sustained oscillation*.[4]

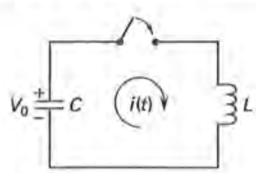


Figure 30.8 Resonant Circuit

The characteristic equation for the circuit in Fig. 30.8 is

$$s^2 + \frac{1}{LC} = 0 \qquad 30.19$$

The solution is

$$s_1, s_2 = \pm j\sqrt{\frac{1}{LC}} = \pm j\omega_0 \qquad 30.20$$

No damping term, σ_1, exists and the oscillations can continue unfettered. This unimpeded flow of energy naturally occurs at the frequency ω_0 and is called *resonance*.

[4]This type of circuit is more than academic and can be realized, or nearly so, with superconducting inductors.

In a real circuit, resistive components are present and will thus yield a damping term, σ_1, in the solution to the circuit equation. The result is that an external source of energy is required to make up the loss. Resonance is then defined as a phenomenon of an AC circuit whereby relatively large currents occur near certain frequencies, and a relatively unimpeded oscillation of energy from potential to kinetic occurs. The frequency at which this occurs is called the *resonant frequency*. At the resonant frequency the capacitive reactance equals the inductive reactance, that is, $X_C = X_L$, the current phase angle difference is zero ($\phi = 0$), and the power factor equals one (pf = 1).

10. RESONANT CIRCUITS

A *resonant circuit* has a zero current phase angle difference. This is equivalent to saying the circuit is purely resistive (i.e., the power factor is equal to one) in its response to an AC voltage. The frequency at which the circuit becomes purely resistive is the *resonant frequency*.

For frequencies below the resonant frequency, a series *RLC* circuit will be capacitive (leading) in nature; above the resonant frequency, the circuit will be inductive (lagging) in nature.

For frequencies below the resonant frequency, a parallel *GLC* circuit will be inductive (lagging) in nature; above the resonant frequency, the circuit will be capacitive (leading) in nature.

Circuits can become resonant in two ways. If the frequency of the applied voltage is fixed, the elements must be adjusted so that the capacitive reactance cancels the inductive reactance (i.e., $X_L - X_C = 0$). If the circuit elements are fixed, the frequency must be adjusted.

As Figs. 30.9 and 30.10 illustrate, a circuit approaches resonant behavior gradually. ω_1 and ω_2 are the *half-power points* (*70 percent points* or *3 dB points*) because at those frequencies, the power dissipated in the resistor is half of the power dissipated at the resonant frequency.

$$Z_{\omega 1} = \sqrt{2}R \qquad 30.21$$

$$I_{\omega 1} = \frac{V}{Z_{\omega 1}} = \frac{V}{\sqrt{2}R} = \frac{I_0}{\sqrt{2}} \qquad 30.22$$

$$P_{\omega 1} = I^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \tfrac{1}{2}P_0 \qquad 30.23$$

The frequency difference between the half-power points is the *bandwidth*, BW, a measure of circuit selectivity. The smaller the bandwidth, the more selective the circuit.

$$\text{BW} = f_2 - f_1 \qquad 30.24$$

The *quality factor*, Q, for a circuit is a dimensionless ratio that compares the reactive energy stored in an inductor each cycle to the resistive energy dissipated.[5] Figure 30.9 illustrates the effect the quality factor has on the frequency characteristic.

$$Q = 2\pi\left(\frac{\text{maximum energy stored per cycle}}{\text{energy dissipated per cycle}}\right)$$
$$= \frac{f_0}{(\text{BW})_{\text{Hz}}} = \frac{\omega_0}{(\text{BW})_{\text{rad/s}}}$$
$$= \frac{f_0}{f_2 - f_1} = \frac{\omega_0}{\omega_2 - \omega_1} \quad \begin{bmatrix}\text{parallel}\\ \text{or series}\end{bmatrix} \qquad 30.25$$

Then, the energy stored in the inductor of a series RLC circuit each cycle is

$$U = \frac{I_m^2 L}{2} = I^2 L = Q\left(\frac{I^2 R}{2\pi f_0}\right) \qquad 30.26$$

The relationships between the half-power points and quality factor are

$$f_1, f_2 = f_0\left(\sqrt{1 + \frac{1}{4Q^2}} \mp \frac{1}{2Q}\right)$$
$$\approx f_0 \mp \frac{f_0}{2Q} = f_0 \mp \frac{\text{BW}}{2} \qquad 30.27$$

Various resonant circuit formulas are tabulated in App. 30.A.

11. SERIES RESONANCE

In a resonant series RLC circuit,

- impedance is minimum
- impedance equals resistance
- current and voltage are in phase
- current is maximum
- power dissipation is maximum

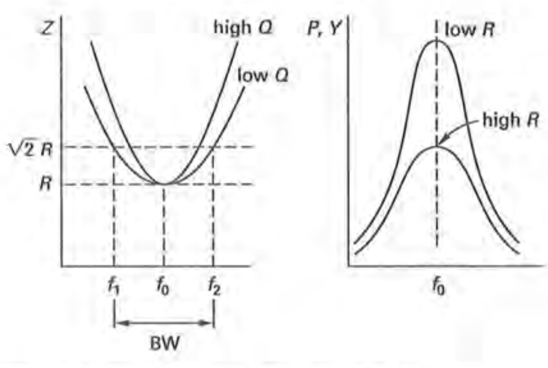


Figure 30.9 Series Resonance (Band-Pass Filter)

[5]The name *figure of merit* refers to the quality factor calculated from the inductance and internal resistance of a coil.

The total impedance (in rectangular form) of a series RLC circuit is $R + j(X_L - X_C)$. At the resonant frequency, $\omega_0 = 2\pi f_0$,

$$X_L = X_C \quad [\text{at resonance}] \qquad 30.28$$
$$\omega_0 L = \frac{1}{\omega_0 C} \qquad 30.29$$
$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \qquad 30.30$$

The power dissipation in the resistor is

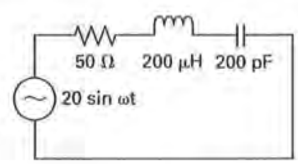
$$P = \tfrac{1}{2} I_m^2 R = \frac{V_m^2}{2R}$$
$$= I^2 R$$
$$= \frac{V^2}{R} \qquad 30.31$$

The quality factor for a series RLC circuit is

$$Q = \frac{X}{R} = \frac{\omega_0 L}{R}$$
$$= \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
$$= \frac{\omega_0}{(\text{BW})_{\text{rad/s}}} = \frac{f_0}{(\text{BW})_{\text{Hz}}}$$
$$= G\omega_0 L = \frac{G}{\omega_0 C} \qquad 30.32$$

Example 30.5

A series RLC circuit is connected across a sinusoidal voltage with peak of 20 V. (a) What is the resonant frequency in rad/s? (b) What are the half-power points in rad/s? (c) What is the peak current at resonance? (d) What is the peak voltage across each component at resonance?



Solution

(a) Equation 30.30 gives the resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(200 \times 10^{-6}\ \text{H})(200 \times 10^{-12}\ \text{F})}}$$
$$= 5 \times 10^6\ \text{rad/s}$$

(b) The half-power points are given by

$$\omega_1, \omega_2 = \omega_0 \mp \frac{\text{BW}}{2} = \omega_0 \mp \frac{\omega_0}{2Q} = \omega_0 \mp \frac{R}{2L}$$
$$= 5 \times 10^6 \mp \frac{50\ \Omega}{(2)(200 \times 10^{-6}\ \text{H})}$$
$$= 5.125 \times 10^6,\ 4.875 \times 10^6\ \text{rad/s}$$

(c) The total impedance is the resistance at resonance. The peak resonant current is

$$I_0 = \frac{V_m}{Z_0} = \frac{V_m}{R} = \frac{20\angle 0^\circ \text{V}}{50\ \Omega} = 0.4\angle 0^\circ\ \text{A}$$

(d) The peak voltages across the components are

$$V_R = I_0 R = (0.4\angle 0^\circ \text{A})(50\ \Omega) = 20\angle 0^\circ\ \text{V}$$

$$V_L = I_0 X_L = I_0 j\omega_0 L = j(0.4\angle 0^\circ \text{A})\left(5\times 10^6\ \frac{\text{rad}}{\text{s}}\right)(200\times 10^{-6}\ \text{H}) = 400\angle 90^\circ\ \text{V}$$

$$V_C = I_0 X_C = \frac{I_0}{j\omega_0 C} = \frac{0.4\angle 0^\circ\ \text{A}}{j\left(5\times 10^6\ \frac{\text{rad}}{\text{s}}\right)(200\times 10^{-12}\ \text{F})} = 400\angle -90^\circ\ \text{V}$$

12. PARALLEL RESONANCE

In a resonant parallel GLC circuit,

- impedance is maximum
- impedance equals resistance
- current and voltage are in phase
- current is minimum
- power dissipation is minimum

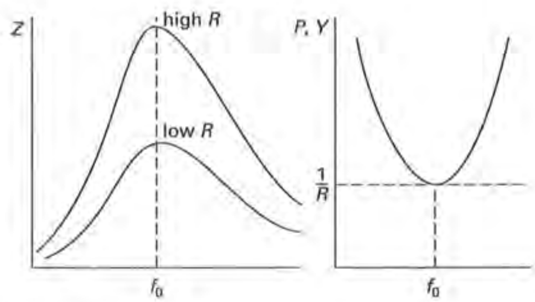


Figure 30.10 *Parallel Resonance (Band-Reject Filter)*

The total admittance (in rectangular form) of a parallel GLC circuit is $G + j(B_C - B_L)$. At resonance,

$$X_L = X_C \qquad 30.33$$

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad 30.34$$

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \qquad 30.35$$

The power dissipation in the resistor is

$$P = \tfrac{1}{2} I_m^2 R = \frac{V_m^2}{2R} = I^2 R = \frac{V^2}{R} \qquad 30.36$$

The quality factor for a parallel RLC circuit is

$$Q = \frac{R}{X} = \omega_0 RC = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}} = \frac{\omega_0}{(\text{BW})_{\text{rad/s}}} = \frac{f_0}{(\text{BW})_{\text{Hz}}} = \frac{\omega_0 C}{G} = \frac{1}{G\omega_0 L} \qquad 30.37$$

Example 30.6

A parallel RLC circuit containing a 10 Ω resistor has a resonant frequency of 1 MHz and a bandwidth of 10 kHz. To what should the resistor be changed in order to increase the bandwidth to 20 kHz without changing the resonant frequency?

Solution

From Eqs. 30.25 and 30.37,

$$Q_{\text{old}} = \frac{f_0}{\text{BW}} = 2\pi f_0 RC$$

$$C = \frac{1}{2\pi R(\text{BW})} = \frac{1}{(2\pi)(10\ \Omega)(10\times 10^3\ \text{Hz})} = \frac{1\times 10^{-5}}{2\pi}\ \text{F}$$

The new quality factor is

$$Q_{\text{new}} = \frac{f_0}{\text{BW}} = \frac{10^6\ \text{Hz}}{20\times 10^3\ \text{Hz}} = 50$$

From Eq. 30.37, the required resistance is

$$R = \frac{Q}{2\pi f_0 C} = \frac{50}{(2\pi)(10^6\ \text{Hz})\left(\frac{1\times 10^{-5}}{2\pi}\ \text{F}\right)} = 5\ \Omega$$

43 Electronic Components

Nomenclature

Å	angstrom	10^{-10} m
A	constant	$K^{-3}m^{-3}$ or $K^{-3}cm^{-3}$
A	anode	–
A	gain	–
B	base or substrate	–
c	speed of light	3.00×10^8 m/s
C	capacitance	F
C	collector	–
C_{iss}	gate channel capacitance	F
C_{oss}	output capacitance	F
C_{tc}	collector junction transition capacitance	F
C_{te}	emitter junction transition capacitance	F
CB	common base	–
CC	common collector	–
CE	common emitter	–
D	diffusion constant	m^2/s
D	drain	–
E	emitter	–
E	energy	J or eV
g_m	mutual conductance or transconductance	S
G	gate	–
h	Planck's constant	6.626×10^{-34} J·s
$h_{i,o,f,r}$ and e,c,b	hybrid parameter[1]	–
$h_{I,O,F,R}$ and E,C,B	hybrid parameter[2]	–
h_{fe}	CE small signal (AC) forward current transfer ratio or gain	–
h_{ie}	CE small signal (AC) input impedance	Ω
h_{oe}	CE small signal (AC) open-circuit output admittance	S
h_{re}	CE small signal (AC) reverse voltage transfer ratio or gain	–
i	instantaneous current	A

[1]The first subscript refers to the AC or small signal input (i), output (o), forward (f), or reverse (r) value. The second subscript refers to the configuration, that is, e for common emitter (CE), c for common collector (CC), b for common base (CB).
[2]The first subscript refers to the DC or static input (I), output (O), forward (F), or reverse (R) value. The second subscript refers to the configuration, that is, E for common emitter (CE), C for common collector (CC), B for common base (CB).

I	effective or DC current	A
I_{CBO}	DC collector cutoff current, emitter open[3]	A
I_{DSS}	zero-gate-voltage drain current	A
I_{ZK}	keep-alive current	A
K	cathode	–
n	electron concentration	m^{-3} or cm^{-3}
n, N	concentration	m^{-3} or cm^{-3}
p	hole concentration	m^{-3} or cm^{-3}
q	charge	1.602×10^{-19} C
Q	quiescent point	–
r	small-signal resistance[4]	Ω
$r_{bb'}$	base spreading resistance	Ω
$r_{b'c}$	feedback resistance	Ω
$r_{b'e}$	base input resistance	Ω
r_{ce}	output resistance	Ω
R	resistance	Ω
S	apparent power	voltampere (volt-amps or VA)
S	source	–
tc	temperature coefficient	K^{-1}
T	temperature	K
v	instantaneous voltage	V
V	effective or DC voltage	V
V_T	voltage equivalent of temperature	V

Symbols

α	CB forward current ratio or gain	–
β	CE forward current ratio or gain	–
E	electric field strength	V/m
η	empirical constant	–
κ	Boltzmann's constant	1.381×10^{-23} J/K or 8.621×10^{-5} eV/K
λ	wavelength	m
μ	mobility	$m^2/V{\cdot}s$
ρ	charge density	C/m^3
σ	conductivity	S/m
ω	angular frequency	rad/s

[3]This is essentially the reverse-saturation current between the collector and the base.

[4]This is also known as the *dynamic resistance* or the *instantaneous resistance*. Any resistance represented with a lowercase letter is the small-signal value.

Subscripts

0	at 0K
0	barrier or source value
A	acceptor
ac	alternating current or small signal
b, B	base
BB	base supply or base biasing
BE	base to emitter
BO	breakover
(BR)R	breakover, reverse
c	conduction band or collector
cf	corner frequency or cutoff frequency
co	cutoff
C	collector
CB	collector to base
CBO	collector cutoff current
CC	collector supply
CE	collector to emitter
CQ	Q-point collector current
d	dynamic or diffusion
D	donor, diode, or drain
DC	direct current or static
DS	drain to source
DSS	drain to source saturation
DSQ	drain to source at Q-point
e, E	emitter
EB	emitter to base
f	forward (AC or instantaneous)
F	forward (DC component) or total
G	gap, gate
GD	gate to drain
GS	gate to source
H	hold
i	intrinsic or input (AC or instantaneous)
iss	input short-circuit common source
I	input (DC or total)
L	load
m	maximum or mutual
n	electrons or n-type
o	saturation or output (AC or instantaneous)
O	output (DC or total)
p	holes or p-type
pn	p to n
P	pinchoff or power
$P0$	pinchoff at 0 V
r	reverse (AC or instantaneous)
R	reverse (DC component or total)
s	saturation
S	source
SD	source to drain
SG	source to gate
t	transition
T	thermal
v	valence band
v, V	voltage
Z	zener
ZK	zener knee

1. OVERVIEW

Electronics involves charge motion through materials other than metals, such as a vacuum, gases, or semiconductors. The focus in this chapter will be on semiconductor materials. Because an understanding of the electron structure is vital to understanding electronics, a periodic table of the elements is given in App. 43.A.

An *electronic component* is one able to amplify, control, or switch voltages or currents without mechanical or other nonelectrical commands. The charge in metals is carried by the electron, with a charge of -1.60×10^{-19} C. In semiconductor materials, the charge is carried both by the electron and by the absence of the electron in a covalent bond, which is referred to as a *hole*[5] with a charge of $+1.60\times10^{-19}$ C. The concentration of electrons, n, and holes, p, is given by the *mass action law*, Eq. 43.1.

$$n_i^2 = np \qquad 43.1$$

The term n_i is the concentration of carriers in a pure (*intrinsic*) semiconductor.[6] In intrinsic semiconductor materials, the number of electrons equals the number of holes, and the mass action law is stated as in Eq. 43.1. Nevertheless, the mass action law applies for intrinsic and extrinsic semiconductor materials. *Extrinsic* semiconductor materials are those that have had impurities deliberately added to modify their properties, normally their conductivity. For extrinsic semiconductors, the mass action law is probably better understood as indicating that the product np remains constant regardless of position in the semiconductor or doping level. Carriers, either n or p, in a semiconductor are constantly generated due to thermal creation of electron-hole pairs and constantly disappearing due to recombination of electron-hole pairs. The carriers generated diffuse due to concentration gradients in the semiconductor, a phenomenon that does not occur in metals.

Semiconductor devices are inherently nonlinear. Nevertheless, they are commonly analyzed over ranges in which their behavior is approximately linear. Such an analysis is called *small-signal analysis*. The DC or effective value of the electrical parameters in the models used for analysis will be represented by uppercase letters. AC or instantaneous values will be represented by lowercase letters. Equivalent parameters in the models will use lowercase letters. For convenience, the lowercase letter t is omitted for functions of time. For example $v(t)$ is written simply as v. The subscripts on currents indicate the terminals into which current flows. Subscripts on voltages indicate the terminals across which the voltage appears. Subscripts indicating biasing voltages are capitalized. For example, V_{BB} is the base biasing voltage while v_{be} is the instantaneous voltage signal applied to the base-emitter junction. A list of common designations is given in App. 43.B.

[5]The concept of holes explains the conduction of electricity without free electrons. The hole is considered to behave as a free positive charge—quantum mechanics justifies such an interpretation. (The *Hall voltage* is experimental confirmation.) The calculation of total charge motion in semiconductors is simplified as a result. The same equations used for electron movement can be used for hole movement with a change of sign and a change of values for some terms, such as mobility.

[6]Intrinsic means natural. Pure silicon is silicon in its natural state, with no doping, though it will have naturally occurring impurities. The terms pure and intrinsic are used interchangeably.

Electronic components are often connected in an array known as an *integrated circuit* (IC). An integrated circuit is a collection of active and passive components on a single semiconductor substrate (*chip*) that function as a complete electronic circuit. *Small-scale integration* involves the use of less than 100 components per chip. *Medium-scale integration* involves between 100 and 999 components per chip. *Large-scale integration* involves between 1000 and 9999 components per chip. *Very large-scale integration* involves more than 10,000 components per chip.

The nonlinearity of electronic devices makes them attractive for use as amplifiers. An *amplifier* is a device capable of increasing the amplitude or power of a physical quantity without distorting the wave shape of the quantity. The amplifying properties of transistors are covered in this chapter. Topics peculiar to amplifiers and amplifier types are covered in Chs. 44 and 45.

Example 43.1

Silicon has an intrinsic carrier concentration of 1.6×10^{10} cm^{-3}. What is the concentration of holes?

Solution

Regardless of whether the material is intrinsic or extrinsic, the law of mass action applies. However, in intrinsic materials, the concentration of electrons and the concentration of holes are equal. Thus, Eq. 43.1 can be used as follows.

$$n_i^2 = np = p^2$$

$$n_i = p = 1.6\times10^{10}\ \text{cm}^{-3}$$

2. SEMICONDUCTOR MATERIALS

The most common semiconductor materials are silicon (Si) and germanium (Ge). Both are in group IVA of the periodic table and contain four valence electrons. A *valence electron* is one in the outermost shell of an atom. Both materials use covalent bonding to fill the outer shell of eight when they create a crystal lattice. Semiconductor materials ($N \approx 10^{10}$ to 10^{13} electrons/m^3) are slightly more conductive than insulators ($N \approx 10^7$ electrons/m^3) but less conductive than metals ($N \approx 10^{28}$ electrons/m^3). The conductivity, σ, of semiconductor materials can be made to vary from approximately 10^{-7} to 10^5 S/m.

The semiconductor crystal lattice has many defects (i.e., free electrons and their corresponding holes). The formation of free electrons and holes is driven by the temperature and is called *thermal carrier generation*. The

Electronics

density of these electron-hole pairs in intrinsic materials is given by

$$n_i^2 = A_0 T^3 e^{-\frac{E_{G0}}{\kappa T}} \qquad 43.2$$

The term A_0 is a constant independent of temperature and related to the density states at the bottom edge of the conduction band and the top edge of the valence band. T is the absolute temperature. E_{G0} is the energy gap at 0K, that is, the energy between the conduction band and the valence band. This is the energy required to break the covalent bond. The energy gap for silicon at 0K is approximately 1.21 eV. The energy gap for germanium is approximately 0.78 eV. The term κ is Boltzmann's constant.[7] At a given temperature at thermal equilibrium, Eq. 43.2 becomes

$$n_i^2 = N_c N_v e^{-\frac{E_G}{\kappa T}} \qquad 43.3$$

The terms N_c and N_v are the effective density states in the conduction band and valence band, respectively. The energy gap for silicon at room temperature (300K) is 1.12 eV. The energy gap for germanium at room temperature is 0.80 eV.

When minor amounts of impurities called *dopants* are added, the materials are termed *extrinsic semiconductors*.[8] If the impurities added are from group IIIA, with three valence electrons, an additional hole is created in the lattice. These dopants are called *acceptors*, and semiconductors with such impurities are called *p-types*. The *majority carriers* in p-type semiconductors are holes, and the *minority carriers* are electrons. The majority of the charge movement takes place in the valence band. Typical dopants are indium (In) and gallium (Ga). If the impurities added are from group VA, with five valence electrons, an additional electron is provided to the lattice.[9] These dopants are called *donors*, and semiconductors with such impurities are called *n-types*. The majority carriers in n-type semiconductors are electrons, and the minority carriers are holes. The majority of charge movement takes place in the conduction band. Typical dopants are phosphorus (P), arsenic (As), and antimony (Sb).

The conductivity of the semiconductor is determined by the carriers. The mobility of electrons is higher than that of holes.[10] The total conductivity in any semiconductor is a combination of the movement of the electrons and holes and is given by

$$\sigma = q\,(n\mu_n + p\mu_p) \qquad 43.4$$

[7]The symbology for Boltzmann's constant often varies with the units of the energy gap. If eV is used as the energy unit, Boltzmann's constant may be seen as either κ or $\bar{\kappa}$ to distinguish between eV/K and J/K.

[8]A minor amount of dopant material is on the order of 10 parts per billion.

[9]The additional electron exists because the outer shell octet is satisfied by the covalent bonding of the first four electrons in the impurity.

[10]The mobility of electrons is a factor of 2.5 higher in silicon and about 2.1 higher in germanium.

The law of mass action, Eq. 43.1, applies to extrinsic semiconductors. With the addition of dopants, the law of electrical neutrality given by Eq. 43.5 also applies. The concentration of acceptor atoms is N_A, each contributing one positive charge to the lattice. (Do not confuse N_A with Avogadro's number.) The concentration of donor atoms is N_D, each contributing one negative charge. Since neutrality is maintained, the result is[11]

$$N_A + n = N_D + p \qquad 43.5$$

In a p-type material, the concentration of donors is zero, ($N_D = 0$). Additionally, the concentration of holes is much greater than the number of electrons ($p \gg n$). Thus, for a p-type material, Eq. 43.5 can be rewritten as $N_A \approx p$. Using the law of mass action, Eq. 43.1, the concentration of electrons in a p-type material is

$$n = \frac{n_i^2}{N_A} \qquad 43.6$$

In an n-type material, the concentration of acceptors is zero ($N_A = 0$). Additionally, the concentration of electrons is much greater than the number of holes ($n \gg p$). Thus, for an n-type material, Eq. 43.5 can be rewritten as $N_D \approx n$. Using the law of mass action, Eq. 43.1, the concentration of holes in an n-type material is

$$p = \frac{n_i^2}{N_D} \qquad 43.7$$

Example 43.2

The energy gap for silicon at room temperature (300K) is 1.12 eV. The energy density states are $N_c = 2.8 \times 10^{19}$ cm^{-3} and $N_v = 1.02 \times 10^{19}$. What is the intrinsic carrier concentration?

Solution

The intrinsic carrier concentration, assuming thermal equilibrium, is given by Eq. 43.3.

$$\begin{aligned} n_i^2 &= N_c N_v e^{\frac{-E_G}{\kappa T}} = (2.8 \times 10^{19}\ \text{cm}^{-3}) \\ &\quad \times (1.02 \times 10^{19}\ \text{cm}^{-3}) \left(e^{-\frac{1.12\ \text{eV}}{\left(\frac{8.621 \times 10^{-5}\ \text{eV}}{\text{K}}\right)(300\text{K})}} \right) \\ &= 4.452 \times 10^{19}\ \text{cm}^{-6} \\ n_i &= 6.67 \times 10^{9}\ \text{cm}^{-3} \\ &\approx 0.7 \times 10^{10}\ \text{cm}^{-3} \end{aligned}$$

This number differs slightly from the number determined experimentally and more exact calculations (1.6×10^{10} cm^{-3}).

[11]Neutrality is maintained because each atom of an added impurity removes one intrinsic atom.

3. DEVICE PERFORMANCE CHARACTERISTICS

Electronic components function on some variation of *pn* junction principles. Amplifiers function on some variation of transistor principles, that is, *pnp* or *npn* junction principles. Most electronic components can be modeled as two-port devices with two variables—current and voltage—for each port. The relationship between the variables depends on the type of device. The relationship can be expressed mathematically, as for MOSFETs, modeled in equivalent circuits, as with transistor *h*-parameters, or described graphically, as with BJT characteristic curves.

Semiconductor devices are inherently nonlinear. The performance of such devices is analyzed in a linear fashion over small portions of the characteristic curve(s). Such analysis is called *small-signal analysis*. A small signal is one that is much less than the average, that is, steady-state, value for the device, which usually is the biasing value. The models used in each linear portion of the characteristic curve are termed *piecewise linear models*. If the operation is outside the linear region or if the input signal is large compared to the average value, the device distorts the input signal. Such operation is called *nonlinear operation*.

The characteristic curve for an ideal transistor operating as a current-amplifying device is shown in Fig. 43.1. (The shape of the curve is the same for a diode, that is, a *pn* junction.)

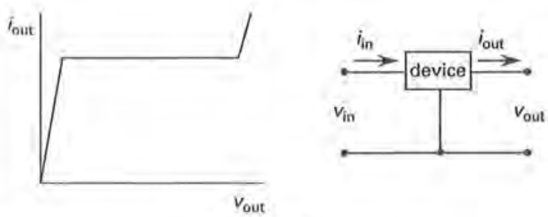


Figure 43.1 *Typical Semiconductor Performance*

The voltage-current graph is divided into various regions known by names such as *saturation* (or *on*), *cutoff* (or *off*), *active*, *breakdown*, *avalanche*, and *pinchoff* regions. The locations of these regions depend on the type of transistor—for example, BJT or FET—and its polarity. Operation is normally in the linear active region, but applications for operation in other regions exist in digital and communications (radio frequency) applications.

4. BIAS

Bias is a DC voltage applied to a semiconductor junction to establish the operating point, also called the *quiescent* (*no-signal*) *point*. *Biasing* establishes the operating point with no input signal.[12] Biasing, then, is the process of establishing the DC voltages and currents (the bias) at the device's terminals when the input signal is zero (or nearly so).

For *pn* junctions, *forward bias* (or *on condition*) is the application of a positive voltage to the *p*-type material or, equivalently, the flow of current from the *p*-type to the *n*-type material. In a small semiconductor device, forward bias results in current in the milliampere range.

Reverse bias (or *off condition*) is the application of a negative voltage to the *p*-type material, or equivalently, the flow of current from the *n*-type to the *p*-type material. In a small semiconductor device, reverse bias results in current in the nanoampere range.

Self-biasing is the use of the amplifier's output voltage, rather than a separate power source, as the supply for the input bias voltage. This negative feedback control regulates the output current and voltage against variations in transistor parameters.

5. AMPLIFIERS

An *amplifier* produces an output signal from the input signal. The input and output signals can be either voltage or current. The output can be either smaller or larger (the usual case) than the input in magnitude. While most amplifiers merely scale the input voltage or current upward, the amplification process can include a sign change, a phase change, or a complete phase shift of 180°.[13]) The ratio of the output to the input is known as the *gain* or *amplification factor*, A. A *voltage amplification factor*, A_V, and *current amplification factor*, A_I or β, can be calculated for an amplifier.

Figure 43.2 illustrates a simplified current amplifier with current amplification factor β. The additional current leaving the amplifier is provided by the bias battery, V_2.

$$i_{out} = \beta i_{in} \qquad 43.8$$

A capacitor, C, is placed in the output terminal to force all DC current to travel through the *load resistor*, R_L. Kirchhoff's voltage law for loop abcd is

$$V_2 = i_{out}R_L + V_{ac} = \beta i_{in}R_L + V_{ac} \qquad 43.9$$

If there is no input signal (i.e., $i_{in} = 0$), then $i_{out} = 0$ and the entire battery voltage appears across terminals ac ($V_{ac} = V_2$). If the voltage across terminals ac is zero, then the entire battery voltage appears across R_L so that $i_{out} = V_2/R_L$.

[12]Bias is used both as a verb and a noun in electronics.

[13]An *inverting amplifier* is one for which $v_{out} = -A_V v_{in}$. For a sinusoidal input, this is equivalent to a phase shift of 180° (i.e., $\mathbf{V}_{out} = A_V V_{in} \angle{-180°}$.

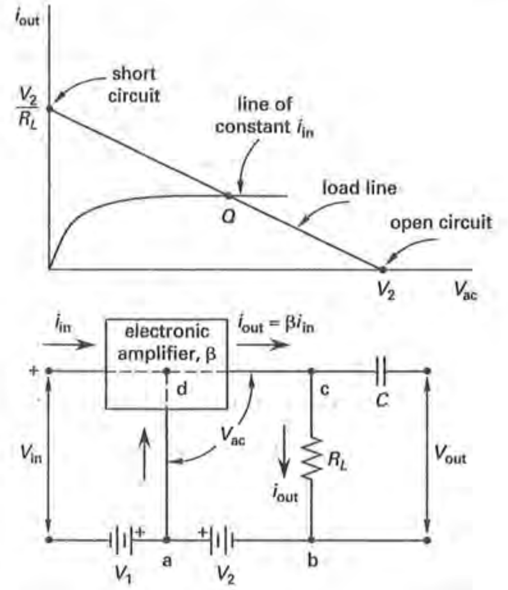


Figure 43.2 General Amplifier

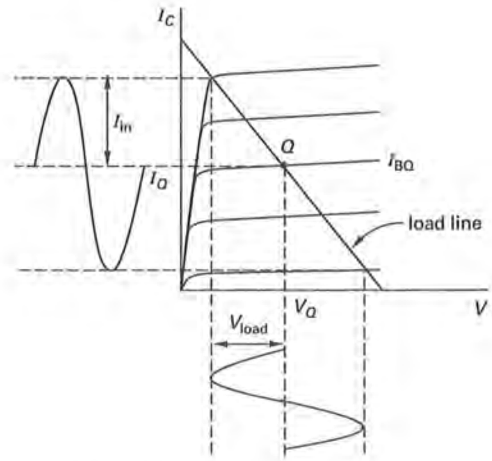


Figure 43.3 Class A Amplifier

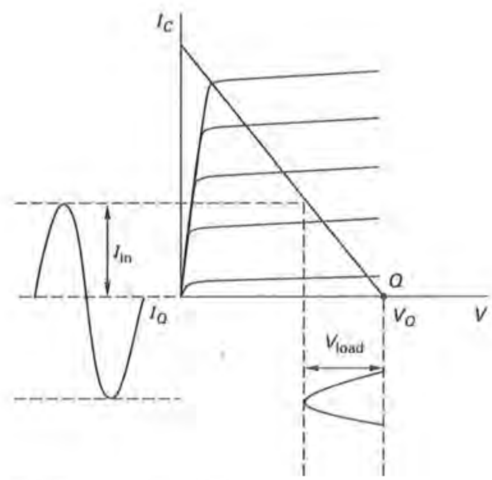


Figure 43.4 Class B Amplifier

6. AMPLIFIER CLASSIFICATION

Amplifiers are classified on the basis of how much input is translated into output. A sinusoidal input signal is assumed. The output of an amplifier depends on the bias setting, which in turn establishes the quiescent point.

A *Class A amplifier*, as shown in Fig. 43.3, has a quiescent point in the center of the active region of the operating characteristics. Class A amplifiers have the greatest linearity and the least distortion. Load current flows throughout the full input signal cycle. Since the load resistance of a properly designed amplifier will equal the Thevenin equivalent source resistance, the maximum power conversion efficiency of an ideal Class A amplifier is 50%.

For *Class B amplifiers*, as shown in Fig. 43.4, the quiescent point is established at the cutoff point. A load current flows only if the signal drives the amplifier into its active region, and the circuit acts like an amplifying half-wave rectifier. Class B amplifiers are usually combined in pairs, each amplifying the signal in its respective half of the input cycle. This is known as *push-pull operation*. The output waveform will be sinusoidal except for the small amount of crossover distortion that occurs as the signal processing transfers from one amplifier to the other. The maximum power conversion efficiency of an ideal Class B push-pull amplifier is approximately 78%.

The intermediate *Class AB amplifier* has a quiescent point somewhat above cutoff but where a portion of the input signal still produces no load current. The output current flows for more than half of the input cycle. AB amplifiers are also used in push-pull circuits.

Class C amplifiers, as shown in Fig. 43.5, have quiescent points well into the cutoff region. Load current flows during less than one-half of the input cycle. For a purely resistive load, the output would be decidedly nonsinusoidal. However, if the input frequency is constant, as in radio frequency (rf) power circuits, the load can be a parallel LRC tank circuit tuned to be resonant at the signal frequency. The LRC circuit stores electrical energy, converting the output signal to a sinusoid. The power conversion efficiency of an ideal Class C amplifier is 100%.

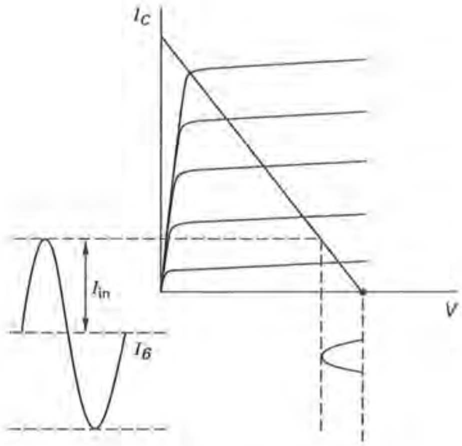


Figure 43.5 Class C Amplifier (Resistive Load)

7. LOAD LINE AND QUIESCENT POINT CONCEPT

The i_{in}-v_{out} curves of Secs. 43.5 and 43.6 illustrate how amplification occurs. The two known points, $(v_{out}, i_{out}) = (V_2, 0)$ and $(v_{out}, i_{out}) = (0, V_2/R_L)$, are plotted on the voltage-current characteristic curve. The straight *load line* is drawn between them. The change in output voltage (the horizontal axis) due to a change in input voltage (parallel to the load line) can be determined. Equation 43.10 gives the voltage *gain* (*amplification factor*).[14,15]

$$A_V = \frac{\partial v_{out}}{\partial v_{in}} \approx \frac{\Delta v_{out}}{\Delta v_{in}} \qquad 43.10$$

Usually, a nominal current (the *quiescent current*) flows in the abcd circuit even when there is no signal. The point on the load line corresponding to this current is the *quiescent point* (*Q-point* or *operating point*). It is common to represent the quiescent parameters with uppercase letters (sometimes with a subscript Q) and to write instantaneous values in terms of small changes to the quiescent conditions.

$$v_{in} = V_Q + \Delta v_{in} \qquad 43.11$$

$$v_{out} = V_{out} + \Delta v_{out} \qquad 43.12$$

$$i_{out} = I_{out} + \Delta i_{out} \qquad 43.13$$

Since it is a straight line, the load line can also be drawn if the quiescent point and any other point, usually $(V_{BB}, 0)$, are known.

The ideal voltage amplifier has an infinite *input impedance* (so that all of v_{in} appears across the amplifier and no current or power is drawn from the source) and zero *output impedance* so that all of the output current flows through the load resistor.

Determination of the load line for a generic transistor amplifier is accomplished through the following steps.

step 1: For the configuration provided, label the x-axis on the *output characteristic curves* with the appropriate voltage. (For a BJT, this is V_{CE} or V_{CB}. For a FET, this is V_D.)[16]

step 2: Label the y-axis as the output current. (For a BJT, this is I_C. For a FET, this is I_D.)

step 3: Redraw the circuit with all three terminals of the transistor open. Label the terminals. (For a BJT, these are base, emitter, and collector. For a FET, these are gate, source, and drain.) Label the current directions all pointing inward, toward the amplifier. (For a BJT, these are I_B, I_E, and I_C. For a FET, these are I_D and I_S.)

step 4: Perform KVL analysis in the output loop. (For a BJT, this is the collector loop. For a FET, this is the drain loop.) The transistor voltage determined is a point on the x-axis with the output current equal to zero. Plot the point.

step 5: Redraw the circuit with all three terminals of the transistor shorted. Label as in step 3.

step 6: Use Ohm's law, or another appropriate method, in the output loop to determine the current. (For the BJT, this is the collector current. For the FET, this is the drain current.) The transistor current determined is a point on the y-axis with the applicable voltage in step 1 equal to zero. Plot the point.

step 7: Draw a straight line between the two points. This is the DC load line.[17]

Electronics

[14]Gain can be increased by increasing the load resistance, but a larger biasing battery, V_2, is required. The choice of battery size depends on the amplifier circuit devices, size considerations, and economic constraints.

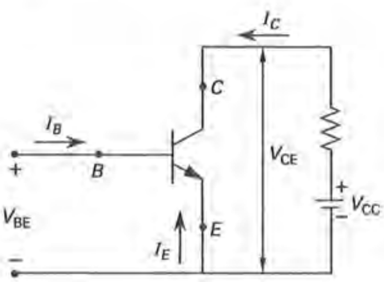
[15]A *high-gain amplifier* has a gain in the tens of hundreds of thousands.

[16]The output characteristic curves for the BJT are also called the *collector characteristics* or the *static characteristics*. Figure 43.2 is an example.

[17]AC load lines are determined in the same manner, but active components, that is, inductors and capacitors, are accounted for in the analysis.

Example 43.3

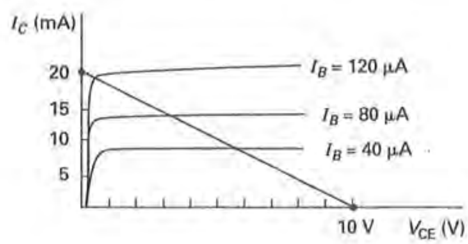
Consider the common emitter (CE) shown.



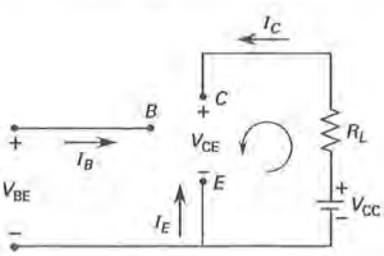
If the load resistance is 500 Ω and the collector supply voltage, V_{CC}, is 10 V, determine and draw the load line.

Solution

The x- and y-axes are drawn as shown.



Redrawing the circuit and labeling gives



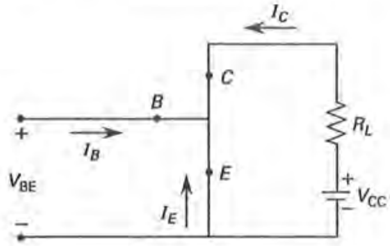
Write KVL around the indicated loop.

$$V_{CC} - I_C R_L - V_{CE} = 0$$

With the terminals open-circuited, $I_C = 0$. Substituting and rearranging gives

$$V_{CE} = V_{CC} = 10 \text{ V}$$

Plot this point (10,0) on the x-axis. Redraw and label the circuit with the terminals shorted.



Using Ohm's law and the given values,

$$I_C = \frac{V_{CC}}{R_L} = \frac{10 \text{ V}}{500 \ \Omega} = 20 \text{ mA}$$

Plot this point on the y-axis. Draw a straight line between the two points. The load line is shown superimposed on the output characteristic curves in the first drawing.

8. *pn* JUNCTIONS

The *pn junction* forms the basis of diode and transistor operation. The *pn junction* is constructed of a p-type material (the anode) and an n-type material (the cathode) bonded together as shown in Fig. 43.6(a). Some of the acceptor atoms are shown as ions with a minus sign because after an impurity atom accepts an electron, it becomes negatively charged. Some of the donor atoms are shown as ions with a plus sign because after an impurity atom gives up an electron it is positively charged. (Overall, the law of charge neutrality holds, and the *pn* junction is neutral.)

Because of the concentration gradient across the junction, holes diffuse to the right and electrons diffuse to the left. As a result, the concentration of holes on the p-side near the junction is depleted and a negative charge exists. The concentration of electrons on the n-side near the junction is depleted as well and a positive charge exists. The result of this diffusion is shown in Fig. 43.6(b), the shape of which is determined by the level of doping.[18] The diffusion process continues until the electrostatic field set up by the charge separation is such that no further charge motion is possible. The net electric field intensity is shown in Fig. 43.6(c). The net result is a small region in which no mobile charge carriers exist. This region is called the *space-charge region*, *depletion region*, or *transition region*. Holes have a potential barrier they must overcome to move from left to right just as electrons have an energy barrier they must overcome to move from right to left, as shown in Figs. 43.6(d) and (e).

[18] A *step-graded junction* and a *linearly graded junction* are two possible types.

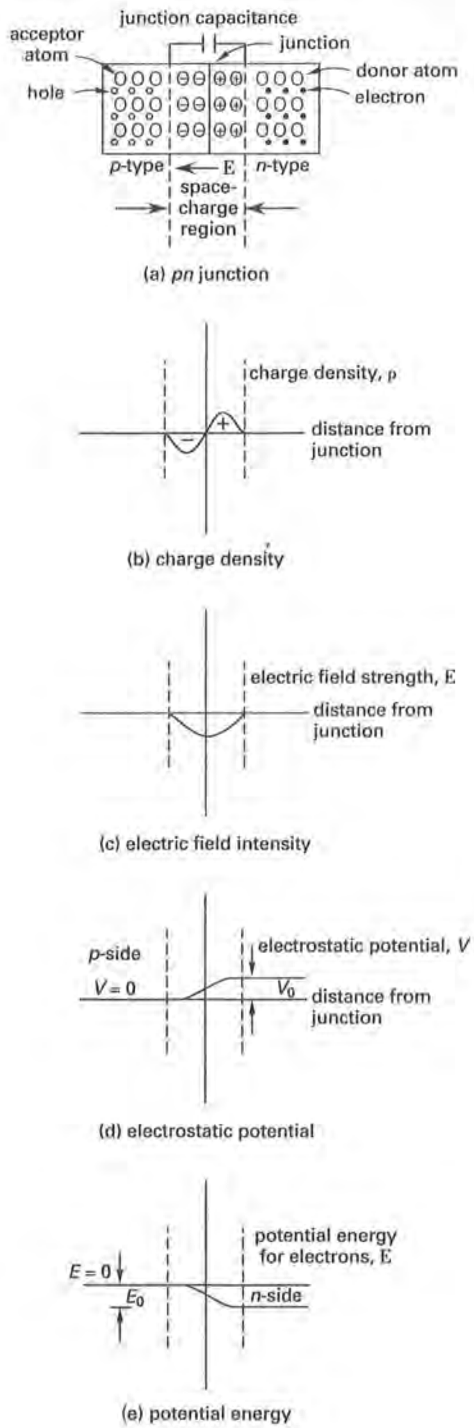


Figure 43.6 *pn Junction Characteristics*

The flow of carriers due to the concentration gradient is called the *diffusion current*, $I_{\text{diffusion}}$, also called the *recombination current* or the *injection current*. The flow of carriers due to the established electric field is called the *drift current*, I_s, also called the *saturation current*, *thermal current*, or *reverse saturation current*.[19] The movement of carriers due to recombination (diffusion) and drift (saturation) is continual, though at thermal equilibrium, without any applied voltage, the net current is zero.

$$I_{\text{junction}} = I_{\text{diffusion}} + I_s = 0 \quad \text{[algebraic sum]} \qquad 43.14$$

A summary of the movement of the carriers is shown in Fig. 43.7.

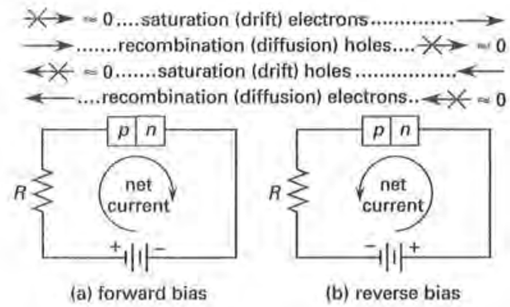


Figure 43.7 *pn Junction Carrier Movement*

The width of the space-charge region at equilibrium is shown in Fig. 43.8(a).

When an external voltage is applied, that is, a biasing voltage, the width of the space-charge region and the barrier height change. When the p-type material is connected to a positive potential, that is, forward biased, holes are repelled across the junction into the n-type material, and electrons are repelled across the junction into the p-type material. The width of the space-charge region is reduced and the barrier height for p to n current flow is reduced as shown in Fig. 43.8(b). A DC forward bias voltage, V_F, of approximately 0.5 to 0.7 V for silicon and 0.2 to 0.3 V for germanium is required to overcome the barrier voltage. Once the barrier is overcome, the junction current increases significantly due to an increase in the diffusion current. That is, holes cross the junction into the n-type material, where they are considered injected minority carriers. Electrons cross the junction into the p-type material, where they too are injected minority carriers. Since hole movement in one direction and electron movement in the opposite direction constitute a current in the same direction, the total current is the sum of the hole and electron minority currents.

[19]Numerous symbols are used, among them I_o and I_{CO}.

Electronics

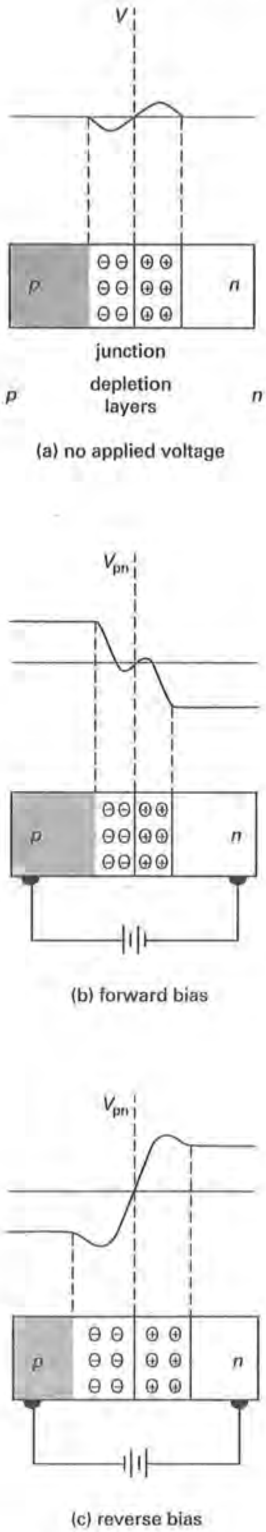


Figure 43.8 *pn Junction Space-Charge Region*

When the *p*-type material is connected to a negative potential, that is, reverse biased, holes and electrons move away from the junction. The width of the space-charge region is increased and the barrier height for *p* to *n* current flow is increased as shown in Fig. 43.8(c). The process nominally stops when the holes in the *p*-type material are depleted. However, a few holes in the *n*-type material are thermally generated, as there are electrons in the *p*-type material. These minority carriers thermally diffuse into the depletion region and are swept across by the electric field. The effect is constant for a given temperature and independent of the reverse bias. This is the reverse saturation current, I_s.[20] The reverse saturation current is small ($\approx 10^{-9}$ A). An ideal *pn* junction, excluding the breakdown region, is governed by[21]

$$I_{\text{pn}} = I_s\left(e^{\frac{qV_{\text{pn}}}{\kappa T}} - 1\right) \qquad 43.15$$

Breakdown is a large, abrupt change in current for a small change in voltage. When a *pn* junction is reverse biased, the saturation current is small up to a certain reverse voltage, where it changes dramatically. Two mechanisms can cause this change. The first is *avalanche breakdown*. Avalanche occurs when thermally generated minority carriers are swept through the space-charge region and collide with ions. If they possess enough energy to break a covalent bond, an electron-hole pair is created. The same effect may occur for these newly generated carriers, resulting in an avalanche effect. Avalanche breakdown occurs in lightly doped materials at greater than 6 V reverse bias. The second breakdown mechanism is *zener breakdown*. Zener breakdown occurs through the disruption of covalent bonds due to the strength of the electric field near the junction. No collisions are involved. The additional carriers created by the breaking covalent bonds increase the reverse current. Zener breakdown occurs in highly doped materials at less than 6 V reverse bias.[22]

9. DIODE PERFORMANCE CHARACTERISTICS

A *diode* is a two-electrode device. The diode is designed to pass current in one direction only. An *ideal diode*, approximated by a *pn* junction, has a zero voltage drop, that is, no forward resistance, and acts as a short circuit when forward biased (on). When reversed biased (off), the resistance is infinite and the device acts as an open circuit. Diode construction and theory is that of a *pn* junction (see Sec. 43-8).

[20]The reverse saturation current also accounts for any current leakage across the surface of the semiconductor.

[21]The subscript *pn* is used here for clarification. Standard diode voltage and current directions are defined in Sec. 43-9, after which the subscript is no longer used.

[22]The name zener is commonly used regardless of the breakdown mechanism.

The characteristics and symbology for a typical real semiconductor diode are shown in Fig. 43.9. The *reverse bias voltage* is any voltage below which the current is small, that is, less than 1% of the maximum rated current. The *peak inverse* (*reverse*) *voltage*, PIV or PRV, is the maximum reverse bias the diode can withstand without damage. The forward current is also limited due to heating effects. Maximum forward current and peak inverse voltage for silicon diode rectifiers are approximately 600 A and 1000 V, respectively.

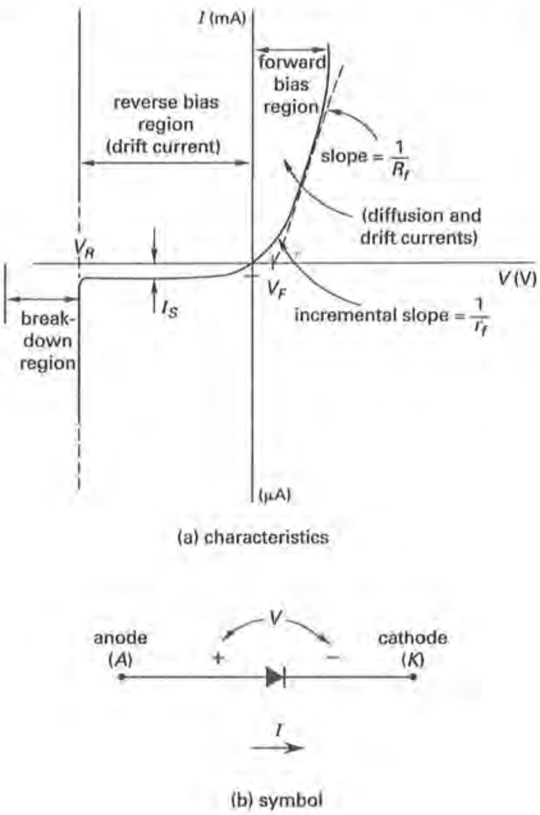


Figure 43.9 *Semiconductor Diode Characteristics and Symbol*

The ideal *pn* junction current was given in Eq. 43.15. For practical junctions (*diodes* or *rectifiers*), the equation becomes

$$I = I_s\left(e^{\frac{qV}{\eta\kappa T}} - 1\right) = I_s\left(e^{\frac{V}{\eta V_T}} - 1\right) \qquad 43.16$$

Equation 43.16, which is based on the *Fermi-Dirac probability function*, is valid for all but the breakdown region (see Fig. 43.9). The term η is determined experimentally. For discrete silicon diodes, $\eta = 2$. For germanium diodes, $\eta = 1$. The saturation current, taken from any value of I with a small reverse bias (for example, between 0 and -1 V) gives $I_s \approx 10^{-9}$ A for silicon and 10^{-6} A for germanium. The term V_T represents the *voltage equivalent of temperature* and is related to the diffusion occurring at the junction.[23]

$$V_T = \frac{\kappa T}{q} = \frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} \qquad 43.17$$

Boltzmann's constant is given by κ. The absolute temperature is T. An electron charge is represented by q. Diffusion constants, measured in m^2/s, are given the symbol D. The mobility, measured in $m^2/V{\cdot}s$, is given by the symbol μ.

The voltage equivalent of temperature is also known as the *thermal voltage*. The value of V_T is often quoted at *room temperature*, which can vary from 293K (20°C) to 300K (27°C) depending upon the reference used. The temperature has the effect of doubling the saturation current every 10°C. Thus,

$$\frac{I_{s2}}{I_{s1}} = (2)^{\frac{T_2 - T_1}{10^\circ C}} \qquad 43.18$$

A *real diode* can be modeled as shown in Fig. 43.10. The real diode model is composed of an ideal diode, a resistor, R_f, and a voltage source, V_F. The voltage source accounts for the barrier voltage. Typically, for silicon $V_F = 0.7$ V, and for germanium $V_F = 0.2$ V.

The resistance, R_f, is the slope of a line approximating the linear portion of the characteristic curve as shown in Fig. 43.9. R_f is the resistance for an ideal diode. It is not the static (average) resistance of the diode, as the average resistance is not a constant. The static (average) resistance is calculated as $R_{\text{static}} = V_D/I_D$. When the diode is forward biased by more than a few tenths of a volt, R_f is equal to the *dynamic forward resistance*, r_f. Disregarding lead contact resistance (less than 2 Ω), the dynamic forward resistance is

$$R_f = r_f = \frac{\eta V_T}{I_D} \qquad 43.19$$

A *dynamic reverse resistance*, r_r, also exists and is the inverse of the slope at a point in the reverse bias region.[24] Since the reverse current is very small, the resistance is often considered infinite. Capacitances associated with the junction are also ignored in most models of the diode. The *diffusion capacitance*, C_d, is associated with the charge stored during forward biased operation. The *transition capacitance*, C_t, is associated with the space-charge region width and thus is the primary capacitance of concern during reverse bias operation. (Diodes designed to take advantage of this voltage-sensitive capacitance are called *varactors*).

[23]This is also called the *Einstein relationship*.

[24]Specifying the reverse current, I_{CO}, is equivalent to specifying the reverse resistance.

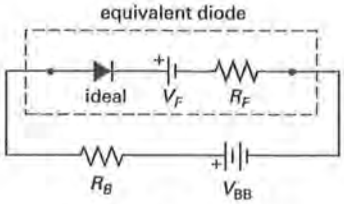


Figure 43.10 Diode Equivalent Circuit

The model of Fig. 43.10 assumes that (1) the reverse bias current is sufficiently small that the diode acts as an open circuit in that direction, (2) the reverse bias voltage does not exceed the breakdown voltage, and (3) the switching time is instantaneous. The *switching time* is the transient that occurs from the time interval of the application of a voltage to forward (reverse) bias and the achievement of the actual condition. The switching time depends upon the speed of movement of minority carriers near the junction and the junction capacitance.

Example 43.4

What is the thermal voltage at a room temperature of 300K?

Solution

The thermal voltage is given by Eq. 43.17 as

$$V_T = \frac{\kappa T}{q} = \frac{\left(1.381 \times 10^{-23}\ \frac{\text{J}}{\text{K}}\right)(300\text{K})}{1.602 \times 10^{-19}\ \text{C}} = 0.026\ \text{V}$$

10. DIODE LOAD LINE

Figure 43.11 shows a forward biased real diode in a simple circuit. V_{BB} is the *bias battery* (hence the subscripts), and R_B is a current-limiting resistor. R_f and V_f are equivalent diode parameters, not discrete components. If R_f is known in the vicinity of the operating point, the diode current, I_D, is found from Kirchhoff's voltage law. The diode voltage is found from Ohm's law: $V_D = I_D R_f$.

$$V_{BB} - V_f = I_D(R_B + R_f) \qquad 43.20$$

The diode current and voltage drop can also be found graphically from the *diode characteristic curve* (Fig. 43.11) and the *load line*, a straight line representing the locus of points satisfying Eq. 43.20.[25] The load line is defined by two points. If the diode current is zero, all of the battery voltage appears across the diode (point $(V_{BB}, 0)$). If the voltage drop across the diode is zero, all of the voltage appears across the current-limiting resistor (point $(0, V_{BB}/R_B)$). (Since the diode characteristic curve implicitly includes the effects of V_f and R_f, these terms should be omitted.) The no-signal *operating point*, also known as the *quiescent point*, is the intersection of the diode characteristic curve and the load line.

The *static load line* is derived assuming there is no signal (i.e., $v_{in} = 0$). With a signal, the *dynamic load line* shifts left or right while keeping the same slope. This is equivalent to solving Eq. 43.20 with an additional voltage source.

$$i_D = I_D + \Delta i_D = \frac{V_{BB} - V_f + v_{in}}{R_B + R_f} \qquad 43.21$$

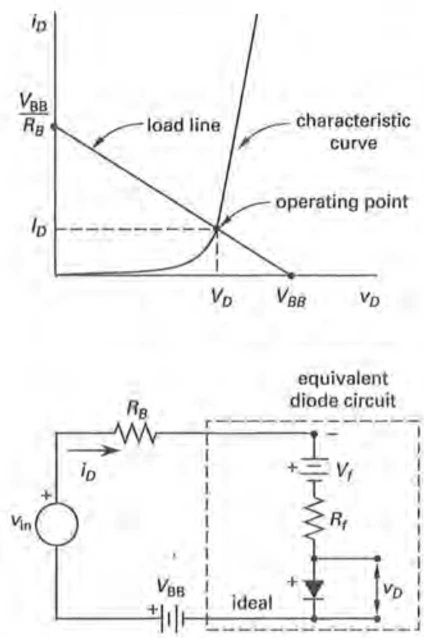


Figure 43.11 Diode Load Line

Although presented in a slightly different manner, the method for determining the load line is similar to that given in Sec. 43-7. That is,

step 1: Open-circuit the electronic component's equivalent circuit and determine the point $(x,0)$.

step 2: Short-circuit the electronic component's equivalent circuit and determine the point $(0,y)$.

step 3: Connect the two points.

The load line superimposed on the characteristics curve is then used to determine the operating point (Q-point) of the overall circuit.

[25] Notice that the horizontal axis voltage is the voltage across the diode (modeled as a resistor).

11. DIODE PIECEWISE LINEAR MODEL

If the voltage applied to a diode varies over an extensive range, a piecewise linear model may be used.[26] For the real diode characteristic shown in Fig. 43.9, three regions are evident from V_R to V_F.

(1) *Reverse breakdown region*, $V_D < V_R$

$$I_D = \frac{V_D + V_R}{R_r} \quad [V_D \text{ and } V_R \text{ are negative}] \qquad 43.22$$

(2) *Off region*, $V_R < V_D < V_F$

$$I_D = 0 \qquad 43.23$$

(3) *Forward bias region*, $V_D > V_F$

$$I_D = \frac{V_D - V_F}{R_f} \qquad 43.24$$

Using ideal diodes, the real diode can be represented by the model in Fig. 43.12 over the entire range of operation.

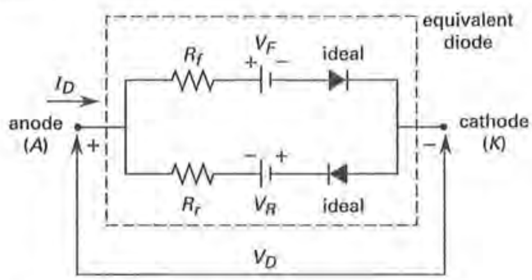
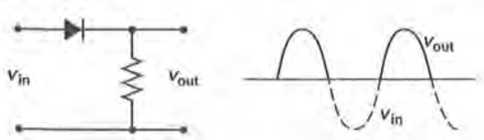


Figure 43.12 *Piecewise Linear Model*

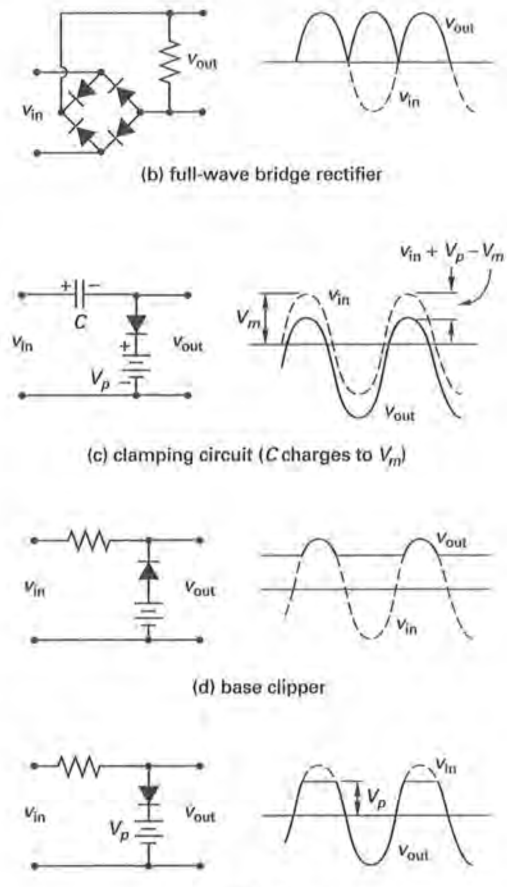
12. DIODE APPLICATIONS AND CIRCUITS

Diodes are readily integrated into *rectifier, clipping*, and *clamping circuits*. A clipping circuit cuts the peaks off of waveforms; a clamping circuit shifts the DC (average) component of the signal. Figure 43.13 illustrates the response to a sinusoid with peak voltage V_m for several simple circuits.



(a) half-wave rectifier

[26]The model in Sec. 43-10 is for the forward biased region only and assumes no reverse current flow.



(b) full-wave bridge rectifier

(c) clamping circuit (C charges to V_m)

(d) base clipper

(e) peak clipper

Figure 43.13 *Output from Simple Diode Circuits*

13. SCHOTTKY DIODES

A *Schottky diode*, also called a *barrier diode* or a *hot-carrier diode*, is a diode constructed with a metal semiconductor contact as shown in Fig. 43.14(a). The Schottky diode symbol is shown in Fig. 43.14(b). The metal semiconductor rectifying junction is similar to the *pn* junction, but the physical mechanisms are somewhat different.

In the forward direction, electrons from the lightly doped semiconductor cross into the metal anode, where electrons are plentiful.[27] The electrons in the metal are majority carriers, whereas in a *p*-type material they are minority carriers. Being majority carriers, they are indistinguishable from other carriers and are thus not stored near the junction. This means that no minority carrier population exists to move when switching

[27]The electrons injected into the metal are above the *Fermi energy level*, determined by the Fermi-Dirac distribution of electron energies, and thus are called *hot carriers*.

occurs from forward to reverse (on to off) bias. Consequently, switching times are extremely short (approximately 10^{-12} s).

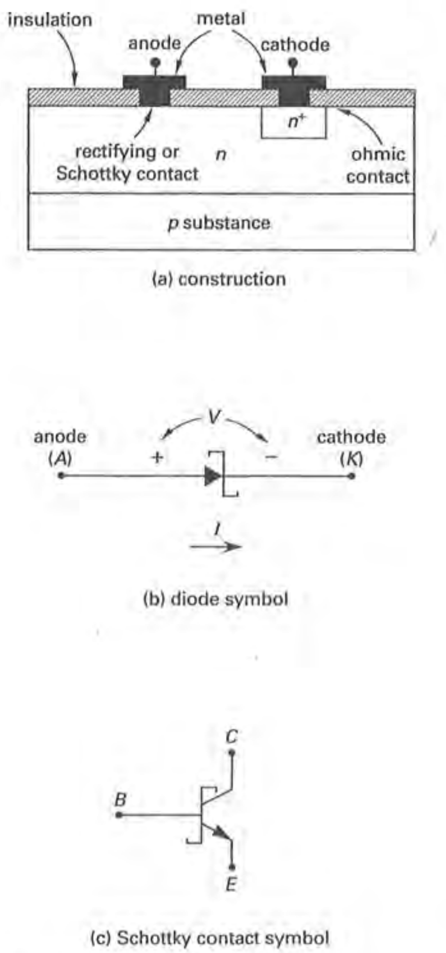


Figure 43.14 *Schottky Diode*

The Schottky symbol is used on any electronic component designed with a *Schottky contact*, that is, a rectifying, rather than an ohmic, contact as shown in Fig. 43.14(c). In the figure, the Schottky contact lies between the base and the collector.

14. ZENER DIODES

A *zener diode* is a diode specifically designed to operate within the breakdown region. The construction and theory of a zener diode is similar to that of a *pn* junction, with the design allowing for greater heat dissipation capabilities. The characteristics and symbology are shown in Fig. 43.15.

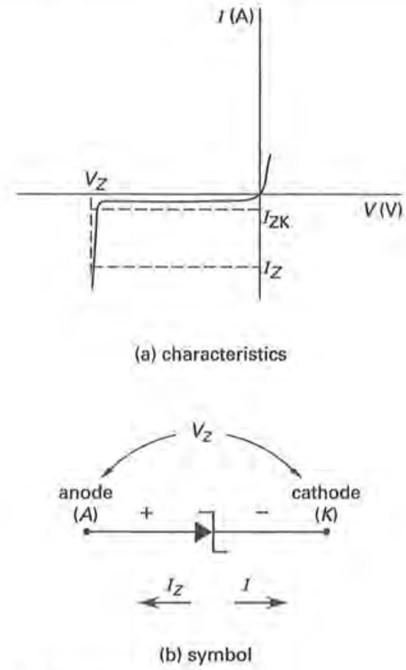


Figure 43.15 *Zener Diode*

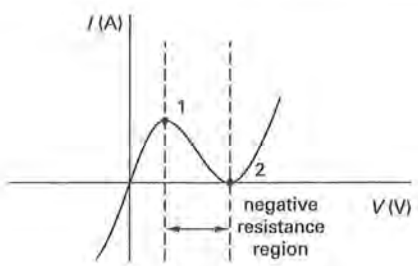
When a reverse voltage, known as the *zener voltage*, V_Z (which is negative with respect to the anode), is applied, a reverse saturation current, I_Z, flows. The voltage-current relationship is nearly linear. Current flows until the reverse saturation current drops to I_{ZK} near the knee of the characteristic curve. This minimum current is the *keep-alive current*, that is, the minimum current for which the output characteristic is linear.

Because the current is large, an external resistor must be used to limit the current to within the power dissipation capability of the diode. Zener diodes are used as voltage regulating and protection devices. The regulated zener voltage varies with the temperature. The *temperature coefficient* is

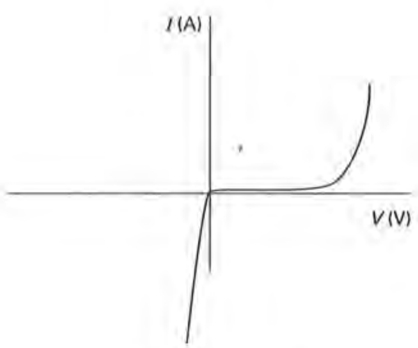
$$\text{tc} = \frac{\Delta V_Z}{V_Z \Delta T} \times 100\% \qquad 43.25$$

15. TUNNEL DIODES

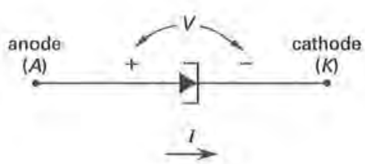
A *tunnel diode* (*Esaki diode*) is a two-terminal device with an extremely thin potential barrier to electron flow, so that the output characteristic is dominated by the quantum-mechanical tunneling process. To make a thin potential barrier (50 to 100 Å), both regions of the diode are heavily doped. The amount of tunneling is limited by the electrons available in the *n*-type material or by the available empty energy states in the *p*-type material to which they can tunnel. The characteristics are shown in Fig. 43.16(a).



(a) tunnel diode characteristics



(b) backward diode



(c) symbol

Figure 43.16 *Tunnel Diode*

Point 1 in Fig. 43.16(a) corresponds to a biasing level that allows for maximum tunneling. (At this point, the minimum electron energy in the n-type material conduction band equals the Fermi level in the p-type material's valence band.) Between points 1 and 2, increases in bias voltage result in a decrease in current as available energy states are filled and the total amount of tunneling drops. This is an area of negative resistance, or negative conductance, and is the primary use for tunnel diodes. If the doping levels are slightly reduced from their typical values of 5×10^{19} cm^{-3}, the forward tunneling current becomes negligible and the output characteristic becomes that of a *backward diode*, Fig. 43.16(b). The symbology for a tunnel diode is shown in Fig. 43.16(c).

16. PHOTODIODES AND LIGHT-EMITTING DIODES

If a semiconductor junction is constructed so that it is exposed to light, the incoming photons generate electron-hole pairs. When these carriers are swept from the junction by the electric field, they constitute a *photocurrent*, which is seen as an increase in the reverse saturation current. The holes generated move to the p-type material and the electrons move to the n-type material due to the electric field that is established whenever p- and n-type semiconductors are joined (see Fig. 43.6(a)). Such devices are used as light sensors and are called *photodiodes*. When the device is designed without a biasing source, it becomes a *solar cell*.

When forward biased, diodes inject carriers across the junction that are above thermal equilibrium. When the carriers recombine, they emit photons from the pn junction area.[28] The photon is due to the recombination of electron-hole pairs. The wavelength depends on the energy band gap and thus on the material used. Gallium arsenide (GaAs) and other binary compounds are commonly used. When the photon is in the infrared region, the mechanism is called *electroluminescence* and the diodes are called *electroluminescent diodes*. If the photons are in the visible region, the devices are called *light-emitting diodes* (LEDs). The emitted wavelength, λ, is given by

$$\lambda = \frac{hc}{E_G} \qquad 43.26$$

Example 43.5

The manufacturer's data sheet for a gallium arsenide diode shows a band gap energy of 1.43 eV. Will such a band gap result in emitted photons within the wavelength of visible light?

Solution

The photon emitted wavelength is given by

$$\begin{aligned}\lambda &= \frac{hc}{E_G}\\ &= \frac{(6.626 \times 10^{-34}\ \text{J·s})\left(3.00 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)}{(1.43\ \text{eV})\left(1.602 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right)}\\ &= 8.68 \times 10^{-7}\ \text{m}\end{aligned}$$

The wavelength of visible light is from approximately 8×10^{-7} to 4×10^{-7} m. Consequently, GaAs is a good choice for a light-emitting diode.

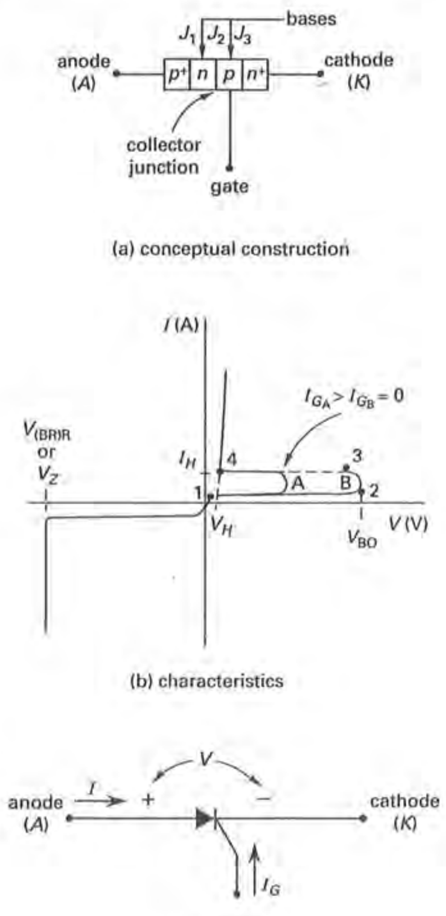
17. SILICON-CONTROLLED RECTIFIERS

A four-layer *pnpn* device with an anode, cathode, and gate terminal is called a *silicon-controlled rectifier*

[28]They can also release the energy as heat, or *phonons*.

(SCR) or *thyristor*.[29] A conceptual construction is shown in Fig. 43.17(a).

When the SCR is reverse biased, that is, when the anode is negative with respect to the cathode, the characteristics are similar to a reverse-biased *pn* junction as shown in Fig. 43.17(b). When the SCR is forward biased, that is, when the anode is positive with respect to the cathode, four distinct regions of operation are evident.



(a) conceptual construction

(b) characteristics

(c) symbol

Figure 43.17 *Silicon-Controlled Rectifier*

From point 1 to point 2, junctions J_1 and J_3 are forward biased. Junction J_2 is reverse biased. The external voltage appears primarily across the reverse-biased junctions. The device continues to operate similarly to a reverse-biased *pn* junction. This is called the *off* or *high-impedance region*.

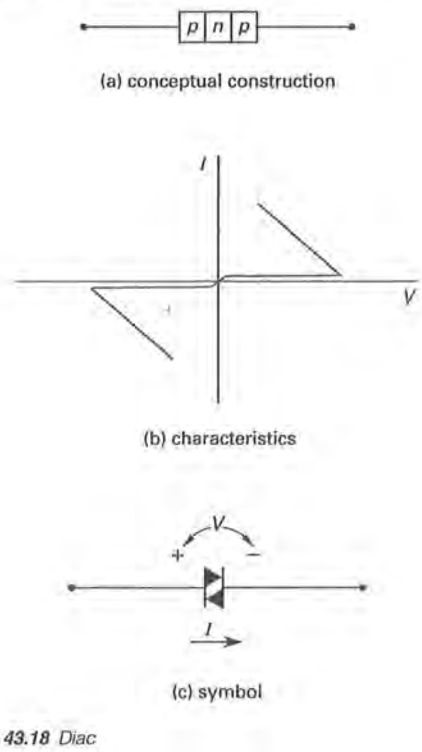
[29]A *thyristor* is defined as a transistor with thyratron-like characteristics. That is, as collector current is increased to a critical value, the alpha (common base current gain) rises above unity and results in a high-speed triggering action.

From point 2 to point 3, the current increases slowly to the *breakover voltage*, V_{BO}. At this point, junction J_2 undergoes breakdown and the current increases sharply.

From point 3 to point 4, the current increases as the voltage decreases. This region is called the *negative resistance region*.

From point 4, junction J_2 is forward biased. The voltage across the device is essentially that of a forward-biased *pn* junction (approximately 0.7 V). If the current through the diode is reduced by the external circuit, the diode remains on until the current falls below the *hold current*, I_H, or the *hold voltage*, V_H. Below this point, the diode switches off, to the high-impedance state.

The gate functions to increase the current at the collector junction of the *npn* transistor, J_2, which is an integral part of the SCR. By increasing the current through the reverse-biased junction, J_2, the anode current is increased. This increases the gain of the two transistors, resulting in a lowering of the forward breakover voltage. This is seen in the difference between paths A and B in Fig. 43.17(b). Consequently, for a given anode to cathode voltage, the gate can be used to turn the SCR on. Once on, however, the SCR must be reverse biased to turn off. A thyristor designed to be turned off by the gate is called a *gate turn-off thyristor*.



(a) conceptual construction

(b) characteristics

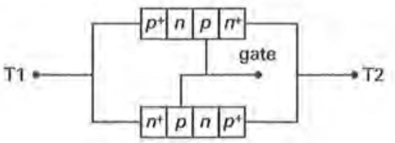
(c) symbol

Figure 43.18 *Diac*

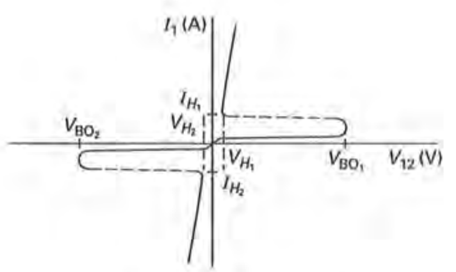
SCRs are used in power applications to allow small voltages and currents to control much larger electrical quantities. The symbology for an SCR is shown in Fig. 43.17(c). Other devices with multiple *pn* junctions using the same principles are the *diac* and *triac*, shown in Figs. 43.18 and 43.19, respectively.

Most electronic components are unable to handle large amounts of current. When properly designed for power dissipation, semiconductors handling large amounts of power are called *power semiconductors*. Such devices constitute a branch of electronics called *power electronics*. Silicon-controlled rectifiers and Schottky diodes are traditional power semiconductors. Newer designs include the *high-power bipolar junction transistor* (HPBT), *power metal-oxide semiconductor field-effect transistor* (MOSFET), *gate turn-off thyristor* (GTO), and *insulated gate bipolar transistor* (IGBT), sometimes called a *conductivity-modulated field-effect transistor* (COMFET).

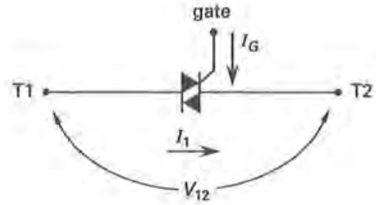
Power semiconductors are classified as either trigger or control devices. *Trigger devices*, such as GTOs, start conduction by some trigger input and then behave as diodes. *Control devices* are normally BJTs and FETs used in full-range amplifiers.



(a) conceptual construction



(b) characteristics

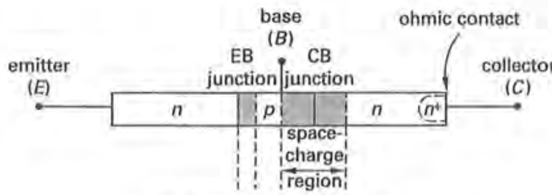


(c) symbol

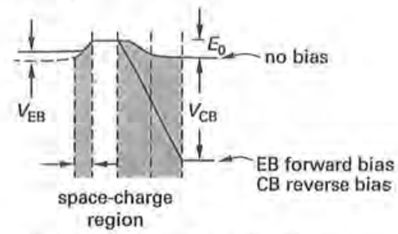
Figure 43.19 *Triac*

18. TRANSISTOR FUNDAMENTALS

A *transistor* is an active device comprised of semiconductor material with three electrical contacts (two rectifying and one ohmic). Two major types of transistors exist: *bipolar junction transistors* (BJTs) and *field-effect transistors* (FETs). A BJT uses the base current to control the flow of charges from the emitter to the collector. Field-effect transistors use an electric field, that is, voltage, established at the gate (equivalent to the base) to control the flow of charges from the source to the drain (equivalent to the emitter and collector). Thus, the controlling variable in a BJT is the current at the base; in a FET it is the voltage at the gate. An *npn* bipolar junction transistor (BJT) is shown in Fig. 43.20(a).

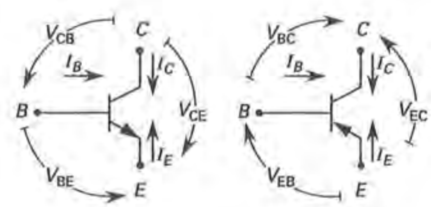


(a) conceptual construction



(space-charge regions shown for active region biasing)

(b) electron energy level



npn *pnp*

(all currents shown in positive direction)
(all voltages shown for active region biasing)

(c) symbol

Figure 43.20 *Bipolar Junction Transistor*

The transistor has three operating regions: *cutoff*, *saturation*, and *active* or *linear*. In the cutoff region, both the base-emitter junction and the collector-base junction are reverse biased. In the saturation region, both the base-emitter junction and the collector-base junction are forward biased. Small amounts of current injected into the base control the movement of charges in the output (from emitter to collector) in a manner similar in theory to that of an SCR, specifically, the triac (see Sec. 43-17). When operating from the cutoff to the saturation region, the transistor is a switch.

In the active, or linear, region, the base-emitter junction is forward biased and the collector-base junction is reverse biased. When so biased, the electron potential energy is of the shape shown in Fig. 43.20(b). Small amounts of current injected into the base lower the potential barrier across the emitter-base junction. As carriers move from the emitter to the base, most are swept across the narrow base by the relatively large electric field established by the reverse-biased collector-base junction. (See the large space-charge, or depletion region, shown in Fig. 43.20(b).) When operating in the active (linear) region, the transistor is an amplifier.

The symbology for BJTs is shown in Fig. 43.20(c). Positive currents are defined as those flowing into the associated terminals. Actual current flows follow the arrows.[30] Majority carriers are always injected into the base by the *emitter* (E). The thin controlling center of a transistor is the *base* (B). The majority carriers are gathered by the *collector* (C).

When operated over all three ranges—cutoff, saturation, and active—the transistor is modeled in a piecewise linear fashion. This is *large-signal analysis*. Operated as a switch, between the cutoff and saturation regions, the transistor is modeled as an open circuit and a short circuit. This is *digital circuit operation*. Operated as an amplifier, in the active region, the transistor is modeled as equivalent parameters, usually h-parameters. This is *analog circuit operation*. Analog analysis occurs in two phases. First, the operating point, or *Q-point*, is determined. Determining the Q-point is a combination of determining the load line for the biasing used and setting the base current by using the values of the biasing components (see Secs. 43-22 and 43-23). This is often a reiterative process. (For a FET, the items determined are the load line and gate voltage.) Second, the incremental AC performance is determined using *small-signal analysis*, that is, using h-parameters to model the transistor and replacing independent sources with the appropriate models (see Sec. 43-26). Real sources are replaced with internal resistance. Ideal voltage sources are replaced with short circuits. Ideal current sources are replaced with open circuits.

19. BJT TRANSISTOR PERFORMANCE CHARACTERISTICS

When the base-emitter junction is forward biased and the collector-base junction is reverse biased, the transistor is said to be operating in the *active region*.

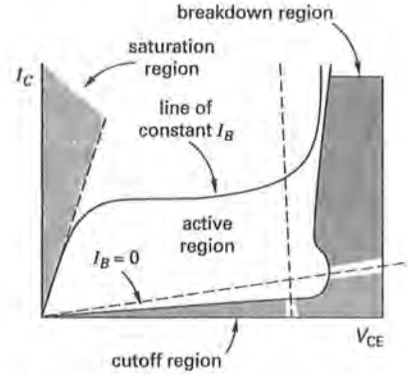


Figure 43.21 BJT Operating Regions

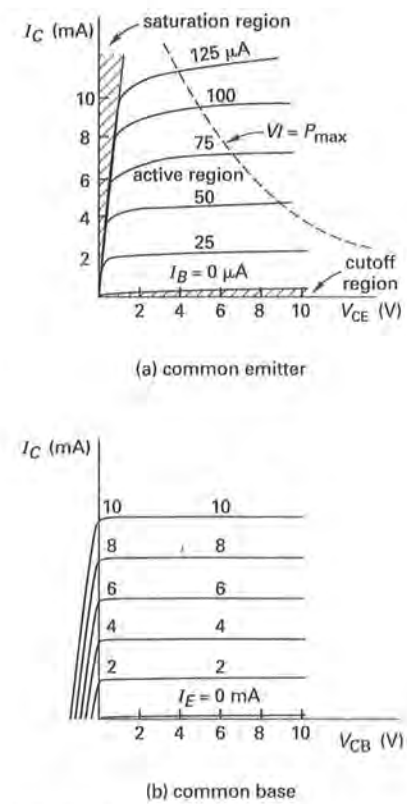


Figure 43.22 BJT Output Characteristics

When the base-emitter junction is not forward biased (as when V_{BB} is zero), the base current will be nearly zero and the transistor acts like a simple switch. This is known as being *off* or *open*, and operating in the *cutoff mode*.[31] Also, the collector and emitter currents are zero

[30] That is, conventional current flow follows the arrows. Electrons flow opposite to the arrows.

[31] Except for digital and switching applications, this condition usually results from improper selection of circuit resistances.

when the transistor operates in the *cutoff region*. However, a very small input voltage (in place of V_{BB}) will forward bias the base-emitter junction which, because I_B is so low, instantly forces the transistor into its saturation region. This results in a large collector current.

When the collector-emitter voltage is very low (usually about 0.3 V for silicon and 0.1 V for germanium), the transistor operates in its *saturation region*. Regardless of the collector current, the transistor operates as a closed switch (i.e., a short circuit between the collector and emitter). This is known as being *on* or *closed*.

$$V_{CE} \approx 0 \quad \text{[saturation]} \qquad 43.27$$

The BJT operating regions are shown in Fig. 43.21. The output characteristics for the common emitter and common base configurations are shown in Fig. 43.22. (A sample maximum power curve is shown as a dashed line. The Q-point must be to the left and below the power curve.)

20. BJT TRANSISTOR PARAMETERS

Equation 43.28 is Kirchhoff's current law, taking the transistor as a node. Usually, the collector current is proportional to, and two or three orders of magnitude larger than, the base current, I_B. Thus, a small change in base current of, for example, 1 mA, can produce a change in collector current of, for example, 100 mA. The *current (amplification) ratio*, β_{DC}, is the ratio of collector-base currents.

$$I_E = I_C + I_B \qquad 43.28$$

$$\beta_{DC} = \frac{I_C}{I_B} = \frac{\alpha_{DC}}{1-\alpha_{DC}} \qquad 43.29$$

$$\alpha_{DC} = \frac{I_C}{I_E} = \frac{\beta_{DC}}{1+\beta_{DC}} \qquad 43.30$$

Both α_{DC} and β_{DC} are for DC signals only. The corresponding values for small signals are designated α_{ac} and β_{ac}, respectively, and are calculated from differentials. (The difference between β_{ac} and β_{DC} is very small, and the two are not usually distinguished.)

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{i_C}{i_B} \qquad 43.31$$

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E} = \frac{i_C}{i_E} \qquad 43.32$$

Thermal (saturation) current is small but always present and can be included in Eq. 43.28. I_{CBO} is the thermal current at the collector-base junction.

$$I_C = I_E - I_B \qquad 43.33$$

$$I_C = \alpha I_E - I_{CBO} \approx \alpha I_E \qquad 43.34$$

Transistors are manufactured from silicon and germanium, although silicon transistors have a higher temperature operating range. While the collector cutoff current is very small at room temperature, it doubles every 10°C, rendering germanium transistors useless around 100°C. Silicon transistors remain useful up to approximately 200°C. While germanium has a lower collector-emitter saturation voltage and may outperform silicon in high-speed and high-frequency devices, silicon is nevertheless the material used for most semiconductor devices and integrated circuit systems.

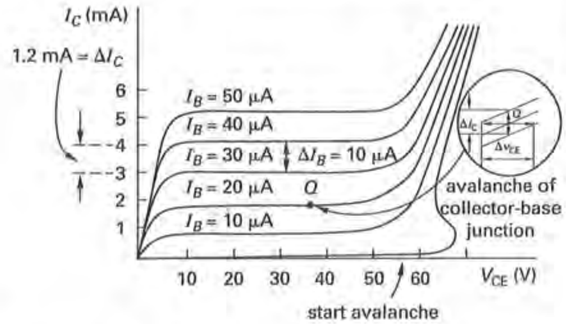


Figure 43.23 *Small Signal Terms*

Figure 43.23 illustrates a family of curves for various base currents. The DC amplification factor, β_{DC}, can be found (for a wide range of V_{CE} values) by taking a point on any line in the active (horizontal line) region and calculating the ratio of the coordinates, I_C/I_B. The small-signal amplification factor, β_{ac}, is calculated as the difference in the I_C between two I_B lines divided by the differences in I_B.

Many of the parameters used have multiple symbols. When given in manufacturers' data sheets or as equivalent parameters, h-parameter symbols are more common. The equivalence is given in Eqs. 43.35 through 43.38.

$$\alpha_{DC} = h_{FB} \qquad 43.35$$

$$\alpha_{ac} = h_{fb} \qquad 43.36$$

$$\beta_{DC} = h_{FE} \qquad 43.37$$

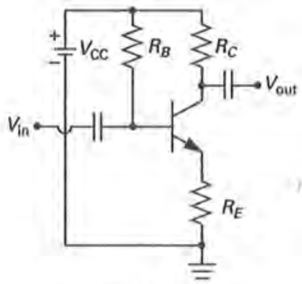
$$\beta_{ac} = h_{fe} \qquad 43.38$$

21. BJT TRANSISTOR CONFIGURATIONS

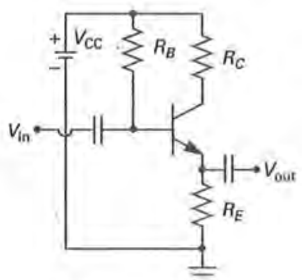
There are six ways a BJT transistor can be connected in a circuit, depending on which leads serve as input and output. Only three configurations have significant practical use, however. The terminal not used for either input or output is referred to as the *common terminal*. For example, in a *common emitter* circuit, the base receives the input signal and the output signal is at the

Electronics

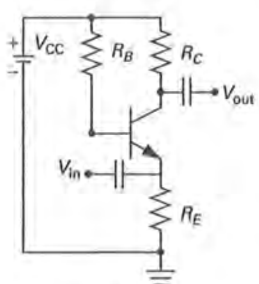
collector. For the *common collector* (also known as an *emitter follower*) circuit, the input is to the base and the output is from the emitter. For a *common base* circuit, the input is to the emitter and the output is from the collector. The various configurations are shown in Fig. 43.24.[32] A summary of the relative properties of the configurations is given in Table 43.1.



(a) common emitter



(b) common collector



(c) common base

Figure 43.24 *Transistor Configurations*

The common emitter configuration is arguably the most versatile and useful. It is the only configuration with a voltage and current gain greater than unity. The common collector is widely used as a buffer stage. That is, it is used between a high-impedance source and a low-impedance load. The common base has the fewest applications, though it is sometimes used to match a low-impedance source with a high-impedance load, or to act as a noninverting amplifier with a voltage gain greater than unity.

[32]Bypass capacitors are not shown.

Table 43.1 *Comparison of Transistor Configuration Properties*

symbol[a]	common emitter[a]	common collector[a]	common base
A_V	high (−100)	low (< 1)	high (+100)
A_I	high (−50)	high (+50)	low (< 1)
R_i	medium (1000 Ω)	high (> 100 kΩ)	low (< 100 Ω)
R_o	high (≈ 50 kΩ)[b]	low (< 100 Ω)	high (≈ 2 MΩ)[b]

[a]Approximate values in parentheses are based on a source and load resistance of 3 kΩ, $h_{ie} \approx 1$ kΩ, and $h_{fe} \approx 50$.
[b]The output resistance is often assumed to be infinite for this configuration, thus simplifying the transistor model.

22. BJT BIASING CIRCUITS

To maximize the transistor's operating range when large swing-signals are expected, the quiescent point should be approximately centered in the active region. The purpose of biasing is to establish the base current and, in conjunction with the load line, the quiescent point. Figure 43.25 illustrates several typical biasing methods: fixed bias, fixed bias with feedback, self-bias, voltage-divider bias, multiple-battery bias, and switching circuit (cutoff) bias. All of the methods can be used with all three common-lead configurations and with both *npn* (shown) and *pnp* transistors.[33] It is common to omit the bias battery (shown in Fig. 43.25(a) in dashed lines) in transistor circuits.

The base current can be found by writing Kirchhoff's voltage law around the input loop, including the bias battery, the external resistances, and the V_{BE} barrier voltage that opposes the bias battery. Since the base is thin, there is negligible resistance from the base to the emitter, and V_{BE} (being less than 1 V) may be omitted as well. For the case of *fixed bias with feedback* (*fixed bias with emitter resistance*) illustrated in Fig. 43.25(b), the base current is found from

$$V_{CC} = I_B R_B + V_{BE} + I_E R_E \quad [v_{in} = 0] \tag{43.39}$$

$$\begin{aligned} I_B &= \frac{V_{CC} - V_{BE} - I_E R_E}{R_B} \quad [v_{in} = 0] \\ &= \frac{V_{CC} - V_{BE}}{R_B + \frac{\beta}{\alpha} R_E} \\ &= \frac{V_{CC} - V_{BE}}{R_B + (1+\beta) R_E} \quad [v_{in} = 0] \end{aligned} \tag{43.40}$$

[33]The polarity of the DC supply voltages must be reversed to convert the circuits shown for use with *pnp* transistors.

SAMSUNG, EXH. 1007, P. 80

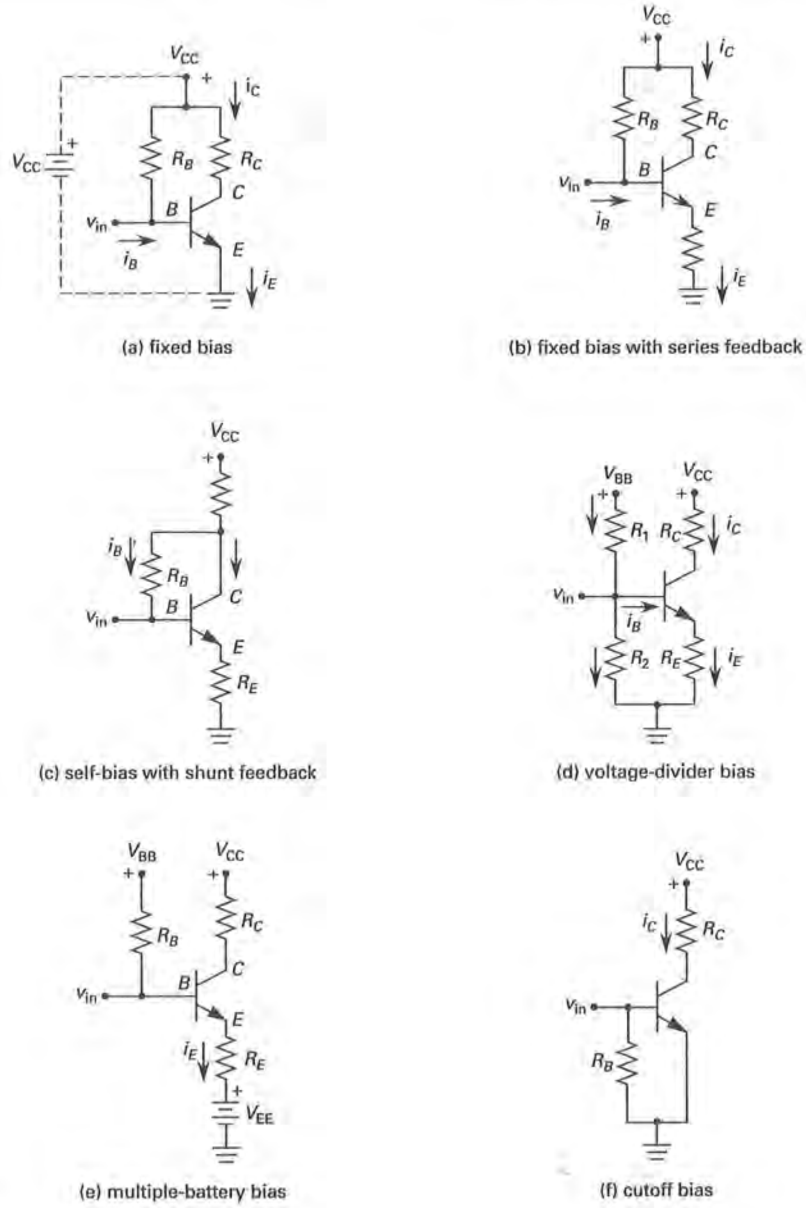


Figure 43.25 *DC Biasing Methods*

The first step in designing a fixed-bias amplifier with emitter resistance is choosing the quiescent collector current, I_{CQ}. R_E is selected so that the voltage across R_E is approximately three to five times the intrinsic V_{BE} voltage (i.e., 0.3 V for germanium and 0.7 V for silicon). Once R_E is found, R_B can be calculated from Eq. 43.39.

$$R_E \approx \frac{3V_{BE}}{I_{CQ}} \qquad 43.41$$

The ***bias stability*** (see the discussion of sensitivity in Sec. 63-5) with respect to any quantity M is given by Eq. 43.42. Variable M commonly represents temperature (T), current amplification (β), collector-base cutoff (I_{CBO}), and base-emitter voltage (V_{BE}).

$$S_M = \frac{\frac{\Delta I_C}{I_{CQ}}}{\frac{\Delta M}{M}} \qquad 43.42$$

The collector-base cutoff current—essentially, the reverse saturation current—doubles with every 10°C rise in temperature as mentioned in Secs. 43-9 and 43-20. The thermal stability of a transistor, then, is affected by this current. The effect is self-reinforcing: as the temperature increases, the saturation current increases, which further increases the temperature. This phenomenon is called *thermal runaway*. The emitter resistor used in the biasing circuits of Fig. 43.25 helps stabilize the transistor against this trend. As the current rises, the voltage drop across the emitter resistor rises in a direction that opposes the forward biased base-emitter junction. This decreases the base current, and so the collector current increases less than it would without the self-biasing resistor R_E.

23. BJT LOAD LINE

Figure 43.26 illustrates part of a simple common emitter transistor amplifier circuit. The bias battery and emitter and collector resistances define the *load line*. If the emitter-collector junction could act as a short circuit (i.e., $V_{CE} = 0$), the collector current would be $V_{CC}/(R_C + R_E)$. If the signal is large enough, it can completely oppose the battery-induced current, in which case the net collector current is zero and the full bias battery voltage appears across the emitter-collector junction. The intersection of the load line and the base current curve defines the *quiescent point*. The load line, base current, and quiescent point are illustrated in Fig. 43.26.

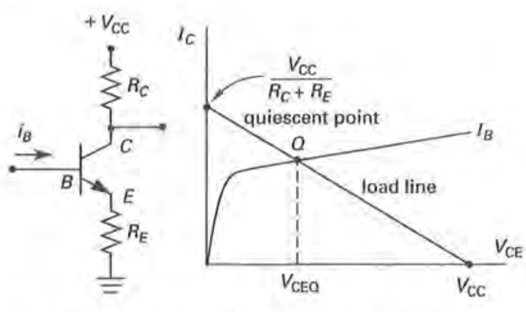


Figure 43.26 *Common Emitter Load Line and Quiescent Point*

Determination of the load line for a BJT transistor is accomplished by the following steps.

step 1: For the configuration provided, label the x-axis on the *output characteristic curves* with the appropriate voltage, V_{CE} or V_{CB}.

step 2: Label the y-axis as the output current, I_C.

step 3: Redraw the circuit with all three terminals of the transistor open. Label the terminals as the base, emitter, and collector. Label the current directions all pointing inward, that is, toward the transistor, I_B, I_E, and I_C.

step 4: Perform KVL analysis in the collector loop. The transistor voltage determined is a point on the x-axis with the output current equal to zero. Plot the point.

step 5: Redraw the circuit with all three terminals of the transistor shorted. Label as in step 3.

step 6: Use Ohm's law, or another appropriate method, in the collector loop to determine the current. The transistor current determined is a point on the y-axis with the applicable voltage in step 1 equal to zero. Plot the point.

step 7: Draw a straight line between the two points. This is the DC load line. The DC load line is used to determine the biasing and Q-point. The AC load line is determined in the same manner, but with active components, that is, inductors and capacitors, included. The AC load line is used to determine the transistor's response to small signals.

24. AMPLIFIER GAIN AND POWER

The *voltage-*, *current-*, *resistance-*, and *power-gain* are

$$A_V = \frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{v_{\text{out}}}{v_{\text{in}}} = \beta A_R \qquad 43.43$$

$$A_I = \frac{\Delta I_{\text{out}}}{\Delta I_{\text{in}}} = \frac{i_{\text{out}}}{i_{\text{in}}} = \beta \qquad 43.44$$

$$A_R = \frac{Z_{\text{out}}}{Z_{\text{in}}} = \frac{A_V}{\beta} \qquad 43.45$$

$$A_P = \frac{P_{\text{out}}}{P_{\text{in}}} = \beta^2 A_R = A_I A_V \qquad 43.46$$

The collector power dissipation, P_C, should not exceed the rated value. (This restriction applies to all points on the load line.) See Fig. 43.22(a) for a sample power restriction.

$$P_C = \tfrac{1}{2} I_C V_{CE} \quad \text{[rms values]} \qquad 43.47$$

25. CASCADED AMPLIFIERS

Several amplifiers arranged so that the output of one is the input to the next are said to be *cascaded amplifiers*.[34] When each *amplifier stage* is properly coupled to the following, the overall gain is

$$A_{\text{total}} = A_{V,1} A_{V,2} A_{V,3} \cdots \qquad 43.48$$

[34] A cascade amplifier should not be confused with a *cascode amplifier* (a high-gain, low-noise amplifier with two transistors directly connected in common emitter and common base configurations).

Capacitors are used in amplifier circuits to isolate stages and pass small signals. This is known as *capacitive coupling*.[35] A capacitor appears to a steady (DC) voltage as an open circuit. However, it appears to a small (AC) voltage as a short circuit, and so input and output signals pass through, leaving the DC portion behind.

26. EQUIVALENT CIRCUIT REPRESENTATION AND MODELS

A transistor can be modeled as any of the equivalent two-port networks described in Sec. 29-31. The different transistor configurations are shown as two-port networks in Fig. 43.27.

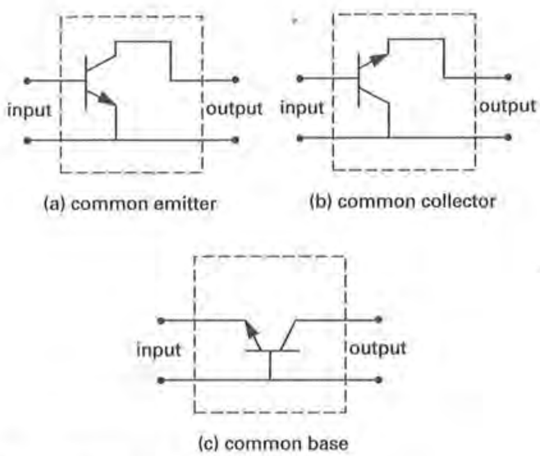


Figure 43.27 *Transistors as Two-Port Networks*

The equivalent parameters most often used are the hybrid parameters, also called the *h-parameters*. The *h*-parameters are defined for any two-port network in terms of Eqs. 43.49 through 43.51.

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \qquad 43.49$$

$$v_1 = h_{11}i_1 + h_{12}v_2 \qquad 43.50$$

$$i_2 = h_{21}i_1 + h_{22}v_2 \qquad 43.51$$

[35]The term *resistor-capacitor coupling* is also used.

Each of the *h*-parameters in Eqs. 43.50 and 43.51 is defined specifically for transistor models in the following way.

h_i = input impedance with output shorted (Ω)

h_r = reverse transfer voltage ratio with input open (dimensionless)

h_f = forward transfer current ratio with output shorted (dimensionless)

h_o = output admittance with input open (S)

Using the *h*-parameters so defined, Eqs. 43.52 and 43.53 become the governing equations for *small-signal circuit models*, also called *AC incremental models*.

$$v_i = h_i i_i + h_r v_o \qquad 43.52$$

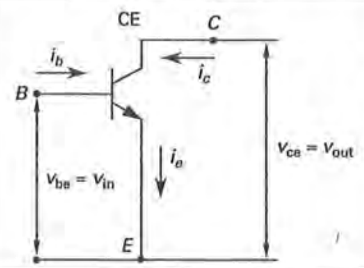
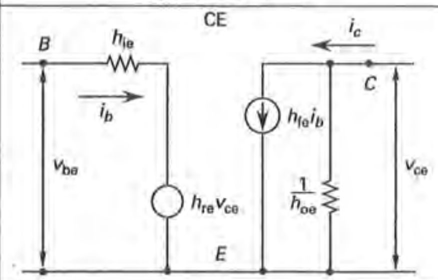
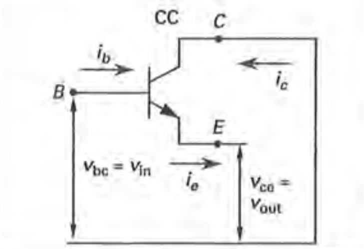
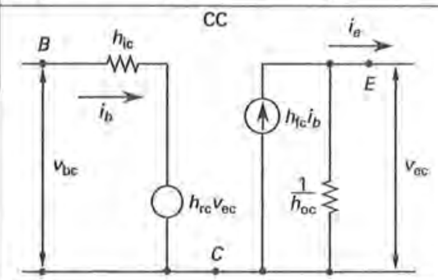
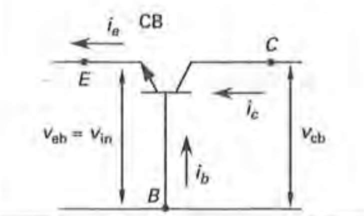
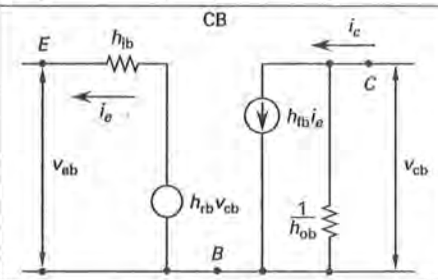
$$i_o = h_f i_i + h_o v_o \qquad 43.53$$

Table 43.2 *Equivalent Circuit Parameters*

symbol	common emitter	common collector	common base
h_{11}, h_{ie}	h_{ie}	h_{ic}	$\frac{h_{ib}}{1+h_{fb}}$
h_{12}, h_{re}	h_{re}	$1 - h_{rc}$	$\frac{h_{ib}h_{ob}}{1+h_{fb}} - h_{rb}$
h_{21}, h_{fe}	h_{fe}	$-1 - h_{fc}$	$\frac{-h_{fb}}{1+h_{fb}}$
h_{22}, h_{oe}	h_{oe}	h_{oc}	$\frac{h_{ob}}{1+h_{fb}}$
h_{11}, h_{ib}	$\frac{h_{ie}}{1+h_{fe}}$	$\frac{-h_{ic}}{h_{fc}}$	h_{ib}
h_{12}, h_{rb}	$\frac{h_{ie}h_{oe}}{1+h_{fe}} - h_{re}$	$h_{rc} - \frac{h_{ic}h_{oc}}{h_{fc}} - 1$	h_{rb}
h_{21}, h_{fb}	$\frac{-h_{fe}}{1+h_{fe}}$	$\frac{-1-h_{fc}}{h_{fc}}$	h_{fb}
h_{22}, h_{ob}	$\frac{h_{oe}}{1+h_{fe}}$	$\frac{-h_{oc}}{h_{fc}}$	h_{ob}
h_{11}, h_{ic}	h_{ie}	h_{ic}	$\frac{h_{ib}}{1+h_{fb}}$
h_{12}, h_{rc}	$1 - h_{re}$	h_{rc}	1
h_{21}, h_{fc}	$-1 - h_{fe}$	h_{fc}	$\frac{-1}{1+h_{fb}}$
h_{22}, h_{oc}	h_{oe}	h_{oc}	$\frac{h_{ob}}{1+h_{fb}}$

Electronics

Table 43.3 BJT Equivalent Circuits

common connection	equivalent circuit	network equations
		common emitter $v_{be} = h_{ie}i_b + h_{re}v_{ce}$ $i_c = h_{fe}i_b + h_{oe}v_{ce}$
		common collector $v_{bc} = h_{ic}i_b + h_{rc}v_{ec}$ $i_e = h_{fc}i_b + h_{oc}v_{ec}$
		common base $v_{eb} = h_{ib}i_e + h_{rb}v_{cb}$ $i_c = h_{fb}i_e + h_{ob}v_{cb}$

Normally the h-parameters are given with two subscripts. The first is defined as in Eqs. 43.52 and 43.53. The second subscript indicates the configuration, such as common emitter (e or E), common collector (c or C), or common base (b or B). The h-parameters are normally specified for the common emitter configuration only. Table 43.2 shows the equivalence between the parameters for the different transistor configurations. The defining equations for the small-signal models are given in Table 43.3. Typical values for a widely used *npn* transistor, the 2N2222A, are given in Table 43.4.

Table 43.4 Typical h-Parameter Values

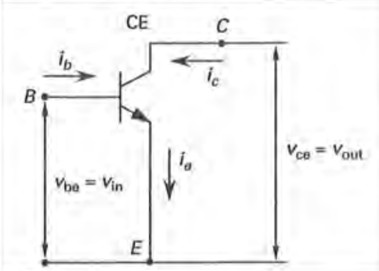
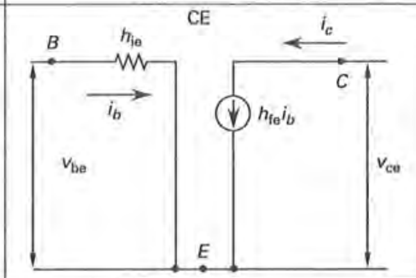
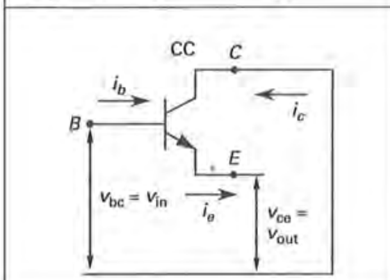
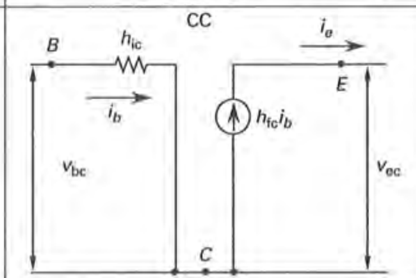
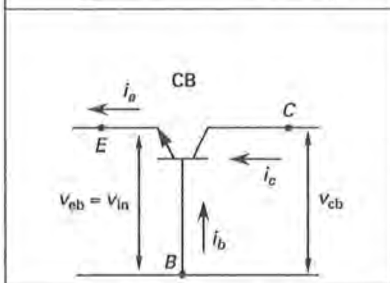
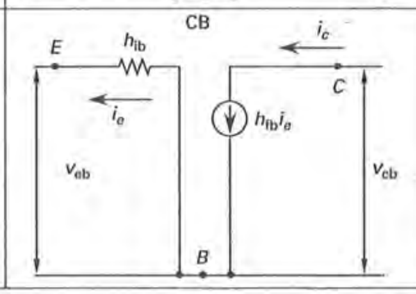
h-parameter	range of values
h_{ie}	0.25×10^3 to 8.0×10^3 Ω
h_{re}	4.0×10^{-4} to 8.0×10^{-4} [max values]
h_{fe}	50 to 375
h_{oe}	5.0×10^{-6} to 200×10^{-6} S

27. APPROXIMATE TRANSISTOR MODELS

The models used in Sec. 43-26 are exact. In many practical applications, sufficiently accurate results can be obtained with simplified models. The values of h_r and h_o are very small. That is, the reverse transfer voltage ratio and the output admittance are insignificant. The simplified models of Table 43.5 are obtained by ignoring these two parameters.

The simplified models assume that the output resistance is infinite, that is, $1/h_o \approx \infty$ (see Table 43.1). The simplified models further assume that the reverse voltage source, $h_r v$, is negligible. The reverse voltage accounts for the narrowing of the base width as the collector-base junction reverse bias increases. As the effectiveness of the base current in controlling the output is minimized, gain decreases. This explains the opposing voltage at the base-emitter junction. This phenomenon is called the *Early effect* (see Fig. 40.20(a)). The simplified models can also be shown with the input impedance, h_i, replaced with a voltage source equal to the barrier voltage (0.7 V for silicon and 0.3 V for germanium).

Table 43.5 BJT Simplified Equivalent Circuits

common connection	equivalent circuit	network equations
		common emitter[a] $v_{be} = h_{ie} i_b \approx 0.7$ V $i_c = h_{fe} i_b$
		common collector $v_{bc} = h_{ic} i_b$ $i_e = h_{fc} i_b$
		common base[a] $v_{eb} = h_{ib} i_e \approx 0.7$ V $i_c = h_{fb} i_e$

[a]Germanium transistors are *pnp* types. $|v_{be}| = |v_{eb}| = 0.3$ for germanium.

28. HYBRID-π MODEL

Though the h-parameter model is common, another significant model is the *hybrid-π* or *Giacoletto* model. The hybrid-π model is shown in Fig. 43.28. The terms used in the model follow.

$r_{bb'}$ = *base spreading resistance*, or the *interbase resistance*, or the *small-signal base bulk resistance* (Ω)

$r_{b'e}$ = *small-signal base input resistance* (Ω)

$r_{b'c}$ = *small-signal feedback resistance* (Ω) This resistance accounts for the Early effect.

r_{ce} = *small-signal output resistance* (Ω)

g_m = transistor *transconductance* (S)

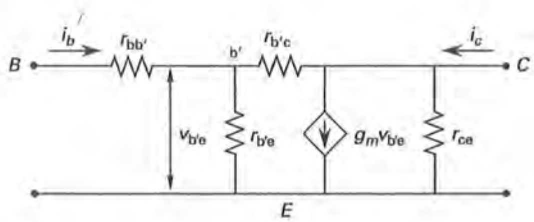


Figure 43.28 Hybrid-π Model

If the common emitter h-parameters are known, the hybrid-π parameters can be calculated from Eqs. 43.54 through 43.58, in the order given.

$$g_m = \frac{|I_C|}{V_T} \qquad 43.54$$

$$r_{b'e} = \frac{h_{fe}}{g_m} \qquad 43.55$$

$$r_{bb'} = h_{ie} - r_{b'e} \qquad 43.56$$

$$r_{b'c} = \frac{r_{b'e}}{h_{re}} \qquad 43.57$$

$$g_{ce} = h_{oe} - \frac{1 + h_{fe}}{r_{b'c}} \qquad 43.58$$

29. TRANSISTOR CIRCUIT LINEAR ANALYSIS

Because the small-signal, low-frequency response of a transistor is linear, it can be obtained analytically rather than graphically by using the models in Sec. 43-26.[36] The procedure for small-signal, or AC incremental, analysis follows.

step 1: Draw the circuit diagram. Include all external components, such as resistors, capacitors, and sources from the network.

step 2: Label the points for the base, emitter, and collector but do not draw the transistor. Maintain the points in the same relative position as in the original circuit.

step 3: Replace the transistor by the desired model. (Several models exist. Exact models were given in Sec. 43-26. Approximate models are given in Sec. 43-27. The hybrid-π model is given in Sec. 43-28.)

step 4: Since small-signal analysis is to be accomplished, only slight changes around the quiescent point are of interest. Therefore, replace each *independent* source by its internal resistance. An ideal voltage source is replaced with a short circuit. An ideal current source is replaced with an open circuit. (If biasing analysis were occurring, that is, large-signal analysis, the sources would remain as drawn.) Additionally, replace inductors with open circuits, and capacitors with short circuits.

step 5: Solve for the desired parameter(s) in the resultant circuit using Kirchhoff's current and voltage laws.

Example 43.6

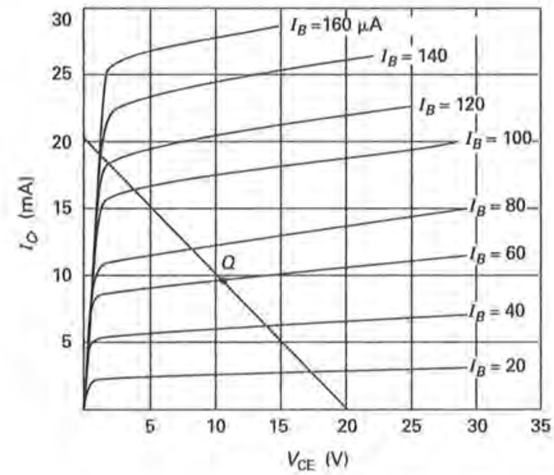
A transistor is used in a common emitter amplifier circuit as shown. Assume the inductor has infinite impedance and the capacitors have zero impedance to AC signals. The transistor h-parameters are $h_{ie} = 750\ \Omega$, $h_{oe} = 9.09 \times 10^{-5}$ S, $h_{fe} = 184$, and $h_{re} = 1.25 \times 10^{-4}$.

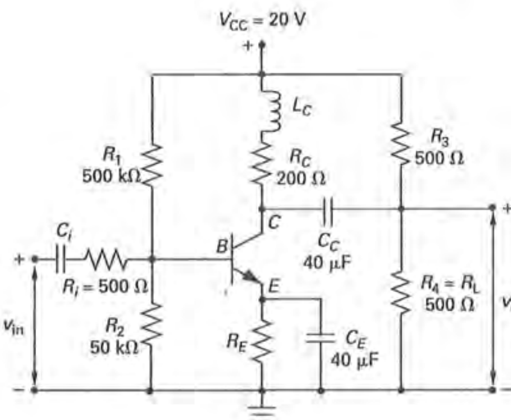
(a) If a quiescent point is wanted at approximately $V_{CE} = 11$ V and $I_C = 10$ mA, what should the emitter resistance, R_E, be?

For parts (b) through (g), assume $R_E = 800\ \Omega$.

[36]The h-parameters must be considered complex functions of the frequency to analyze high-frequency circuits, so the h-parameter model is not used.

(b) Draw the DC load line. (c) If the base current (i_B) is 80×10^{-6} A, what is the collector current (i_C)? (d) What is the AC circuit voltage gain? (e) What is the AC circuit current gain? (f) What is the input impedance? (g) What is the output impedance? (h) Given $v_{in} = 0.25 \sin 1400t$ V, what is v_{out}? (i) What is the purpose of the inductor, L_C?





Solution

(a) Write the voltage drop in the common emitter circuit. Disregard the inductor (which passes DC signals). Use h_{fe} for h_{FE} since they are essentially the same and both are large. Kirchhoff's voltage law is

$$\alpha = \frac{\beta}{1+\beta} \approx \frac{h_{fe}}{1+h_{fe}} = \frac{184}{1+184} \approx 1$$

$$\begin{aligned} V_{CC} &= I_C R_C + I_E R_E + V_{CE} \\ &= I_C R_C + \left(\frac{I_C}{\alpha}\right) R_E + V_{CE} \\ &\approx I_C (R_C + R_E) + V_{CE} \end{aligned}$$

$$R_E = \frac{V_{CC} - V_{CE}}{I_C} - R_C = \frac{20\text{ V} - 11\text{ V}}{10 \times 10^{-3}\text{ A}} - 200\ \Omega \approx 700\ \Omega$$

(b) If $I_C = 0$, then $V_{CE} = V_{CC}$. This is one point on the load line. If $V_{CE} = 0$, then

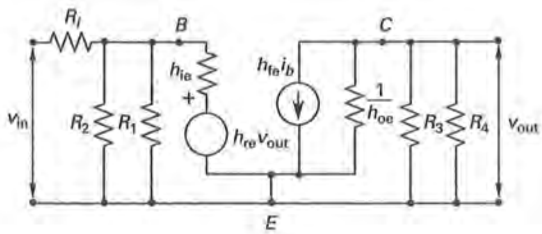
$$I_C = \frac{V_{CC}}{R_C + R_E} = \frac{20\text{ V}}{200\ \Omega + 800\ \Omega} = 0.02\text{ A}$$

These two points define the DC load line.

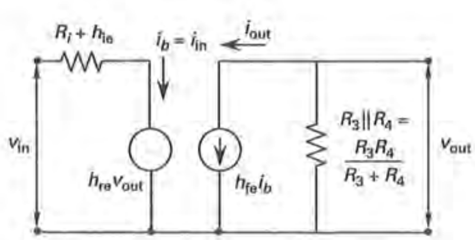
(c) From Eq. 43.29,

$$i_C = \beta i_B = h_{fe} i_B = (184)(80 \times 10^{-6}\text{ A}) = 14.7\text{ mA}$$

(d) To determine the AC circuit voltage gain, it is necessary to simplify the circuit. The bias battery V_{CC} is shorted out. (This is because the battery merely shifts the signal without affecting the signal swing.) Therefore, many of the resistors connect directly to ground. The inductor has infinite impedance, so R_C is disconnected. Both capacitors act as short circuits, so R_E is bypassed.



To simplify the circuit further, recognize that R_1, R_2, and $1/h_{oe}$ are very large and can be treated as infinite impedances.



Continuing with the simplified analysis,

$$v_{out} = h_{fe} i_b \left(\frac{R_3 R_4}{R_3 + R_4}\right) = (184 i_b)\left(\frac{(500)(500)}{500 + 500}\right) = 46{,}000 i_b$$

$$v_{in} = i_b(R_i + h_{ie}) + h_{re} v_{out} = i_b(500 + 750) + (1.25 \times 10^{-4})(46{,}000 i_b) = 1256 i_b$$

(To perform an exact analysis, h_{oe} and h_{re} must be considered. Convert R_i to its Norton equivalent resistance and place it in parallel with R_1, R_2, and h_{ie}. Use the current-divider concept to calculate i_b.)

The voltage gain is

$$A_V = \frac{v_{out}}{v_{in}} \approx \frac{46{,}000 i_b}{1256 i_b} = 36.6$$

(e) The current gain is

$$A_I = \frac{i_{out}}{i_{in}} \approx \frac{h_{fe} i_b}{i_b} = 184$$

(f) The input impedance (resistance) is

$$R_{in} = \frac{v_{in}}{i_{in}} \approx \frac{1256 i_b}{i_b} = 1256\ \Omega$$

(g) The output impedance is effectively the Thevenin equivalent resistance of the output circuit. The load resistance (R_4 in this instance) is removed. The independent source voltage (v_{in}) is shorted, which effectively opens the controlled source $h_{fe} i_b$. The remaining resistance between collector and ground is

$$R_{out} = R_3 = 500\ \Omega$$

(h) The output voltage is

$$v_{out} = A_V v_{in} = (36.6)(0.25)\sin 1400t = 9.15 \sin 1400t\text{ V}$$

(i) There are several possible uses for the inductor. It might be included to limit voltage extremes, such as high-voltage spikes, which could damage the transistor when the amplifier is turned on or off. Alternatively, it might prevent AC current from being drawn across R_C, and, in so doing, hold V_C at a constant value.

30. TRANSISTOR CIRCUIT HIGH-FREQUENCY ANALYSIS

The h-parameter models are useful at low frequencies. For high frequencies, the hybrid-π model is modified to include the effects of various capacitances as shown in Fig. 43.29.

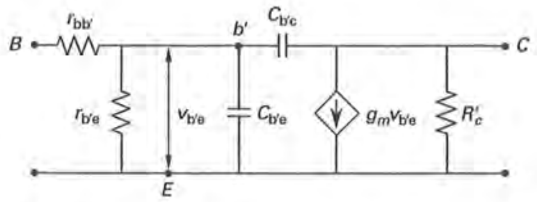


Figure 43.29 *High-Frequency Hybrid-π Model*

The capacitance $C_{b'e}$ is the sum of the diffusion capacitance, C_d, and the transition capacitance, C_t (see Sec. 43-9). The capacitance $C_{b'c}$ is the transition capacitance for the collector junction. The high-frequency model is used whenever the frequency exceeds the *corner frequency*, ω_{cf}, that is, the frequency at which the resistance is 3 dB less than its DC value.[37]

$$\omega_{cf} = \frac{1}{RC} \qquad 43.59$$

The capacitance term, C, in Eq. 43.59 is the total capacitance. The resistance, R, is as seen from the terminals of the total capacitance. (In low-frequency h-parameter models, the bypass capacitors are typically not shown. It is to these capacitors that Eq. 43.59 refers.)

31. UNIJUNCTION TRANSISTORS

A *unijunction transistor* is an electronic device with three contacts, two ohmic and one to a *pn* junction, as shown in Fig. 43.30(a). The output characteristics are illustrated in Fig. 43.30(b), which clearly shows a negative resistance region. The UJT operates in two states: on (low resistance) and off (high resistance). The symbol and equivalent circuit are shown in Figs. 43.30(c) and (d). The device is used in oscillator circuits, as a *UJT relaxation oscillator*, and in pulse generation and delay circuits.

32. DARLINGTON TRANSISTORS

A *Darlington transistor*, more commonly called a *Darlington pair*, is a current amplifier consisting of two separate transistors treated as one. A Darlington transistor has a high input impedance and a significant current gain. For example, the forward current transfer gain, h_{fe}, can be as high as 30,000. A Darlington transistor is illustrated in Fig. 43.31.

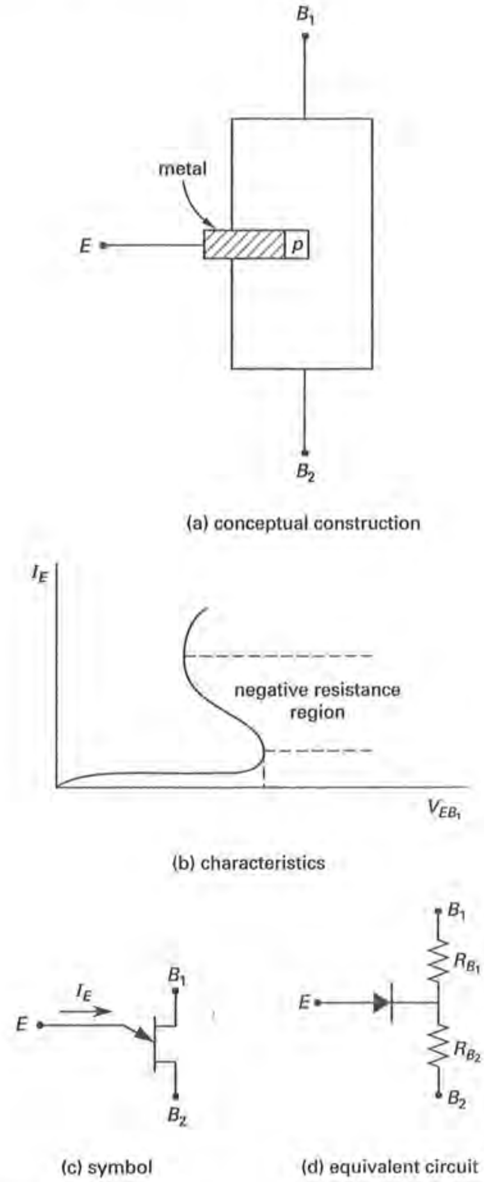


Figure 43.30 *Unijunction Transistor*

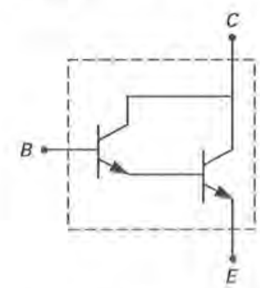


Figure 43.31 *Darlington Transistor*

[37] When the frequency is $10\omega_{cf}$, the impedance is essentially the capacitive reactance. The resistance is considered insignificant at that point.

33. FET FUNDAMENTALS

Field-effect transistors (FETs) are bidirectional devices constructed of an *n*-type channel surrounded by a *p*-type gate, or vice versa.[38] The connections are named in a different manner than BJTs in order to distinguish the two. For a FET (BJT), the connections are the *gate* (base), *source* (emitter), and *drain* (collector). In a BJT, the base current controls the overall operation of the transistor. In a FET, the gate voltage, and thus the corresponding electric field, controls the overall operation of the transistor. The various configurations and the concepts regarding biasing, load lines, and amplifier operation described for BJTs also apply to FETs.

There are two major types of FETs: the *junction field-effect transistor* (JFET) and the *metal-oxide semiconductor field-effect transistor* (MOSFET). Both types are made in *n*- and *p*-channel types, with the *n*-channel types more common. The fundamental difference between the JFET and the MOSFET is that the latter can operate in the enhancement mode (see Sec. 43-37).[39]

Variations in construction result in different names and properties. An *n*-channel MOSFET is sometimes referred to as an NMOS, and the *p*-channel MOSFET is sometimes called a PMOS. A high-power MOSFET is called an HMOS. A MOSFET with increased drain current capacity due to its V-type structure is called a VMOS. A double-diffused MOSFET, which has replaced the VMOS except in high-frequency applications, is called a DMOS. A bipolar transistor using an insulated gate is called an *insulated gate bipolar transistor* (IGBT). An SCR using an insulated gate for control is called a *MOS-controlled thyristor* (MCT). A MOSFET with *p*-channel and *n*-channel devices on the same chip is called a *complementary MOSFET* (CMOS) and is used primarily because of its low power dissipation.

When the conventional MOSFET oxide layer is replaced by a double layer of nitride and oxide, which allows electrons to tunnel from the gate, the device is called a *metal-nitride-oxide-silicon* (MNOS) transistor. The MNOS device is used as a read-only memory (ROM). MOS transistors are also used as memory devices. The MOS transistor is used to control the transfer of the charge from one location to the next. As a result, it is also called a *charge-transfer device* (CTD). When the charge transfer is accomplished on the circuit level by discrete MOS transistors and capacitors, it is called a *bucket-brigade device* (BBD). When the charge transfer takes place on the device level, it is called a *charge-coupled device* (CCD).

34. JFET CHARACTERISTICS

Junction field-effect transistors' (JFETs') construction, characteristics, and symbols are shown in Fig. 43.32.

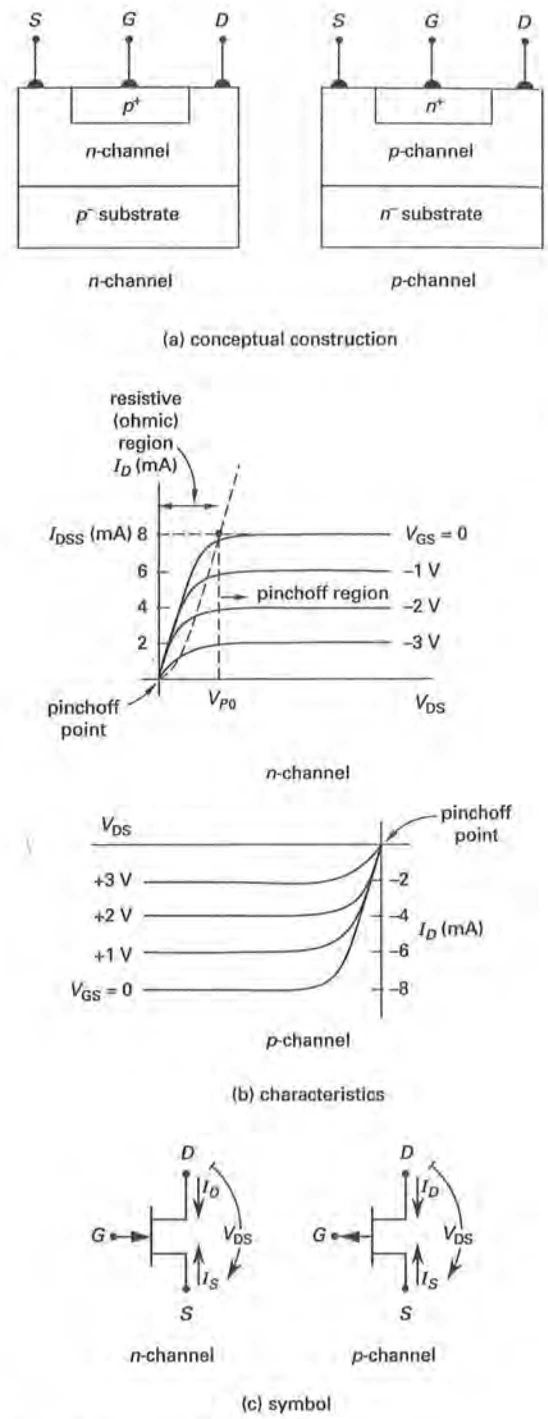


Figure 43.32 *Junction Field-Effect Transistor*

A bidirectional channel for current flow exists between the source and the drain. The current flow is controlled by the reverse biased *pn* junction at the gate, with the

[38]Bidirectional indicates that the current flow direction depends on the potential between the source and the drain.
[39]MOSFETs are also much more susceptible to damage from *electrostatic discharges* (ESD) due to the thin oxide layer at the gate.

depletion width providing the control.[40] For example, when the gate is unbiased, a normal depletion zone is set up at the gate *pn* junction, as shown in Fig. 43.33(a) for an *n*-channel JFET.

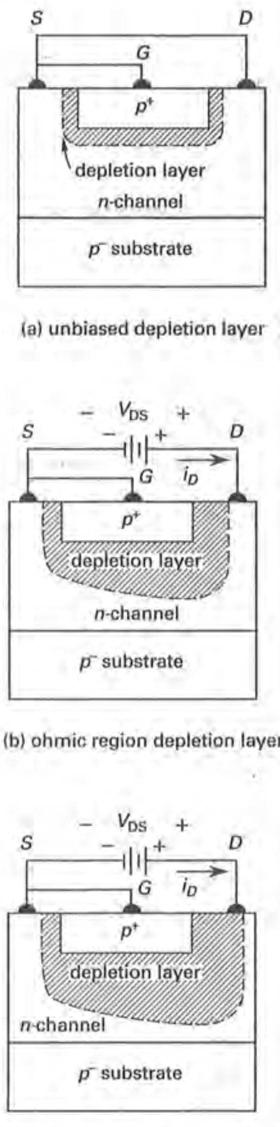


Figure 43.33 *Pinchoff Theory*

Once biased, the depletion layer shifts, and the JFET operates in a relatively linear fashion in the *ohmic region*, also called the *triode region.* The ohmic depletion layer is shown in Fig. 43.33(b). The ohmic region is shown in Fig. 43.32(b). As the drain-source voltage increases, the depletion layer widens until it encompasses the drain terminal. At this point, the current is pinched off, and the JFET operates in a relatively constant manner in the *pinchoff region.* The pinchoff region depletion layer is shown in Fig. 43.33(c). The pinchoff region is shown in Fig. 43.32(b). As the drain-source voltage continues to increase, avalanche breakdown occurs and the voltage remains approximately constant as the current increases dramatically. This is known as the *breakdown* or *avalanche region* (not shown).

For a fixed value of V_{GS}, the drain-source voltage separating the resistive and pinchoff regions is the *pinchoff voltage.* As Fig. 43.32(b) shows, there is a value for V_{GS} for which no drain current flows. This is also referred to as the pinchoff voltage but is designated $V_{GS\,(off)}$. The drain current corresponding to the horizontal part of a curve (for a given value of V_{GS}) is the *saturation current*, represented by I_{DSS}.

The term "pinchoff voltage" and the symbol V_P are ambiguous, as the actual pinchoff voltage in a circuit depends on the gate-source voltage, V_{GS}. When V_{GS} is zero, the pinchoff voltage is represented unambiguously by V_{P0} (where the zero refers the value of V_{GS}). For other values of V_{GS}),

$$V_P = V_{P0} + V_{GS} \qquad 43.60$$

Some manufacturers do not adhere to this convention when reporting the pinchoff voltage for their JFETs. They may give a value for V_{P0} and refer to it as V_P. The absence of a value for V_{GS} implies that the value given is actually V_{P0}.

Some manufacturers do not provide characteristic curves, choosing instead to indicate only I_{DSS} and $V_{GS\,(off)}$. The characteristic curves can be derived, as necessary, from *Shockley's equation*.

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \qquad 43.61$$

The *transconductance*, g_m, is defined for small-signal analysis by Eq. 43.62.

$$\begin{aligned} g_m &= \frac{\Delta I_D}{\Delta V_{GS}} = \frac{i_D}{v_{GS}} \\ &= \left(\frac{-2I_{DSS}}{V_P}\right)\left(1 - \frac{V_{GS}}{V_P}\right) = g_{mo}\left(1 - \frac{V_{GS}}{V_P}\right) \\ &\approx \frac{A_V}{R_{out}} \end{aligned} \qquad 43.62$$

The drain-source resistance can be obtained from the slope of the V_{GS} characteristic in Fig. 43.32(b).

$$r_d = r_{DS} = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{v_{DS}}{i_D} \qquad 43.63$$

[40]The source "emits" the *n*- or *p*-channel carriers, by convention. Thus, the reverse bias is with respect to the gate. FETs are bidirectional, nevertheless.

35. JFET BIASING

JFETs operate with a reverse biased gate-source junction. The quiescent point is established by choosing V_{GSQ}. I_D is then determined by Shockley's equation (Eq. 43.61).

Figure 43.34 shows a JFET self-biasing circuit. Since the gate current is negligible, the load line equation is

$$V_{DD} = I_S(R_D + R_S) + V_{DS}$$
$$= I_D(R_D + R_S) + V_{DS} \qquad 43.64$$

The load line for any JFET is determined using the procedures outlined in Sec. 43-7.

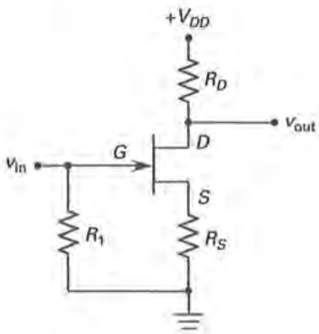


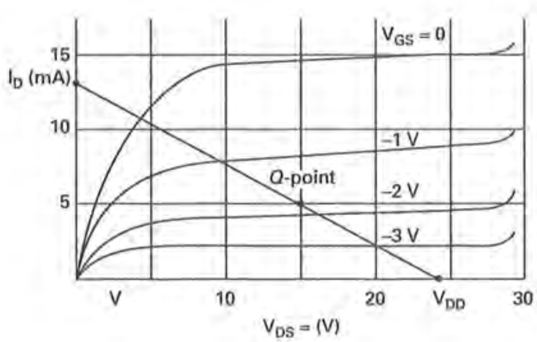
Figure 43.34 Self-Biasing JFET Circuit

At the quiescent point, $V_{in} = 0$. From Kirchhoff's voltage law, around the input loop,

$$V_{GS} = -I_S R_S = -I_D R_S \qquad 43.65$$

Example 43.7

A JFET with the characteristics shown operates as a small-signal amplifier. The supply voltage is 24 V. A quiescent bias source current of 5 mA is desired at a bias voltage of $V_{DS} = 15$ V. Design a self-biasing circuit similar to Fig. 43.34.



Solution

Since the gate draws negligible current, $I_D = I_S$. At the quiescent point, $V_{GS} = -1.75$ V. From Eq. 43.65,

$$R_S = \frac{-V_{GS}}{I_D} = \frac{-(-1.75 \text{ V})}{0.005 \text{ A}} = 350 \ \Omega$$

From Eq. 43.64,

$$R_D = \frac{V_{DD} - V_{DS}}{I_S} - R_S = \frac{24 \text{ V} - 15 \text{ V}}{0.005 \text{ A}} - 350 \ \Omega$$
$$= 1450 \ \Omega$$

36. FET MODELS

A field-effect transistor can be modeled as shown in Fig. 43.35. The model is valid for both the JFET and the MOSFET. Because the drain resistance is very high (see the typical values in Table 43.6), the model can be simplified further by removing r_{DS}, that is, by assuming $r_{DS} \approx \infty$.

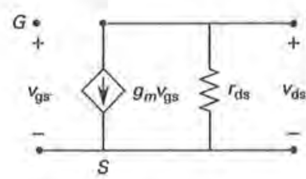


Figure 43.35 FET Equivalent Circuit

Table 43.6 Typical FET Parameter Values

parameter	JFET	MOSFET
g_m	0.1×10^{-3} to 10×10^{-3}S	0.1×10^{-3} to 20×10^{-3} S or greater
r_{ds}	0.1×10^6 to 1×10^6 Ω	1×10^3 to 50×10^3 Ω
C_{ds}	0.1×10^{-12} to 1×10^{-12} F	0.1×10^{-12} to 1×10^{-12} F
C_{gd}, C_{gs}	1×10^{-12} to 10×10^{-12} F	1×10^{-12} to 10×10^{-12} F

At high frequencies the various capacitances associated with the FET must be accounted for as shown in the model in Fig. 43.36.

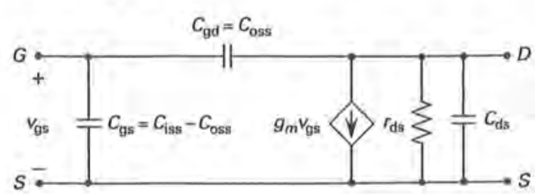


Figure 43.36 FET High-Frequency Model

37. MOSFET CHARACTERISTICS

The *metal-oxide semiconductor field-effect transistor* (MOSFET) is constructed as either a *depletion device* or an *enhancement device* as shown in Fig. 43.37(a). In the depletion device, a channel for current exists with the gate voltage at zero. In order to control the current, a voltage applied to the gate must push channel majority carriers away from the gate, thus pinching off the current. For an n-channel device, the gate voltage must be

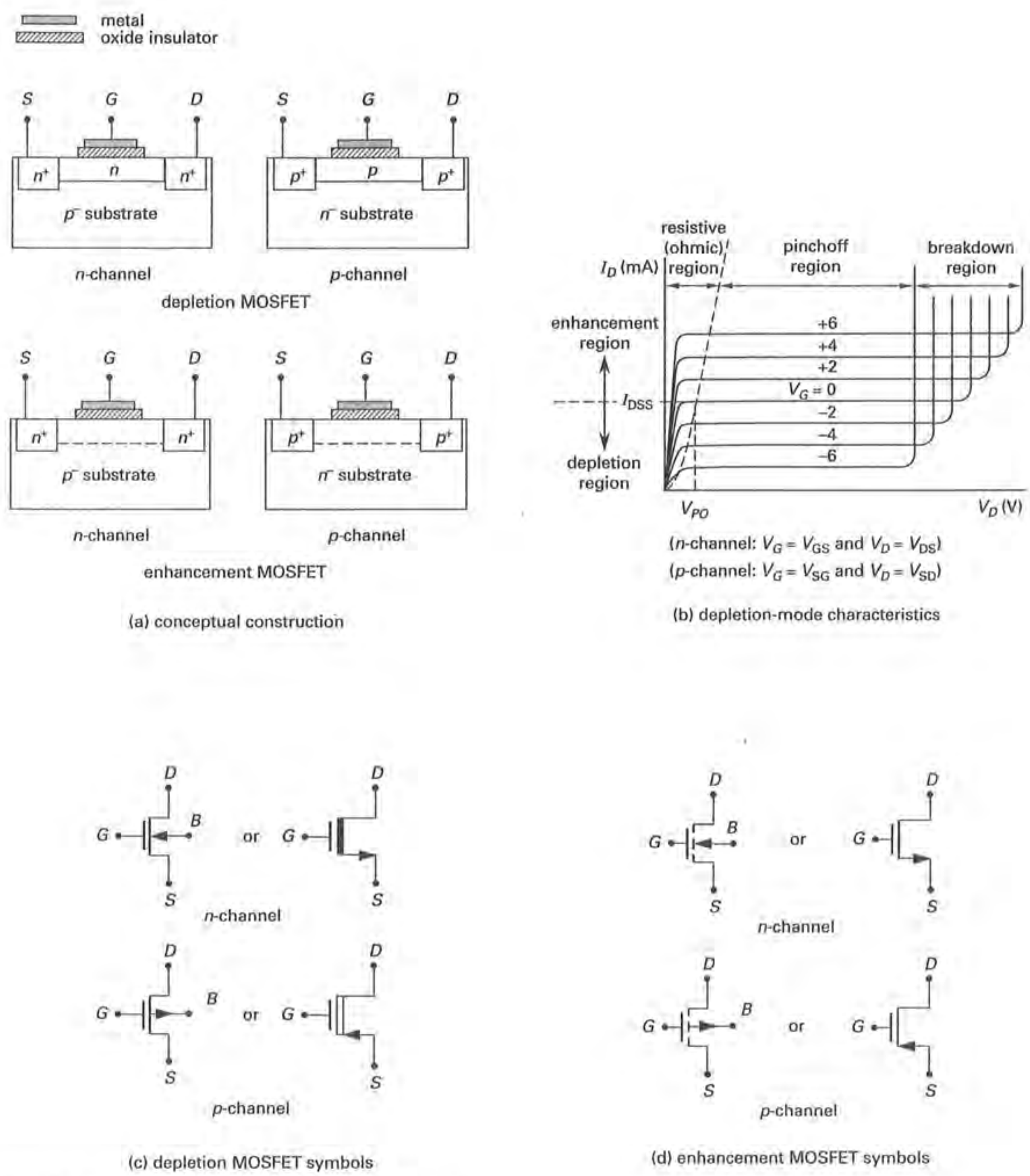


Figure 43.37 *Metal-Oxide Semiconductor Field-Effect Transistor*

negative. For a p-channel device, the gate voltage must be positive.[41] In an enhancement device, a channel for current does not exist with the gate voltage at zero,but must be created. In order to create the channel and control the current, a voltage must be applied to the gate to pull majority carriers toward the gate, thus increasing the current. For an n-channel device, the gate voltage must be positive. For a p-channel device, the gate voltage must be negative. A summary of these characteristics, which are similar to those for JFETs, is shown in Fig. 43.37(b). Three operating regions exist: ohmic, pinchoff, and breakdown. The various symbols used to represent MOSFETs are shown in Fig. 43.37(c) and (d). The MOSFET model is identical to that for the JFET (see Sec. 43-36).

38. MOSFET BIASING

A typical MOSFET biasing circuit is illustrated in Fig. 43.38. Since MOSFETs do not have a pn junction between the gate and channel, biasing can be either forward or reverse. The polarity of the gate-source bias voltage depends on whether the transistor is to operate in the depletion mode or enhancement mode. Since no gate current flows, the resistor R_G is used merely to control the input impedance. When R_G is very large, the input impedance is essentially R_2.

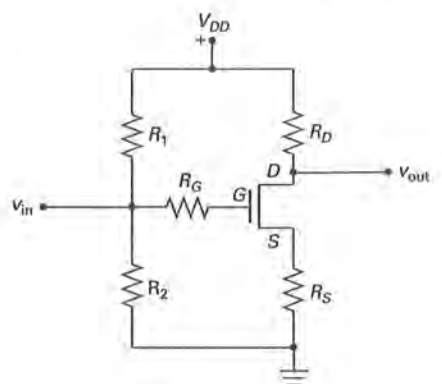


Figure 43.38 *Typical MOSFET Biasing Circuit*

Together, R_1 and R_2 form a voltage divider. Since the gate current is zero, the voltage divider is unloaded. Therefore, the gate voltage is

$$V_G = V_{DD}\left(\frac{R_2}{R_1+R_2}\right) = V_{GS} + I_S R_S \qquad 43.66$$

The load line equation is found from Kirchhoff's voltage law and is the same as for the JFET.

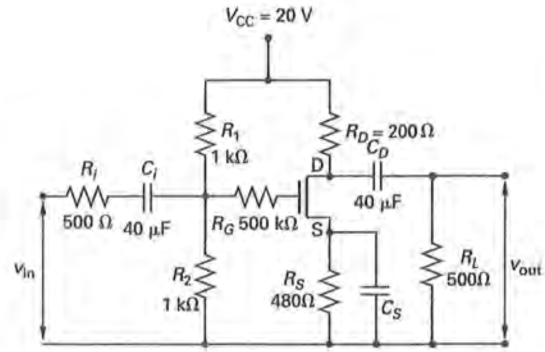
$$V_{DD} = I_S(R_D + R_S) + V_{DS} \qquad 43.67$$

[41]The gate voltage is referenced to the source by convention, since the source is considered the emitter of majority carriers into the channel.

Example 43.8

A MOSFET is used in an amplifier as shown. All capacitors have zero impedance to AC signals. The no-signal drain current (I_D) is 20 mA. The performance of the transistor is defined by

$$I_D = (14 + V_{GS})^2 \qquad \text{[in mA]}$$



(a) What is the V_D (potential) at point D with no signal? (b) What is V_S with no signal? (c) What is V_{DSQ}? (d) If $V_G = 0$ V, what is V_{GS}? (e) What is the input impedance? (f) What is the voltage gain? (g) What is the output impedance?

Solution

(a)
$$V_D = V_{CC} - I_D R_D$$
$$= 20\ V - (20 \times 10^{-3})(200\ \Omega) = 16\ \text{V}$$

(b) The voltage drop between the source and the ground is through R_S. Since the gate draws negligible current, the drain and source currents are the same.

$$V_S = I_S R_S = (20 \times 10^{-3}\ \text{A})(480\ \Omega) = 9.6\ \text{V}$$

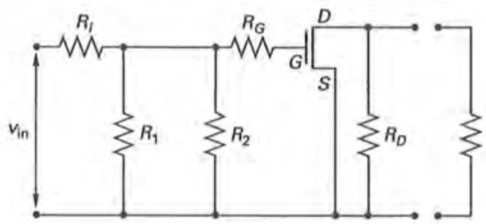
(c) The quiescent voltage drop across the drain-source junction is

$$V_{DSQ} = V_D - V_S = 16\ \text{V} - 9.6\ \text{V} = 6.4\ \text{V}$$

(d) If the gate is at zero potential and the source is at 9.6 V potential, then

$$V_{GS} = V_G - V_S = 0\ V - 9.6\ \text{V} = -9.6\ \text{V}$$

(e) To simplify the circuit, short out the bias battery, V_{CC}. Consider the capacitors as short circuits to AC signals.



Since R_G is so large, it effectively is an open circuit. The input impedance (resistance) is

$$R_{in} = R_i + R_1 \parallel R_2 = 500\ \Omega + \frac{(1000\ \Omega)(1000\ \Omega)}{1000\ \Omega + 1000\ \Omega}$$
$$= 1000\ \Omega$$

(f) A FET has properties similar to a vacuum tube. The voltage gain is normally calculated from the transconductance.

$$A_V \approx g_m R_{out}$$

The transconductance is not known, but it can be calculated from Eq. 43.62 and the performance equation.

$$g_m = \frac{dI_D}{dV_{GS}} = \frac{d(14 + V_{GS})^2 \times 10^{-3}}{dV_{GS}}$$
$$= (2)(14 + V_{GS}) \times 10^{-3}$$

Since $V_{GS} = -9.6$ V at the quiescent point,

$$g_m = (2)(14 - 9.6) \times 10^{-3} = 8.8 \times 10^{-3}\ \text{S}$$

The resistance is a parallel combination of R_D, R_L, and r_d. The drain-source resistance, r_d, is normally very large and, since it was not given in this problem, is disregarded. Then,

$$R = R_D \parallel R_L = \frac{R_D R_L}{R_D + R_L}$$
$$= \frac{(200\ \Omega)(500\ \Omega)}{200\ \Omega + 500\ \Omega} = 143\ \Omega$$
$$A_V = g_m R = (8.8 \times 10^{-3}\ \text{S})(143\ \Omega) = 1.26$$

This is a small gain. If advantage is not being taken of other properties possessed by the circuit, the resistances should be adjusted to increase the voltage gain.

(g) Proceeding as in the solution to Ex. 43.6, part (g), the output impedance is $R_D = 200\ \Omega$.