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Addressing the gap in scheduling research: a review of optimization and heuristic methods in production scheduling

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This paper considers the gap between scheduling theory and scheduling practice. The development and the main results of classical scheduling theory are reviewed and presented in an easily accessible way. Recent trends in scheduling research which attempt to make it more relevant and applicable are described. The nature of the gap between theory and practice is discussed. The failure of classical scheduling theory to address the total environment within which the scheduling function operates is noted. However, scheduling research in operations management and manufacturing systems tends to ignore the rich vein of methods, techniques and results in the classical theory. The need for an integrated scheduling research effort, containing elements of both approaches, is stressed.

1. Introduction

Scheduling may be defined as the allocation of resources over time to perform tasks. The importance of good scheduling strategies in production environments in today's competitive markets cannot be overstressed. The need to respond to market demands quickly and to run plants efficiently gives rise to complex scheduling problems in all but the simplest production environments.

The theory of scheduling has received a lot of attention from OR practitioners, management scientists, production and operations research workers and mathematicians since the early 1950s. A number of books have been published on the subject, e.g. Muth and Thompson (eds) 1963, Conway *et al.* (1967), Elmaghraby (ed) 1973, Baker (1974), Rinnooy Kan (1976), French (1982), Bellman *et al.* (1982). Review articles of varying breadths and depths which survey the development of scheduling theory include Mellor (1966), Lenstra *et al.* (1977), Graham *et al.* (1979), Graves (1981), Frost (1984), Blazewicz *et al.* (1988), Rodammer and White (1988), Buxey (1989), Kovalev *et al.* (1989), White (1990).

The utilization of classical scheduling theory in most production environments is minimal. In many production environments scheduling and plant loading is frequently carried out by first line management. In some sectors it may be delegated to shift leaders, foremen, or chargehands. In many cases there is no appreciation that a body of theory exists which may relate to some or perhaps all of the scheduling problems. More importantly the consequences of poor scheduling strategies on overall company performance is generally not appreciated.

This paper is aimed at researching the gap between scheduling theory and scheduling practice. Firstly, the state-of-the-art in classical scheduling theory is

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reviewed in sections 2 and 3. The emphasis is on clarity and simplicity in outlining the main developments, solution approaches and the most significant results. This review emphasizes major trends and does not attempt fully comprehensive coverage. Secondly, research which attempts to make classical scheduling theory more useful in practice is discussed in section 4. In section 5 the need for an integrated approach to production scheduling research is discussed.

2. Classical scheduling theory and its development

Although the definition of scheduling given at the beginning of the paper is widely applicable it is conventional (and appropriate in this case) to denote the tasks as jobs and the resources as machines. The definitions and assumptions in classical scheduling theory are now outlined. The limitations of these definitions and assumptions are discussed in sections 4 and 5.

2.1. *Definitions and assumptions*

A general scheduling problem may be stated thus:

n jobs $\{J_1, J_2, \ldots, J_n\}$ have to be processed. m machines $\{M_1, M_2, \ldots, M_m\}$ are available. A subset of these machines is required to complete the processing of each job. The flow pattern or order of machines for any job may or may not be fixed for some or all jobs. The processing of job J_j on machine M_i is called an operation, denoted by O_{ij}. For each operation O_{ij}, there is an associated processing time t_{ij}. In addition, there may be a ready time (or release date) r_j associated with each job, at which time J_j is available for processing, and/or a due date d_j, by which time J_j should be completed. A schedule in this context is an assignment of jobs over time onto machines. The scheduling problem is to find a schedule which optimizes some performance measure.

The stated scheduling problem may be generalized further by replacing machines by processing stages which may contain several machines.

The following assumptions appear frequently in scheduling theory literature:

(1) Machines are always available and never break down.
(2) Each machine can process at most one job at any time.
(3) Any job can be processed on at most one machine at any time.
(4) Ready times of all jobs are zero, i.e. all jobs are available at the commencement of processing.
(5) No pre-emption is allowed—once an operation is started it is continued until complete.
(6) Setup times are independent of the schedules and are included in processing times.
(7) Processing times and technological constraints are deterministic and known in advance and similarly for due dates, where appropriate.

In classical scheduling theory the planning framework or time horizon in which a scheduling problem may arise or a schedule be applicable is generally not considered. The implicit assumption is not only that decision-making is short term in a static, deterministic environment but also the fact that researchers have realized that problem complexity increases further if a dynamic environment is considered.

2.2.1. *Classification of scheduling problems*

In defining a scheduling problem both the technological constraints on jobs and the scheduling objectives must be specified.

Technological constraints are determined principally by the flow pattern of the jobs on machines. In this context the following definitions are useful:

(1) *Job shop*: each job has its own individual flow pattern or specific route through the machines which must be adhered to.
(2) *Flow shop*: each job has an identical flow pattern.
(3) *Open shop*: there is no specified flow pattern for any jobs.
(4) *Permutation flow shop*: a flow shop in which the order of processing of jobs on all machines is constrained to be the same.
(5) *Single machine shop*: only one machine is available.

In cases (1), (2) and (3) the schedule may produce a different order of jobs on machines in the shop. When processing stages are considered rather than machines the following definitions are useful.

(6) *Parallel machines*: k identical machines in a single processing stage. Each job needs one and only one of these machines.
(7) *Job shop with duplicate machines*: A job shop in which there are k_i identical machines in each stage ($i = 1, \ldots, m$) and any job requiring that stage needs to be processed on one and only one of these machines.

The diagram in Fig. 1 illustrates schematically the relationship between the different machine environments. At the expense of clarity it could be extended further, e.g. a flow shop with duplicate machines.

Within any of these environments scheduling may be attempted with respect to various objectives. Mellor (1966) lists 27 different objectives. A useful classification for single objective problems was given by Baker (1974). For the jth job define the following measures:

Completion time C_j

Flow time $F_j = C_j - r_j$†

Waiting time $W_j = C_j - r_j - \sum_{i=1}^{m} t_{ij}$

Lateness $L_j = C_j - d_j$

Tardiness $T_j = \max\{0, L_j\}$

Baker (1974) noted three types of decision-making goals prevalent in scheduling and indicated commonly used measures of schedule performance which are associated with them:

- *Efficient utilization of resources*: Maximum completion time (or makespan) $C_{\max}$.
- *Rapid response to demands*: Mean completion time $\bar{C}$, mean flow time $\bar{F}$, or mean waiting time $\bar{W}$.

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† Clearly completion times and flow times are equivalent when ready times are all zero.

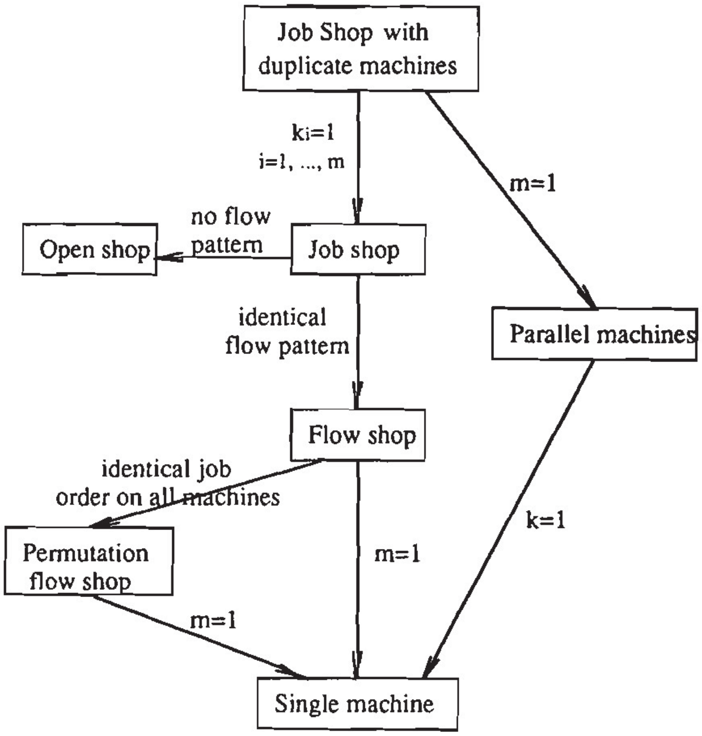

Figure 1. Relationships between machine shop environments.

- *Close conformance to prescribed deadlines*: Mean tardiness $\bar{T}$, maximum tardiness $T_{\max}$, and the number of tardy jobs N_T.

Most other commonly used measures may also be viewed within this framework. Other approaches to measure schedule performance are considered in section 4.

2.2.2. *Notation for scheduling problems*

Conway *et al.* (1967) give a classification scheme for scheduling problems based on four descriptors A/B/C/D which has since been followed by a number of researchers. It is used here in an extended form to include a larger problem set.

Definition	Possible value
A—number of jobs.	any positive integer, usually *n*.
B—number of machines.	any positive integer, frequently *m*.

Note: When parallel machines are considered the value of this descriptor is the number of processing stages and the number of machines at each stage is included in descriptor C.

C—flow pattern and further technological and management constraints. Possible values are:

|| : single machine
J : job shop
F : flow shop
O : open shop
F,perm : permutation flow shop
k-parallel : k-machines in parallel
J,k-parallel : job shop with k parallel machines at each stage

The following abbreviations have been used to represent additional constraints which may occur in more complex scheduling environments:

r_j : jobs with different ready times
str : string jobs
prec : precedence constraints
prmt : pre-emption is allowed
unit : unit processing times
eq : equal processing time for all jobs
depend : dependent jobs
setup : sequence-dependent setup times

D—criteria to be optimized.

Usually minimization of schedule performance measures noted in section 2.2.1, e.g. $C_{\max}$, $\bar{F}$, etc.

For convenience here those problems for which the assumptions in 2.1 hold are referred to as basic problems. These can all be described easily in the above notation. For example, $n/m/J/C_{\max}$ refers to the job shop scheduling problem with n jobs and m machines which attempts to minimize makespan. Non-basic problems are ones where some of the assumptions are not valid and/or extra conditions apply, e.g. $r_j > 0$ for some j. In presenting these problems, descriptor C may become long. Graham *et al.* (1979) introduced another notation based on three descriptors $\alpha/\beta/\gamma$. The first descriptor α defines the flow pattern together with the number of machines. The second descriptor β represents other constraints on jobs. The third descriptor γ defines the scheduling criterion. Although this notation can represent non-basic problems easily and has been used by some authors (e.g. Lawler *et al.* 1989), we use Conway's notation with the refinements defined above in this paper. This notation has been used widely for a long time and is familiar to most manufacturing and scheduling researchers. Converting between the two notations is simple.

2.3. *Historical development of classical scheduling theory*

Like many OR application areas the study of scheduling theory began in the early 1950s. Johnson's article (Johnson 1954) is acknowledged as a pioneering work. It presented an efficient optimal algorithm for $n/2/F/C_{\max}$ and generalized the method for some special cases of $n/3/F/C_{\max}$. Jackson (1955) and Smith (1956) gave various optimal rules for single-machine problems. These early works formed the basis for much of the development of classical scheduling theory.

Later several kinds of general optimization procedures were applied to scheduling problems. These included mixed and pure integer programming formulations (Wagner 1959, Bowman 1959), dynamic programming (Held and Karp 1962), and branch and bound methods (Ignall and Schrage 1965, Lomnicki 1965). Meanwhile heuristic methods were being developed for problems which were known to be computationally difficult (Palmer 1965). By the late 1960s a solid body of scheduling theory had emerged (Conway *et al.* 1967).

In the 1970s theoretical work on problem complexity began. It was found that most problems are NP-hard (Lenstra *et al.* 1977). Fast optimal algorithms are unlikely to exist for these problems. The effectiveness of heuristic algorithms was studied by theoretical analysis and computational experiment. By the late 1970s the theory of scheduling was maturing (French 1982) with a greater understanding of the nature of these problems. The last decade has seen a tendency to emphasize the practical nature of scheduling problems and to try to bridge the gap between theory and practice.

3. Main results of classical scheduling research

This section outlines clearly the main development and solution approaches in classical scheduling theory and presents the most significant methods and results in an easily accessible and digestible form.

3.1. *Results for basic scheduling problems*

A range of methods have been developed to solve scheduling problems. They are mainly of three kinds—efficient optimal methods, enumerative optimal methods, and heuristic methods.

3.1.1. *Efficient optimal methods*

These are methods which generate an optimal schedule with respect to some scheduling criterion in polynomial time, i.e. the time taken to find an optimal solution is a polynomial function of the problem variables (jobs and machines). Specific methods of this type can usually be applied only to specific problems or relatively narrow problem classes. Figure 2 lists the main problems for which such methods have been found. In common with Figs 4–6 the results are presented in order of increasing machine shop complexity with an indication of the type of method and an appropriate reference.

These methods solve their related problems optimally and efficiently even for problem instances with a large number of jobs. Unfortunately this type of algorithm is found only for a small class of problems—mainly single machine and simple flowshop problems. More general problems are inherently more complex.

Results concerning complexity of scheduling problems were given in Garey *et al.* (1976) and Lenstra *et al.* (1977). Figure 3 is a list of problems which have been shown to be NP-hard (Lenstra *et al.* 1977). The main conclusion from this work is that efficient optimal algorithms are unlikely to exist for these problems. To generate good schedules in these cases requires either enumerative or heuristic methods.

3.1.2. *Enumerative optimal methods*

Here we consider more general methods which involve a partial enumeration of the set of all possible schedules. The most general are mathematical programming formulations, followed by branch and bound methods and elimination methods. Figure 4 summarizes results which have been reported using these methods. Note that for some problems the same type of method has been applied by a number of researchers. Formulations for these methods are not unique and may frequently be improved.

Problem	Method	Reference
$n/1//\Sigma C_j$	SPT rule	Smith (1956)
$n/1//\Sigma w_jC_j$	WSPT rule	Smith (1956)
$n/1//L_{\max}$ or $T_{\max}$	EDD rule	Jackson (1955)
$n/1//N_T$	Moore's algorithm and Hodgson's efficient implementation	Moore (1968)
$n/1//k$-parallel/$\bar{F}$		see Baker (1974)
$n/2/F/C_{\max}$	Johnson's algorithm	Johnson (1954)
$n/3/F/C_{\max}$	(special case)	
$n/2/F/C_{\max}$ and $n/3/F/C_{\max}$	alternative implementation of Johnson's algorithm	Kusiak (1986) Chow (1989)
$n/2/J/C_{\max}$	Johnson's algorithm	Johnson (1954) also see Frech (1982)
$n/2/O/C_{\max}$		See Gonzalez & Sahni (1976)

Figure 2. Efficient optimal methods for basic problems.

Problem	Remark†
$n/1/\text{prec.}/\sum C_j$	with precedence constraints
$n/1/r_j \geqslant 0/\sum C_j$	with different ready times
$n/1/r_j \geqslant 0/L_{\max}$	with different ready times
$n/1/r_j \geqslant 0/\sum T_j$	with different ready times
$n/1/r_j \geqslant 0/N_T$	with different ready times
$n/1//\sum w_jT_j$	
$n/1/C_j \leqslant d_j/\sum w_jC_j$	no tardy jobs are allowed
$n/1/2\text{-parallel}/C_{\max}$	
$n/1/2\text{-parallel}/L_{\max}$	
$n/2/F/r_j \geqslant 0/C_{\max}$	with different ready times
$n/2/F/\sum C_j$	
$n/2/F/\sum T_j$	
$n/2/F/L_{\max}$	
$n/2/F/N_T$	
$n/3/F/C_{\max}$	
$n/m/F$, no wait/$C_{\max}$	no waiting between any two stages
$n/2/J$, $n_j \leqslant 3/C_{\max}$	each job can have at most 3 operations
$n/3/J$, $n_j \leqslant 2/C_{\max}$	each job can have at most 2 operations
$n/2/O/L_{\max}$	Lawler *et al.* (1981)
$n/3/O/C_{\max}$	Gonzalez and Sahni (1976)
$n/m/O/\sum C_j$	Gonzalez (1982)

Figure 3. Some NP-hard scheduling problems.

Problem	Method	Reference
$n/1//\sum \gamma_j(C_j)$†	dynamic programming	Held and Karp (1962)
$n/1//\sum w_j T_j$	Branch & Bound	Shwimer (1972)
$n/2/F/\bar{C}$	Branch & Bound	Ignall and Schrage (1965)
$n/3/F/C_{\max}$	Branch & Bound	Ignall and Schrage (1965) Lomnicki (1965) McMahon and Burton (1967)
$2/m/J/C_{\max}$	Graphical approach	Akers (1956)
$2/m/J/C_{\max}$	dynamic programming	Szwarc (1960)
$n/3/F/C_{\max}$	IP formulation	Wagner (1959)
$n/m/F$, perm$/C_{\max}$	Branch & Bound	Lageweg *et al.* (1978)
$n/m/F$, perm$/C_{\max}$ or $\bar{F}$	MIP formulation	Stafford (1988)
$n/m/F$, perm$/C_{\max}$	elimination method	Smith and Dudek (1967) Szwarc (1973) Baker (1975)
$n/m/J/C_{\max}$	MIP formulation	Greenberg (1968)
$n/m/J/C_{\max}$	Branch & Bound	Ashour and Hiremath (1973) Lageweg *et al.* (1977) Barker *et al.* (1985) Carlier and Pinson (1989)

† where γ_js are non-decreasing functions of C_j.

Figure 4. Enumerative methods for basic problems.

For MILP formulations the choice and definition of variables and constraints determines the structure and size of the model. The scheduling criterion is defined by the objective function and the constraints define the machine environment and other features of the problem. Branch and bound methods differ in the choice of bound and the searching strategy. Elimination methods choose different elimination conditions or rules. It is worth noting that the prevalence of methods for the permutation flowshop is more a reflection of the difficulty of the general problem rather than its occurrence in practice.

3.1.3. *Heuristic methods*

A good heuristic strategy attempts to approximate an optimal solution with some degree of closeness in polynomial time. Figure 5 lists heuristic algorithms reported for basic problems. Although it is difficult to abstract overall common strategies for the range of heuristics used, three types may be usefully identified:

(1) Decisions are made each time a machine is released, or when a job arrives in a queue. Priority rules are examples of this type.
(2) A neighbourhood structure is defined and the solution found must be optimal within this neighbourhood structure. For instance, Fry *et al.* (1989) use the well-known adjacent pairwise interchange methodology. Widmer and Hertz (1989) develop taboo search techniques in a neighbourhood structure.
(3) The order of jobs is determined on one machine after another. A good example of this type is the shifting bottleneck procedure for job shop problems given by Adams *et al.* (1988).

It is worth noting that irrespective of the strategy, most heuristic algorithms incorporate branch and bound procedures in which the most promising part of the

Problem	Reference
$n/1//\bar{T}$	Wilkinson and Irwin (1971)
	Fry *et al.* (1989)
$n/1/k$-parallel/$C_{\max}$	see Baker (1974)
$n/m/F/\bar{C}$	Krone and Steiglitz (1974)
$n/m/F/C_{\max}$	Palmer (1965)
	Campbell *et al.* (1970)
	Ashour (1970)
	Gupta (1971)
	Bonney and Gundry (1976)
	Dannenbring (1977)
	Widmer and Hertz (1989)
$n/m/J/C_{\max}$	Ashour (1967)
	Ashour and Hiremath (1973)
	Adams *et al.* (1988)

Figure 5. Heuristic methods for basic problems.

branch tree is searched. For example, MacCarthy and Liu (1991) describe a procedure for a single machine problem which they call limited branch and bound and indicate that the principle could be applied to other problems.

Zanakis *et al.* (1989) have surveyed heuristic methods and applications more generally and have identified scheduling as one of the most important application areas. Most of the heuristic algorithms identified in this paper fall into their more general framework.

3.2. *Methods for non-basic problems*

These can also be classified using the three types of solution methods discussed for the basic problems. Figure 6 lists significant results reported.

Problem	Method	Reference
$n/1/\text{str}/\bar{F}$	efficient	see Conway *et al.* (1967)
$n/1/\text{prec}/\max_j \gamma_j(C_j)$	efficient	Lawler (1973)
$n/1/\text{prec}/\sum w_j C_j$	heuristic	Morton and Dharan (1978)
		Weiss (1981)
$n/1/r_j/N_T$ special case	efficient	Kise *et al.* (1978)
$n/1/r_j/\bar{F}$	B&B	Deogun (1983)
$n/1/r_j/\sum w_j C_j$	B&B	Bianco and Ricciardelli (1982)
		Hariri and Potts (1983)
$n/1/r_j/C_{\max}$	heuristic	Beshara and Magazine (1981)
$n/1/r_j/\sum w_j C_j$		
$n/1/r_j/\sum C_j$	heuristic	Liu and MacCarthy (1991)
$n/1/k$-parallel, prmt/$C_{\max}$	efficient	McNaughton (1959)
$n/1/k$-parallel, depend, eq/$C_{\max}$	efficient	Hu (1961)
$n/m/O$, unit/$\sum T_j$	efficient	Liu and Bulfin (1988)
$n/m/O$, unit/N_T	efficient	
$n/m/F$, set-up/$C_{\max}$	MIP	Srikar and Ghosh (1986)
$n/m/F$, no wait/$C_{\max}$	heuristic	Bonney and Gundry (1976)

Figure 6. methods for non-basic problems.

3.3. *Comparative evaluation of algorithms*

As can be seen from Figs 3–6, there is often more than one algorithm available for a specific problem. Comparative evaluation must be done in order to make an appropriate choice for a given problem. Two types of criteria are important when comparing the performance of algorithms.

- *Effectiveness or optimality*: Any schedule will generate a computable measure of effectiveness in relation to a desired optimization criterion. If the optimal value is known then effectiveness or optimality may be expressed numerically as a simple ratio or as a relative difference. For complex problems optimal values may not be available because of resource constraints. A bounding analysis or worst-case analysis may be necessary.
- *Efficiency* refers to the computational resources necessary to obtain a solution. As most scheduling problems are difficult combinatorial optimization problems the efficiency of an algorithm is very important for practical use. Measures which are often used to reflect this criterion relate to algorithm complexity and running time. For simple algorithms, complexity may be represented using mathematical expressions and then compared. For more complex algorithms, computational experimentation is required.

Both criteria may be considered with respect to problem size and problem structure. Care must be exercised in generating test problems, particularly with respect to problem structure. Clearly, comparative work should be done on the same set of test problems. Independence and objectivity are valuable attributes in this type of study.

Reported comparison studies fall into two categories. The first type reviews algorithms for certain scheduling problems and compares them. Baker (1975) surveys optimal algorithms (branch and bound and elimination algorithms) for the flow shop sequencing problem and compares them in terms of efficiency. Dannenbring (1977) compares optimality and efficiency of heuristic algorithms for the same problem. Both of them use running time as the criterion for efficiency. Giglio and Wagner (1964) compared different kinds of methods for the three machine flow shop sequencing problem.

The second type of comparative work frequently arises when new algorithms are proposed for a scheduling problem. Some comparative work may be done to predict the performance of the new algorithms. Kusiak (1986) and Chow (1989) present efficient implementations of Johnson's algorithm respectively. Both of them give the complexity of their algorithms and compare them with that of Johnson's. It is shown that Chow's implementation is more efficient than Kusiak's, and Kusiak's more efficient than Johnson's.

The paucity of comparative evaluation studies, particularly of the first kind, is noteworthy given the abundance of scheduling research literature and the obvious need to identify the best algorithms. With advances in computer hardware and computational methods the time is ripe for large-scale independent comparative evaluation studies.

3.4. *An illustrative example*

A good example to show the difficulty and different methods for classical scheduling problems is the single machine sequencing problem with ready times where the objective is to minimize mean completion time—$n/1/r_j \geqslant 0/\bar{C}$. This problem

Example group†	Heuristic 1	Heuristic 2	Heuristic 3	Best of the three
1 ($n = 5$)	1	1	0·9908	1
2 ($n = 6$)	1	1	0·9962	1
3 ($n = 8$)	0·9981	0·9981	0·9981	0·9981
4 ($n = 10$)	0·9920	0·9910	0·9588	0·9959
5 ($n = 15$)	0·9804	0·9835	0·9769	0·9994
6 ($n = 20$)	0·9960	0·9998	0·9913	1
7 ($n = 25$)	0·9906	0·9982	0·9796	0·9993
8 ($n = 50$)	0·9924	0·9933	0·9861	0·9995
Average in all	0·9937	0·9955	0·9848	0·9991
Worst case	0·9516	0·9675	0·8459	0·9795
Number of examples optimum	22	26	17	30
% of examples optimum	55	65	42·5	75

† In each group there are five randomly generated problem examples.

Figures for "% of Examples Optimum" indicate the percentage, out of all 40 examples, of the examples for which the heuristic generated an optimal sequence.

Figure 7. Optimality of heuristics (on 40 problem examples).

is important for more complex problem environments. For example, an effective algorithm for this problem may be applied to a bottleneck machine in a multi-machine environment.

Although simple to state this problem has been shown to be NP-hard (Rinnooy Kan 1976). Dessouky and Deogun (1981) presented a branch and bound method for this problem and Deogun (1983) improved the method by introducing an initial partitioning into subproblems. An MILP formulation for this problem was given by Liu and MacCarthy (1991). They also proposed three heuristics for this problem and gave a justification for these heuristics. In order to test the performance of the heuristics 40 example problems were generated in groups of five. The processing and ready times for these examples were sampled randomly from uniform distributions. The range of processing times was from 1 to 20 for all example problems. In each group the range of ready times was 1 to $10n$ in three examples and 1 to 20 in the other two examples. Figure 7 shows the computational performance of the heuristics in terms of a measure of optimality C^*/C where C^* is the optimal mean completion time. Optimal solutions were generated by branch and bound. Figure 8 shows graphically the comparative running time performance for the three solution methods (branch and bound, MILP, and heuristics) on a PC. It is clear that the use of the three heuristics (choosing the best from three) provides a better solution method in terms of effectiveness and efficiency. Almost optimal results are possible in most cases ($\geqslant$97%). It is also clear that MILP becomes computationally expensive for $n > 8$ and branch and bound for $n > 19$.

This example illustrates a number of general points:

(1) Even relatively simple scheduling problems can be NP-hard.

(2) Enumerative methods, particularly MILP, are computationally expensive and for many practical problems prohibitive. However, the study of enumerative methods may lead to good heuristics (e.g. Gelders and Sambandam 1978).

(3) The search for good heuristics should continue. These may result from the

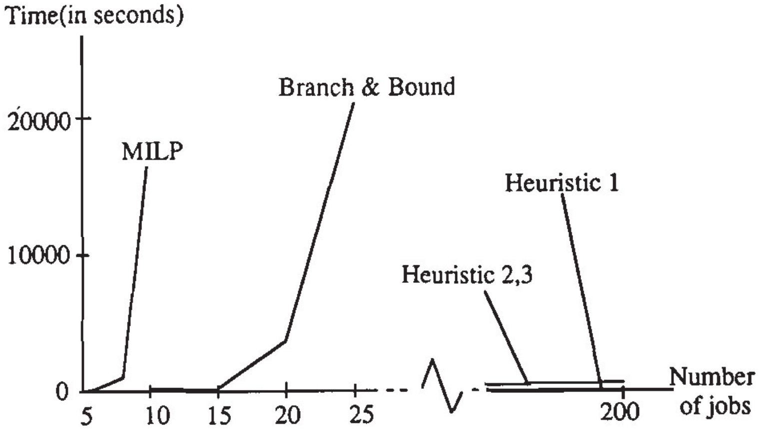

Figure 8. Running times of the algorithms on a PC.

study of enumerative methods as indicated above, from the study of problem structure or from the study of practical situations and experience.

(4) Good heuristics developed for simple problems may be extendible to more complex environments.
(5) For complex problems, testing heuristic methods is difficult and only comparative tests are possible.

4. Trends in scheduling research

Although classical scheduling theory has matured, the theoretical methods which have been developed are still far from being widely used in practice. Scheduling theory has tended to develop a mathematical momentum of its own. Too little emphasis has been placed on practical application. Two problems can be identified:

(1) The theory and solution methods are unknown or not properly understood by practitioners.
(2) The ideal situations assumed by the theory are not sufficiently close to those found in practice.

The first problem is one of education. It is necessary that all prospective practitioners are exposed to scheduling theory in their education disciplines—manufacturing engineers, industrial managers and operations managers in a range of industries and services. Furthermore it is important that senior management in industry realize the difficulty and complexity of many scheduling problems and assign appropriately educated personnel to tackle them.

The second problem will be addressed more fully in this and the next section as it is crucial to the development of scheduling research. Some efforts have been made towards the solution of more practical scheduling problems, especially in recent years. This research is more practical in two ways, namely, the criteria used and the problem environments considered. These aspects are considered in the following two sections. Although flexible manufacturing systems may be thought of as particular types of scheduling environment, this important research area is addressed in a separate section.

4.1. *More practical scheduling criteria*

The results presented in section 3 relate to a single scheduling criterion. Most of these are regular measures of schedule performance, i.e. non-decreasing functions of completion times, such as $\bar{C}$, $\Sigma w_j C_j$, $C_{\max}$, $T_{\max}$, N_T. Each criterion represents some specific cost associated with the problem environment. Frequently, however, more than one cost may be associated with a scheduling decision. This type of problem—multi-criteria decision-making has received a lot of attention in the last decade in the context of general optimization.

Some research has been done in this respect in scheduling. Sidney (1977) and Fry *et al.* (1987) consider the total penalty of earliness and tardiness. Sen and Gupta (1983) choose a linear combination of $\bar{F}$ and $T_{\max}$ as a criterion. Emmons (1975) minimizes $\bar{F}$ subject to a minimum N_T. Selen and Hott (1986) and Willon (1989) give mixed integer goal programming formulations of flow shop scheduling problems which consider $\bar{C}$ and $C_{\max}$. Nelson *et al.* (1986) consider the problem of minimizing the three criteria $\bar{F}$, $T_{\max}$ and N_T and the associated bi-criteria problems.

4.2. *More practical scheduling environments*

The environments which appear most frequently in the classical theory make the assumptions listed in section 2.1. The non-basic problems discussed in section 3.2 make less assumptions or add further constraints. Further problems for which solution methods have been reported include sequence-dependent setup times (Srikar and Ghosh 1986), parallel machines in each stage (Salvador 1973, Gupta 1988), waiting time constraints between process stages (Hodson *et al.* 1985), and variable due dates (Cheng and Gupta 1989).

Sequence-dependent setup times are one of the most frequent additional complications in scheduling problems. Many situations occur where the machine setup time is dependent on what the machine is currently producing. These situations are often found in process industries and are frequently associated with the problem of lot sizing. Selen and Heuts (1990) consider such a problem in a chemical processing environment with storage capacity constraints and develop a heuristic procedure to solve the simultaneous lot sizing-sequencing problem. Leong and Oliff (1990) give a heuristic approach for scheduling jobs on parallel machines with sequence-dependent changeover cost and apply it to a fibreglass company. Both of these papers are applications orientated. Srikar and Ghosh (1986) introduce an MILP formulation for the flowshop sequencing problem with sequence-dependent setup times. They employ a clever formulation which halves the number of binary variables. However the emphasis is on formulation rather than the solution of large practical problems.

Parallel machines in flow shop and job shop environments are very common. Even when the number of stages is small, parallel machines in some or all stages complicates the problem. Salvador (1973) considered a flow shop problem in a nylon plant with parallel facilities at a number of stages. Some success was claimed for a moderately sized problem using an MILP formulation. Gupta (1988) shows that the two stage flow shop problem with parallel machines in at least one stage is NP-hard and gives a heuristic algorithm for the case in which there is only one machine in the second stage.

Hodson *et al.* (1985) considered heuristic approaches for a flow shop with limited time between processes, a constraint which is real in a number of industries. A related type of flow shop problem is considered by Leisten (1990) in which the capacities of buffer storages between machines are limited.

The problems addressed so far involving tardiness measures are appropriate for environments where due dates are specified. Many environments involve some discussion between the customer and the producer to assign a delivery date which is mutually acceptable. This is an important problem with scheduling implications (Viviers 1983) but will not be considered here. Cheng and Gupta (1989) have recently reviewed the area.

From a wider perspective the lack of use of scheduling theory in practice has frequently been noted (Melnyk *et al.* 1986, Buxey 1989). The reality of production management environments is discussed by Melnyk *et al.* (1986). Real pressures on schedulers such as the dynamic nature of many environments, capacity planning and load balancing, etc., are often ignored. However, these aspects are essential for schedules to be feasible in practice. The dynamic nature of production scheduling is emphasized in the survey by Rodammer and White (1988). Buxey (1989) examines these aspects in depth, emphasizing the capacity planning problem and exploring more generally the role of analytical methods in modern manufacturing planning and control philosophies such as MRP, OPT and JIT. These are important considerations and are discussed further in section 5.

A further area is that of automated manufacturing systems. Effective scheduling strategies are vital to obtain the benefits from these systems. In the following section we will give a brief review of production scheduling in flexible manufacturing systems.

4.3. *Scheduling in flexible manufacturing systems*

Flexible manufacturing systems (FMS) contain machining centres and automated handling facilities rather than just simple machines (Van Vliet and Van Wassenhove 1989). Scheduling in an FMS is quite different from that in a conventional job shop. FMS scheduling may involve all the following aspects simultaneously:

- Routeing of parts through machine centres.
- Sequencing the parts on each machine centre.
- Tool changing of the machine centres.

Moreover, FMS scheduling problems have to consider additional constraints on resources such as storage, transport devices, and tool change facilities. These factors make FMS scheduling intractable, even for single-machine problems such as the one discussed by Bard (1988). As a result the complexity of FMS scheduling problems is greater than in classical scheduling problems. In fact most FMS scheduling problems are NP-hard (Blazewicz *et al.* 1988).

Three kinds of approaches are mainly used for FMS Scheduling:

- *Heuristics, dispatching rules and simulation*: Montazer and Van Wassenhove (1990) compare 14 dispatching rules using simulation in a specific FMS environment. The performance criteria include makespan, waiting time of parts, and utilization of machines, buffers, and carriers. They conclude that no single scheduling rule is the best on all performance measures and it is up to the user to choose one or more of the rules according to the performance measures prevailing in the particular application. Frese (1987) gives a heuristic-based simple simulation approach to schedule an FMS in which the processing times vary as the parts are sequenced in different ways. Bard (1988) uses a heuristic to solve the sequencing and tool changing problem on a single flexible machine.
- *Mathematical programming formulations*: Mathematical programming formu-

lations may give a very clear representation of the scheduling problems. Hutchinson *et al.* (1989) give an MILP formulation and solution using branch and bound. Van Vliet and Van Wassenhove (1989) also give mathematical programming formulations as examples of the application of operational research methods in FMS analysis.

- *AI techniques*: Many FMS scheduling systems use artificial intelligence (AI) techniques, especially expert systems. For example, Sauve and Collinot (1987) built an expert system for scheduling an FMS containing two parts: one for off-line daily scheduling and the other for on-line control of production disturbances. Walburn and Powner (1990) give a knowledge-based approach to FMS scheduling. An FMS model constructed using an object-orientated approach schedules one job at a time. A knowledge-based analyser and a knowledge-based rescheduler are used to handle the 'exceptions'.

Due to the high complexity, even when FMS scheduling problems are representible as MILP formulations, heuristics are still needed to solve the formulations. Heuristics or AI techniques seem to be unavoidable for FMS scheduling problems.

5. Discussion and future research directions

Section 1 of this paper has stressed the gap between scheduling theory and scheduling practice. When viewed from the perspective of combinatorial optimization, the classical theory appears a mature subject. The more complex problem environments discussed in section 4 are potentially useful in moving towards more applicable research.

There is considerable scope for development, particularly with respect to sequence-dependent setup times, parallel machines at different processing stages and systems incorporating both these features. These are particularly relevant to the process industries. It is likely that significant results will be heuristic in nature. This raises a more general need for theoretical analysis of heuristic methods. Research is required to show how and why good heuristics perform well and to indicate how to develop good heuristic strategies for new problems. This may involve further study of enumerative methods as many heuristic approaches apply partial enumeration strategies.

The lack of good objective comparative studies has been highlighted in section 3.3. There is a clear need for a programme of work in this area. The classical approach to scheduling has its greatest potential in highly automated systems. In particular as FMS's develop and become more widespread it is likely that theoretical scheduling research will influence the design of FMS control systems.

When scheduling research is viewed from a manufacturing systems perspective the subject appears far from mature. The classical theory would seem to have little relevance or impact. It is necessary to identify the reasons for this divergence before a more integrated strategy can be presented.

Classical scheduling theory has tended to consider scheduling problems in isolation from the higher levels of the production planning and control function. The interaction between scheduling and higher level decision making is not taken into account. Modern manufacturing systems tend to be designed and managed using a 'top-down' approach. Within these structures scheduling problems do not arise in isolation as discrete activities divorced from any contextual environment. In fact many different elements of the total business environment may influence the nature of the scheduling function: the objectives, criteria and constraints within the function and the reality of scheduling practice. These aspects must be acknowledged and addressed in serious practical scheduling approaches.

At the most general level the nature of the business and the competitive and market environments must be considered. Product sectors have similarities with respect to technological aspects and organizational structures and relative complexity. Varying customs and life-styles in different market environments frequently influence ordering systems and methods, order quantities and batch sizes and expected lead times. However, it must be noted that strategy and policy have a major influence on the nature of the business and at an operational level on the nature of the scheduling function. Researchers attempting multi-criteria approaches must be aware of the different environments at this level and the complexity of criteria emanating from them.

Knowledge-based integrated approaches to production scheduling attempt to overcome some of the drawbacks of traditional approaches based on simple mathematical models. These may have the potential to represent the interactions within the production planning and control function. An overview of the area is given by Kanet and Adelsberger (1987). ISIS (Fox and Smith 1984) was one of the first knowledge-based systems for job shop scheduling problems. It uses a knowledge-based constraint-directed search approach. Full details of this system can be found in Fox (1987). Another early system was ISA (Orciuch and Frost 1984). Further discussion of scheduling systems using knowledge-based approaches can be found in the recent survey by Noronha and Sarma (1991). Knowledge-based approaches are also important for scheduling problems in FMS as can be seen in section 4.3.

Classical scheduling theory is frequently criticized for its static deterministic assumptions. The problems of stochastic occurrences which affect resources or cause quality problems are well known in practice. However, most planners and schedulers will assume deterministic conditions and account for stochastic occurrences in capacity and utilization calculations. The assumption of a static planning environment is more problematic. Most systems must be reactive over some future time horizon. A feature which is often overlooked is that business policy may affect the scheduling which has to be done—in particular where the company positions itself in terms of response to market demands. The planning and control function must also be considered in this respect.

The nature of the planning and control function directly influences the scheduling function. It defines the time framework for the scheduling function. The systems used vary tremendously and will determine to a large extent the period over which inputs to scheduling may be assumed to be static and the frequency with which schedules are updated. Most classical scheduling research is unclear where it interfaces with the planning and control function and how relevant or applicable the results are in different environments.

Many manufacturing firms utilize planning and control methods and philosophies such as MRP, MRPII, JIT, and OPT. The nature of short-term scheduling in these environments has not been adequately researched. Neither has short-term scheduling been adequately addressed in organizational strategies such as group technology.

Human involvement in the scheduling function is another area which raises important issues which are not frequently addressed. Although more important in smaller systems, the human factor in larger computer-based systems can be significant. Interesting research in this area includes work by Haider *et al.* (1981) and Nakamura and Salvendy (1988).

This paper has not attempted to review currently available scheduling software systems. However, effective scheduling software is essential in complex manufacturing environments. Many systems are good at modelling real scheduling environments, providing good user interfaces and links with other production planning and control software. However, many software systems are limited in their approaches to scheduling and employ only simple scheduling rules.

Technology now allows the generation of rapid, if not real-time, information in manufacturing systems. Computer systems, combined with sensor and automatic identification technologies, allow much more information on status to be presented to managers. In the presence of this information, scheduling research has the potential to be a major contributor to the development of effective and efficient manufacturing systems. However this will require a sustained integrated research effort if this is to be achieved.

This discussion has highlighted two approaches to scheduling research. The classical approach is pursued mainly by applied mathematicians and theoretical operational research workers. The manufacturing systems perspective is favoured by operations management and production research workers. It is likely that these two views of the research area will persist in the short and medium term. To begin to bridge this gap and integrate the area of practical scheduling research there is a need to change focus in both camps:

(1) Where possible, research in classical scheduling theory should be more applications oriented, understanding and addressing all the significant issues within the total business environment. It would help if applications oriented published research clearly stated the relevance and applicability of results in the context of modern manufacturing systems.
(2) Where possible, researchers in manufacturing systems must attempt to tap the rich vein of methods, techniques and results developed in the classical theory. It is hoped that the mode of presentation in this paper will aid this and allow the development of a methodology to link the two areas to identify and select appropriate results.
(3) An important practical development to bridge the gap between research and practice in production scheduling is the development of effective scheduling software. More powerful software systems may be developed by utilizing algorithms and results in classical scheduling theory, where appropriate, and incorporating advances in scheduling research as they occur.

6. Conclusions

This paper has reviewed the state-of-the-art in classical scheduling theory. Major trends and solution approaches have been identified and significant results have been presented in an easily accessible way.

The nature of the gap between scheduling theory and scheduling practice has been explored. Classical scheduling theory has a very limited view of the total environment in which scheduling decisions are made and within which the scheduling function operates. Operations management and manufacturing systems research ignores the rich vein of methods and results in classical scheduling theory. This paper has stressed the need for an integrated approach to scheduling research, combining the elements of both approaches for the development of effective and efficient manufacturing systems.

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