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IF SCIENTISTS were as fair as Humpty Dumpty in paying words extra for overtime work, the ubiquitous Q would come near the top of the payroll. It all started with K. S. Johnson.* Many scientists have contributed new words to our vocabulary. To only one, however, has come the distinction of elevating a letter of the alphabet into a word of everyday use in many and diverse fields. Little did Johnson dream, when he first used the symbol Q to represent the ratio of reactance to effective resistance in a coil or a condenser, that within a span of some 30 years this same symbol would be commonly used to describe an attribute of such dissimilar things as a resonant circuit, a spectral line, a mechanical vibrator, and a bouncing ball. The story of this expanding usage of the 17th letter of the alphabet makes an interesting study for the scientific etymologist.

Coils and Condensers

The tale begins in the teens of this century. It was then the usual practice, in appraising the quality of the devices which were then known as coils, and which engineers have now become educated to call inductors, to use the ratio of effective resistance to reactance as a sort of figure of merit. Because it was related to dissipation, this ratio was often designated d, and in fact it is now commonly referred to as the dissipation factor. Strictly speaking, d is not a figure of merit but a figure of demerit, since the normally desirable condition of minimum losses occurs as the value of d moves towards zero.

As early as 1914, Johnson came to realize that a ratio of greater utility for many purposes than the one in vogue was its reciprocal. Johnson was aware that the ratio d is convenient for certain mathematical computations, since it permits the combining of different sources of loss by direct addition. He observed, however, that in practical cases d would usually involve one or more zeros preceding the significant figures, whereas the reciprocal could usually be taken as a whole number. The same sort of logic, which leads to the common use of impedance, and avoidance of admittance, argued for putting reactance in the numerator of the ratio.

For a time Johnson designated the ratio of reactance to effective resistance of a coil by the symbol K (1). It was in 1920, while working on the practical application of the wave filter which G. A. Campbell had invented some years before, that he for the first time employed the sym-

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bol Q for his parameter [2]. His reason for choosing Q was quite simple. He says that it did not stand for "quality factor" or anything else, but since the other letters of the alphabet had already been pre-empted for other purposes, Q was all he had left.

Initially Johnson used a capital Q for coils, and a small q for the corresponding ratio in condensers (now renamed capacitors). Before long, however, he began to apply capital Q to both coils and condensers, using subscripts where differentiation was needed. The first printed use of Q seems to be in Johnson's U.S. patent No. 1,628,983, where it is applied to the coils in an electrical network. In Johnson's classic treatise on "Transmission Circuits for Telephonic Communication" [3] the symbol Q appears in a number of places to designate the parameter which he called the "coil dissipation constant." Subsequently this was shortened to "dissipation constant," applying to both coils and condensers [4]. The terms "coil constant" and "condenser constant" also were used to some extent. Later on, V. E. Legg coined the apt name of "quality factor," while others tried to introduce such terms as "storage factor" and "figure of merit." But none of these appellations could prevail over the terse and trenchant Q.

Another measure which has frequently been used for a reactive element is the power factor, i.e., the ratio of active power to total volt-amperes. At any two terminals the power factor is the cosine of the phase angle of the impedance, whereas Q is the tangent of the phase angle, neglecting sign. Thus for the common case where reactance is large compared with resistance, the power factor is substantially equal to the dissipation factor d. Power engineers, who are accustomed to using power factor to designate the ratio of active power to total volt-amperes, might occasionally, if they experience a need for a ratio greater than unity, find it advantageous to borrow Q from the communication engineer.

Others before Johnson had made use of the ratio of reactance to resistance for either an inductor or a capacitor (to use modern parlance). Johnson's role was to popularize this ratio and to assign to it the contagious symbol Q. He did not intend to apply Q to anything except the ratio of reactance to resistance, whether of an inductor, a capacitor, or any two-terminal network. In fact, he was somewhat disturbed, as originators of terminology often are, when others began to extend his usage—an extension which has gone so far that a few modernists would even like to ban Johnson's original meaning.

Resonant Circuits

What happened next? First was the discovery that Q was a convenient symbol to apply to a resonant circuit. It was noted that the high-frequency losses in a well-constructed capacitor were ordinarily negligible in comparison with those of an inductor. Hence the high-frequency resistance of the usual inductor and capacitor resonant circuit could be assumed equal to the inductor resistance, and the Q of the resonant circuit could therefore be assumed the same as the Q of the inductor. In those cases where the capacitor resistance could not be neglected, the Q of the resonant circuit was $Q_L Q_C/(Q_L+Q_C)$, where Q_L and Q_C are the Q's of the inductor and capacitor, respectively, at the resonant frequency. It is noteworthy that this use of Q for a resonant network is uniquely related to the resonant frequency, whereas Q when applied to an impedance is a property at any specified frequency.

At this point it became apparent that Q as applied to a resonant circuit was an already recognized parameter which for want of a better name had previously been called "sharpness of resonance" [5]. This permitted the establishment of several relationships which today are elementary. Curves like those of Figure 1 could be drawn to show current *versus* frequency as a function of Q for a series resonant circuit, and analogous curves for the impedance of a parallel resonant circuit.

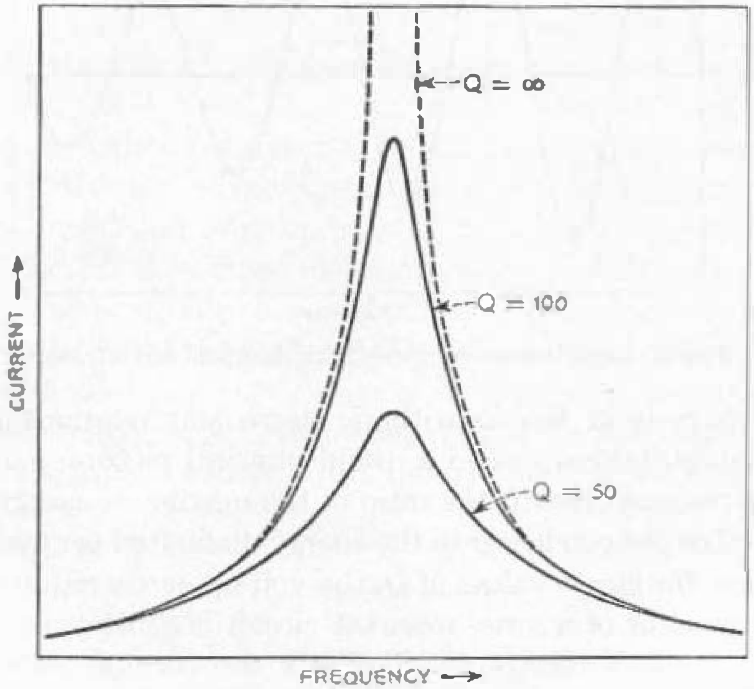

FIG. 1. Series resonance.

Once identified with sharpness of resonance, Q was seen to bear a close relationship to a familiar parameter of an oscillatory wave train of continuously decreasing amplitude. This parameter was the logarithmic decrement, which was defined as the natural logarithm of the ratio of two successive maxima in a damped wave train. Thus in Figure 2 the logarithmic decrement δ is equal to $\log_e (AB/CD)$.

Solution of the differential equation for a resonant circuit comprising resistance, inductance, and capacitance in series gives for the logarithmic decrement δ the value $\pi R/\omega L$, or $\pi R\omega C$. Hence Q equals π/δ. While much importance attached to logarithmic decrement in the earlier days of radio, in connection with the damped waves produced in a spark transmitter by the sudden discharge of a condenser through a spark gap, Q was so much better adapted to continuous wave techniques that today the term logarithmic decrement is all but forgotten.

Many relations previously established for the logarithmic decrement were restated in terms of Q. Thus the number of complete oscillations necessary to reach a given ratio ρ of initial amplitude to final amplitude is Q/π times $\log_e \rho$. From this we learn that for a Q of 100, for example, the number of oscillations necessary to reach one per cent of the initial value is 146, while for a Q of 200 twice as many oscillations would be required.

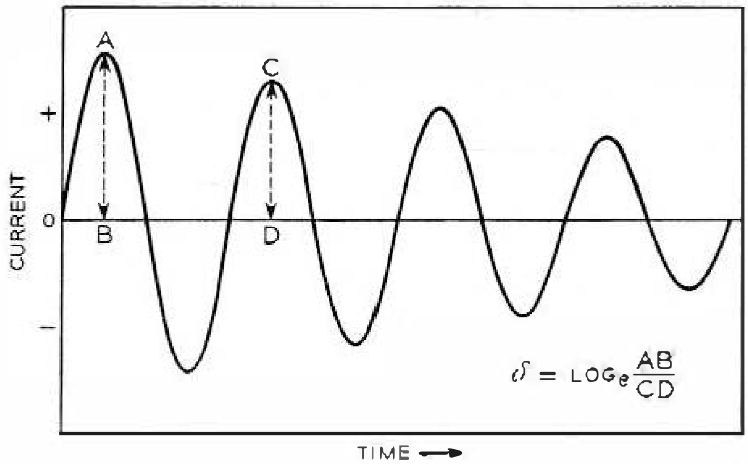

FIG. 2. Logarithmic decrement for damped wave-train.

Through pursuit of the logarithmic decrement relationship, simple algebraic manipulations yielded a useful physical picture, namely, that for a simple resonant circuit the ratio of the maximum energy stored in either the coil or the condenser to the energy dissipated per cycle is equal to $Q/2\pi$. Also, for larger values of Q, the voltage across either the inductor or the capacitor of a series resonant circuit is substantially equal to Q times the applied voltage [6]. Similarly the current through either the inductor or the capacitor of a parallel resonant circuit is equal to Q times the total current.

An even more interesting relationship was found between Q and the shape of the resonance curve [7]. Through derivations too detailed for inclusion here, it turns out that for a curve showing magnitude of impedance or admittance of a resonant circuit *versus* frequency, Q is approximately equal to the ratio of the resonant frequency to the width of the resonance curve between the points, on either side of resonance, where the ordinate is, respectively, $1/\sqrt{2}$ times the maximum or $\sqrt{2}$ times the minimum ordinate.

The groundwork was now complete, and anyone could take off in any direction. A natural extension from the inductor and capacitor resonant circuit was to apply Q to any resonant structure or device. For this purpose the definition of Q in terms of energy storage and dissipation was directly applicable, while the relation for the shape of the resonance curve was broadened by stating it in terms of response. Thus Q became equal to the ratio of the resonant frequency to the bandwidth between those frequencies on opposite sides of resonance (known as "half-power points") where the response of the resonant structure differs by 3 db from that at resonance. The use of Q with such connotations for tuning forks, piezoelectric resonators, magnetostrictive rods, and the like soon became commonplace.

Resonant Transmission Lines

Resonant transmission lines came next. The standing wave patterns for open-circuited or short-circuited lines, exhibiting maxima and minima at "resonance" points located at quarter-wave multiples from the terminating end, were, of course, familiar from classical derivations. The trend to higher frequencies, especially for radio communciation, made it increasingly advantageous to utilize such resonant-line phenomena for oscillator frequency control, voltage step-up, impedance inversion, and the like. Since the curve of line impedance in the vicinity of resonance is essentially similar to that of a resonant circuit, it was a natural step to apply the factor Q to a resonant transmission line. F. E. Terman [8] showed that the Q of such a line is equal to $\pi f/\alpha V$, where f is the resonant frequency, α is the real part of the propagation constant, and V is the group velocity, i.e., the velocity with which signals are transmitted.

Cavity Resonators

At frequencies upward from about 1000 mc (commonly referred to as microwaves) resonant transmission lines usually give way to cavity resonators. The cavity may be cylindrical, parallelepipedal, spherical, or some other shape, depending on end use. Regardless of shape, a cavity resonator has an infinity of resonant frequencies, starting at a minimum value and becoming more closely spaced with increasing frequency. Each resonance corresponds to a particular standing wave pattern of the electromagnetic field, which is called a resonant mode, and for which the cavity may be considered as a single tuned circuit (with L and C not defined). The Q of a cavity resonator for any mode is therefore definable in terms of losses or bandwidth, and turns out to be a function of the ratio of internal volume to internal area. A general expression for the Q

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