EXERCISES

8.1 Consider a region segmentation where regions are of two types: (1) filled in and (2) with holes. Relate the number of junctions, boundaries, and filled-in regions to the Euler number.

8.2 Write a procedure for finding where two chain codes intersect.

8.3 Devise algorithms to intersect and union two regions in the y-axis representation.

8.4 Show that the number of intersections of the curves under a clear strip intersection is odd.

8.5 Modify Algorithm 8.4 to work with strip trees with varying numbers of sons.

8.6 Derive Eq. (8.9) from Eq. (8.7).

8.7 Show that Eqs. (8.12) and (8.13) are equivalent.

8.8 Given two points $\mathbf{x}_1$ and $\mathbf{x}_2$ and slopes $\phi(\mathbf{x}_1)$ and $\phi(\mathbf{x}_2)$, find the ellipse with major axis a that fits the points.

8.9 Write a procedure to intersect two regions represented by quad trees, producing the quad tree of the intersection.

8.10 Determine the shape numbers for (a) a circle and (b) an octagon. What is the distance between them?

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Representations of Three-Dimensional Structures 9

9.1 SOLIDS AND THEIR REPRESENTATION

We consider three general classes of representations for rigid solids:

1. Surface or boundary
2. Sweep (in general, generalized cylinders)
3. Volumetric (in general, constructive solid geometry)

The semantics of solid representations is intuitively clear but sometimes mathematically tricky. The representations have different computational properties, and readers should keep this in mind when assessing a representation for possible use. As a simple example, a surface representation can describe how an object looks; a volumetric version, which expresses the solid as a combination of subparts, may not explicitly contain information about the surface of the object. However, the solid representation may be better for matching, if it can be structured to reflect functional subparts.

Certainly we believe, as do others, that model-based vision will ultimately have to confront the issues of geometric modeling in three dimensions [Nishihara 1979]. Ultimately, nonrigid as well as rigid solids will have to be represented. The characterization of nonrigid solids presents very challenging problems.

Nonrigid solids are often a useful way to model time-varying aspects of objects. Here, again, the kind of model that is best depends heavily on the domain. For example, a useful mammal model may be one with a piecewise rigid linkage (for the skeleton) and some elastic covering (for the flesh). Computer vision in the domain of mammals, either static in various positions or actually moving, might be based on generalized cylinders (Section 9.3). However, another nonrigid domain is that of heart chambers, that change through time as the heart beats. Here the skeleton is a much less intuitive notion, so a different model of nonrigidity may apply. In most cases, nonrigid objects are modeled as parameterized rigid objects. In

the example of the human figure, the parameters may be joint angles for linkages representing the skeleton.

The last part of this chapter deals with understanding line drawings, an influential and well-publicized subfield of computer vision. This seemingly simple and accessible domain avoids many of the problems involving early processing and segmentation, yet it is important because it has furnished several important algorithmic and geometric insights. An important breakthrough in this domain was a move from "image understanding" in two dimensions to to an approach based on the three-dimensional world and laws governing three-dimensional solids.

9.2 SURFACE REPRESENTATIONS

The *enclosing surface*, or *boundary*, of a well-behaved three-dimensional object should unambiguously specify the object [Requicha 1980]. Since surfaces are what is seen, these representations are important for computer vision. Section 9.2.1 considers mainly planar polyhedral surface representations. More complex "sculptured surfaces" [Forrest 1972; Barnhill and Riesenfeld 1974; Barnhill 1977] are treated in Section 9.2.2. Some useful surfaces are defined as functions of three-dimensional directions from a central point of origin. Two of these are mentioned in Section 9.2.3.

9.2.1 Surfaces with Faces

Figure 9.1 shows the solid representation scheme most familiar to computer scientists. Solids are represented by their boundaries, or enclosing surfaces, which are represented in terms of such primitive entities as unbounded mathematical surfaces, curves, and points which together may be used to define "faces."

In general, a boundary is made up of a number of faces; faces are represented by mathematical surfaces and by information about their own boundaries (consisting of edges and possibly vertices). A closed surface such as the sphere or a spherical harmonic surface of Section 9.2.3 may be thought of as having only one face.

To specify a boundary representation, one must answer several important questions of representation design. What is a face, and how are faces represented? What is an edge, and how are edges represented? How much extra information (i.e., useful but redundant relationships and geometric data) should be kept?

What is a face? "Face" is an initially appealing but imprecise notion; it is at its clearest in the context of planar polyhedra. A face should probably always be a subset of the boundary of an object; presumably, it should have area but no dangling edges or isolated points, and the union of all the faces should make up the boundary or the object. Beyond this little can be said. For many purposes it makes sense to have faces overlap; it may be elegant to consider the letter on an alphabet block a special kind of face on the block that is a subset of the face making up the side of the block. On the other hand, it is easy to imagine applications in which faces should not overlap in area (then one easily can compute the surface area of a solid from its faces). In some objects, just what the faces are is purely a matter of

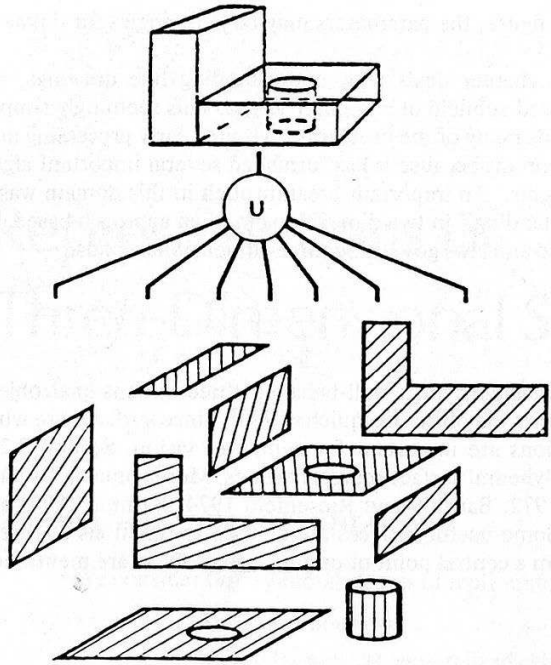


Fig. 9.1 A volume and the faces of a boundary representation.

opinion (Fig. 9.2). In short, any single definition of face is likely to be inadequate for some important application.

The availability of explicit representations of edges, faces, and vertices makes boundary representations quite useful in computer vision and graphics. The computational advantages of polyhedral surfaces are so great that they are often pressed into service as approximate representations of nonpolyhedra (Fig. 9.3).

An influential system for using face-based representations for planar polyhedral objects is the "winged edge" representation [Baumgart 1972]. Included in the system is an editor for creating complex polyhedral objects (such as that of Fig. 9.3) interactively. The system uses rules for construction based on the theorem of Euler that if V is the number of vertices in a polyhedron, E the number of edges, and F the number of faces, then $V - E + F = 2$. In fact, the formula can be extended to deal with non-simply connected bodies. The extended relation is $V - E + F = 2(B - H)$, with B being the number of bodies and H being the

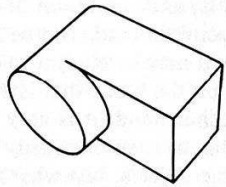


Fig. 9.2 What are the faces?

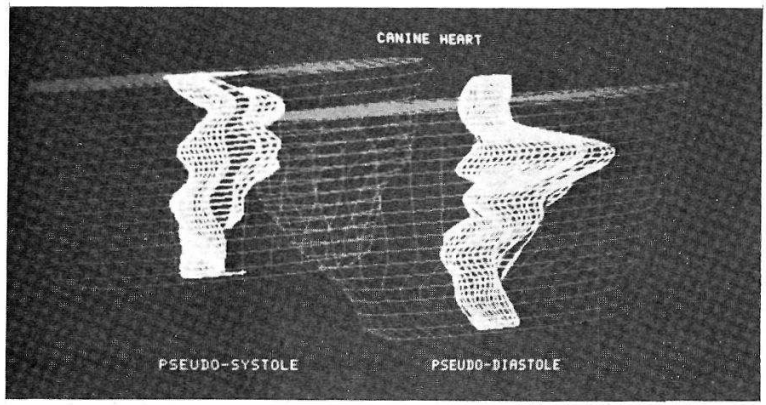


Fig. 9.3 A polyhedral approximation to a portion of a canine heart at systole and diastole. Both exterior (coarse grid) and interior surfaces (fine grid) are shown.

number of holes, or "handles," each resulting from a hole through a body [Lakatos 1976]. Baumgart's system uses these rules to oversee and check certain validity conditions on the constructions made by the editor.

The "winged edge" polyhedron representation achieves many desiderata for boundary representations in an elegant way. This representation is presented below to give a flavor of the features that have been traditionally found useful. Given as primitives the vertices, edges, faces, and polyhedra themselves, and given various relations between these primitives, one is naturally thinks of a record and pointer (relational) structure in which the pointers capture the binary relations and the records represent primitives and contain data about their locations or parameters.

In the winged edge representation, there are data structure records, or nodes, which contain fields holding data or links (pointers) to other nodes. An example using this structure to describe a tetrahedron is shown in Fig. 9.4. There are four kinds of nodes: vertices, edges, faces, and bodies. To allow convenient access to these nodes, they are arranged in a circular doubly linked list. The body nodes are actually the heads of circular structures for the faces, edges, and vertices of the body. Each face points to one of its perimeter edges, and each vertex points to one of the edges impinging on it. Each edge node has links to the faces on each side of it, and the vertices at either end.

Figure 9.4 shows only the last-mentioned links associated with each edge node. The reader may notice the similarity of this data structure with the data structure for region merging in Section 5.4. They are topologically equivalent. Each edge also has associated four links which give the name "winged edge" to the representation. These links specify neighboring edges in order around the two faces which are associated with the edge. The complete link set for an edge is shown in Fig. 9.5, together with the link information for bodies, vertices, and faces. To allow unambiguous traversal around faces, and to preserve the notion of

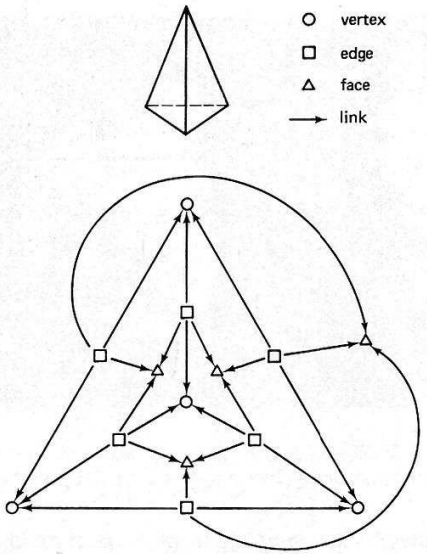


Fig. 9.4 A subset of edge links for a tetrahedron using the "winged edge" representation.

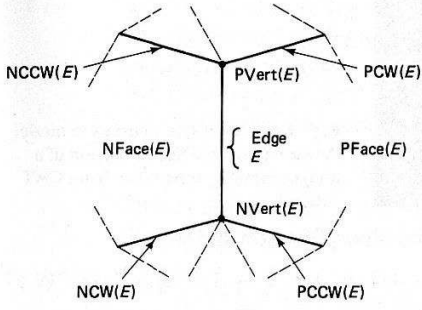
interior and exterior of a polyhedron, a preferential ordering of vertices and lines is picked (counterclockwise, say, as seen from outside the polyhedron).

Data fields in each vertex allow storage of three-dimensional world coordinates, and also of three-dimensional perspective coordinates for display. Each node has fields specifying its node type, hidden line elimination information, and other general information. Faces have fields for surface normal vector information, surface reflectance, and color characteristics. Body nodes carry links to relate them to a tree structure of bodies in a scene, allowing for hierarchical arrangement of subbodies into complex bodies. Thus body node data describe the scene structure; face node data describe surface characteristics; edge node data give the topological information needed to relate faces, edges, and vertices; and vertex node data describe the three-dimensional vertex location.

This rich and redundant structure lends itself to efficient calculation of useful functions involving these bodies. For instance, one can easily follow pointers to extract the list of points around a face, faces around a point, or lines around a face. Winged edges are not a universal boundary representation for polyhedra, but they do give an idea of the components to a representation that are likely to be useful. Such a representation can be made efficient for accessing all faces, edges, or vertices; for accessing vertex or edge perimeters; for polyhedron building; and for splitting edges and faces (useful in construction and hidden-line picture production, for instance).

9.2.2 Surfaces Based on Splines

The natural extension of polyhedral surfaces is to allow the surfaces to be curved. However, with an arbitrary number of edges for the surface, the interpolation of



(a)

Boundary Representation Node Accessing Functions

1. To enter and traverse Face ring of a body:
   NextFace, PreviousFace: Body or Face → Face
2. To enter and traverse Edge ring of a body:
   NextEdge, PreviousEdge: Body or Edge → Edge
3. To enter and traverse Vertex ring of a body:
   NextVert, PreviousVert: Body or Vertex → Vertex
4. First Edge of a Face:
   FirstEdge: Face → Edge
5. FirstEdge of a Vertex:
   FirstEdge: Vertex → Edge
6. Faces of an Edge: [see diagram in (a)]
   N(ext) Face, P(revious) Face: Edge → Face
7. Vertices of an Edge: [see diagram in (a)]
   N(extVert, P(revious) Vert: Edge → Vertex
8. Neighboring Wing Edges of an Edge: [see diagram in (a)]
   NCW, NCCW: Edge → Edge (NFace Edge Clockwise, NFace Edge Counterclockwise)
   PCW, PCCW: Edge → Edge (PFace Edge Clockwise, PFace Edge Counterclockwise

(b)

Fig. 9.5 (a) Node accessing functions. (b) Semantics of winged edge functions.

interior face points becomes impractically complex. For that reason, the number of edges for a curved face is usually restricted to three or four.

A general technique for approximating surfaces with four-sided surface *patches* is that of Coons [Coons 1974]. Coons specifies the four sides of the patch with polynomials. These polynomials are used to interpolate interior points. Although this is appropriate for synthesis, it is not so easy to use for analysis. This is because of the difficulty of registering the patch edges with image data. A given surface will admit to many patch decompositions.

An attractive representation for patches is splines (Fig. 9.6). In general, two-dimensional spline interpolation is complex: For two parameters u and v interpolate with

$$\mathbf{x}(u, v) = \sum_i \sum_j V_{ij} B_{ij}(u, v) \tag{9.1}$$

similar to Eq. (8.4). However, for certain applications a further simplification can be made. In a manner analogous to (8.9) define a grid of knot points $\mathbf{v}_{ij}$ corresponding to $\mathbf{x}_{ij}$ and related by

$$\mathbf{x}_{ij} = M\mathbf{v}_{ij} \tag{9.2}$$

Now rather than interpolating in two dimensions simultaneously, interpolate in one direction, say t, to obtain

$$\mathbf{x}_{ij}(t) = [t^3 \quad t^2 \quad t \quad 1][C][\mathbf{v}_{i-1,j_0}, \mathbf{v}_{i,j_0}, \mathbf{v}_{i+1,j_0}, \mathbf{v}_{i+2,j_0}]^T \tag{9.3}$$

for each value of j. Now compute $\mathbf{v}_{ij}(t)$ by solving

$$\mathbf{x}_{ij}(t) = M\mathbf{v}_{ij}(t) \tag{9.4}$$

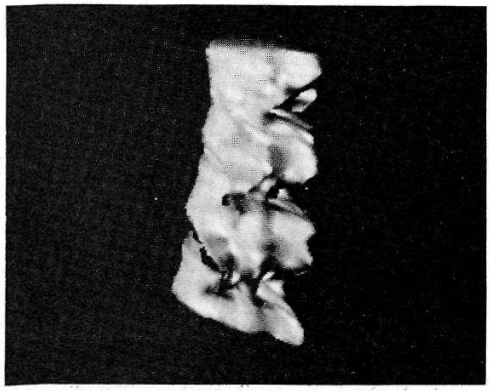


Fig. 9.6 Using spline curves to model the surface of an object: a portion of a human spinal column taken from CAT data.

for each value of t. Finally, interpolate in the other direction and solve:

$$\mathbf{x}_{ij}(s, t) = [s^3 \quad s^2 \quad s \quad 1][C][\mathbf{v}_{i-1,j}(t), \mathbf{v}_{i,j}(t), \mathbf{v}_{i+1,j}(t), v_{i+2,j}(t)] \quad (9.5)$$

This is the basis for the spline filtering algorithm discussed in Section 3.2.3.

Some advantages of spline surfaces for vision are the following.

1. The spline representation is economical: the space curves are represented as a sparse set of knot points from which the underlying curves can be interpolated.
2. It is easy to define splines interactively by giving the knot points; reference representations may be built up easily.
3. It is often useful to search the image in a direction perpendicular to the model reference surface. This direction is a simple function of the local knot points.

9.2.3 Surfaces That Are Functions on the Sphere

Some surfaces can be expressed as functions on the "Gaussian sphere." (the distance from the origin to a point on the surface is a function of the direction of the point, or of its longitude and latitude if it were radially projected on a sphere with the center at the origin.) This class of surfaces, although restricted, is useful in some application areas [Schudy and Ballard 1978, 1979]. This section explores briefly two schemes for representation of these surfaces. The first specifies explicitly the distance of the surface from the origin for a set of vector directions from the origin. The second is akin to Fourier descriptors; an economically specified set of coefficients characterizes the surface with greater accuracy as the number of coefficients increases.

Direction–Magnitude Sets

One approximation to a spherical function is to specify a number of three-dimensional direction vectors from the origin and for each a magnitude. This is equivalent to specifying a set of (θ, ϕ, ρ) points in a spherical coordinate system (Appendix 1). These points are on the surface to be represented; connecting them yields an approximation.

It is often convenient to represent directions as points on the unit (Gaussian) sphere centered on the origin. The points may be connected by straight lines to form a polyhedron with triangular, hexagonal or rhomboidal faces. Moving the points on the sphere out (or in) by their associated magnitude distorts this polyhedron, moving its vertices radically out or in.

The spherical function determines the distance of face vertices from the origin. Resolution at the surface increases with the number of faces. An approximately isotropic distribution of directions over the surface may be obtained by placing the face vertices (directions) in accordance with "geodesic dome"–like calculations which make the faces approximately equilateral triangles [Clinton 1971].

Although the geodesic tesselation of the sphere's surface is more complex than a straightforward (latitude and longitude, say) division, its pleasant properties of isotropy and display [Brown 1979a; 1979b; Schudy and Ballard 1978] sometimes recommend it. Some example shapes indicating the range of representable surfaces are given in Fig. 9.7. Methods for tesselating the sphere are given in Appendix 1.

Spherical Harmonic Surfaces

In two dimensions, Fourier coefficients can give approximations to certain curved boundaries (Section 8.3.4). Analogously in three dimensions, a set of orthogonal functions may be used to express a closed boundary as a set of coefficients when the boundary is a function on the sphere. One such decomposition is *spherical harmonics.* Low order coefficients capture gross shape characteristics; higher order coefficients represent surface shape variations of higher spatial frequency. The function with $m = 0$ is a sphere, the three with $m = 1$ represent translation about the origin, the five with $m = 2$ are similar to prolate and oblate spheroids, and so forth, the lobedness of the surfaces increasing with m. A sample three dimensional shape and its "description" is shown in Fig. 9.8.

Spherical harmonics are analogs on the sphere of Fourier functions on the plane; like Fourier functions, they are smooth and continuous to every order. They may be parameterized by two numbers, m and n; thus they are a doubly infinite set of functions which are continuous, orthogonal, single-valued, and complete on the

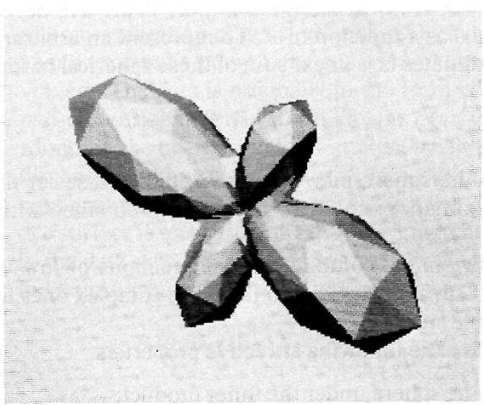


Fig. 9.7 Sample surfaces described by some 320 triangular facets in a geodesic tesselation.

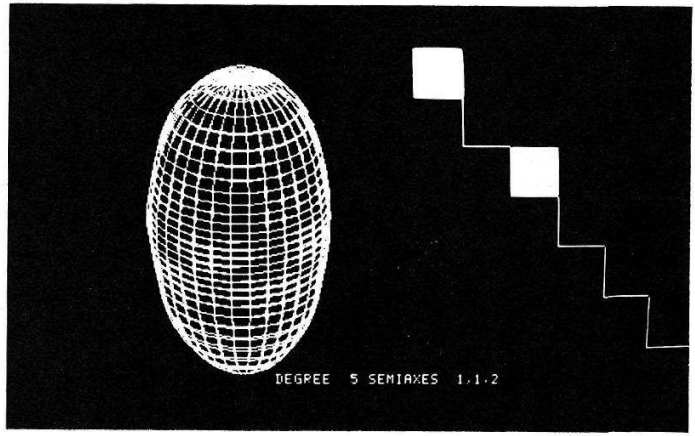


Fig. 9.8 A spherical harmonic function description of an ellipsoid. Coefficients are displayed on the right as grey levels in the matrix format

$$\begin{array}{cccc} u_{00} & & & \\ u_{01} & v_{11} & & \\ u_{11} & u_{02} & v_{12} & v_{22} \\ & u_{12} & & \\ & u_{21} & & \end{array}$$

sphere. In combination, the harmonics can thus produce all "well-behaved" spherical functions.

The spherical harmonic functions $U_{mn}(\theta, \phi)$ and $V_{mn}(\theta, \phi)$ are defined in polar coordinates by:

$$U_{mn}(\theta, \phi) = \cos(n\theta) \sin^n(\phi) P(m, n, \cos(\phi)) \tag{9.6}$$

$$V_{mn}(\theta, \phi) = \sin(n\theta) \sin^n(\phi) P(m, n, \cos(\phi)) \tag{9.7}$$

with $m = 0, 1, 2, \ldots, M$; $n = 0, 1, \ldots, m$. Here $P(m, n, x)$ is the nth derivative of the mth Legendre polynomial as a function of x. To represent an arbitrary shape, let the radius R in polar coordinates be a linear sum of these spherical harmonics:

$$R(\theta, \phi) = \sum_{m=0}^{M} \sum_{n=0}^{m} A_{mn} U_{mn}(\theta, \phi) + B_{mn} V_{mn}(\theta, \phi) \tag{9.8}$$

Any continuous surface on the sphere may be represented by a set of these real constants; reasonable approximations to heart volumes are obtained with $m \leqslant 5$ [Schudy and Ballard 1979].

Figure 9.9 shows a few simple combinations of functions of low values of (m, n). The sphere, or $(0, 0)$ surface, is added to the more complex ones to ensure positive volumes and drawable surfaces.

Spherical harmonics have the following attractive properties.

1. They are orthogonal on the sphere under the inner product;

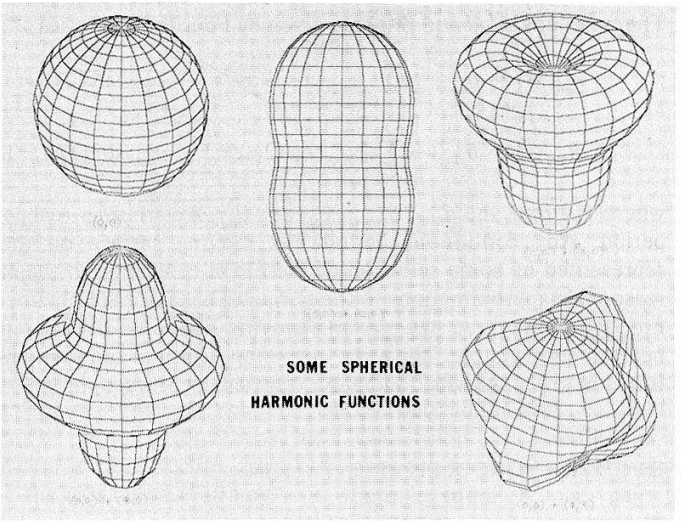


Fig. 9.9 Simple combinations of functions.

$$(u, v) = \int uv \sin\phi \, d\theta \, d\phi$$

2. The functions are arranged in increasing order of spatial complexity.
3. The whole set is complete; any twice-differentiable function on the sphere can be approximated arbitrarily closely.

Spherical harmonics can provide compact, nonredundant descriptions of surfaces that are useful for analysis of shape, but are less useful for synthesis. The principal disadvantages are that the primitive functions are not necessarily related to the desired final shape in an intuitive way, and changing a single coefficient affects the entire resulting surface.

An example of the use of spherical harmonics as a volume representation is the representation of heart volume [Schudy and Ballard 1978, 1979]. In extracting a volume associated with the heart from ultrasound data, a large mass of data is involved. The data is originally in the form of echo measurements taken in a set of two-dimensional planes through the heart. The task is to choose a surface surrounding the heart volume of interest by optimization techniques that will fit three dimensional time-varying data. The optimization involved is to find the best coefficients for the spherical harmonics that define the surface. The goodness of fit of a surface is measured by how well it matches the edge of the volume as it appears in the data slices. To extend spherical harmonics to time-varying periodic data, let the radius R in polar coordinates be a linear sum of these spherical harmonics:

$$R(\theta, \phi, t) = \sum_{m=0}^{M} \sum_{n=0}^{m} A_{mn}(t) U_{mn}(\theta, \phi) + B_{mn}(t) V_{mn}(\theta, \phi) \qquad (9.9)$$

The functions $A(t)$ and $B(t)$ are given by Fourier time series:

$$A_{mn}(t) = a_{mno} + \sum_{i=1}^{I} a_{mni} \cos(2\pi t/\tau) + b_{mni} \sin(2\pi t/\tau) \tag{9.10}$$

$$B_{mn}(t) = b_{mno} + \sum_{i=1}^{I} c_{mni} \cos(2\pi t/\tau) + d_{mni} \sin(2\pi t/\tau) \tag{9.11}$$

where t is time, the a_{mni}, b_{mni}, c_{mni}, and d_{mni} are arbitrary real constants, and τ the period. Any continuous periodically moving surface on the sphere may be represented by some selection of these real constants; in the cardiac application, reasonable approximations to the temporal behavior are obtained with $t \leqslant 3$. Figure 9.10 shows three stages from a moving-harmonic-surface representation of the heart in early systole. The atria, at the top, contract and pump blood into the ventricles below, after which there is a ventricular contraction.

9.3 GENERALIZED CYLINDER REPRESENTATIONS

The volume of many biological and manufactured objects is naturally described as the "swept volume" of a two-dimensional set moved along some three-space curve. Figure 9.11 shows a "translational sweep" wherein a solid is represented as the volume swept by a two-dimensional set when it is translated along a line. A "rotational sweep" is similarly defined by rotating the two-dimensional set around an axis. In "three-dimensional sweeps," volumes are swept. In a "general" sweep scheme, the two-dimensional set or volume is swept along an arbitrary space curve, and the set may vary parametrically along the curve [Binford 1971; Soroka and Bajcsy 1976; Soroka 1979a; 1979b; Shani 1980]. General sweeps are quite a popular representation in computer vision, where they go by the name *generalized cylinders* (sometimes "generalized cones").

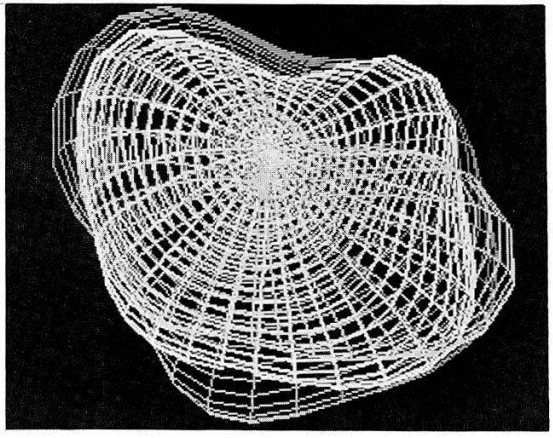


Fig. 9.10 Three stages from a moving harmonic surface (*see text and color insert*).

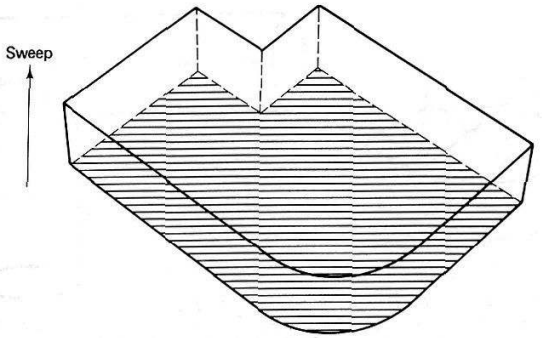


Fig. 9.11 A translational sweep.

A *generalized cylinder* (*GC*) is a solid whose axis is a 3-D space curve (Fig. 9.12a). At any point on the axis a closed cross section is defined. A usual restriction is that the axis be normal to the cross section. Usually it is easiest to think of an axis space curve and a cross section point set function, both parameterized by arc length along the axis curve. For any solid, there are infinitely many pairs of axis and cross section functions that can define it.

Generalized cylinders present certain technical subtleties in their definition. For instance, can it be determined whether any two cross sections intersect, as they would if the axis of a circular cylinder were sharply bent (Fig. 9.12b)? If the solid is defined as the volume swept by the cross section, there is no conceptual or computational problem. A problem might occur when computing the surface of such an object. If the surface is expressed in terms of the axis and cross-section functions (as below), the domain of objects must be limited so that the boundary formula indeed gives only points on the boundary.

Generalized cylinders are intuitive and appealing. Let us grant that "pathological" cases are barred, so that relatively simple mathematics is adequate for representing them. There are still technical decisions to make about the representation. The axis curve presents no difficulties, but a usable representation for the cross-section set is often not so straightforward. The main problem is to choose a usable coordinate system in which to express the cross section.

9.3.1 Generalized Cylinder Coordinate Systems and Properties

Two mathematical functions defining axis and cross section for each point define a unique solid with the "sweeping" semantics described above. In a fixed Cartesian coordinate system x, y, z, the axis may be represented parametrically as a function of arc length s:

$$\mathbf{a}(s) = (x(s), y(s), z(s)) \tag{9.12}$$

It is convenient to have a local coordinate system defined with origin at each point of $\mathbf{a}(s)$. It is in this coordinate system that the cross section is defined. This system may change in orientation as the axis winds through space, or it may be most natural for it not to be tied to the local behavior of the axis. For instance, imagine tying a knot in a solid rubber bar of square cross section. The cross section

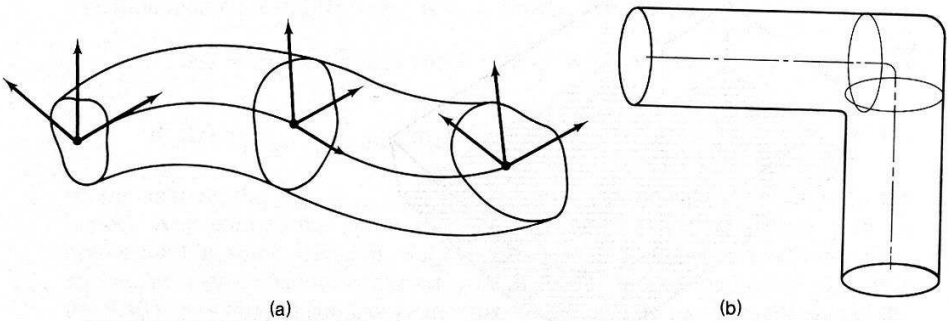


Fig. 9.12 (a) A generalized cylinder and some cross-sectional coordinate systems. (b) A possibly "pathological" situation. Cross sections may be simply described as circles centered on the axis, but then their intersection makes volume calculations (for instance) less straightforward.

will stay approximately a square, and (this is the point) will remain approximately fixed in a coordinate system that twists and turns through space with the axis of the bar. On the other hand, imagine bolt threads. They can be described by a single cross section that stays fixed in a coordinate system that rotates as it moves along the straight axis of the bolt. There is no a priori reason to suppose that such a useful local coordinate system should twist along the GC axis.

A coordinate system that mirrors the local behavior of the GC axis space curve is the "Frenet frame," defined at each point on the GC axis. This frame provides much information about the GC-axis behavior. The GC axis point forms the origin, and the three orthogonal directions are given by the vectors $(\boldsymbol{\xi}, \boldsymbol{\nu}, \boldsymbol{\zeta})$, where

$\boldsymbol{\xi}$ = unit vector tangent axis

$\boldsymbol{\nu}$ = unit vector direction of center of curvature of axis normal curve

$\boldsymbol{\zeta}$ = unit vector direction of center of torsion of axis

Consider the curve to be produced by a point moving at constant speed through space; the distance the point travels is the parameter of the space curve [O'Neill 1966]. Since ξ is of constant length, its derivative measures the way the GC axis turns in space. Its derivative ξ' is orthogonal to ξ and the length of ξ' measures the curvature κ of the axis at that point. The unit vector in the direction of ξ' is ν. Where the curvature is not zero, a binormal vector ζ orthogonal to ξ and ν is defined. This binormal ζ is used to define the torsion τ of the curve. The vectors ξ, ν, ζ obey Frenet's formulae:

$$\begin{aligned} \boldsymbol{\xi}' &= \kappa\boldsymbol{\nu} \\ \boldsymbol{\nu}' &= -\kappa\boldsymbol{\xi} + \tau\boldsymbol{\zeta} \\ \boldsymbol{\zeta}' &= -\tau\boldsymbol{\nu} \end{aligned} \tag{9.13}$$

where

$$\kappa = \text{curvature} = -\boldsymbol{\nu}' \cdot \boldsymbol{\xi} = \boldsymbol{\nu} \cdot \boldsymbol{\xi}' \tag{9.14}$$

$$\tau = \text{torsion} = \boldsymbol{\nu}' \cdot \boldsymbol{\zeta} = -\boldsymbol{\nu} \cdot \boldsymbol{\zeta}' \tag{9.15}$$

The Frenet frame gives good information about the axis of the GC, but it has certain problems. First, it is not well defined when the curvature of the GC axis is zero. Second, it may not reflect known underlying physical principles that generate the cross sections (as in the bolt thread example). A solution, adopted in [Agin 1972, Shani 1980], is to introduce an additional parameter that allows the cross section to rotate about the local axis by an arbitrary amount. With this additional degree of freedom comes an additional problem: How are successive cross sections registered? Figure 9.13 shows two solutions in addition to the Frenet frame solution.

The cross sectional curve is usually defined to be in the $\boldsymbol{\nu}$–$\boldsymbol{\zeta}$ plane, normal to $\boldsymbol{\xi}$, the local GC axis direction. The cross section may be described as a point set in this plane, using inequalities expressed in the $\boldsymbol{\nu}$–$\boldsymbol{\zeta}$ coordinate system. The cross section boundary (outline curve) may be used instead, parameterized by another parameter r. Let this curve be given by

$$\text{cross section boundary} = (x(r, s),\ y(r, s))$$

The dependence on s reflects the fact that the cross section shape may vary along the GC axis. The expression above is in world coordinates, but should be moved to

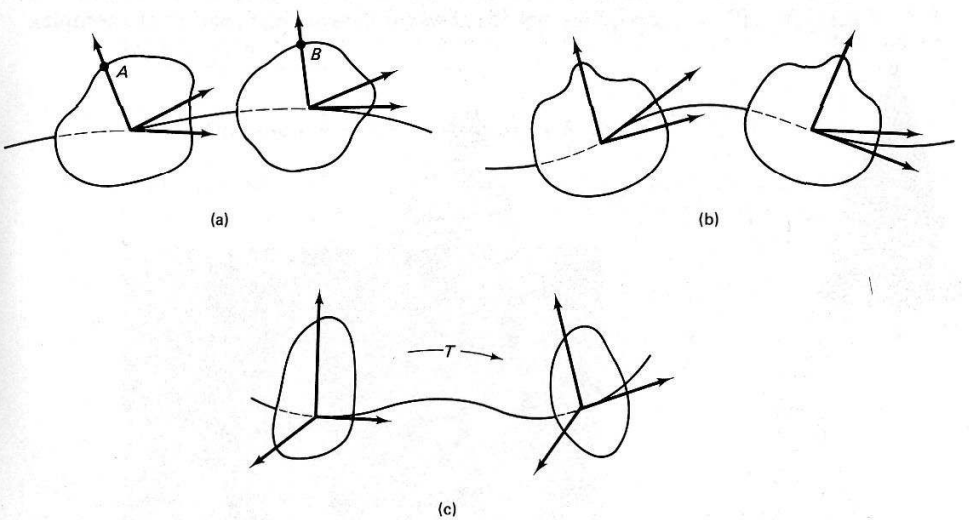


Fig. 9.13 (a) Local coordinates are the Frenet frame. Points A and B must correspond. (b) Local coordinates are determined by the cross sectional shape. (c) Local coordinates are determined by a heuristic transformation from world coordinates.

the local coordinates on the GC axis. A transformation of coordinates allows the GC boundary to be expressed (if the GC is well behaved) as

$$B(r, s) = \mathbf{a}(s) + x(r, s)\,\boldsymbol{\nu}(s) + y(r, s)\,\boldsymbol{\zeta}(s) \tag{9.16}$$

One of the advantages of the generalized cylinder representation is that it allows many parameters of the solid to be easily calculated.

- In matching the GC to image data it is often necessary to search perpendicular to a cross section. This direction is given from $x(r, s)$, $y(r, s)$ by $((dy/ds)\boldsymbol{\nu}, -(dx/ds)\boldsymbol{\zeta})$.
- The area of a cross section may be calculated from Eq. (8.16).
- The volume of a GC is given by the integral of: the area as a function of the axis parameter multipled by the incremental path length of the GC axis, i.e.,

$$\text{volume} = \int_0^L \text{area}(s)\; ds$$

9.3.2 Extracting Generalized Cylinders

Early work in biological form analysis provides an example of the process of fitting a GC to real data and producing a description [Agin 1972]. One of the goals of this work was to infer the stick figure skeleton of biological forms for use in matching models also represented as skeletons. In Fig. 9.14 the process of inferring the axis from the original stripe three-dimensional data is shown; the process iterates toward a satisfactory fit, using only circular cross sections (a common constraint with "generalized" cylinders). Figure 9.15 shows the data and the analysis of a complex

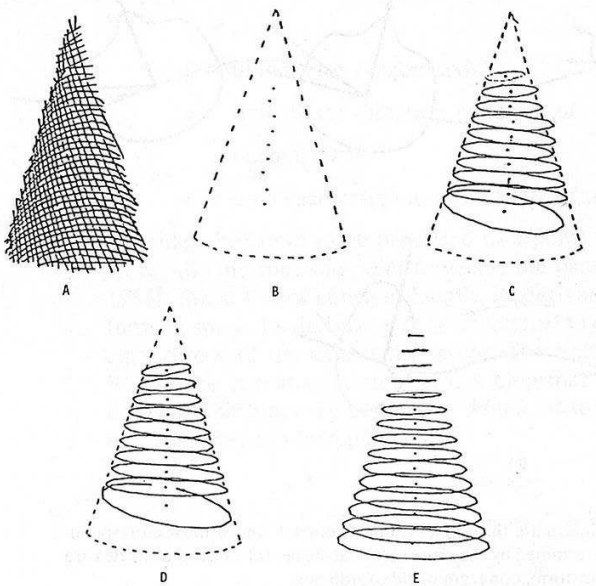


Fig. 9.14 Stages in extracting a generalized cylinder description for a circular cone. (a) Front view. (b) Initial axis estimate. (c) Preliminary center and axis estimate. (d) Cone with smoothed radius function. (e) Completed analysis.

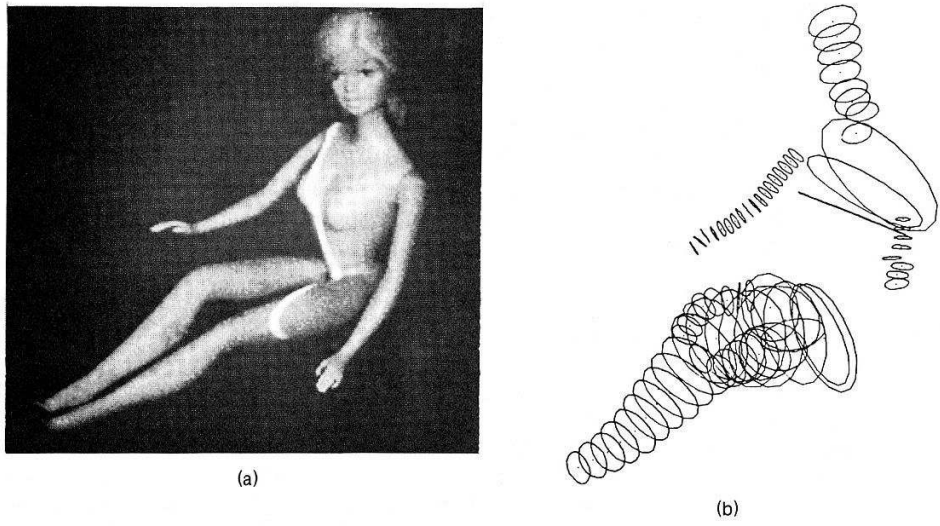


Fig. 9.15 (a) TV image of a doll. (b) Completed analysis of doll.

biological form. In real data, complexly interrelated GCs are hard to decompose into satisfactory subparts. Without that, the ability to form a satisfactory articulated skeleton is severely restricted.

In later work, GCs with spline-based axes and cross sections were used to model organs of the human abdomen [Shani 1980]. Figure 9.16 shows a rendition of a GC fit to a human kidney.

9.3.3 A Discrete Volumetric Version of the Skeleton

An approximate volume representation that can be quite useful is based on an articulated wire frame skeleton along which spheres (not cross sections) are placed.

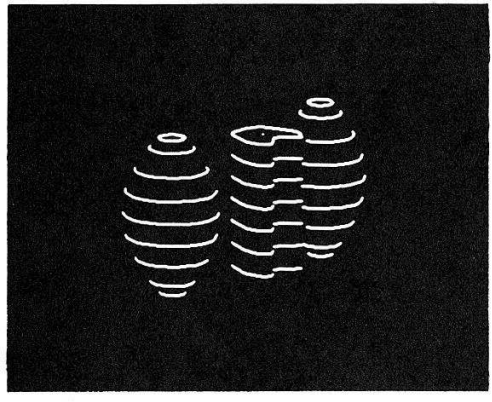


Fig. 9.16 Generalized cylinder representation of two kidneys and a spinal column. This coarse, nominal model is refined during examination of CAT data (see Fig. 9.6).

This representation has some of the flavor of an approximate sweep representation. An example of the use of such a representation and a figure are given in Section 7.3.4. This representation was originally conceived for graphics applications (the spheres look the same from any viewpoint) [Badler and Bajcsy 1978]. Collision detection is easy, and three-dimensional objects can be decomposed into spheres automatically [O'Rourke and Badler 1979]. From the spheres, the skeleton may be derived, and so may the surface of the solid. This representation is especially apt for many computer vision applications involving nonrigid bodies if strict surface and volumetric accuracy is not necessary [Badler and O'Rourke 1979].

9.4 VOLUMETRIC REPRESENTATIONS

Most world objects are solids, although usually only their surfaces are visible. A representation of the objects in terms of more primitive solids is often useful and can have pleasant properties of terseness, validity, and sometimes ease of computation. The representations given here are presented in order of increasing generality; constructive solid geometry includes cell decomposition, which in turn includes spatial occupancy arrays.

Algorithms for processing volume-based representations are often of a different flavor than surface-based algorithms. We give some examples in Section 9.4.4. Objects represented volumetrically can be depicted on raster graphics devices by a "ray-casting" approach in which a line of sight is constructed through the viewing plane for a set of raster points. The surface of the solid at its intersection with the line of sight determines the value of the display at the raster point. Ray casting can produce hidden-line and shaded displays; graphics is only one of its applications (Section 9.4.4).

9.4.1 Spatial Occupancy

Figure 9.17 shows that three-dimensional spatial occupancy representations are the three-dimensional equivalent of the two-dimensional spatial occupancy representations of Chapter 8. Volumes are represented as a three-dimensional array of cells which may be marked as filled with matter or not. Spatial occupancy arrays can require much storage if resolution is high, since space requirements increase as the cube of linear resolution. In low-resolution work with irregular objects, such as arise in computer-aided tomography, spatial occupancy arrays are very common. It is sometimes useful to convert an exact representation into an approximate spatial occupancy representation. Slices or sections through objects may be easily produced. The spatial occupancy array may be run-length encoded (in one dimension), or coded as blocks of different sizes; such schemes are actually cell-decomposition schemes (Section 9.4.2).

With the declining cost of computer memory, explicit spatial occupancy arrays may become increasingly common. The improvement of hardware facilities for parallel computation will encourage the development of parallel algorithms to compute properties of solids from these representations.

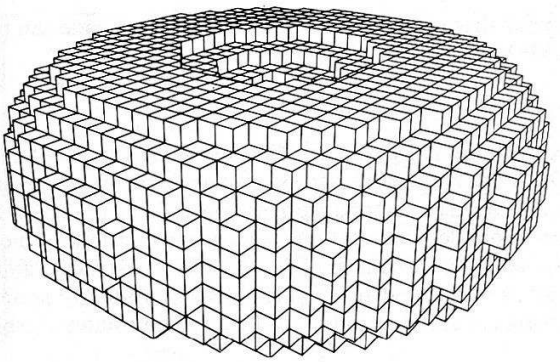


Fig. 9.17 A solid (the shape of a human red blood cell) approximated by a volume occupancy array.

9.4.2 Cell Decomposition

In cell decomposition, cells are more complex in shape but still "quasi-disjoint" (do not share volumes), so the only combining operation is "glue" (Fig. 9.18). Cells are usually restricted to have no holes (they are "simply connected"). Cell decompositions are not particularly concise; their construction (especially for curved cells) is best left to programs. It seems difficult to convert other representations exactly into cell decompositions. Two useful cell decompositions are the "oct-tree" [Jackins and Tanimoto 1980] and the kd-tree [Bentley 1975]. They both can be produced by recursive subdivision of volume; these schemes are the three-dimensional analogs of pyramid data structures for two dimensional binary images.

The quasi-disjointness of cell-decomposition and spatial-occupancy primitives may be helpful in some algorithms. Mass properties (Section 9.4.4) may be computed on the components and summed. It is possible to tell whether a solid is connected and whether it has voids. Inhomogeneous objects (such as human anatomy inside the thorax) can be represented easily with cell decomposition and spa-

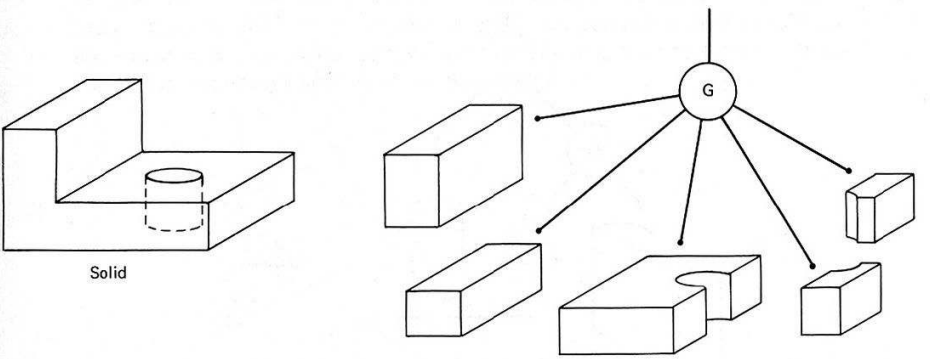


Fig. 9.18 A volume and its cell decomposition.

tial occupancy. The CT number (transparency to x-rays) or a material code can be kept in a cell instead of a single bit indication of "solid or space."

9.4.3 Constructive Solid Geometry

Figure 9.19 shows one constructive solid geometry (CSG) scheme [Voelcker and Requicha 1977; Boyse 1979]. Solids are represented as compositions, via set operations, of other solids which may have undergone rigid motions. At the lowest level are primitive solids, which are bounded intersections of closed half-spaces defined by some $F(x, y, z) \geqslant 0$, where F is well-behaved (e.g., analytic). Usually, primitives are entities such as arbitrarily scaled rectangular blocks, arbitrarily scaled cylinders and cones, and spheres of arbitrary radius. They may be positioned arbitrarily in space.

Figure 9.20 shows a parameterized representation [Marr and Nishihara 1978; Nishihara 1979] based on shapes (here cylinders) that might be extracted from an image.

A CSG representation is an expression involving primitive solid and set operators for combination and motion.

<CSG*Rep*> ::= <primitive solid> |
MOVE <CSG Rep> BY <Motion Params> |
<CSG Rep> <Combine Op> <CSG Rep>

The combining operators are best taken to be *regularized* versions of set union, intersection, and difference (the complement is a possible operator, but it allows unbounded solids from bounded primitives).

Regularity is a fundamental property of any set of points that models a solid. In a given space, a set X is regular if $X = kiX$, where k and i denote the *closure* and *interior* operators. Intuitively, a regular set has no isolated or dangling boundary points. The regularization r of a set X is defined by $rX = kiX$. Regularization informally amounts to taking what is inside a set and covering that with a tight skin. Regular sets are not closed under conventional set operations, but *regularized*

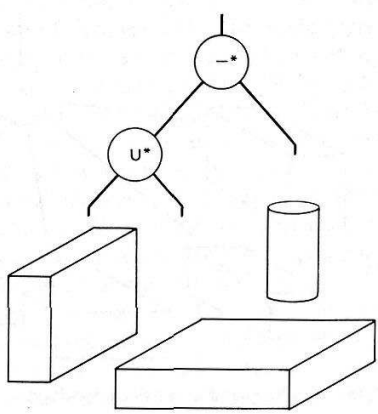


Fig. 9.19 Constructive solid geometry for the volume of Fig. 9.18.

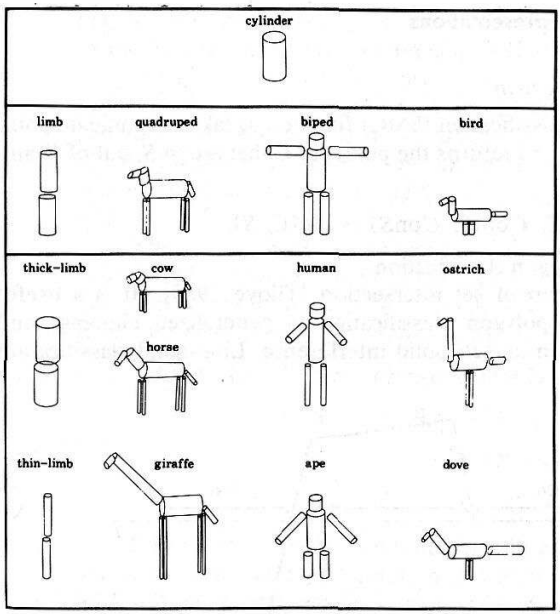


Fig. 9.20 A parameterized constructive representation for animal shapes.

operators do preserve regularity. Regularized operators are defined by

$$X \ <\text{OP}>^* \ Y = r(X \ <\text{OP}> \ Y)$$

Regularity and regularized set operators provide a natural formalization of the dimension-preserving property exhibited by many geometric algorithms, thus obviating the need to enumerate many annoying "special cases." Figure 9.21 illustrates conventional versus regularized intersection of two sets that are regular in the plane.

If the primitives are unbounded, checking for boundedness of an object can be difficult. If they are bounded, any CSG representation is a valid volume representation. CSG can be inefficient for some geometric applications, such as a line drawing display. (Converting the CSG representation to a boundary representation is the one way to proceed; see Section 9.4.4.)

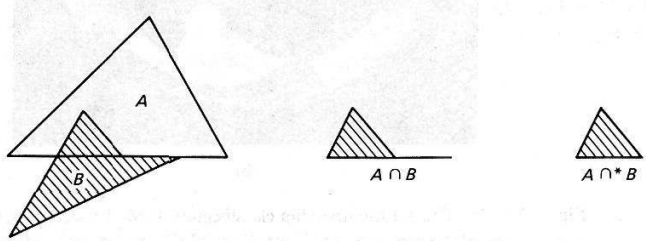


Fig. 9.21 Conventional ($\cap$) and regularized ($\cap^*$) polygon intersection.

9.4.4 Algorithms for Solid Representations

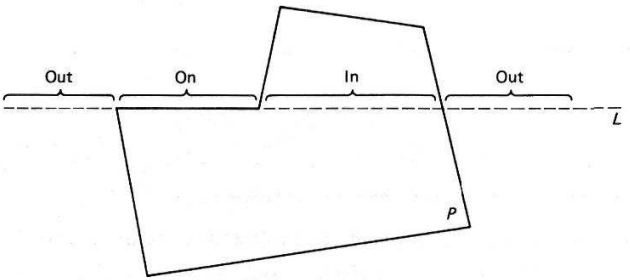
Set Membership Classification

The set membership classification (SMC) function M takes a candidate point set C and a reference set S, and returns the points of C that are in S, out of S, and on the boundary of S.

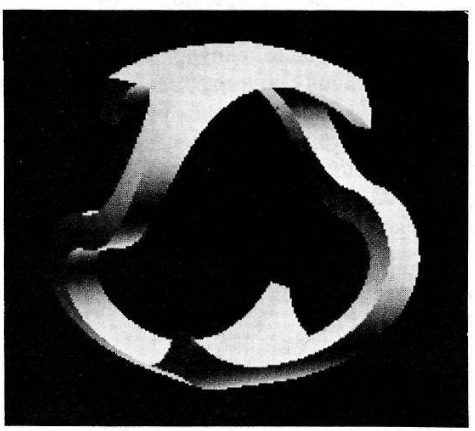
$$(C\text{in}S,\ C\text{out}S,\ C\text{on}S) := M(C, S)$$

Figure 9.22a shows line–polygon classification.

SMC is a generalization of set intersection [Tilove 1980]. It is a useful geometric utility; polygon–polygon classification is generalized clipping, and volume–volume classification detects solid interference. Line–solid classification



(a)



(b)

Fig. 9.22 (a) The set membership classification (SMC) function M(L, P) finds the portions of the candidate set L (here a line) that are in, on, and out of a reference set (here a polygon) P. (b) Image produced by ray casting, a special case of SMC.

may be used for ray casting visualization techniques to generate images of a known three-dimensional representation (Fig. 9.22b).

An algorithm for SMC illustrates a "divide and conquer" approach to computing on CSG. Recall that CSG is like a tree of set operations, whose leaves are primitive sets which usually are simple solids such as cylinders, spheres, and blocks. Presumably classification can be more easily computed with these simple sets as reference than with complex unions, intersections, and differences as reference.

The idea is that the classification of a set C with respect to a complex object S defined in CSG may be determined recursively. Any internal node S in the CSG tree is an operation node. It has left and right arguments and an operation OpofS. Each subtree is itself a CSG subtree or a primitive.

$M(X, S)$ = IF $S$is a primitive THEN prim−$M(X, S)$
ELSE Combine($M(X,$ left−subtree(S),
$M(X,$ right−subtree(S),
OPof$S$);

Prim-M is the easily computed classification with respect to a simple primitive solid. The Combine operation is a nontrivial calculation that combines the subresults to produce a more complex classification. It is illustrated in two dimensions for line classification in Fig. 9.23. Having classified the line L against the polygon $P1$ and $P2$, the classifications can be combined to produce the classification for $P1 \cap P2$. Precise rules for combine may be written for (regularized) union, intersection, and set difference. An important point is that when a point is in the "on" set of S_1 and in the "on" set of S_2, the result of the combination depends on extra information. In Fig. 9.23, segments X and Y both result from this ON–ON case of combine, but segment X is OUT of the boundary of the intersection and Y is IN the intersection. The ambiguity must be resolved by keeping "neighborhood information" (local geometry) attached to point sets, and combining the neighborhoods along with the classifications. The technical problems surrounding combine can be solved, and SMC is basic in several solid geometric modeling systems [Boyse 1979; Voelcker et al. 1978; Brown et al. 1978].

Mass Properties

The analog of many two-dimensional geometric properties is to be found in "mass properties," which are defined by volume integrals over a solid. The four types of mass properties commonly of interest are:

Volume: $V = \int_S du$

Centroid: e.g. $GC_x = \dfrac{\int_S x\, du}{V}$

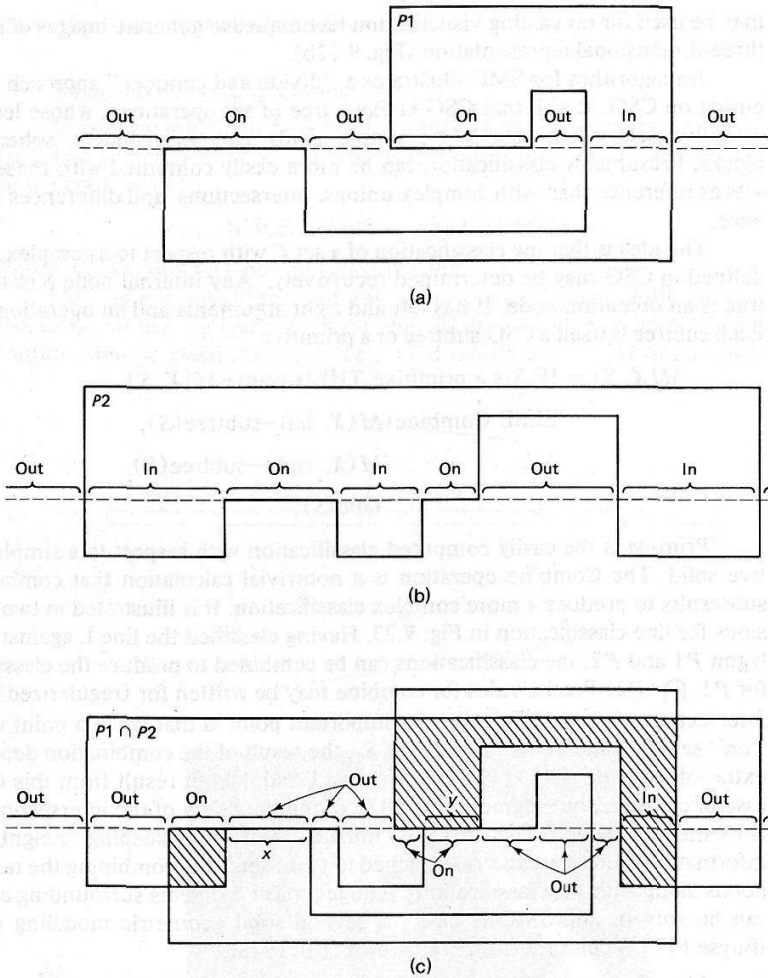


Fig. 9.23 Combining line-polygon classifications (a) and (b) must produce the classification (c).

Moment of Inertia: e.g. $I_{xx} = m \int_s (y^2 + z^2)\, du$ (9.17)

Product of Inertia: e.g. $P_{xy} = m \int_s xy\, du$

where m is a density measure, du the volume differential, and integrals are taken over the volume.

Measures such as these are not necessarily easy to compute from a given representation. The calculation of mass properties of solids from various representations is discussed in [Lee and Requicha 1980]. The approaches suggested by the representations are shown in Fig. 9.24.

One method is based on decomposing the solid into quasi-disjoint cells. An integral property of the cell decomposition is just the sum of the property for each of the cells. Hence if computing the property for the cells is easy, the calculation is easy for the whole volume. One is invited to decompose the body into simple cells, such as columns or cubes, as shown in Fig. 9.25. The resulting calculations, performed to reasonable error bounds on fairly complex volumes, take unacceptably long for the pure spatial occupancy enumeration, but are acceptable for the column and block decompositions. (The column decomposition corresponds to a ray casting approach.) The block decomposition method can be programmed using octtrees or kd-trees in a manner reminiscent of the Warnock hidden-line algorithm [Warnock 1969], in which the blocks are found automatically, and their size diminishes as increased resolution is needed in the solid. In calculating from a constructive solid geometry representation, the same divide-and-conquer strategy that is useful for SMC may be applied. Again, it recursively solves subproblems induced by the set operators (Fig. 9.26). The strategy is less appealing here since the number of subproblems can grow exponentially in the worst case.

In boundary representations, one can perhaps directly integrate over the boundary in a three-dimensional version of the polygon area calculation given in Chapter 8. This method is often impossible for curved surfaces, which, however, may be approximated by planar faces. An alternative is to use the divergence

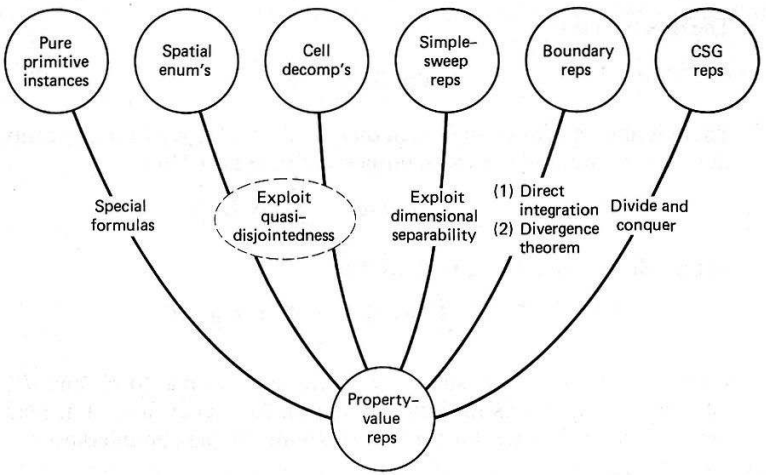


Fig. 9.24 "Natural" approaches to computing mass properties from several representations.

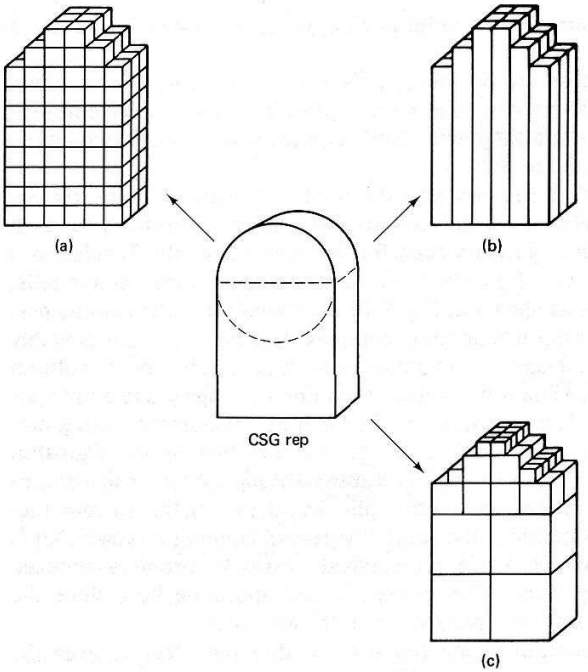


Fig. 9.25 Cell decompositions for mass properties.

theorem (Gauss's theorem). The *divergence* is a scalar quantity defined at any point in a vector field by writing the vector function as

$$\boldsymbol{G}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k}. \tag{9.18}$$

The divergence is

$$\text{div } \mathbf{G} = \frac{P}{x} + \frac{Q}{y} + \frac{R}{z} \tag{9.19}$$

There is always a function $\mathbf{G}$ such that div $\mathbf{G} = f(x, y, z)$ for any continuous function f (f computes the integral property of interest.) Thus

$$\int_S f \, dv = \int_S \text{div } \mathbf{G} \, dv \tag{9.20}$$

But the divergence theorem states that

$$\int_S \text{div } \mathbf{G} \, dv = \sum_i \int_{F_i} \mathbf{G}\mathbf{n}_i \, dF_i \tag{9.21}$$

where F_i is a face of the solid S, $\mathbf{n}_i$ is the unit normal to F_i, and dF_i the surface differential. Again this formula works well for planar faces, but may require approximation techniques for curved faces with complex boundaries.

Boundary Evaluation

The calculation of a face-based surface (boundary) representation from a

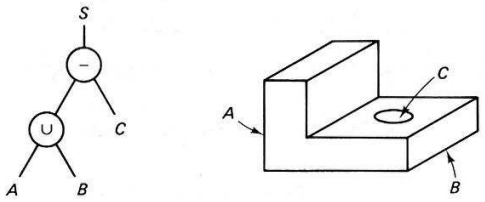
• Divide and conquer

Reduction formula

$$\int_{A \cup B} = \int_A + \int_B - \int_{A \cap B}$$

$$\int_{A-B} = \int_A - \int_{A \cap B}$$

Example



$$I_S = I_A + I_B - I_{A \cap B} - \underbrace{I_{A \cap C}}_{\emptyset} - I_{B \cap C} + \underbrace{I_{A \cap B \cap C}}_{\emptyset}$$

Fig. 9.26 Recursive problem decomposition for mass property calculation.

CSG representation is called *boundary evaluation.* It is an example of *representation conversion.* Both the CSG and boundary are usually unambiguous representations of a volume; a CSG expression (a solid) has just one boundary, but a boundary (representing a solid) usually has many CSG expressions. Since a solid may be put together from primitives in many ways, the mapping back from boundary to CSG is not usually attempted (but see [Markovsky and Wesley 1980, Wesley and Markovsky 1981]).

One style of boundary evaluation is based on the following observations [Voelcker and Requicha 1980; Boyse 1979].

- Boundaries of composite objects may be computed from certain set-theoretic formulae. For (regularized) intersection of two objects S and T, the formula is

$$b(S \cap^* T) = (bS \cap^* iT) \cup^* (iS \cap^* bT) \cup^* (bS \cap^* bT \cap^* ki(S \cap^* T)) \tag{9.22}$$

  where $\cap^*$ and $\cup^*$ are regularized intersection and union: b, i, and k are the boundary, interior, and closure operators. (Recall that ki is r, the regularization operator).
- Faces of composite objects can arise only from faces of primitives.
- Faces are either bounded by edges or are self-closing (as is the sphere).

These observations and the existence of the classification operation motivate the grand strategy that follows (ignoring several important details and concentrating on the core of the algorithm.)

1. Find all possible ("tentative") edges for each face of each primitive in the composite.
2. Classify each tentative edge with respect to the composite solid.
3. The ON portions of those edges must be enough to define the boundary.

Given the grand strategy, several algorithms of varying sophistication are possible, depending on what edges should be classified (how to generate tentative edges), in what order they should be classified, and how classification is done. The following algorithm is very simple (but very inefficient); useful algorithms are rather more complex.

Algorithm 9.1: CSG to Boundary Conversion (top-level control loop)

Input: Solid defined by CSG expression of regularized set operations applied to primitive solids.

Output: "Bfaces" in the object boundary. Bfaces are represented by their bounding edges. They may have little relation to the "intuitive faces" of the boundary; they may overlap each other, and a Bface may be disconnected (specify more than one region). Edges may appear many times. The Bface-oriented boundary may be processed to remove repetition and merge Bfaces into more intuitively appealing boundary faces.

BEGIN

Form a list PFaces of all ("intuitive") faces of primitive solids involved in the CSG expression, and an initially empty list BFaces to hold the output faces.

For every PFace $F1$ in PFaces:

Create a B-Face called ThisBFace, initially with no edges in it.

For every PFace $F2$ after $F1$ in the PFaces list (this generates all distinct pairs of PFaces just once):

Intersect $F1$ and $F2$ to get TEdges, a set of edges tentatively on the boundary of the solid. If $F1$ and $F2$ do not intersect or intersect only in a point, TEdges is empty. If they intersect in a line, TEdges is the single resulting edge. If they intersect in a two-dimensional region, TEdges contains the bounding edges of the intersection region.

Classify every TEdge in TEdges with respect to the whole solid (the CSG expression). Put TEdges that are ON the solid boundary into ThisBFace.

If ThisBFace is not empty, put it into BFaces.

End Inner Loop

End Outer Loop

END

Algorithms such as this involve many technical issues, such as merging coplanar faces, stitching edges together into faces, regularization of faces, removing multiple versions of edges. Boundary evaluation is inherently rather complex, and depends on such things as the definition and representation of faces as well as the geometric utilities taken as basic [Voelcker and Requicha 1981]. Boundary evaluation is an example of exact conversion between significantly different representations. Such conversions are useful, since no single representation seems convenient for all geometric calculations.

9.5 UNDERSTANDING LINE DRAWINGS

"Engineering" line drawings have been (and to a great extent are still) the main medium of communication between human beings about quantitative aspects of three-dimensional objects. The line drawings of this section are only those which are meant to represent a simple domain of polyhedral or simply curved objects. Interpretation of "naturalistic" drawings (such as a sketchmap [Mackworth 1977]) is another matter altogether.

Line drawings (even in a restricted domain) are often ambiguous; interpreting them sometimes takes knowledge of everyday physics, and can require training. Such informed interpretation means that even drawings that are strictly nonsense can be understood and interpreted as they were meant. Missing lines in drawings of polyhedra are often so easy to supply as to pass unnoticed, or be "automatically supplied" by our model-driven perception.

Generalizing the line drawing to three dimensions as a list of lines or points is not enough to make an unambiguous representation, as is shown by Fig. 9.27,

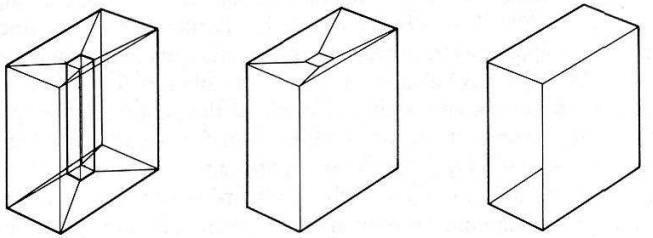


Fig. 9.27 An ambiguous (wireframe) representations of a solid with two of three possible interpretations.

which illustrates that a set of vertices or edges can define many different solids. (It is possible, however, to determine algorithmically all possible polyhedral boundaries described by a three-dimensional wireframe [Markowsky and Wesley 1980].). A line drawing nevertheless does convey three-dimensional information. For any set of N projection specifications (e.g., viewpoint and camera transform), a wire-frame object may be constructed that is ambiguous given the N projections. However, for a given object, there is a maximum number of projections that can determine the object unambiguously. The number depends on the number of edges in the object [Shapira 1974]. Reconstruction of all solids represented by projections is possible [Wesley and Markowsky 1981].

Line drawings were a natural early target for computer vision for the following reasons:

1. They are related closely to surface features of polyhedral scenes.
2. They may be represented exactly; the noise and incomplete visual processing that may have affected the "line drawing extraction" can be modelled at will or completely eliminated.
3. They present an interpretation problem that is significant but seems approachable.

The understanding of simple engineering (3-view) drawings was the first stage in a versatile robot assembly system [Ejiri et al. 1971]. This application underlined the fact that heuristics and conventions are indispensible in engineering drawing understanding. This section deals with the problem of "understanding" a single-view line drawing representation of scenes containing polyhedral and simple curved objects like those in Fig. 9.28.

Our exposition follows a historical path, to show how early heuristic programs in the middle 1960s evolved into more theoretical insights in the early 1970s.

The first real computer vision program with representations of a three-dimensional domain appeared around 1963 [Roberts 1965]. This system, ambitious even by today's standards, was to accept a digitized image of a polyhedral scene and produce a line drawing of the scene as it would appear when viewed from any requested viewpoint. This work addressed basic issues of imaging geometry, feature finding, object representation, matching, and computer graphics.

Since then, several systems have appeared for accomplishing either the same or similar results [Falk 1972; Shirai 1975; Turner 1974]. The line drawings of this section can appear as intermediate representations in a working polyhedral vision system, but they have also been studied in isolation. This topic took on a life of its own and provides a very pretty example of the general idea of going to the three-dimensional world of physics and geometry to understand the appearance of a two-dimensional image. The later results can be used to understand more clearly the successes and failures of early polyhedral vision systems. One form of understanding (line labelling) provided one of the first and most convincing demonstrations of parallel constraint propagation as a control structure for a computer vision process.

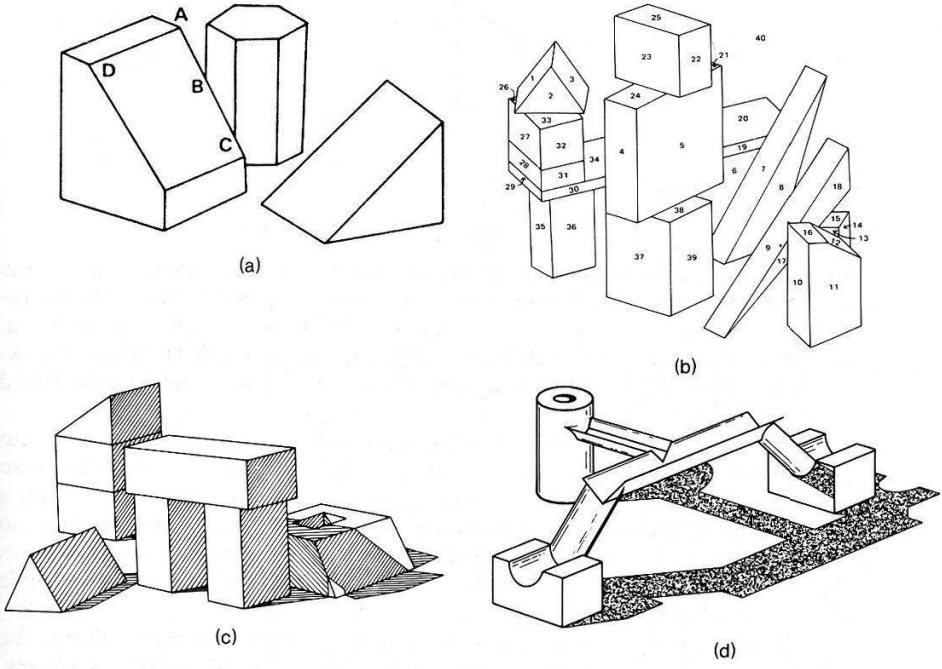


Fig. 9.28 Several typical line drawing scenes for computer understanding.

9.5.1 Matching Line Drawings to Three-dimensional Primitives

Roberts desires to interpret a line drawing such as Fig. 9.28a in terms of a small set of three polyhedral primitives, shown in Fig. 9.29. A simple polyhedron in a scene is regarded as an instance of a transformed primitive, where a transform may involve scaling along the three coordinate axes, translation, and rotation. Compound polyhedra, such as Fig. 9.28a, are regarded as simple polyhedra "glued together." (A cell-decomposition representation is thus used for compound polyhedra.) The program is first to derive from the scene the identity of the primitive objects used to construct it (including details of the construction of compound polyhedra). Next, it is to discover the transformations applied to the primitives to obtain the particular incarnations making up the scene. Finally, to demonstrate its understanding, it should be able to construct a line drawing of the scene from any viewpoint, using its derived description.

To understand a part of the scene, the program first decides which primitive it comes from, and then derives the transformation the primitive underwent to appear as it does in the scene. Identifying primitives is done by matching "topological" features of the line drawing (configurations of faces, lines, and vertices) with those of the model primitives; matching features induce a match between scene and model points. At least four noncoplanar matching points are needed to derive

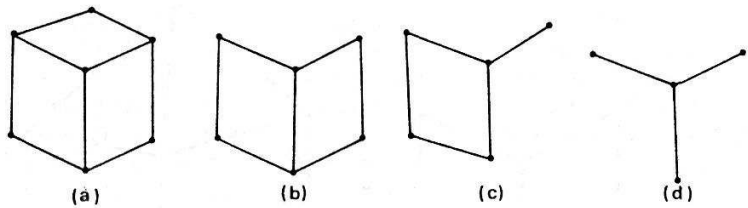


Fig. 9.31 Topological match structures of Roberts.

The idea once again is to accumulate local evidence from the scene, and then to group polygons on the basis of this evidence. The evidence takes the form of "links" which link two regions if they may belong to the same body; links are planted around vertices, which are classified into types, each type always planting the same links (Fig. 9.32). No links are made with the background region.

Scenes are interpreted by grouping according to regions/links, using fairly complex rules, including "inhibitory links" that preclude two neighboring regions from being in the same body.

The final form of the program performs reasonably well on scenes without accidents of visual alignment, but it is a maze of special cases and exceptions, and seems to shed little light on what is going on in known polyhedral line-drawing perception. One might well ask where the links come from; no justification of why they are correct is given. Further ([Mackworth 1973]), Guzman can accept as one body the two regions in Fig. 9.33a. Finally, one feels a little dissatisfied with a scheme that just answers "one body" to a scene like Fig. 9.33b, instead of answering "pyramid on cube" or "two wedges," for example.

Guzman's method is correct for a world of convex isolated trihedral polyhedra: it is extended by ad hoc adjustments based on various potentially conflicting items of evidence from the line drawing. Ultimately it performs adequately with a much increased range of scenes, albeit not very elegantly. Further progress in the line drawing domain came about when attention was directed at the three-dimensional causes of the different vertex types.

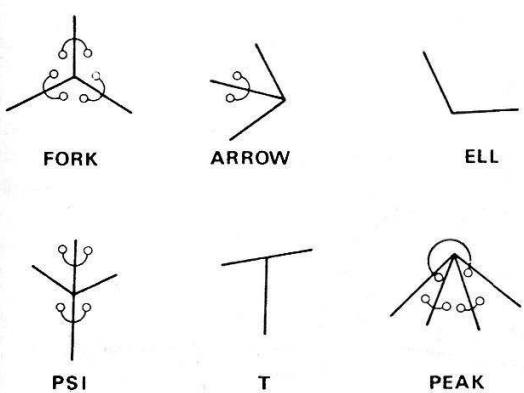


Fig. 9.32 Links around vertices.

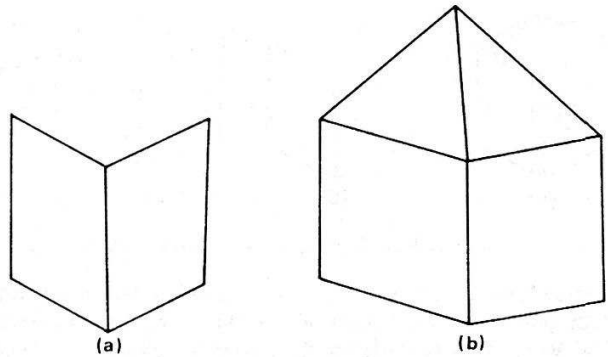


Fig. 9.33 (a) Non-polyhedral scene. (b) Two wedges or a pyramid on cube.

9.5.3 Labeling Lines

Huffman and Clowes independently concerned themselves with scenes similar to Guzman's, not excluding non-simply connected polyhedra, but excluding accidents of alignment [Huffman 1971; Clowes 1971]. They desired to say more about the scene than just which regions arose from single bodies; they wanted to ascribe interpretations to the lines. Figure 9.34 shows a cube resting on the floor; lines labeled with a + are caused by a convex edge, those labeled with a − are caused by a concave edge, and those labeled with a > are caused by matter occluding a surface behind it. The occluding matter is to the right of the line looking in the direction of the >, the occluded surface is to the left. If the cube were floating, one would label the lowest lines with < instead of with −. The shadow line labels (arrows) were not used by Huffman.

A systematic investigation can find the types of lines possibly seen around a trihedral corner; such corners can be classified by how many octants of space are filled by matter around them (one for the corner of a cube, seven for the inside corner of a room, etc.). By considering all possible trihedral corners as seen from

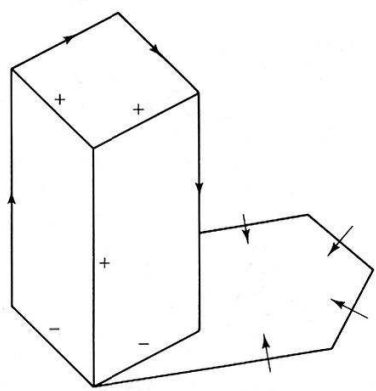


Fig. 9.34 A block resting on its bottom surface.

all possible viewpoints, Huffman and Clowes found that without occlusion, just four vertex types and only a few of the possible labelings of lines meeting at a vertex can occur. Figure 9.35 shows views of one- and three-octant corners which give rise to all possible vertices for these corner types. The vertices appear in the first two rows of Table 9.1, which is a catalogue of all possible vertices, including those arising from occlusion, in this restricted world of trihedral polyhedra. It is easy to imagine extending the catalog to include vertices for other corner types.

It is important to note that there are four possible labels for each line ($+ - > <$), and thus $4^3 = 64$ possible labels for the fork, arrow, and T and 16 possible labels for the ell. In the catalog, however, only 3/64, 3/64, 4/64, and 6/16, respectively, of the possible labels actually occur. Thus only a small fraction of possible labels can occur in a scene.

The main observation that lets line-labeling analysis work is the coherence rule: In a real polyhedral scene, *no line may change its interpretation (label) between vertices.* For example, what is wrong with scenes like Fig. 9.36 is that they cannot be coherently labeled; lines change their interpretation within the impossible object. Perhaps the lines in drawings of real scenes can be interpreted quickly because the small percentage of meaningful labelings interacts with the coherence rule to reduce drastically the number of explanations for the scene.

How does line labeling relate to Guzman? A labeled-line description clearly indicates the grouping of regions into bodies, and also rejects scenes like Fig. 9.33a, which cannot be coherently labeled with labels from the catalog. The origin of Guzman's links can be explained this way: consider again the world of convex polyhedra; the only labels from the catalog that are possible are shown in Fig. 9.37a. Further, it is clear that a convex edge has two faces of the same body on either side of it, and an occluding edge has faces from two different bodies on either side of it. A convex label means the regions on either side of it should be linked; this is Guzman's link-planting rule (Fig. 9.37b). The inhibition rules are a further corollary of the labels; they are to suppress links across an edge if evidence that it

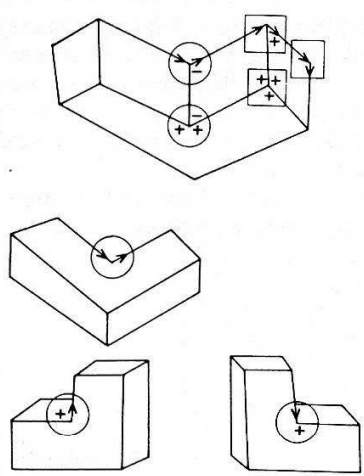
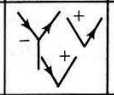
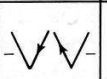


Fig. 9.35 Different views of various corner types.

Table 9.1
VERTEX CATALOGUE

Visible surfaces / Octants filed	3	2	1	0
1				—
3				—
5			—	—
7		—	—	—
Occlusion				

must be occluding is supplied by the vertex at its other end (Fig. 9.37c). When vertices at both ends of a line agree that the line is convex, Guzman would have planted two links; this is in fact the strongest evidence that the regions are part of the same body. If just one vertex gives evidence that the edge has a link, a decision based on heuristics is made; the coherence rule is being used implicitly by Guzman. The same physical and geometric reality is driving both his scheme and that of Huffman.

The labeling scheme explained here still has problems: syntactically nonsensical scenes are coherently labeled (Fig. 9.38a); scenes are given geometrically impossible labels (Fig. 9.38b); and scenes that cannot arise from polyhedra are easily labelled (Fig. 9.38c). It is very hard to see how a labeling scheme can detect the illegality of scenes like (Fig. 9.38c); the problem is not that the edges are incorrectly labeled, but that the faces cannot be planar.

Concern with this last-mentioned problem led to a program (see the next section) that can obtain information about a polyhedral scene equivalent to labeling it,

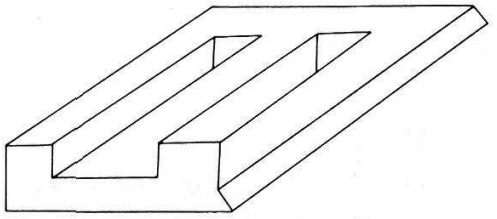


Fig. 9.36 An impossible object.

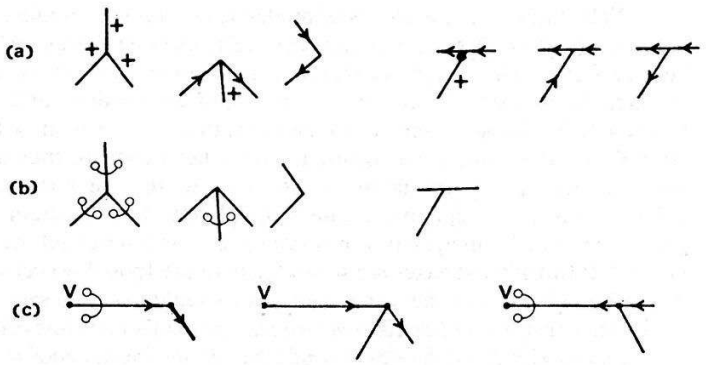


Fig. 9.37 The relation of links to labels. (a) Line labels. (b) Link planting vertices. (c) Inhibitory links.

and also can reject non-polyhedra as impossible. There has also been an exciting denouement to the line-labeling idea [Waltz 1975; Turner 1974].

Waltz extends the line labels to include shadows, three illumination codes for each face on the side of an edge, and the separability of bodies in the scene at cracks and concave edges; this brings the number of line labels possible up to just below 100. He also extends the possible vertex types, so that many vertices of four lines occur. He can deal with scenes such as the one shown in Fig. 9.28c.

The combinatorial consequence of these extensions is clear; the possible vertex labelings multiply enormously. The first interesting thing Waltz discovered was that despite the combinatorics, as more information is coded into the lines, the smaller becomes the percentage of geometrically meaningful labels for a vertex. In his final version, only approximately 0.03 percent of the possible arrow labels can occur, and for some vertices the percentage is approximately 0.000001.

The second interesting thing Waltz did was to use a constraint-propagating labeling algorithm which very quickly eliminates labels for a vertex that is impossible given the neighboring vertices and the coherence rule, which places *constraints* on labelings. The small number of meaningful labels for a vertex imposes severe constraints on the labeling of neighboring vertices. By the coherence rule, the constraints may be passed around the scene from each vertex to its neighbors; eliminating a label for a vertex may render neighboring labels illegal as well, and so on recursively.

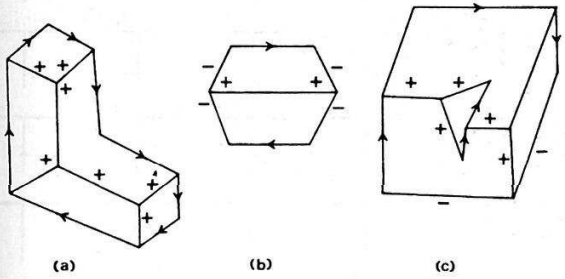


Fig. 9.38 Nonsense labelings and nonpolyhedra.

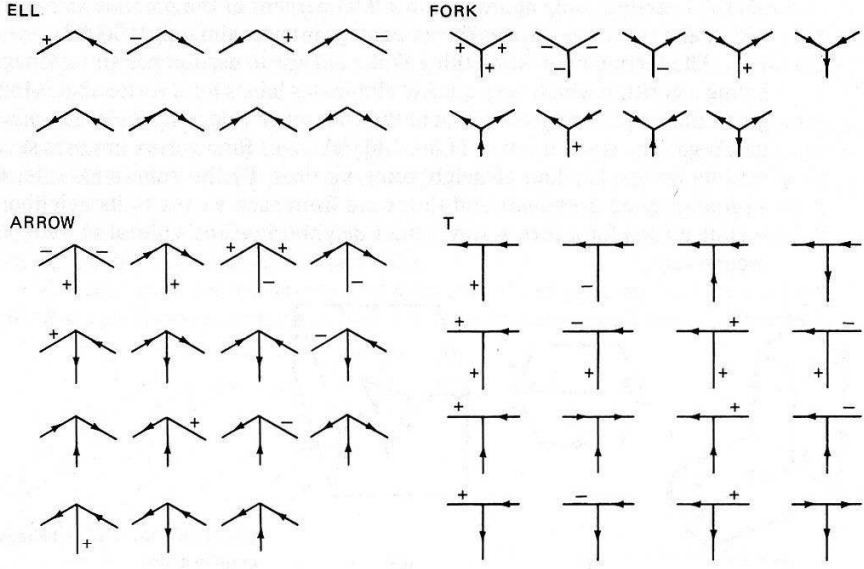
Waltz found that for scenes of moderate complexity, eliminating all impossible labelings left only one, the correct one. The labeling process, which might have been expected to involve much search, usually involved none. This constraint propagation is an example of parallel constraint satisfaction, and is discussed in Chapter 12 in a broader context. In the event that a vertex is left with several labels after all junction coherence constraints have been applied, they all participate in *some* legal labeling. At this point one can resort to tree search to find the explicit labelings, or one can apply more constraints. Many such constraints, heuristic and geometric, may be imagined. For instance, a constraint could involve color edge profiles. If two aligned edges are separated by some (possibly occluding) structure, but still divide faces of the same color, they should have the same label. Another important constraint concerns how face planarity constrains line orientations.

Scenes with missing lines may be labeled; one merely adds to the legal vertex catalog the vertices that result if lines are missing from legal vertices. This idea has the drawbacks of increasing the vertex catalog and widening the notion of consistency, but can be useful.

Another extension to line labeling is that of [Kanade 1978]. This extension considers not only solid polyhedra but objects (including nonclosed "shells") made up of planar faces. This extension has been called *origami world* after the art of making objects from folded (mostly planar) paper. An example from origami world is the box in Fig. 9.39a. A quick check shows that this cannot be labeled with the Huffman-Clowes label set. It can be labeled using the origami world label set (Table 9.2) and its interpretation is shown in Fig. 9.39b.

Table 9.2

EXPANDED JUNCTION TABLE



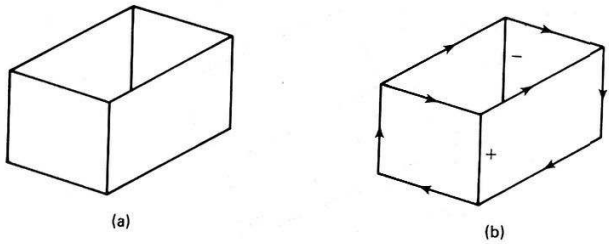


Fig. 9.39 (a) Box. (b) Labeled edges according to origami world label set.

The vertex labels may be extended to include scenes with cylinders, cones, spheres, tori, and other simple curves. In expanded domains the notion of "legal line drawing" becomes very imprecise. In any event the number of vertex types and labels grow explosively, and the coherence rule must be modified to cope with the fact that lines can change their interpretation between vertices and can tail off into nothing, and that one region can attain all three of Waltz's illumination types [Turner 1974, Chakravarty 1979]. The domain is of scenes such as appear in Fig. 9.28d.

9.5.4 Reasoning About Planes

The deficiencies in the scene line-labeling algorithms prompted a consideration of the geometrical foundations of the junction labels [Mackworth 1973, Sugihara 1981]. This work seeks to answer the same sorts of questions as do labeling programs, but also to take account of objects that cannot possibly be planar polyhedra, such as those of Fig. 9.40. Neither approach uses a catalog of junction labels, but relies instead on ideas of geometric coherence. The basis is a plane-oriented formulation rather than a line-oriented one.

Gradient Space

Mackworth's program relies heavily on the relation of polyhedral surface gradients to the lines in the image (recall section 3.5.2). Image information from orthographic projections of planar polyhedral scenes may be related to gradient information in a useful way. An image line L is the projection of a three-space line M arising from the intersection of two faces lying in distinct planes Π_1 and Π_2 of gradients (p_1, q_1) and (p_2, q_2). With the (p, q) coordinate system superimposed on the image (x, y) coordinate system, there is the following constraint. The orientation of L constrains the gradients of Π_1 and Π_2; specifically, the line L is perpendicular to the line G between (p_1, q_1) and (p_2, q_2) (Fig. 9.41).

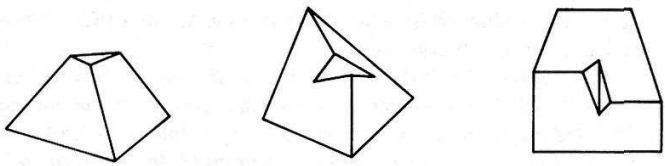


Fig. 9.40 Labelable but not planar polyhedra.

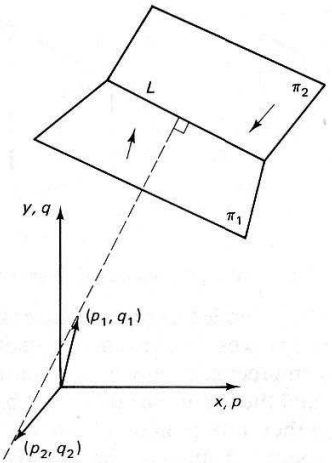


Fig. 9.41 Gradient space constraint.

The result is easily shown. With orthographic projection, the origin may be moved of the image plane to be in L without loss of generality. Then L is defined by its direction vector $(\lambda, \mu) = (\cos\theta, \sin\theta)$. The three-space point on Π_1 corresponding to $(0,0)$ may be expressed as $(0,0,k_1)$, and at (λ, μ) the corresponding point is $(\lambda, \mu, \lambda p_1 + \mu q_1 + k_1)$. Thus moving along M (which is in Π_1) from $(x, y) = (0,0)$ to $(x, y) - (\lambda, \mu)$ moves along $-z$ by $\lambda p_1 + \mu q_1$. The coordinates of a unit vector on L can then be expressed as $(\lambda, \mu, \lambda p_1 + \mu q_1)$. But L is also in Π_2, and this argument may be repeated for Π_2, using p_2 and q_2. Thus

$$\lambda p_1 + \mu q_1 = \lambda p_2 + \mu q_2 \tag{9.23}$$

or

$$(\lambda, \mu) \cdot (p_2 - p_1,\ q_2 - q_1) = 0 \tag{9.24}$$

Equation (9.24) is a dot product set equal to zero, showing that its two vector operands are orthogonal, which was to be shown.

Every picture line results from the intersection of two planes, and so it has a line associated with it in gradient space which is perpendicular to it. Furthermore, if the gradients of the surfaces are on the same side of the picture line as their surfaces, the edge was convex; if the gradients are on opposite sides of the line from their causing surfaces, the edge was concave (Fig. 9.42). For every junction in the image there are just two ways the gradients can be arranged to satisfy the perpendicularity requirement (Fig. 9.43). In the first, all edges are convex, in the second, concave. Switching interpretations from one to the other by negating gradients is the psychological "Necker reversal."

Notice that if an image junction is a three-space polyhedral vertex, each edge of the vertex is the intersection of two face planes. If the corresponding gradients are connected, a "dual" (p, q) space representation of the (x, y) space junction is formed. The connected (p, q) gradient points form a polygon whose edges are perpendicular to the junction lines in (x, y) space. The polygon is larger if the three-

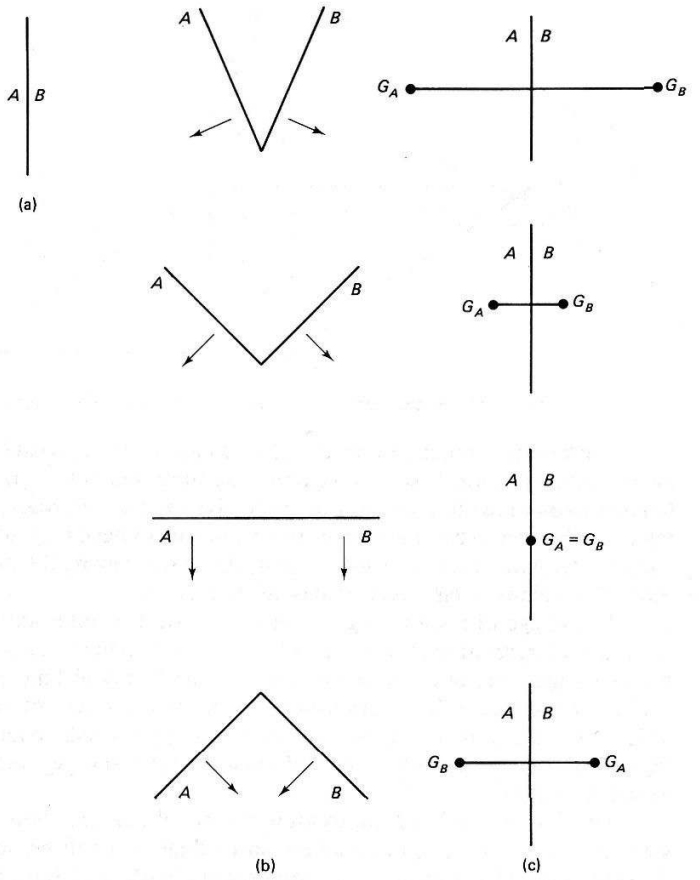


Fig. 9.42 Relation of gradients, image and world structures. (a) Image. (b) World. (c) Gradients.

dimensional corner is sharper, and shrinks toward the junction point as the corner gets blunter.

Interpreting Drawings

It is possible to use these geometric results to interpret the lines in orthogonally projected polyhedral scenes as being "connect" (i.e., as being between two connected faces) or occluding. It can also be determined if connect edges are convex or concave, and for occluding edges which surface is in front. Hidden parts of the scene may sometimes be reconstructed. The orientation of each surface and edge in the scene may be found. Thus a program can determine that input such as Fig. 9.40 is not a planar-faced polyhedron [Mackworth 1973]. Sugihara's work generalizes Mackworth's; it does not use gradient space and does not rely on orthographic projection.

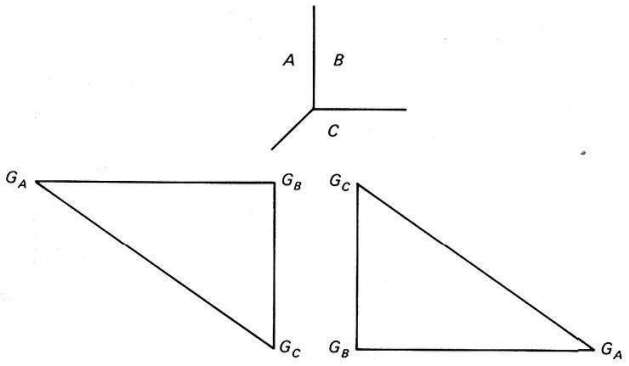


Fig. 9.43 A scene junction and two resulting triangles in gradient space.

Mackworth's procedure to establish connect edges produces the most connected interpretation first (a nonconnected interpretation is just a collection of floating faces which line up by accident to give the line drawing). The background region is the first to be interpreted; that is, means to have its gradient fixed in gradient space. After a region is interpreted, the region having the most lines in common with regions so far interpreted is interpreted next.

The image of a scene is given in Fig. 9.44a; it is interpreted as follows. No coherent interpretation is possible with five or four connect edges. Trying for three connect edges, the program interprets A by arbitrarily picking a gradient for the surface A represents (the background). It picks the origin of gradient space. In order to be able to reason about lines in the image, it needs to have an interpreted region on either side of the line, so it must interpret another region. It picks B (C would be as good).

The lines bounding B are examined to see if they are connect. Line 1 is considered. If it is connect, the gradient space dual of it will be perpendicular to it through the gradient space point representing surface A (i.e., the origin). Now another arbitrary choice: The gradient corresponding to surface B is placed at unit distance from the origin, thus "imagining" the second gradient in a row. From now on, the gradients are more strongly located. The arbitrary scaling and point of origin imposed by these first two choices can be changed later if that is important.

In gradient space, the situation is now shown in Fig. 9.44b. Now consider line 2; to establish it as a connect edge, $\boldsymbol{G}_B = (p_B, 1_B)$ (the gradient space point corresponding to the surface B) must lie on a line perpendicular to 2 through $\boldsymbol{G}_A$ (Fig. 9.44c). This cannot happen; the situation with 1 and 2 both connect is incoherent. Thus, with a line 1 connect edge, 2 must be occluding. This sort of incoherency result was what kept the program from finding four or five edges connect. Further interpretation involves assigning gradients and vertices into the developing diagram in a noncontradictory, maximally connected manner (Fig. 9.44d).

The next part of the program determines convexity or concavity of the lines. The final part of the program looks at occlusion. It also suggests hidden surfaces

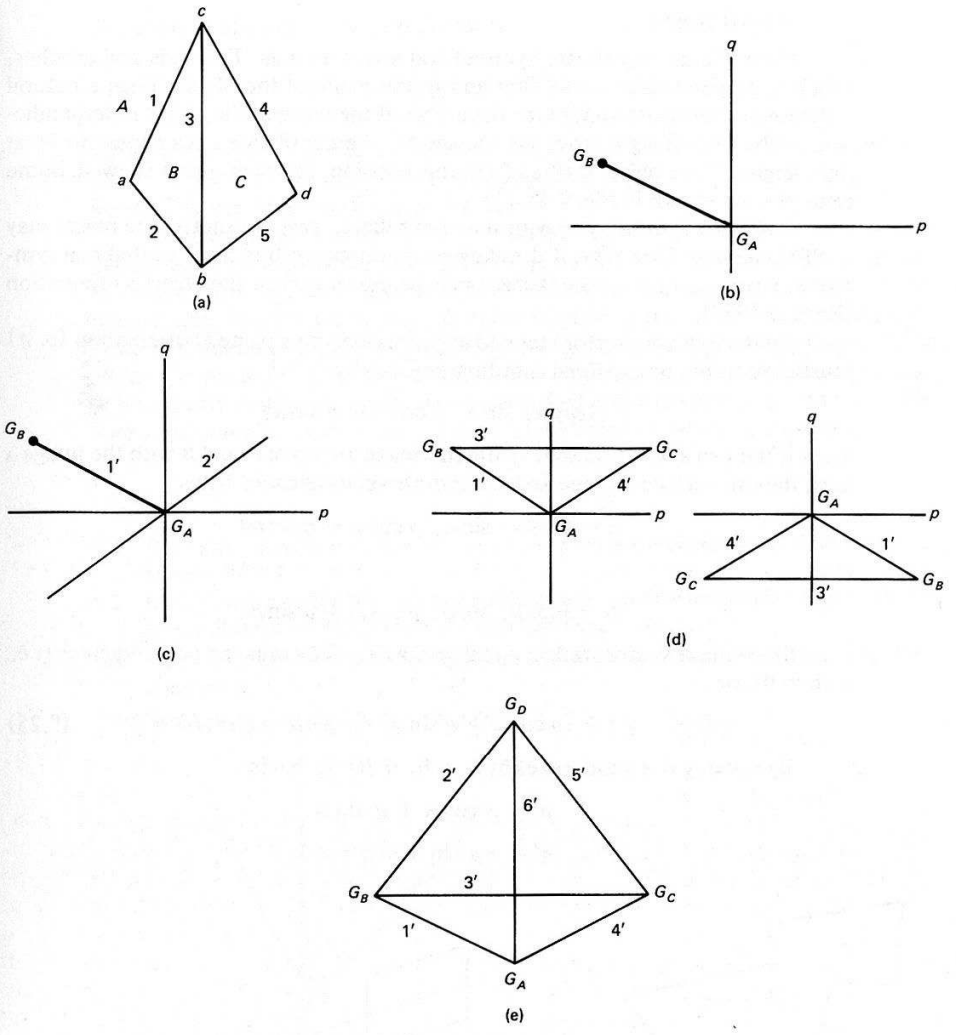


Fig. 9.44 (a) Polyhedral scene considered by Mackworth. (b) Partial interpretation. (c) Continued interpretation. (d) Occluding and connect interpretations. (e) Final interpretation.

and thus hidden lines that are consistent with the interpretation (Fig. 9.44e). This figure in gradient space resembles a tetrahedron, as well it might; it is formed in the same way as the graph-theoretic dual (point per face, edge per edge, face per point) which defines dual graphs and dual polyhedra; the tetrahedron is self-dual. The arbitrary choices of gradient reflect degrees of freedom in the drawing that are also identified by Sugihara.

Skewed Symmetry

Many planar objects are symmetrical about an axis. This axis and another, which is perpendicular to the first and in the plane of the object, form a natural orthogonal coordinate system for the object. If the plane of the object is perpendicular to the line of sight from the viewpoint, the coordinate axes appear to be at right angles. If the object is tilted from this position, the axes appear skewed. Some examples are shown in Fig. 9.45.

A skewed symmetry may or may not reflect a real symmetry; the object may itself be skewed. However, if the skewed symmetry results from a tilted real symmetry, a constraint in gradient space may be developed for the object's orientation [Kanade 1979].

An imaged unit vector inclined at α inscribed on a plane at orientation (p, q) must have three-dimensional coordinates given by

$$(\cos\alpha,\ \sin\alpha,\ p\cos\alpha + q\sin\alpha)$$

Thus if the two axes of skewed symmetry make angles of α and β with the image x axis, the two vectors in three-space a and b must have coordinates

$$\mathbf{a} = (\cos\alpha,\ \sin\alpha,\ p\cos\alpha + q\sin\alpha)$$

and

$$\mathbf{b} = (\cos\beta,\ \sin\beta,\ p\cos\beta + q\sin\beta)$$

Since these vectors reflect a real symmetry, they must be perpendicular (i.e., $\mathbf{a}\cdot\mathbf{b} = 0$), or

$$\cos(\alpha - \beta) + (p\cos\alpha + q\sin\alpha)\ (p\cos\beta + q\sin\beta) = 0 \qquad (9.25)$$

By rotating the p and q axes by $\lambda = (\alpha + \beta)/2$, that is

$$p' = p\ \cos\lambda + q\ \sin\lambda$$

$$q' = -p\ \sin\lambda + q\ \cos\lambda$$

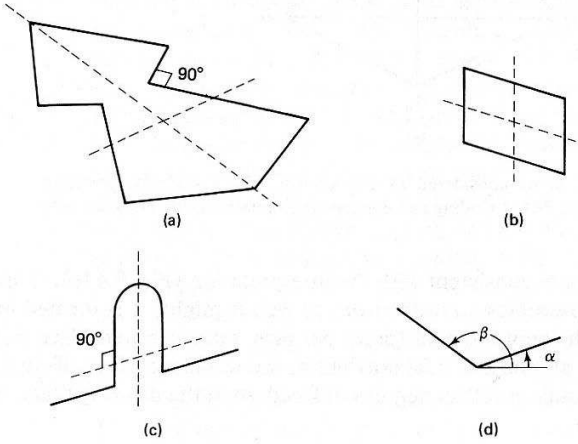


Fig. 9.45 Skewed symmetries. (a,b,c) are examples. (d) Each skewed symmetry defines two axes.

Equation (9.25) can be put into the form

$$p'^2 \cos^2\left[\frac{\gamma}{2}\right] - q'^2 \sin^2\left[\frac{\gamma}{2}\right] = -\cos(\gamma)$$

where $\gamma = \alpha - \beta$. Thus the gradient of the object must lie on a hyperbola with axis tilted λ from the x axis, and with asymptotes perpendicular to the directions of α and β. This constraint is shown in Fig. 9.46.

To show how skewed symmetry can be exploited to interpret objects with planar faces, reconsider the example of Fig. 9.43. In that example the three convex edges constrained the gradients of the corresponding faces to be at the vertices of a triangle, but the size or position of the triangle in gradient space was unknown. However, skewed symmetry applied to each face introduces three hyperbola upon which the gradients must lie. The only way that both the skewed symmetry constraint and triangle constraint can be satisfied simultaneously is shown in Fig. 9.47—the combined constraints have uniquely determined the face orientations.

EXERCISES

9.1 Derive an expression for the volume of an object represented by spherical harmonics of order $M = 1$.

9.2 Derive an expression for the perpendicular to the surface of an object represented by spherical harmonics in terms of the appropriate derivatives.

9.3 Derive an expression for the angle centroid of each of the spherical harmonic functions for $M \leqslant 2$.

9.4 Label the lines in the objects of Fig. 9.48.

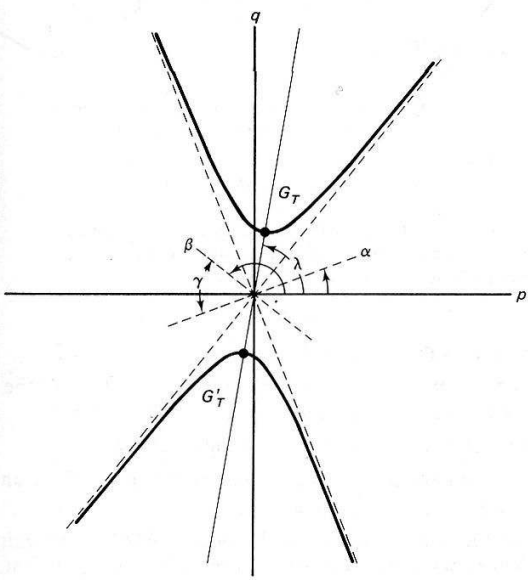


Fig. 9.46 Skewed symmetry constraint in gradient space.

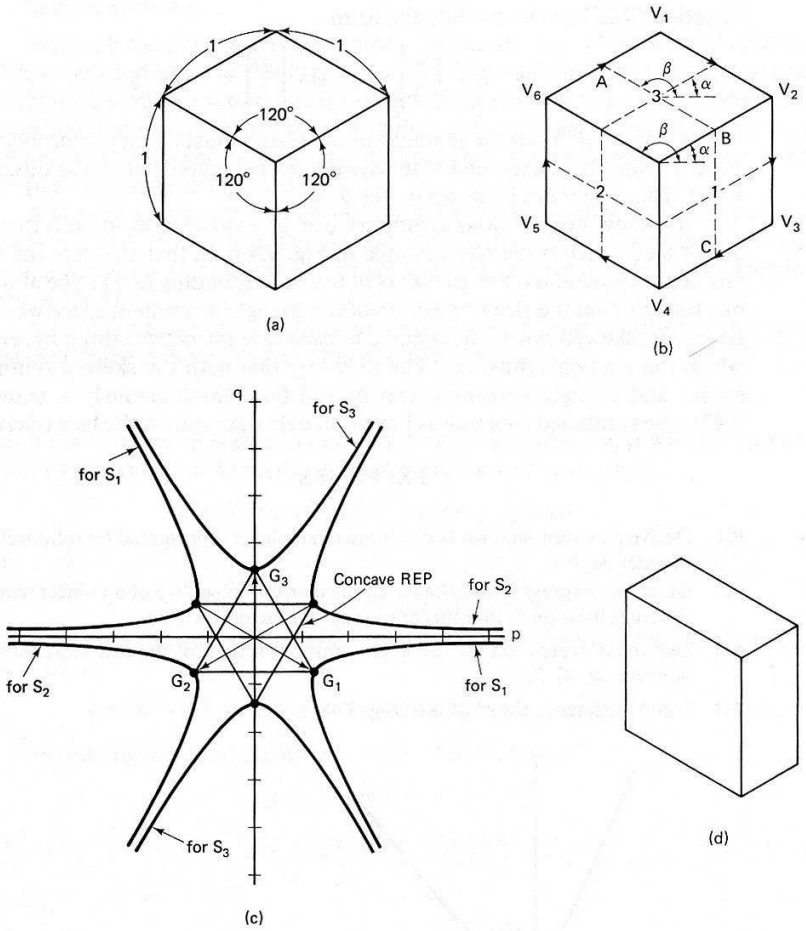


Fig. 9.47 Using skewed symmetry to orient the faces of a cube. (a) The cube. (b) Skewed symmetries. (c) skewed symmetries and junction constraint plotted in gradient space. (d) another possible object obeying the constraints.

9.5 Give two sets of CSG primitives with same domain.

9.6 Show that the dual of the plane of interpretation for a line and the duals of the two planes that meet in the edge causing the line are all on the dual of the edge.

9.7 Prove (Section 9.3.1) that in the Frenet frame ξ' is perpendicular to ξ.

9.8 Write the precise rules for combining classification results for $\cup^*$, $\cap^*$, and $-$ operations.

9.9 Find two interpretations of the tetrahedron of Fig. 9.44a that differ in convexity or concavity of lines. (Hint: The concave interpretation has an accident of alignment.)

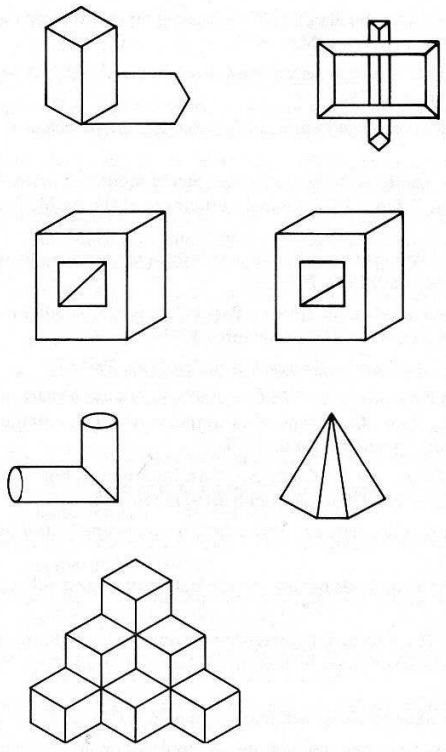


Fig. 9.48 Objects for labeling.

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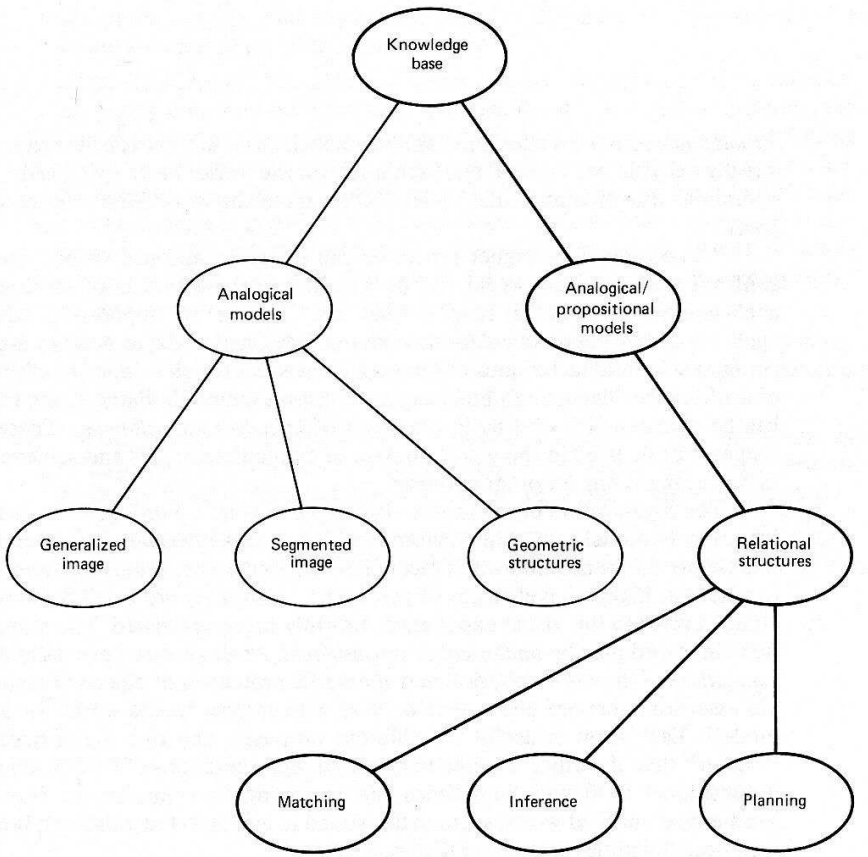
RELATIONAL STRUCTURES IV



Visual understanding relates input and its implicit structure to explicit structure that already exists in our internal representations of the world. More specifically, vision operations must maintain and update *beliefs* about the world, and achieve specific *goals.*

To consider how higher processes can influence and use vision, one must confront the nonvisual world and powers of reasoning that have more general applicability. The world models that are capable of supporting advanced application-dependent calculations about objects in the visual domain are quite complex. General techniques of *knowledge representation* developed in other fields of artificial intelligence can be brought to bear on them. Similarly, much research has been invested in the basic processes of *inference* and *planning.* These techniques may be used in the visual domain to manipulate beliefs and achieve goals, as well as reasoning for other purposes.

The organization of a complex visual system (Fig. 1.5 or Fig. 10.1), is a loose hierarchy of models of world phenomena. The *relational models* that concern us in this chapter are removed from direct perceptual experience—they are used mainly for the last, highest-level stages of perception. Also, they are used for knowledge attained prior to the visual experience currently being processed. The representations involved may be *analogical* or *propositional.* Analogical representations allow *simulations* of important physical and geometric properties of objects. Propositions are assertions that are either true or false with respect to the world (or a world model). Each form is useful for different purposes, and one is not necessarily "higher" than the other. The techniques and representations of Part IV are mainly propositional in flavor. Sometimes the reasoning they implement (say about geometrical entities) would seem better suited to analogical calculations; however, technical difficulties can render that impossible.

Part IV is concerned with techniques for making the "motivation" and "world view" of a vision system explicit and available. Such explicit models would

be interesting from a scientific standpoint even if they were not directly useful. But explicitly available models are decidedly useful. They are useful to the system designer who desires to reconfigure or extend a system. They are useful to the system itself, which can use them to reason about its own actions, flexibly control its own resources in accordance with higher goals, dynamically change its goals, recover from mistakes, and so forth.

We organize the major topics of Part IV as follows.

1. Knowledge representation (Chapter 10). *Semantic nets* are an important technique for structuring complex knowledge, and can be used as a knowledge representation formalism in their own right.
2. Matching (Chapter 11). *Matching* puts a derived representation of an image into correspondence with an existing representation. This style of processing representations is more pronounced as domain-dependent knowledge, idiosyncratic goals, and experience begin to dominate the ultimate use (or understanding) of the visual input.
3. Inference (Chapter 12). Classical *logical inference* (a technique for manipulating purely propositional knowledge representations) is a well-understood and elegant reasoning technique. It has good formal properties, but occasionally seems restricted in its power to duplicate the range of human processing. *Extended inference* techniques such as *production systems* are those in which the inference process as well as the propositions may contribute materially to the derived knowledge. *Labeling* techniques can "infer" consistent or likely interpretations for an input from given rules about the domain. Inference can be used for both problem solving and belief-maintenance activity.
4. Planning (Chapter 13). *Planning* techniques are useful for problem solving, and are especially tailored to integrating vision with real-world *action.* Planning can be used for resource allocation and attentional mechanisms.
5. Control (Chapter 10; Appendix 2). Control *strategies* and *mechanisms* are of vital concern in any complex artificial intelligence system, and are particularly important when the computation is as expensive as that of vision processing.

Learning is missing from the list above. Disappointing as it is, at this writing the problem of learning is so difficult that we can say very little about it in the domain of vision.

Knowledge Representation and Use

10

10.1 REPRESENTATIONS

An internal representation of the world can help an intelligent system plan its actions and foresee their consequences, anticipate dangers, and use knowledge acquired in the past. In Part IV we investigate the creation, maintenance, and use of a *knowledge base*, an abstract representation of the world useful for computer vision. Chapter 1 introduced a layered organization for the knowledge base and divided its contents into "analogical" and "propositional" models. In this section we consider this high-level division more deeply.

The outside world is accessible to a computer vision program through the imaging process. Otherwise, the program is manipulating its internal representations, which should correspond to the world in understood ways. In this sense, the knowledge base of generalized images, segmented images, and geometric entities contains "models" of the phenomena in the world. Another more abstract sense of "model" is high-level, prior expectations about how the world fits together. Such a high-level model is often much more complex than the lower-level representations, often has a large "propositional" component, and is often manipulated by "inference-like" procedures. Explicit knowledge and belief structures are a relatively new phenomenon in computer vision, but are playing an increasingly important role.

The goals of this chapter are three.

1. To develop in more depth some issues of high-level models (Section 10.1).
2. To describe *semantic nets*—an important and general tool for both organizing and representing models (Sections 10.2 and 10.3).
3. To address issues of *control*, at both abstract and implementational levels (Section 10.4 augmented by Appendix 2).

10.1.1 The Knowledge Base—Models and Processes

Figure 10.1 shows the representational layers in the knowledge base as we have developed it through the book, and shows the place of important processes. This organization might be compared with that in [Barrow and Tenenbaum 1981].

The knowledge base organization is mirrored in the organization of the book. Parts I to III dealt with analogical models and their construction; Part IV is concerned with propositional and complex analogical models. In Chapters 11 to 13, the emphasis moves from the structure of models to the processes (matching, inference, and planning) needed to manipulate and use them.

The knowledge base should have the following properties.

- Represent analogical, propositional, and procedural structures
- Allow quick access to information
- Be easily and gracefully extensible
- Support inquiries to the analogical structures
- Associate and convert between structures
- Support belief maintenance, inference, and planning

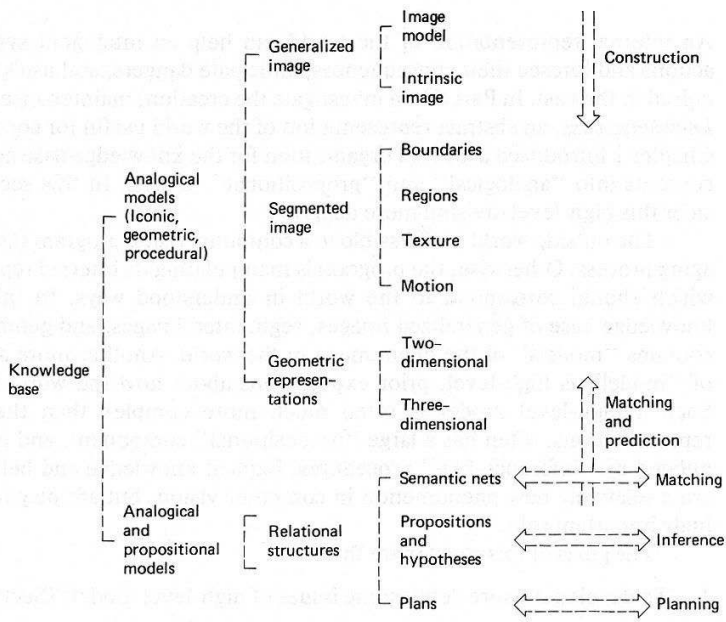


Fig. 10.1 The knowledge base and associated processes in a computer vision system.

The highest levels of the knowledge base contain both *analogical* and *propositional* models. Analogical tools do not exist for many important activities, and when they do exist they are often computationally intensive. A three-dimensional geometric modeling system for automatic manufacturing has very complex data structures and algorithms compared to their elegant and terse counterparts in a propositional model that may be used to plan the highest-level actions. In general it makes sense to do some computation at the analogical level and some at the propositional. This multiple-representation strategy seems more efficient than translating all problems into one representation or the other.

The computations in a vision system should be organized so that information can flow efficiently and unnecessary computation is kept to a minimum. This is the function of the *control* disciplines that allocate effort to different processes. Even the simplest biological vision systems exhibit sophisticated control of processing.

Constructive processes dominate the activity in building lower-level models, and *matching* processes become more important as prior expectations and models are brought into play. Chapter 11 is devoted to the process of matching.

We postulate that an advanced vision system is engaged in two sorts of high-level activity: *belief maintenance* and *goal achievement.* The former is a more or less passive, data-driven, background activity that keeps beliefs consistent and updated. The latter is an active, knowledge-driven, foreground activity that consists of planning future activities. Planning is a problem-solving and simulation activity that anticipates future world states; in computer vision it can determine how the visual environment is expected to change if certain actions are performed. Planning can occur with symbolic, propositional representations (Chapter 13) or in a more analogical vein with such simulations as trajectory planning [Lozano-Perez and Wesley 1979]. Planning is useful as an implementational mechanism even in contexts that are not analogous to human "conscious" problem solving [Garvey 1976]. Helmholtz likened the results of perception to "unconscious conclusions" [Helmholtz 1925]. Similarly even "primitive" vision processes (computer or biological) may use planning techniques to accomplish their ends.

Inference and planning are both classical subfields of artificial intelligence. Neither has seen much application in computer vision. Inference seems useful for belief maintenance. Extended inference can deal with inconsistent beliefs and with beliefs that are maintained with various strengths. We treat inference in Chapter 12. Applications of planning to vision [Garvey 1976; Bolles 1977] show good promise. Planning is treated in Chapter 13.

10.1.2 Analogical and Propositional Representations

Our division of the internal knowledge base into "analogical" and "propositional" reflects a similar division in theories of how human beings represent the world [Johnson-Laird 1980]. Psychological data are not compelling toward either pure theory; there are indications that human beings use both forms of representation. We introduce the division in this book because we find it conceptually useful in the

following way. Low-level representations and processes tend to be purely analogical; high-level representations and processes tend to be both analogical and propositional.

Analogical representations have the following characteristics [Kosslyn and Pomerantz 1977; Shepard 1978; Sloman 1971; Kosslyn and Schwartz 1977, 1978; Waltz and Boggess 1979].

1. *Coherence.* Each element of a represented situation appears once, with all its relations to other elements accessible.
2. *Continuity.* Analogous with continuity of motion and time in the physical world; these representations permit continuous change.
3. *Analogy.* The structure of the representation mirrors (and may be isomorphic to) the relational structure of the represented situation. The representation is a description of the situation.
4. *Simulation.* Analogical models are interrogated and manipulated by arbitrarily complex computational procedures that often have the flavor of (physical or geometric) simulation.

Propositional representations have the following characteristics [Anderson and Bower 1973; Palmer 1975; Pylyshyn 1973].

1. *Dispersion.* An element of a represented situation can appear in several propositions. However, the propositions can be represented in a coherent manner by using semantic nets.
2. *Discreteness.* Propositions are not usually used to represent continuous change. However, they may be made to approximate continuous values arbitrarily closely. Small changes in the representation can thus be made to correspond to small changes in the represented situation.
3. *Abstraction.* Propositions are true or false. They do not have a geometric resemblance to the situation; their structure is not analogous to that of the situation.
4. *Inference.* Propositional models are manipulated by more or less uniform computations that implement "rules of inference" allowing new propositions to be developed from old ones.

Each sort of model derives its "meaning" differently; the distinctions are interesting, because they can point out weaknesses in each theory [Johnson-Laird 1980; Schank 1975; Fodor, et al. 1975]. Especially in computer implementations, the two representations only differ essentially in the last two points. It is often possible to transform one representation to another without loss of information.

Some examples are in order. A generalized image (Part I) is an analogical model: to find an object above a given object, a procedure can "search upward" in the image. An unambiguous three-dimensional model of a solid (Chapter 9) is analogical. It may be used to calculate many geometric properties of the solid, even those unimagined by the designer of the representation. A set of predicate calculus clauses (Chapter 12) is a propositional model. Closely related models can be used to solve problems and make plans [Nilsson 1971, 1980; Chapter 13].

A short digression: It is interesting that people do not seem to perform syllogistic inference (formal propositional deduction) in a "mechanical" way. Given two clauses such as "Some appliances are telephones" and "All telephones are black," we are much more likely to conclude "Some appliances are black" than the equally valid "Some black things are appliances." There is not a satisfying theory of the mental processes underlying syllogistic inference. An interesting speculation [Johnson-Laird 1980] is that inference is primarily done through analogical mental models (in which, for example, a population of individuals is conjured up and manipulated). Then syllogistic inference techniques may have arisen as a bookkeeping mechanism to assure that analogical reasoning does not "miss any cases."

10.1.3 Procedural Knowledge

Procedures as explicit elements in a model pose problems because they are not readily "understood" by other knowledge base components. It is very hard to tell what a procedure does by looking at its code.

In our taxonomy we think of "procedural" knowledge as being analogical. The sequential nature of a program's steps is analogous to an ordering of actions in time that can only be clumsily expressed in current propositional representations. Knowledge about "how-to" perform a complex activity is most propitiously represented in the form of explicit process descriptions. Descriptions not involving the element of time may be naturally represented as passive (analogical or propositional) structures.

There have been several attempts to organize chunks of procedural knowledge by associating with the procedure a description of what it is to accomplish. For example, procedural knowledge can be stored in the internal model structure (knowledge base) indexed under *patterns* that correspond to the arguments of the procedure. *Pattern-directed invocation* involves going to the knowledge base for a procedure that matches the given pattern, matching pattern elements to bind arguments, and invoking the procedure. Several advantages accrue in pattern-directed invocation, such as not having to know the "proper names" of procedures, only their descriptions (what they claim to do). Also, when several procedures match a pattern, one either gets nondeterminism or a chance to choose the best. Often system facilities include a procedure to run to choose the best procedure dynamically. Similar pattern matching is involved in resolution theorem provers and production systems (Chapter 12).

As an example, in a program to locate ribs in a chest radiograph [Ballard 1978], procedures to find ribs under different circumstances are attached to nodes in a mixed analogic and propositional model of the ribcage as shown in Fig. 10.2. Each procedure has an associated description which determines whether it can be run. For example, some programs require instances of neighboring ribs to be located before they can run, whereas others can run given only rudimentary scaling information. When invoked, each procedure tries to find a geometric structure corresponding to the associated rib in a radiograph. Instead of searching for ribs in a mechanical order, descriptors allow a choice of order and procedures and hence a

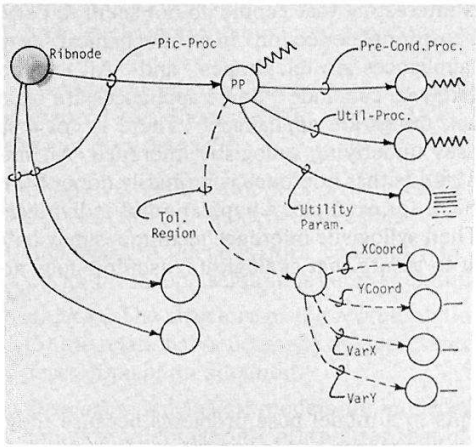


Fig. 10.2 A portion of a ribcage model (see text). Procedural attachment to a model is denoted by jagged lines.

more flexible, efficient and robust program (Appendix 2).

The representation and use of procedural knowledge is an important topic [Schank and Abelson 1977; Winograd 1975; Freuder 1975]. We expect it to be increasingly important for computer vision.

10.1.4 Computer Implementations

A computer implementation can (and often does) obscure the sharp divisions imposed by pure philosophical differences between analogical and propositional models. A propositional representation need not be an unordered set of clauses, but may have a coherent structure; the coherent versus dispersed distinction is thus blurred. A geometry theorem prover or a block-stacking program may manipulate diagrams or simulate physical phenomena such as gravitational stability and wobble in the manipulator [Gelernter 1963; Fahlman 1974; Funt 1977]. "Nonstandard inference" is an important tool that extends classical inference techniques. Although techniques such as production systems and relaxation labeling algorithms (Chapter 11) bear little superficial resemblance to predicate logic, both may be naturally used to manipulate propositional models.

Propositions may be implemented as procedures. If a proposition "evaluates" to true or false, it is perhaps most naturally considered a function from a world (or world model) to a truth value. This is not to say that all such functions exist or are evaluated when the proposition is "brought to mind"; perhaps "understanding a proposition" is like compiling a function and "verifying a proposition" is like evaluating it. The function may be implicit in an evaluation (inference) mechanism or more explicit, as in a "procedural" semantics such as that of the programming languages PLANNER and CONNIVER [Hewitt 1972; Sussman and McDermott 1972; Winograd 1978]. A proposition may thus be encoded as an (analogical!) procedural recipe for establishing the proposition. An example might

be this representation of the fact "In California, Grass and Trees produce green regions."

```
(To-Establish (GreenRegion x)
  Establish (AND (InCalifornia())
    (OR (Establish (Grass x))
    (Establish (Trees x)))))
```

This might mean: To infer that x is a green region, establish that you are in California and then try to establish that x arose from grass. Should the grass inference fail, try to establish that x arose from trees. Since the full power of the programming language is available to an Establish statement, it can perform general computations to establish the inference.

The important point here: Rather than a set of clauses whose application must be organized by an interpreter, propositions may be represented by an explicit control sequence, including procedure calls to other programs. In the example, (Grass x) and (Trees x) may be procedures which have their own complicated control structures.

To say that in a computer "everything is propositions" is a truism; any program can be reduced to a Turing machine described by a finite set of "propositions" with a very simple rule of "inference." The issue is at what level the program should be described. A program may be doing propositional resolution theorem proving or analogical trajectory planning with three-dimensional models; it is not helpful to blur this basic functional distinction by appealing to the lowest implementational level.

10.2 SEMANTIC NETS

10.2.1 Semantic Net Basics

Semantic nets were first introduced under that name as a means of modeling human associative memory [Quillian 1968]. Since then they have received much attention [Nilsson 1980; Woods 1975; Brachman 1976; Findler 1979]. We are concerned with three aspects of semantic nets.

1. Semantic nets can be used as a data structure for conveniently accessing both analogical and propositional representations. For the latter their construction is straightforward and based solely on propositional syntax (Chapter 12).
2. Semantic nets can be used as an analogical structure that mirrors the relevant relations between world entities.
3. Semantic nets can be used as a propositional representation with special rules of inference. Both classical and extended inference can be supported, but it is a challenging enterprise to design net structure that provides the properties of formal logic [Schubert 1976; Hendrix 1979].

A semantic network represents objects and relationships between objects as a graph structure of *nodes* and (labeled) *arcs.* The arcs usually represent relations between nodes and may be "followed" to proceed from node to node. A directed arc with label L between nodes X and Y can signify that the predicate $L(X, Y)$ is true. If, in addition, it has a value V, the arc can signify that some function or relation holds: $L(X, Y) = V$.

The *indexing property* of a network is one of its useful aspects. The network can be constructed so that objects that are often associated in computations, or are especially relevant or conceptually close to each other, may be represented by nodes in the network that are near each other in the network (as measured by number of arcs separating them). Figure 10.3 shows these ideas: (a) nodes can be associated by searching outward along arcs and (b) nodes near a specified node are readily available by following arcs. Semantic networks are especially attractive as analogical representations of spatial states of affairs. If we restrict ourselves to binary spatial relations ("above," and "west of," for example), physical objects or parts of objects may be represented by nodes, and their positions with respect to each other by arcs.

Let us look at a semantic net and make some basic observations. Figure 10.4 is meant to be an analogical representation of an arrangement of chairs around a table. The LEFT-OF and RIGHT-OF relations are directed arcs, the ADJACENT relation is undirected; there can be several such undirected arcs between nodes. Note here that the LEFT-OF and RIGHT-OF relations do not behave in their normal way. If they are transitive, as is normal, then every chair is both LEFT-OF and

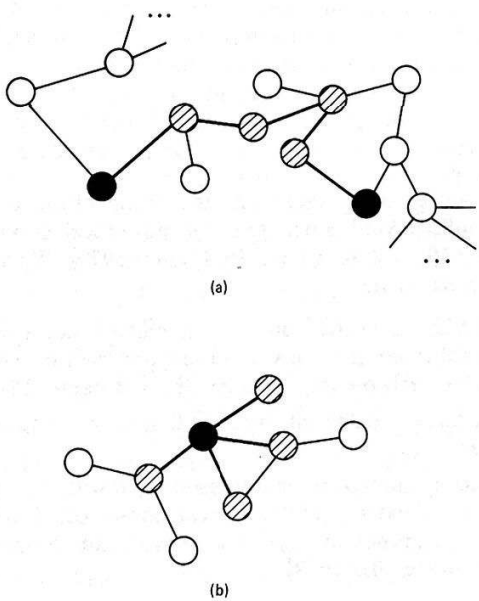


Fig. 10.3 Semantic networks as structures for associative search. (a) Associating two nodes. (b) Retrieving nearby nodes.

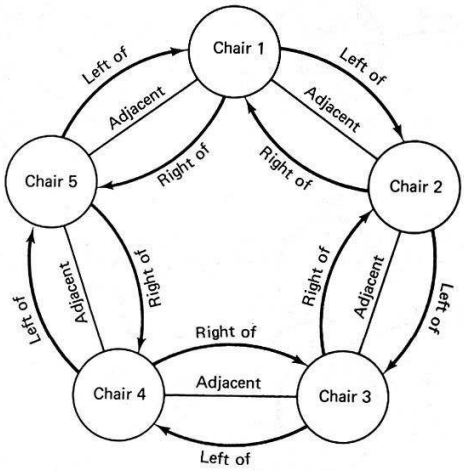


Fig. 10.4 A representation of chairs at a table.

RIGHT-OF every other chair. Flexible treatment of this sort of phenomenon is sometimes difficult in propositional representations.

A simple but basic point: The net of Fig. 10.4 seems to say interesting things about furniture in a scene. But notice that merely by rewriting labels the same net could be "about" modular arithmetic, a string of pearls, or any number of things. There are two morals here. First, a sparsely connected representation (analogical or propositional) may have several equally good interpretations. Second, a net without any interpretation procedures essentially represents nothing [McDermott 1976].

Now consider three neighboring chairs described by the following relations.

1. CHAIR(Armchair), CHAIR(Highchair), CHAIR(Stool)
2. WIDE(Armchair)
3. HIGH(Highchair)
4. LOW(Stool)
5. LEFT-OF(Armchair, Highchair)
6. LEFT-OF(Highchair, Stool)
7. BETWEEN(Highchair, Armchair, Stool)

The relations include four properties (relations with "one argument"), a two-argument and a three-argument relation. One way to encode this information in a net is shown in Fig. 10.5a. Nodes represent individuals, and properties are kept as node contents. The directed arcs represent only binary relations, and "betweenness" is left implicit. Properties can equally well be represented as labeled arcs (Fig. 10.5b).

Relations are encoded as nodes in Fig. 10.6. Here the BETWEEN relation is encoded asymmetrically: it is not possible to tell by arcs emanating from the stool that it is in a "between" relationship.

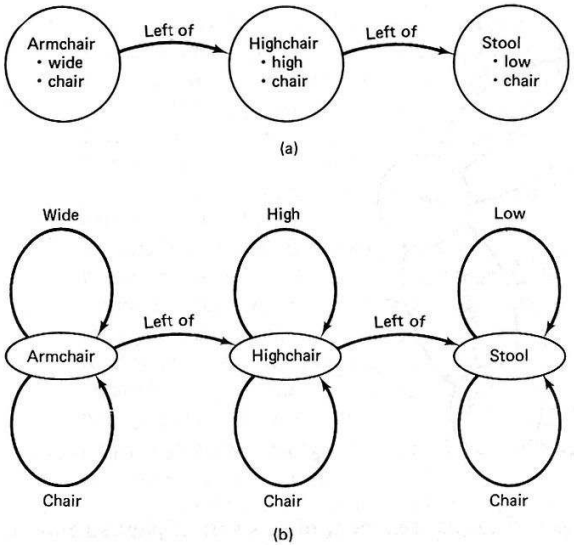


Fig. 10.5 (a) A simple semantic net. (b) An equivalent net.

The three-place relation is treated more symmetrically in Fig. 10.7. In general, *n*-place relations may be "binarized" this way; create a node for the "relation instance" and new (relation) nodes for each distinct argument role in the *n*-ary relation.

An important point: Arcs and nodes had a uniform semantics in Fig. 10.4. This property was lost in the succeeding nets; nodes are either "things" or relations, and arcs leading into relations are not the same as those leading out. For such nets to be useful, the net interpreter (a program that manipulates the net) must keep these things straight. It is possible but not easy to devise a rich and uniform network semantics [Brachman 1979].

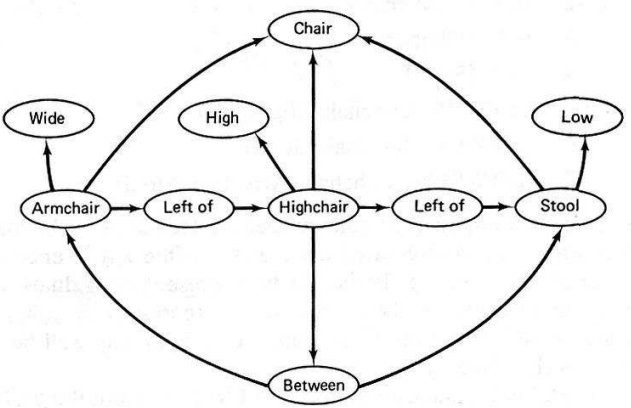


Fig. 10.6 A net with more explicit information.

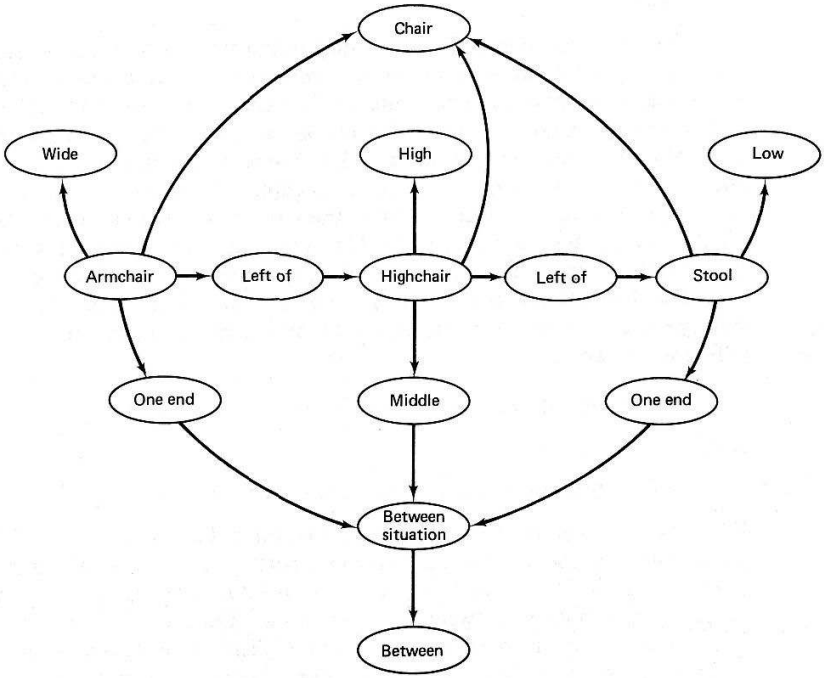


Fig. 10.7 A net with yet more explicit information.

10.2.2 Semantic Nets for Inference

This section explores some further important issues in the semantics of semantic nets. In Chapter 12 semantic nets are used as an indexing mechanism in predicate calculus theorem proving. In some applications an inference system with provably good formal properties may be too restrictive. Some formal properties (such as maintaining consistency by not deducing contradictions) may be considered vital, however. How can "good behavior" be obtained from a representation that may contain "inconsistent" information?

One example of an "inconsistent" representation is the net of Fig. 10.3, with its LEFT-OF and RIGHT-OF problem. Another example is a net version of the propositions "All birds fly," "Penguins are birds," "Penguins do not fly." The generalization is useful "commonsense" knowledge, but the rare exceptions may be important, too. Network interpreters can cope with these sorts of problems by a number of methods, such as only accessing a consistent subnetwork, making deductions from the particular toward the general (this takes care of penguins), and so forth. All these techniques depend on the structure imposed by the net.

Some more subtle aspects of net representations appear below.

Nodes

The basic notation of Fig. 10.4 may tempt us to produce a net such as that shown in Fig. 10.8. Consider the object node sky in Fig. 10.8. Does it stand for the generic sky concept or for a particular sky at a particular time and location? Clearly both meanings cannot be embodied in the same node because they are used in such different ways in reasoning. The standard solution is to use nodes to differentiate between a *type*, or generic concept, and a *token*, or instance of it. Figure 10.9 shows this modification using the *e* (element of) relation to relate the individual to the generic concept. In this simple case, the node sky stands for the *type*, and the empty node stands for a *token*, or instance of the sky concept.

The distinction between type and token is related to the distinction between *intensional* and *extensional* concepts. In analyzing an aerial image there is a difference between

"All bridges span roads or rivers." (10.1)

and

"All bridges (found so far) span roads or rivers." (10.2)

If "bridges" in (10.1) means *any* bridge that might be found, "bridges" is used in an *intensional* sense. If "bridges" means a particular set, it is used it in an *extensional* sense. Normally relations between *type* nodes are used in an intensional sense and relationships between *token* nodes have the extensional sense.

Virtual nodes are objects that are not explicitly represented as object nodes. The need for them arises in expressing complex relations. For example, consider

"The bridge that is at the intersection of road 57
and river 3 is near building 30." (10.3)

which may be represented as shown in Fig. 10.10. The node labeled *x* is the result of intersecting a particular road with a particular river. It is not represented explicitly as an instance of any generic concept; it is a *virtual node*. Virtual nodes can be eliminated by introducing very complex relations, but this would sacrifice an important property of networks, the ability to build up a very large number of com-

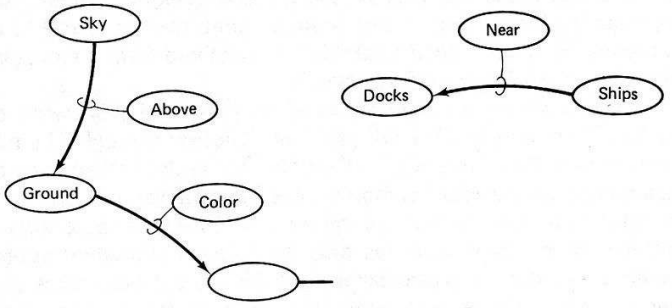


Fig. 10.8 Type or token nodes?

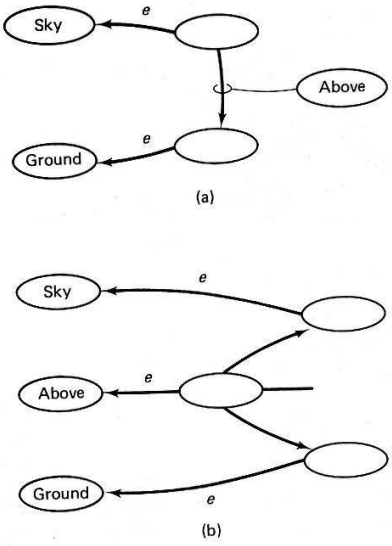


Fig. 10.9 Distinguishing between types and tokens: (a) Tokenizing an instance. (b) Tokenizing an assertion.

plex relations from a small set of primitives. Virtual nodes enhance this ability by referring to portions of complicated relations.

Nodes in the network can also be used as *variables*. These variables can match other nodes which represent constants. In Fig. 10.11, x and y are variables and the rest of the nodes are constants. If node x is matched to the "telephone" node, then x can be regarded as a "telephone" node.

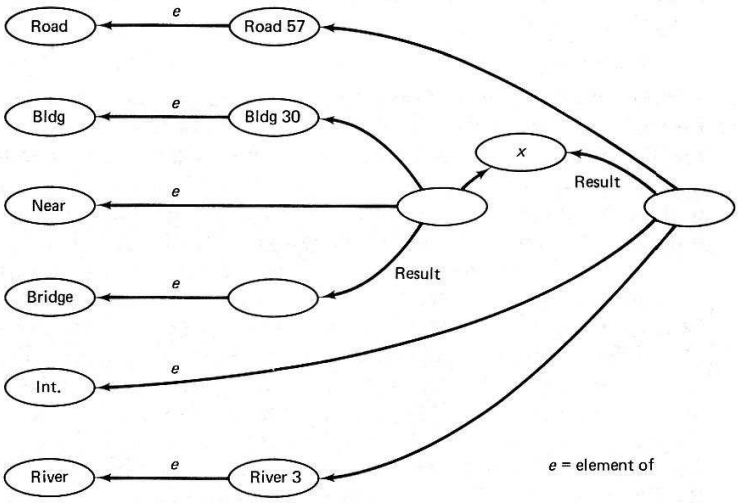
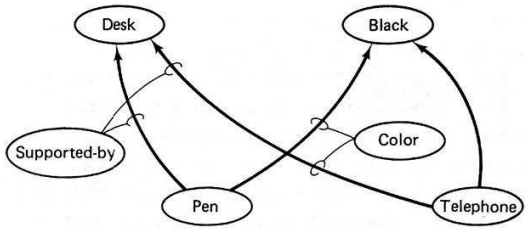
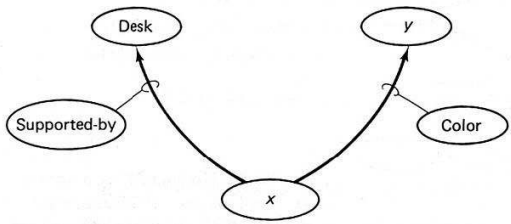


Fig. 10.10 Virtual nodes.



(a)



(b)

Fig. 10.11 Nodes as variables. (a) Black telephone and pen on desk. (b) Object denoted by variable x with variable color y.

Often, it is useful to have numerical values as node properties. This can extend the discrete representation of nodes and arcs to a continuous one. For example, in addition to "color of x is red37" we may also associate the particular value of red that we mean with node red37. A special kind of value is a *default value.* If a value can be found for the node in the course of matching other nodes with values or by examining image data, then that value is used for the node value. Otherwise, the default is used.

Relations

Complex relations of many arguments are not uncommon in the world, but for the bulk of practical work, relations of only a few arguments seem to suffice. Semantic nets can clearly represent two-argument relations through their nodes and arcs. More complex relations may be dealt with by various devices. The links to multiple arguments may be ordered within a relation node, or new nodes may be introduced to label the roles of multiple arguments (Fig. 10.7).

If inference mechanisms are to manipulate semantic nets, certain important relations deserve special treatment. One such relation is the "IS-A" relation. The basic issue addressed by this relation is *property inheritance* [Moore 1979]. That is, if Fred IS-A Camel and a Camel IS-A Mammal, then presumably Fred has the properties associated with mammals. It often seems necessary to differentiate between various senses of "IS-A." One basic sense of "X IS-A Y" is "X is an element of the set Y "; others are "X denotes Y," "X is a subset of Y," and "Y is an abstraction of X." Notice that each sense depends on differently "typed" arguments; in the first three cases X is, respectively, an individual, a name, and a set. Deeper