

# Design of Large Air-Gap Transformers for Wireless Power Supplies

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**Abstract**—This paper addresses the design of two-dimensional large air gap transformers for wireless power transmission. A theoretical analysis of the optimum shape and arrangement of the primary coils is presented. The magnetic field created by such a system is discussed and optimised. The effects of shielding of the secondary coils are discussed. The optimum coil arrangement for one example design is derived.

## I. INTRODUCTION

Novel power supplies for use in applications such as robotics, automated production machines, and applications with high insulation requirements where wired energy transfer is not suitable have recently been proposed [1], [2], [3]. Some of these supplies use unconventional transformers with large air-gaps to supply energy to the load via magnetic fields over distances up to several meters and provide for the wireless supply of power to devices such as sensors, communication devices, or actuators. Figure 1 depicts the main transformer components being used. The primary coils define a system with the coils lying in several different planes. The secondary coil(s) are wound around a ferrite core and placed inside the box formed by the primary coil(s).

For a reliable operation of the power supply the transformer must be designed and powered in such a way as to transfer the energy to secondary coils that may be shifting in position and may be magnetically shielded by metallic objects within the operating volume. The field created by the primary coils should be as uniform as possible over the greatest possible volume in order to provide adequate power to the secondary coils. Apart from the two dimensional system shown in Figure 1, which can be regarded as a practical compromise between performance and complexity, one and three dimensional systems can be conceived as being formed by one or several coils in each plane.<sup>1</sup>

This paper explores various design options for the primary side of the transformer with respect to uniformity of power transfer, utilization of the power per volume unit, and ability to overcome the problems associated with shielding of the magnetic field. An example one-dimensional system using a current of 24A and a magnetic field intensity at the center of the system of 4A/m is described.

<sup>1</sup> Patents pending

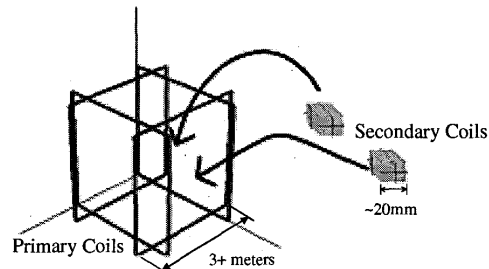


Figure 1 - Power supply using a large air-gap transformer

## II. DESIGN OF SECONDARY COILS

Energy is transferred from the primary to the secondary coil(s) via a magnetic field. If the magnetic field lines created by the primary coil(s) do not pass through the area enclosed by a receiving (secondary) coil(s), no voltage will be induced in those coils. In order to minimize the effect of the position, shielding, and orientation of the secondary coil on the transformer's energy transfer characteristics, three secondary coils are designed such that they form a three-dimensional orthogonal coil system. The coils are arranged at a 90° separation around a cube-shaped ferrite core as indicated in Figure 1.

## III. BASIC SHAPES OF PRIMARY COIL

The size, shape, and uniformity of the field created by the primary coils depend significantly on the coil configuration. Square, rectangular, and circular coils are investigated.

### A. Magnetic Fields due to Rectangular Coils

*Single rectangular coil:* The intensity of the magnetic field created at an arbitrary point in space  $P$  when a current is applied to a straight current carrying wire of length  $2L$  is described in the near field ( $2\pi r/\lambda \ll 1$ , where  $\lambda$  is the wavelength of the current) by equation (1).

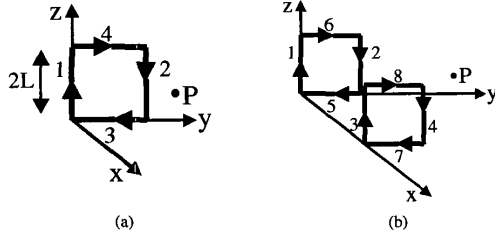


Figure 2 – (a) rectangular coil with field intensity described by (2) (b) rectangular coil system with field intensity described by (3)

$$\vec{H}_P = \hat{a}_x \left( -\sin\phi \frac{IL}{2\pi r \sqrt{L^2 + r^2}} \right) + \hat{a}_y \left( \cos\phi \frac{IL}{2\pi r \sqrt{L^2 + r^2}} \right) \quad (1)$$

where  $I$  is the current in the wire,  $r$  is the perpendicular distance between the point  $P$  at which the field is being calculated and the current carrying wire,  $\phi$  is the angle between the imaginary line of length  $r$  connecting the field point with the wire and the  $x$ -axis.

The magnetic field intensity at an arbitrary point in space due to a rectangular coil can be expressed as the vector sum of the fields created by each of the straight wires. A rectangular coil lying in the  $y$ - $z$  plane (Figure 2a) where wires 1 and 2 are parallel to the  $z$ -axis and wires 3 and 4 are parallel to the  $y$ -axis will produce

$$\begin{aligned} \vec{H}_{rect} = & \hat{a}_x \left( -\sin\phi_1 \frac{IL_1}{2\pi r_1 \sqrt{L_1^2 + r_1^2}} + \sin\phi_2 \frac{IL_2}{2\pi r_2 \sqrt{L_2^2 + r_2^2}} \right. \\ & \left. -\cos\phi_3 \frac{IL_3}{2\pi r_3 \sqrt{L_3^2 + r_3^2}} + \cos\phi_4 \frac{IL_4}{2\pi r_4 \sqrt{L_4^2 + r_4^2}} \right) \\ & + \hat{a}_y \left( \cos\phi_1 \frac{IL_1}{2\pi r_1 \sqrt{L_1^2 + r_1^2}} - \cos\phi_2 \frac{IL_2}{2\pi r_2 \sqrt{L_2^2 + r_2^2}} \right) \\ & + \hat{a}_z \left( \sin\phi_3 \frac{IL_3}{2\pi r_3 \sqrt{L_3^2 + r_3^2}} - \sin\phi_4 \frac{IL_4}{2\pi r_4 \sqrt{L_4^2 + r_4^2}} \right) \end{aligned} \quad (2)$$

**Multiple Rectangular Coils:** The intensity of the field at an arbitrary point created by a set of rectangular coils can be expressed as the vector sum of the fields created by each of the coils. Numbering the sides of the rectangular coils as in Figure 2(b),

$$\begin{aligned} \vec{H} = & \sum_{i=1}^{2s} [(-1)^i \sin\phi_i X_i \hat{a}_x + (-1)^{i+1} \cos\phi_i X_i \hat{a}_y] \\ & + \sum_{i=2s+1}^{4s} [(-1)^{i+1} \sin\phi_i X_i \hat{a}_z + (-1)^i \cos\phi_i X_i \hat{a}_x] \end{aligned} \quad (3)$$

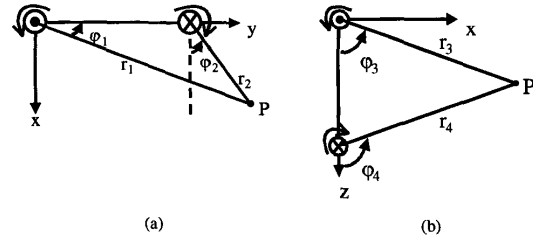


Figure 3 – (a) angles used to calculate field strength due to a single coil ( $xy$ -axis). (b) angles used to calculate field strength due to a single coil ( $xz$ -axis)

where  $X_i = \frac{IL_i}{2\pi r_i \sqrt{L_i^2 + r_i^2}}$ ,  $s$  = number of coils

On the axis of square coils, the equation describing the magnetic field simplifies greatly to

$$\vec{H} = \hat{a}_x 4s \left( -\sin\left(\frac{\pi}{4}\right) \frac{IL}{2\pi r \sqrt{L^2 + r^2}} \right) \quad (4)$$

As an example, Figure 4 shows the intensity of the magnetic field created by two square coils 3m x 3m in dimension carrying 24A. The surface shown covers all points where the value of  $H$  is greater than 4A/m. Figure 5 shows a cross section of the field intensity at  $z=1.5$  m.

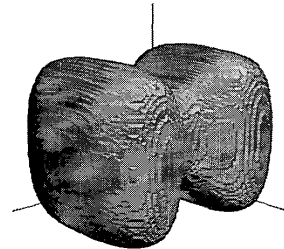


Figure 4 - Area of field strength greater than or equal to 4A/m created by two square coils carrying 24A

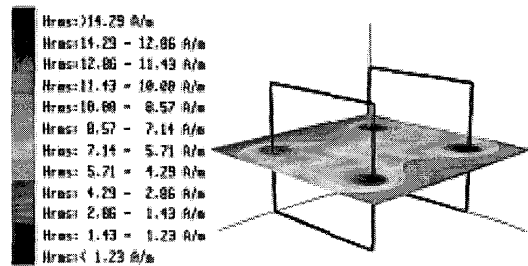


Figure 5 - Cross section (at  $z=1.5$ m) showing field intensity created by two square coils carrying 24A

B. Magnetic Field due to Circular Coils

Single circular coil: The equations describing the field created by a single circular coil are somewhat more complex. For one circular coil centered on the z axis (Figure 6a) the magnetic field intensity can be shown to be:

$$|H| = \frac{I}{2\pi\sqrt{(a+\rho)^2+z^2}} \left[ \frac{z^2 \left[ \frac{a^2+\rho^2+z^2}{\rho^2} E(k) - K(k) \right]^2}{\left[ \frac{a^2-\rho^2-z^2}{(a-\rho)^2+z^2} E(k) + K(k) \right]^2} \right]^{1/2} \quad (5)$$

where  $K(k)$  and  $E(k)$  are a complete elliptic integral of the first kind and second kind, respectively.

$$K(k) = \int_0^{\pi/2} \frac{d\Phi}{\sqrt{1-k^2 \sin^2 \Phi}}$$

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \Phi} d\Phi$$

$$k = 2\sqrt{\frac{a\rho}{(a+\rho)^2+z^2}} \quad (k \in [0,1])$$

On the axis of a circular loop, the equation simplifies to

$$\bar{H} = \hat{a}_z \frac{Ia^2}{2(z^2+a^2)^{3/2}} \quad (6)$$

Multiple Circular Coils: The magnetic field created by two circular current carrying coils is the vector sum of the field components created by each single coil. The magnitude of the field created by two coils numbered 1 and 2 at an arbitrary point P is

$$|H_P| = \sqrt{(H_{\rho 1} + H_{\rho 2})^2 + (H_{z 1} + H_{z 2})^2} \quad (7)$$

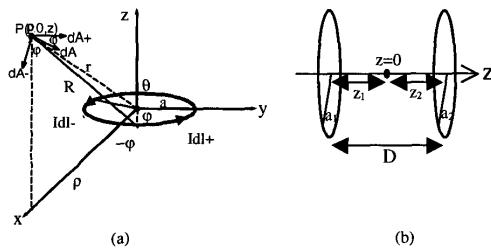


Figure 6 - (a) circular coil in xy-plane, (b) two circular coils with equal radius

IV. OPTIMUM SHAPE OF PRIMARY COILS

Fields can be created using circular, square, or rectangular coils. It is beneficial to examine some differences between the fields created by coils of each shape.

Obtaining a configuration in which the greatest possible volume of stable (unchanging) magnetic field exists between two coils may be of interest for certain applications. For two equal coils of any shape and size, the optimum distance  $d_H$  (also known as Helmholtz spacing) by which the two coils should be separated to achieve the greatest possible area of stable magnetic field can be determined by finding the distance at which the gradient of the magnetic field in the center of the coils is equal to zero [4], [5], [6].

Expanding (6) to find the magnetic field at the center of two circular coils (Figure 6b) lying in a plane perpendicular to the z-axis with equal radius  $a_1 = a_2 = a$  and a distance  $D = d_H$  apart. The z-axis passes through the center of the coils and the origin of the z-axis is at the point midway between the coils.

$$\bar{H} = \frac{I}{2} \left[ \frac{a^2}{\left[ a^2 + \left( \frac{D}{2} - z_1 \right)^2 \right]^{3/2}} + \frac{a^2}{\left[ a^2 + \left( \frac{D}{2} - z_2 \right)^2 \right]^{3/2}} \right] \quad (8)$$

where  $z$  is the distance from each coil to the point in question.  $z_1 = z_2$  for a point in the center of the coils. Finding the gradient of the magnetic field

$$\frac{\partial^2 H}{\partial z^2} = \frac{Ia^2}{2} \left( \frac{-3}{2} \right) \left[ a^2 + \left( \frac{D}{2} - z \right)^2 \right]^{-5/2} + \left[ a^2 + \left( \frac{D}{2} - z \right)^2 \right]^{-7/2} (-5) \left( \frac{D}{2} + z \right) + \left[ a^2 + \left( \frac{D}{2} + z \right)^2 \right]^{-5/2} + \left[ a^2 + \left( \frac{D}{2} + z \right)^2 \right]^{-7/2} (-5) \left( \frac{D}{2} + z \right) \quad (9)$$

It is now clear that  $\frac{\partial^2 H}{\partial z^2} \Big|_{z=0} = 0$  when  $D = a$ . For circular coils  $d_H$  is always equal to the radius. For square coils  $d_H$  is always equal to 0.5445 times the length of one side [4], [5], [6]. For rectangular coils  $d_H$  varies with the dimensions of the rectangle as shown in Figure 7.

The total volume contained within the box created by two square coils at Helmholtz spacing is 1.3866 times larger than the total volume contained within the cylinder created by two circular coils also at Helmholtz spacing. However, this result does not provide adequate information about the volume of stable magnetic field created by these configurations.

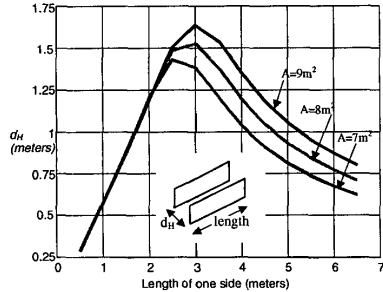


Figure 7- Helmholtz distance vs. side length for rectangles of area=7,8, and 9m<sup>2</sup>

In order to further investigate general differences between coil shapes, the volume of a sphere centered on the origin of the system (Figure 8) in which the magnitude of the magnetic field vector varies by up to a certain percentage from the value at the center of the system is plotted for circular and square coil configurations. The radii of the coils are set to a value of one (the “radii” of the square coils are assumed to be one-half the length of one side of one coil) [7].

Figure 9 and Figure 12 show that while the volume is maximized at or near Helmholtz spacing in all cases, spacing becomes less critical as the percentage of allowable deviation is increased. These results are confirmed by [8]. Figure 10 shows that for coils with the same radius at the same distance of separation, square coils have a larger volume of stable magnetic field for all investigated percentages of deviation from the center field values. Figure 13 shows the trade off between a larger total system volume and a larger volume of stable magnetic field. Although square coils provide the same volume of stable magnetic field with a smaller distance between them than do circular coils, circular coils provide a larger volume of stable magnetic field as a percentage of total volume taken up by the system. However, square coils can in practice be more easily assembled into a modular system [2]. This is a clear advantage when developing scalable systems for industrial use.

The most important criteria in choosing a coil configuration will involve finding the distance of separation at which the coils will produce the largest possible volume of specified minimum field strength within the system volume while maintaining field values outside the system volume which do not exceed minimum safety standards. An approximation of this volume can be found easily for square coils.

For example two coils 3m x 3m each carrying 24A, the field strength parallel to the x-axis and through the center of the coils for distances of separation from 0.7 meters to 3.7 meters at steps of 0.085 meters is shown in Figure 11 for field strengths greater than 4 A/m. All of the distances of separation considered lead to a field strength of 4A/m or more along the entire path. Figure 14 shows the field strength parallel to the z-axis at a distance of D/2 for distances of separation from 0.7 to 3.7 meters.

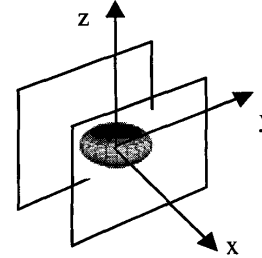


Figure 8- Sphere centered at the origin of a system of square coils

In this direction the field strength does not remain at 4A/m or higher along the entire path for all distances of separation.

Because of the symmetry of the field created by square coils, the field strength in the y-direction will be the same as that in the z-direction.

Figure 15 shows a plot of the approximate volume in which the field strength is 4A/m or higher inside the cube volume for varying values of D. For 3m x 3m coils carrying 24A, the distance of separation at which the maximum volume within the cube created by the coil pair has a field strength of greater than 4 A/m is approximately 2.8m.

The optimum distance of separation for a coil system can be found most easily by first calculating Helmholtz spacing and then varying the distance slightly to achieve the maximum volume of desired field strength. Although the example above showed a one-dimensional system, this method is also useful when designing multi-dimensional systems.

## V. POWER TRANSFER

One-dimensional systems create a pulsating magnetic field vector and do not provide constant power to the secondary. Two-dimensional systems with coil sets positioned 90° apart spatially (Figure 1) create a rotating field vector of constant length and provide constant power to the secondary when the current applied to the first set of coils is phase shifted by 90° from the current applied to the second set. Simplifying equation (4) shows that the magnitude of the resulting vector is constant.

$$\vec{H} = \hat{a}_x k I_x \sin \omega t \quad (10)$$

$$\vec{H} = \hat{a}_y k I_y \cos \omega t \quad (11)$$

$$\text{with } k = 4s \left( -\sin\left(\frac{\pi}{4}\right) \frac{IL}{2\pi r \sqrt{L^2 + r^2}} \right) \quad (12)$$

$$|H| = \sqrt{H_x^2 + H_y^2} = \sqrt{(k + I_x)^2 + (k + I_y)^2}$$

The rotating field is also beneficial in that it mitigates the problem of shielding.

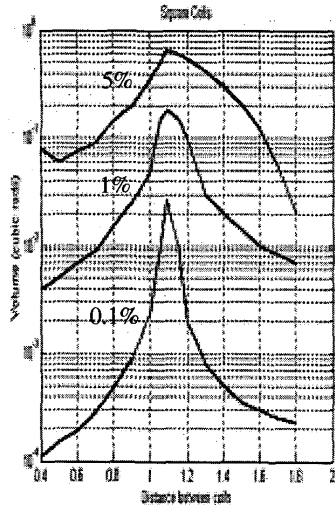


Figure 9 - Volume of specified field strength as a function of spacing for a pair of square coils

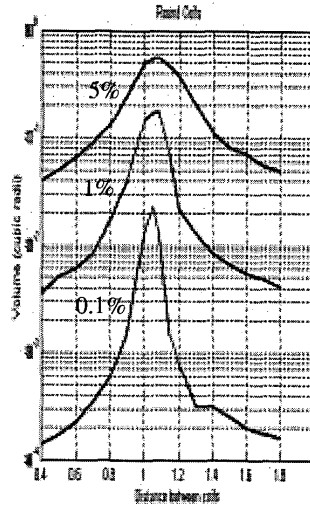


Figure 12 - Volume of specified field strength as a function of spacing for a pair of circular coils

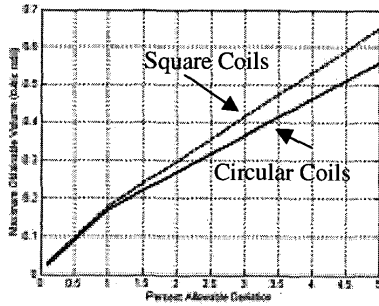


Figure 10 - Maximum obtainable sphere volume vs. percent allowable deviation for circular and square coils

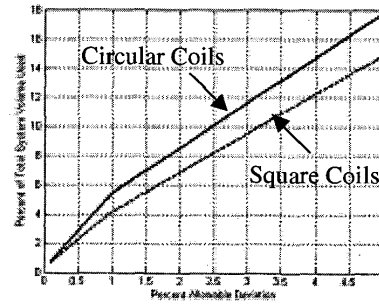


Figure 13 - Percent of total system volume utilized vs. percent allowable deviation for circular and square coils

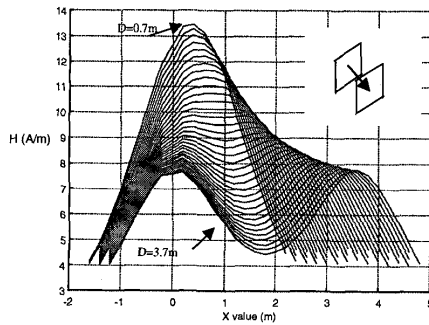


Figure 11 - Field strength along the axis of a 3m x 3m square coil pair

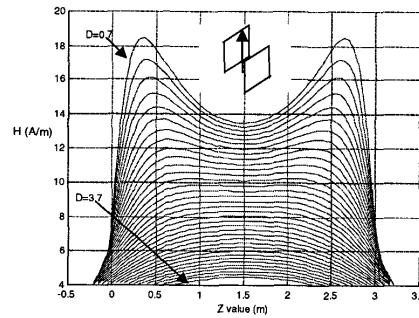


Figure 14 - Field strength perpendicular to the axis of a 3m x 3m square coil pair

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