PHYSICS of the Life Sciences

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2. TORQUE AND FORCE ON A MAGNETIC DIPOLE

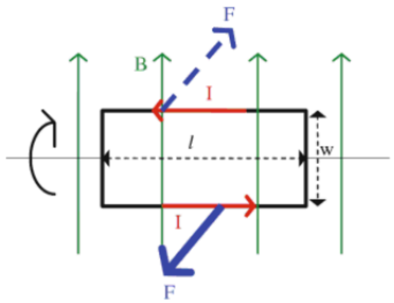

FIGURE 17.11 *A current loop in a uniform magnetic field. The two forces shown are perpendicular to the plane of the paper as determined by the right-hand rule for Equation (17.6).*

At the end of the last section we considered the magnetic force on a straight current-carrying wire in a uniform magnetic field. Another important geometry of current flow, the *current loop*, is worthy of its own discussion. A current loop is a generic term for a simple circuit with a single closed loop, regardless of the exact trajectory of the current. Its importance lies not only in actual conducting wire circuits, but also in its use as a model for understanding the magnetic properties of matter through atomic electron current loops.

In Section 4 we show that a current loop, or in fact any current carrying wire, generates its own characteristic magnetic field. Here we wish to examine the forces acting on a current loop placed in an external uniform magnetic field. Consider the rectangular current loop in Figure 17.11 lying in a region of uniform B field as shown. In this orientation, the two edges that are parallel to the magnetic field have no force acting on them, whereas the other two edges perpendicular to the B field each have a force on them given by Equation (17.6). Because the current direction is opposite in those two wire segments, the corresponding forces act in opposite directions to create a couple (the torque due to equal and opposite forces) about the horizontal axis shown in the figure. There is no net force acting on the loop but the net torque acting will tend to produce a rotation of the loop as shown.

Using the dimensions of the loop shown, we can calculate the net torque acting on the current loop about its central axis in the orientation shown in Figure 17.11 to be

$$\tau = I\ell B\frac{w}{2} + I\ell B\frac{w}{2} = I\ell wB = IAB, \tag{17.7}$$

where $w/2$ is the lever arm and $A = \ell w$ is the area of the loop. If the loop is able to rotate, the couple will produce a rotation of the loop about the axis of rotation as shown. Equation (17.7) gives the maximum torque acting on the loop because, as can be seen in the side view shown in Figure 17.12, the lever arm distance changes with the orientation of the loop. With θ equal to the angle between the B field and the normal to the plane of the loop, the lever arm can be written as

$$r_\perp = \frac{w}{2}\sin\theta,$$

so that in general the torque on a current loop in a uniform B field becomes a function of the rotation angle

$$\tau = \mu B\sin\theta, \tag{17.8}$$

where we have introduced the *magnetic dipole moment* $\mu = IA$.

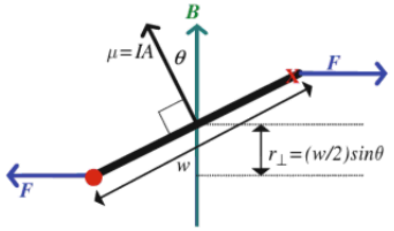

FIGURE 17.12 *Side view of a current loop in a uniform magnetic field. The normal to the loop makes an angle θ with respect to the B field. The two forces that produce a net torque are shown with the moment arm $r_\perp$ as well as the magnetic dipole moment $\mu = IA$ along the normal to the loop. The net torque tends to align the magnetic dipole moment with the magnetic field.*

The magnetic dipole moment is a vector quantity, just as is the electric dipole moment, and we choose its direction to be perpendicular to the plane of the current loop. A simple second right-hand rule indicates which of the two directions perpendicular to the current loop plane is correct: if the fingers of your right hand are curled along the direction of current flow in a wire loop, your thumb will point in the proper direction of the magnetic dipole moment. Of course, if the current direction reverses so does the direction of the magnetic dipole moment, in accord with this right-hand rule. Note that if, instead of a single loop, we have a circuit with a tightly wound helical loop of N turns, we can replace this with N identical loops each having the same area and current so that the magnetic dipole moment of the circuit is $\mu = NIA$. Also note that Equation (17.8) is very similar to the equation for the torque on an electric dipole moment in an electric field (Equation (15.13))

$$\tau = pE\sin\theta,$$