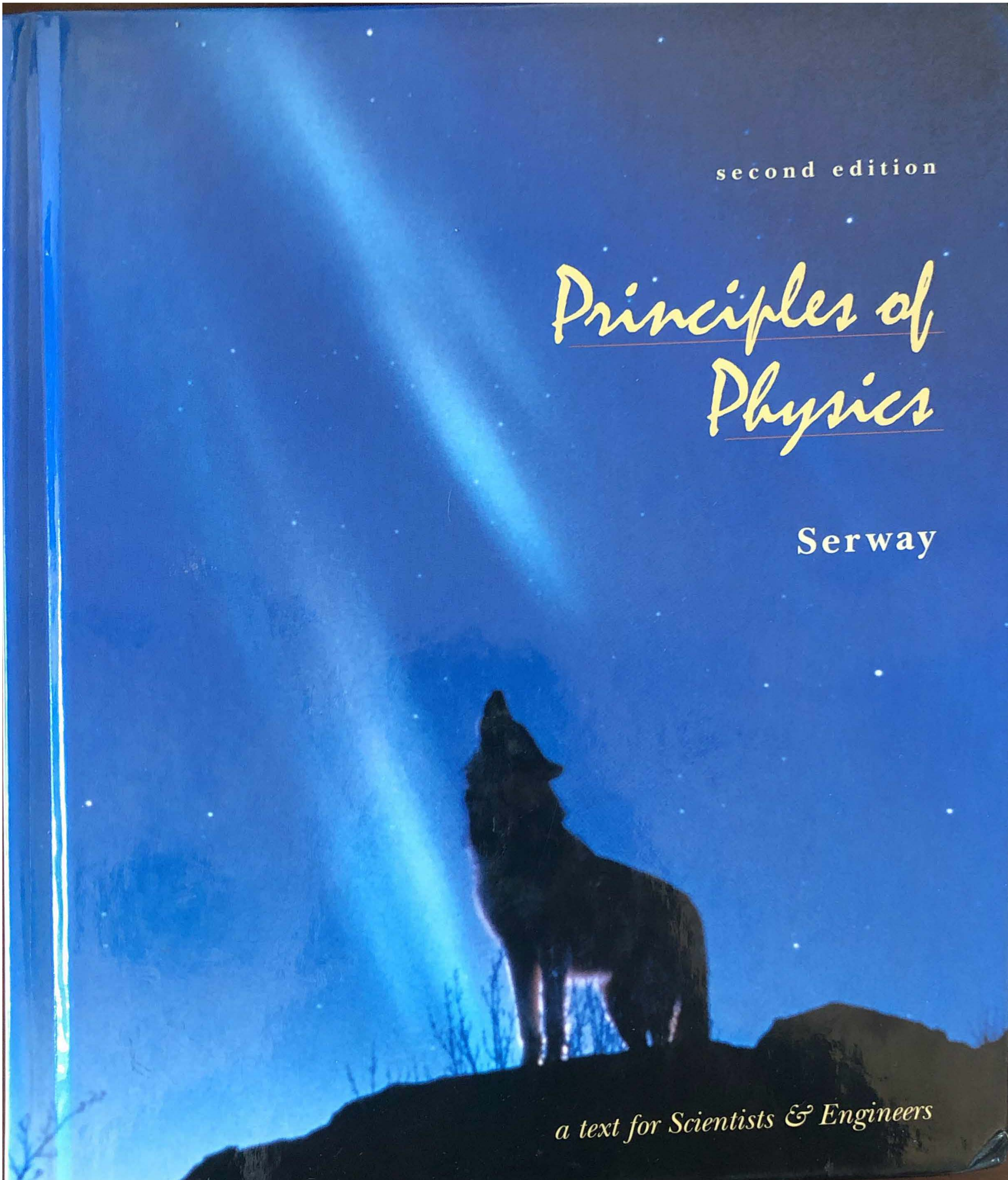
second edition
Principles of Physics
Serway
a text for Scientists & Engineers

Principles *of* Physics

SECOND EDITION

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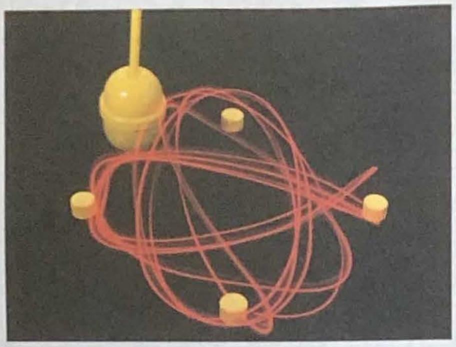

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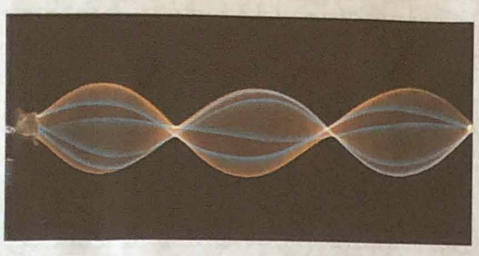

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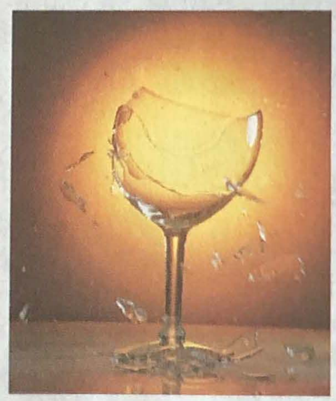

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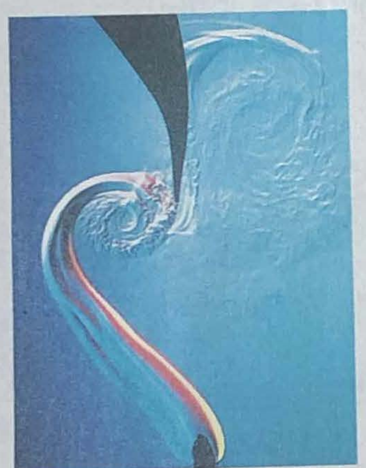

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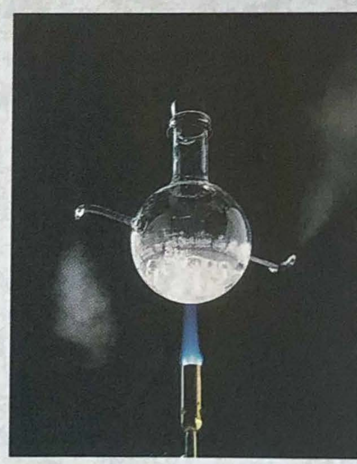

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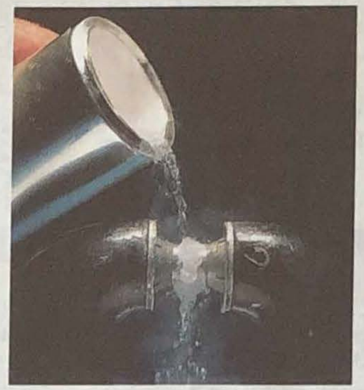

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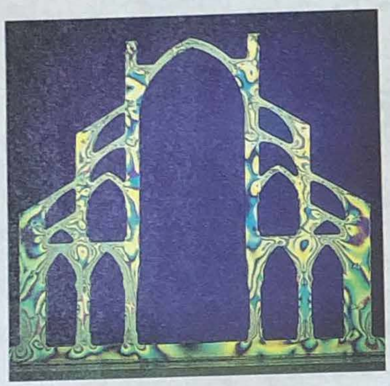

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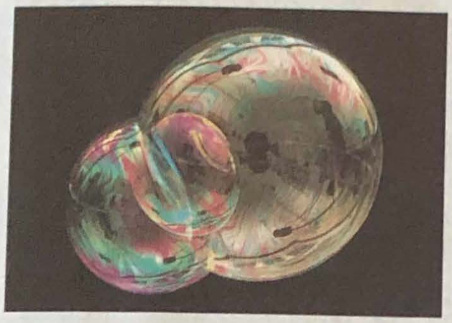

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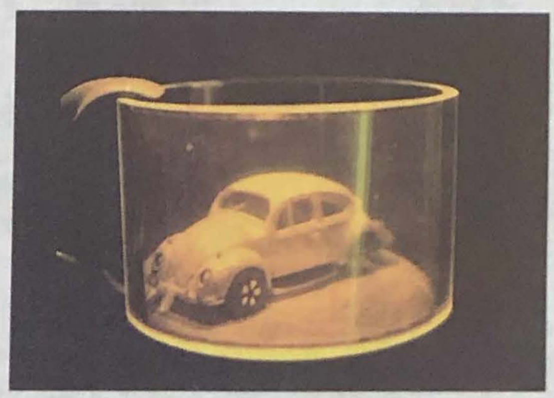

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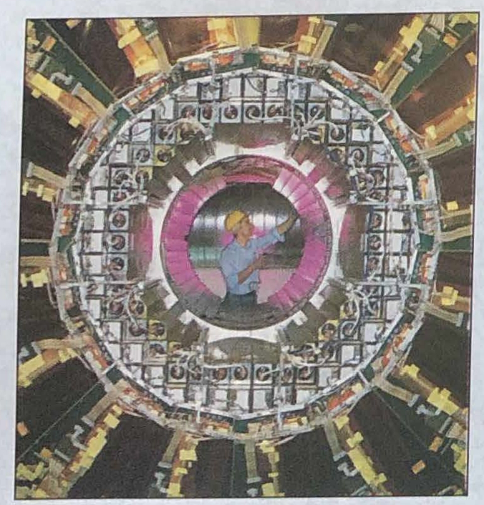

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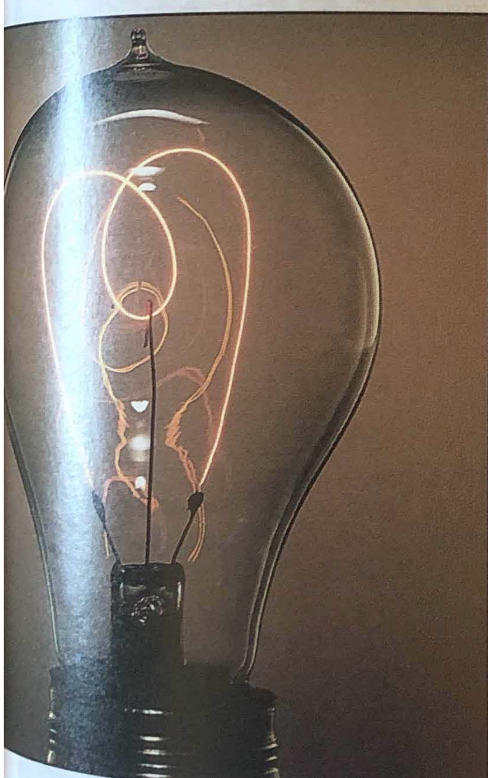
21

Current and Direct Current Circuits

CHAPTER OUTLINE

Thus far, our discussion of electrical phenomena has been confined to charges at rest, or electrostatics. We shall now consider situations involving electric charges in motion. The term *electric current,* or simply *current,* is used to describe the rate of flow of charge through some region of space. Most practical applications of electricity involve electric currents. For example, the battery of a flashlight supplies current to the filament of the bulb when the switch is turned on. In these common situations, the flow of charge takes place in a conductor, such as a copper wire. It is also possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

In this chapter we shall first define current and current density. A microscopic description of current will be given, and some of the factors that contribute to resistance to the flow of charge in conductors will be discussed. Mechanisms responsible for the electrical resistances of various



◀ **Photograph of a carbon filament incandescent lamp. The resistance of such a lamp is typically 10 Ω, but its value changes with temperature. Most modern lightbulbs use tungsten filaments, the resistance of which increases with increasing temperature.**
(Courtesy of Central Scientific Co.)

LG Display Co., Ltd.
Exhibit 1018
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materials depend on the materials' compositions and on temperature. A classical model is used to describe electrical conduction in metals; we shall point out some of the limitations of this model.

This chapter is also concerned with the analysis of some simple circuits the elements of which include batteries, resistors, and capacitors in varied combinations. The analysis of these circuits is simplified by the use of two rules known as *Kirchhoff's rules,* which follow from the laws of conservation of energy and conservation of charge. Most of the circuits analyzed are assumed to be in *steady state,* which means that the currents are constant in magnitude and direction. We shall close with a discussion of circuits containing resistors and capacitors, in which the current varies with time.

21.1 • ELECTRIC CURRENT

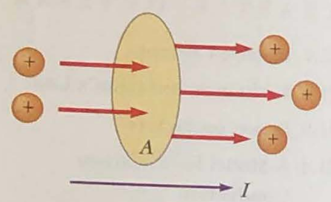

Figure 21.1 Charges in motion through an area *A*. The time rate of flow of charge through the area is defined as the current *I*. The direction of the current is the direction in which positive charge would flow if free to do so.

Whenever there is a net flow of charge, a **current** is said to exist. To define current more precisely, suppose the charges are moving perpendicular to a surface of area *A*, as in Figure 21.1. (This area could be the cross-sectional area of a wire, for example.) **The current is the rate at which charge flows through this surface.** If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current, I_{av}, is the ratio of the charge to the time interval:

$$I_{av} = \frac{\Delta Q}{\Delta t} \qquad [21.1]$$

If the rate at which charge flows varies in time, the current also varies in time. We define the **instantaneous current *I*** as the differential limit of the preceding expression:

Electric current •

$$I \equiv \frac{dQ}{dt} \qquad [21.2]$$

The SI unit of current is the **ampere** (A):

$$1 \text{ A} = 1 \text{ C/s} \qquad [21.3]$$

That is, 1 A of current is equivalent to 1 C of charge passing through a surface in 1 s.

When charges flow through a surface as in Figure 21.1, they can be positive, negative, or both. **It is conventional to give the current the same direction as the flow of positive charge.** In a common conductor such as copper, the current is due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, **the direction of the current is opposite the direction of flow of electrons.** However, if one considers a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—gases and electrolytes, for example—the current is the result of the flow of both positive and negative charges. It is common to refer to a moving charge (whether it is positive or negative) as a mobile **charge carrier.** For example, the charge carriers in a metal are electrons.

The direction of the current •

It is instructive to relate current to the motions of the charged particles. To illustrate this point, consider the current in a conductor of cross-sectional area *A*

(Fig. 21.2). The volume of an element of the conductor of length Δx is $A\,\Delta x$. If n represents the number of mobile charge carriers per unit volume, then the number of carriers in the volume element is $nA\,\Delta x$. Therefore, the charge ΔQ in this element is

$$\Delta Q = \text{number of carriers} \times \text{charge per carrier} = (nA\,\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a speed of v_d, the distance they move in the time Δt is $\Delta x = v_d\,\Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d\,\Delta t)q$$

If we divide both sides of this equation by Δt, we see that the current in the conductor is

$$I = \frac{\Delta Q}{\Delta t} = nqv_dA \qquad [21.4]$$

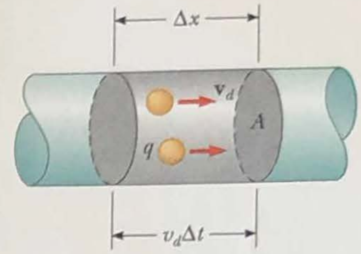

Figure 21.2 A section of a uniform conductor of cross-sectional area A. The charge carriers move with a speed v_d, and the distance they travel in a time Δt is given by $\Delta x = v_d\,\Delta t$. The number of mobile charge carriers in the section of length Δx is given by $nAv_d\,\Delta t$, where n is the number of mobile carriers per unit volume.

The speed of the charge carriers, v_d, is an average speed called the **drift speed.** To understand its meaning, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated, these electrons undergo random motion similar to that of gas molecules. When a potential difference is applied across the conductor (say, by means of a battery), an electric field is set up in the conductor, which creates an electric force on the electrons, accelerating them, and hence producing a current. In reality, the electrons do not simply move in straight lines along the conductor. Instead, they undergo repeated collisions with the metal atoms, and the result is a complicated zigzag motion (Fig. 21.3). The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor. However, despite the collisions, the electrons move slowly along the conductor (in a direction opposite **E**) with the drift velocity, $\mathbf{v}_d$. One can think of the collisions within a conductor as being an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool.

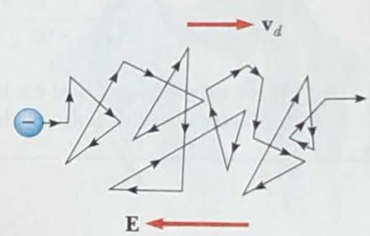

Figure 21.3 A schematic representation of the zigzag motion of a charge carrier in a conductor. The changes in direction are due to collisions with atoms in the conductor. Note that the net motion of electrons is opposite the direction of the electric field. The zigzag paths are actually parabolic segments.

The **current density** J in the conductor is defined to be the current per unit area. Because $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \qquad [21.5]$$

where J has the SI units amperes per square meter. In general, the current density is a *vector quantity.* That is,

• *Current density*

$$\mathbf{J} \equiv nq\mathbf{v}_d \qquad [21.6]$$

From this definition, we see that the current density vector is in the direction of motion of positive charge carriers and opposite the direction of motion of negative charge carriers. Because the drift velocity is proportional to the electric field **E** in the conductor, we conclude that the current density is also proportional to **E**.

Thinking Physics 1

Suppose a current-carrying wire has a cross-sectional area that gradually becomes smaller along the wire, so that the wire has the shape of a very long cone. How does the drift velocity of electrons vary along the wire?

Reasoning Every portion of the wire is carrying the same amount of current. Thus, as the cross-sectional area decreases, the drift velocity must increase to maintain the constant value of the current. This increased drift velocity is a result of the electric field lines in the wire being compressed into a smaller area, thus increasing the strength of the field.

Example 21.1 Drift Speed in a Copper Wire

A copper wire of cross-sectional area $3.00 \times 10^{-6}\ \mathrm{m}^2$ carries a current of 10.0 A. Find the drift speed of the electrons in this wire. The density of copper is $8.95\ \mathrm{g/cm^3}$.

Solution From the periodic table of the elements in Appendix C, we find that the atomic mass of copper is 63.5 g/mol. Recall that one atomic mass of any substance contains Avogadro's number of atoms, 6.02×10^{23} atoms. Knowing the density of copper enables us to calculate the volume occupied by 63.5 g of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \mathrm{g}}{8.95\ \mathrm{g/cm^3}} = 7.09\ \mathrm{cm^3}$$

If we now assume that each copper atom contributes one free electron to the body of the material, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \mathrm{cm^3}}$$
$$= 8.48 \times 10^{22}\ \text{electrons/cm}^3$$
$$= \left(8.48 \times 10^{22}\ \frac{\text{electrons}}{\mathrm{cm^3}}\right)\left(10^6\ \frac{\mathrm{cm^3}}{\mathrm{m^3}}\right)$$
$$= 8.48 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 21.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$
$$= \frac{10.0\ \mathrm{C/s}}{(8.48 \times 10^{28}\ \mathrm{m^{-3}})(1.60 \times 10^{-19}\ \mathrm{C})(3.00 \times 10^{-6}\ \mathrm{m^2})}$$
$$= 2.46 \times 10^{-4}\ \mathrm{m/s}$$

Example 21.1 shows that typical drift speeds are very small. In fact, the drift speed is much smaller than the average speed between collisions. For instance, electrons traveling with this speed would take about 68 min to travel 1 m! In view of this low speed, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of 10^7 m/s.

EXERCISE 1 If a current of 80.0 mA exists in a metal wire, how many electrons flow past a given cross section of the wire in 10.0 min? Answer 3.0×10^{20} electrons

21.2 • RESISTANCE AND OHM'S LAW

When a voltage (potential difference) ΔV is applied across the ends of a metallic conductor, as in Figure 21.4, the current in the conductor is found to be proportional to the applied voltage; that is, $I \propto \Delta V$. If the proportionality is exact, we can write $\Delta V = IR$, where the proportionality constant R is called the resistance of the

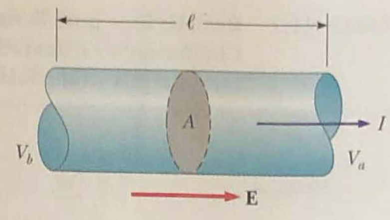

Figure 21.4 A uniform conductor of length ℓ and cross-sectional area A. A potential difference $V_b - V_a$ maintained across the conductor sets up an electric field **E** in the conductor, and this field produces a current I that is proportional to the potential difference.

conductor. In fact, we define this **resistance** as the ratio of the voltage across the conductor to the current it carries:

$$R \equiv \frac{\Delta V}{I} \quad [21.7]$$

• ***Resistance***

Resistance has the SI units volts per ampere, called **ohms (Ω).** Thus, if a potential difference of 1 V across a conductor produces a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source carries a current of 6 A, its resistance is 20 Ω.

It is useful to compare the concepts of electric current, voltage, and resistance with the flow of water in a river. As water flows downhill in a river of constant width and depth, the flow rate (water current) depends on the angle of flow and the effects of rocks, the river bank, and other obstructions. Likewise, electric current in a uniform conductor depends on the applied voltage and the resistance of the conductor caused by collisions of the electrons with atoms in the conductor.

For many materials, including most metals, experiments show that **the resistance is constant over a wide range of applied voltages.** This statement is known as **Ohm's law** after Georg Simon Ohm (1787–1854), who was the first to conduct a systematic study of electrical resistance.

Ohm's law is *not* a fundamental law of nature, but an empirical relationship that is valid only for certain materials. Materials that obey Ohm's law, and hence have a constant resistance over a wide range of voltages, are said to be **ohmic.** Materials that do not obey Ohm's law are **nonohmic.** Ohmic materials have a linear current-voltage relationship over a large range of applied voltages (Fig. 21.5a). Nonohmic materials have a nonlinear current-voltage relationship (Fig. 21.5b). One

Georg Simon Ohm (1787–1854). *(Courtesy of North Wind Picture Archives)*

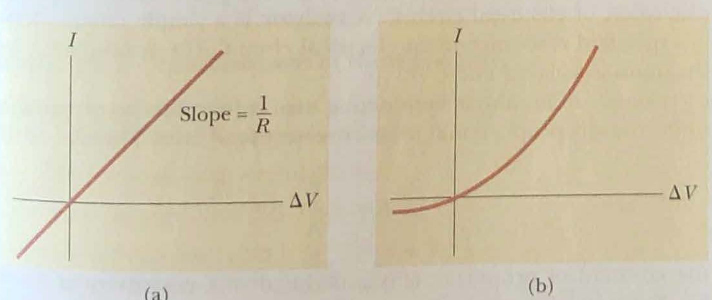

Figure 21.5 (a) The current-voltage curve for an ohmic material. The curve is linear, and the slope gives the resistance of the conductor. (b) A nonlinear current-voltage curve for a semiconducting diode. This device does not obey Ohm's law.

TABLE 21.1 Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[b]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	$10^{10} - 10^{14}$	—
Hard rubber	$\sim 10^{13}$	—
Sulfur	10^{15}	—
Quartz (fused)	75×10^{16}	—

[a] All values at 20°C.

[b] A nickel–chromium alloy commonly used in heating elements.

common semiconducting device that is nonohmic is the diode. Its resistance is small for currents in one direction (positive ΔV) and large for currents in the reverse direction (negative ΔV). Most modern electronic devices, such as transistors, have nonlinear current-voltage relationships; their operation depends on the particular ways in which they violate Ohm's law.

We can express Equation 21.7 in the form

$$\Delta V = IR \tag{21.8}$$

where R is understood to be independent of ΔV. We shall use this expression later in the discussion of electrical circuits. A **resistor** is a simple circuit element that provides a specified resistance in an electrical circuit. The symbol for a resistor in circuit diagrams is a zigzag line (-WW-).

The resistance of an ohmic conducting wire is found to be proportional to its length and inversely proportional to its cross-sectional area. That is,

$$R = \rho \frac{\ell}{A} \tag{21.9}$$

where the constant of proportionality ρ is called[1] the **resistivity** of the material, which has the unit ohm-meter ($\Omega \cdot m$). To understand this relationship between

[1]The symbol ρ used for resistivity should not be confused with the same symbol used earlier in the text for mass density and charge density.

resistance and resistivity, note that every ohmic material has a characteristic resistivity, a parameter that depends on the properties of the material and on temperature. However, as you can see from Equation 21.7, the resistance of a conductor depends on size and shape as well as on resistivity. Table 21.1 provides a list of resistivities for various materials measured at 20°C.

The inverse of the resistivity is defined[2] as the **conductivity, σ.** Hence, the resistance of an ohmic conductor can also be expressed in terms of its conductivity as

$$R = \frac{\ell}{\sigma A} \qquad [21.10]$$

where $\sigma(=1/\rho)$ has the unit $(\Omega \cdot \text{m})^{-1}$.

Equation 21.10 shows that the resistance of a cylindrical conductor is proportional to its length and inversely proportional to its cross-sectional area. This is analogous to the flow of liquid through a pipe. As the length of the pipe is increased, the resistance to liquid flow increases because of a gain in friction between the fluid and the walls of the pipe. As its cross-sectional area is increased, the pipe can transport more fluid in a given time interval, so its resistance drops.

Thinking Physics 2

We have seen that an electric field must exist inside a conductor that carries a current. How is this possible in view of the fact that in electrostatics we concluded that the electric field is zero inside a conductor?

Reasoning In the electrostatic case in which charges are stationary, the internal electric field must be zero because a nonzero field would produce a current (by interacting with the free electrons in the conductor), which would violate the condition of static equilibrium. In this chapter we deal with conductors that carry current, a nonelectrostatic situation. The current arises because of a potential difference applied between the ends of the conductor, which produces an internal electric field. So there is no paradox.

CONCEPTUAL PROBLEM 1

Newspaper articles often have statements such as, "10 000 volts of electricity surged through the victim's body." What is wrong with this statement?

Example 21.2 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\text{m}$ (Table 21.1). Thus, we can use Equation 21.9 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = 4.6\ \Omega/\text{m}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 21.7 gives

[2] Again, do not confuse the symbol σ for conductivity with the same symbol used for surface charge density.

$$I = \frac{\Delta V}{R} = \frac{10\text{ V}}{4.6\ \Omega} = 2.2\text{ A}$$

Note from Table 21.1 that the resistivity of Nichrome wire is about 100 times that of copper. Therefore, a copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied voltage of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

EXERCISE 2 What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does it carry when connected to a 120-V source? Answer 28 Ω; 4.3 A

EXERCISE 3 Calculate the current density and electric field in the wire assuming that it carries a current of 2.2 A. Answer 6.7×10^6 A/m^2; 10 N/C

Change in Resistivity with Temperature

Resistivity depends on a number of factors, one of which is temperature. For most metals, resistivity increases approximately linearly with increasing temperature over a limited temperature range, according to the expression

Variation of ρ with temperature

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad [21.11]$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually 20°C), and α is called the **temperature coefficient of resistivity.** From Equation 21.11, we see that α can also be expressed as

Temperature coefficient of resistivity

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \qquad [21.12]$$

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The resistivities and temperature coefficients of certain materials are listed in Table 21.1. Note the enormous range in resistivities, from very low values for good conductors, such as copper and silver, to very high values for good insulators, such as glass and rubber. An ideal, or "perfect," conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Because resistance is proportional to resistivity according to Equation 21.9, the temperature variation of the resistance can be written

$$R = R_0[1 + \alpha(T - T_0)] \qquad [21.13]$$

Precise temperature measurements are often made using this property, as shown in Example 21.3.

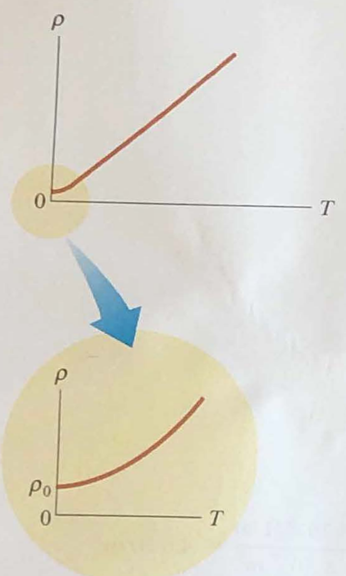

Figure 21.6 Resistivity versus temperature for a normal metal, such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature. As T approaches absolute zero (insert), the resistivity approaches a finite value ρ_0.

CONCEPTUAL PROBLEM 2

Aliens with strange powers visit Earth and double every linear dimension of every object on the surface of the earth. Does the electrical cord from the wall socket to your floor lamp now have more resistance than before, less resistance, or the same resistance? Does the lightbulb filament glow more brightly than before, less brightly, or the same? (Assume the resistivities of materials remain the same.)

CONCEPTUAL PROBLEM 3

When incandescent bulbs burn out, they usually do so just after they are switched on. Why?

Example 21.3 A Platinum Resistance Thermometer

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. What is the melting point of indium?

Solution Solving Equation 21.13 for ΔT and obtaining α from Table 21.1, we get

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (^\circ\text{C})^{-1}](50.0\ \Omega)} = 137^\circ\text{C}$$

Because $T_0 = 20.0^\circ$C, we find that $T =$ 157°C.

For several metals, resistivity is nearly proportional to absolute temperature, as shown in Figure 21.6. In reality, however, there is always a nonlinear region at very low temperatures, and the resistivity usually approaches some finite value near absolute zero (see the magnified inset in Fig. 21.6). This residual resistivity near absolute zero is due primarily to collisions of electrons with impurities and to imperfections in the metal. In contrast, the high-temperature resistivity (the linear region) is dominated by collisions of electrons with the metal atoms. We shall describe this process in more detail in Section 21.4.

Semiconductors, such as silicon and germanium, have intermediate resistivity values. Their resistivity generally decreases with increasing temperature, corresponding to a negative temperature coefficient of resistivity (Fig. 21.7). This is due to the increase in the density of charge carriers at the higher temperatures. Because the charge carriers in a semiconductor are often associated with impurity atoms, the resistivity is very sensitive to the type and concentration of such impurities. A **thermistor** is a semiconducting thermometer that makes use of the large changes in its resistivity with temperature.

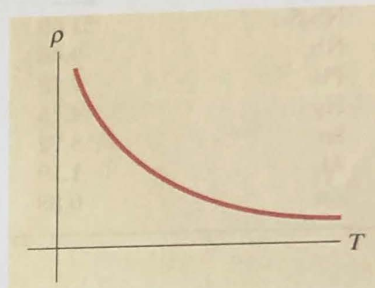

Figure 21.7 Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

EXERCISE 4 If a silver wire has a resistance of 10 Ω at 20°C, what resistance does it have at 40°C? Neglect any change in length or cross-sectional area due to the change in temperature.
Answer 10.8 Ω

21.3 • SUPERCONDUCTORS

There is a class of metals and compounds the resistances of which go to zero below certain *critical temperatures,* T_c. These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures greater than T_c (Fig. 21.8). When the temperature is equal to or less than T_c, the resistivity drops suddenly to zero. This phenomenon was discovered by the Dutch physicist Heike Kamerlingh Onnes in 1911 as he worked with mercury, which is a superconductor below 4.2 K. Recent measurements have shown that the resistivities of superconductors below T_c are less than $4 \times 10^{-25}\ \Omega \cdot$m, which is around 10^{17} times smaller than the resistivity of copper and considered to be zero in practice.

Today thousands of superconductors are known. Such common metals as aluminum, tin, lead, zinc, and indium are superconductors. Table 21.2 lists the critical temperatures of several superconductors. The value of T_c is sensitive to chemical composition, pressure, and crystalline structure. It is interesting to note that copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

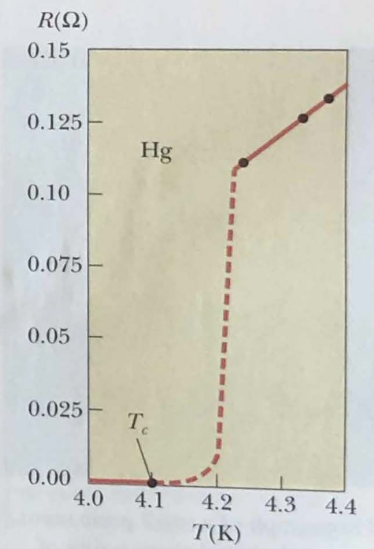

Figure 21.8 Resistance versus temperature for mercury. The graph follows that of a normal metal above the critical temperature, T_c. The resistance drops to zero at the critical temperature, which is 4.2 K for mercury.

TABLE 21.2
Critical Temperatures for Various Superconductors

Material	T_C (K)
$YBa_2Cu_3O_7$	92
Bi-Sr-Ca-Cu-O	105
Tl-Ba-Ca-Cu-O	125
$HgBa_2Ca_2Cu_3O_8$	134
Nb_3Ge	23.2
Nb_3Sn	21.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

One of the truly remarkable features of superconductors is the fact that once a current is set up in them, it persists *without any applied voltage* (because $R = 0$). In fact, steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important recent development in physics that has created much excitement in the scientific community has been the discovery of high-temperature copper-oxide-based superconductors. The excitement began with a 1986 publication by Georg Bednorz and K. Alex Müller, scientists at the IBM Zurich Research Laboratory in Switzerland, in which they reported evidence for superconductivity at a temperature near 30 K in an oxide of barium, lanthanum, and copper. Bednorz and Müller were awarded the Nobel Prize in 1987 for their remarkable discovery. Shortly thereafter, a new family of compounds was open for investigation, and research activity in the field of superconductivity proceeded vigorously. In early 1987, groups at the University of Alabama at Huntsville and the University of Houston announced the discovery of superconductivity at about 92 K in an oxide of yttrium, barium, and copper ($YBa_2Cu_3O_7$). Late in 1987, teams of scientists from Japan and the United States reported superconductivity at 105 K in an oxide of bismuth, strontium, calcium, and copper. More recently, scientists have reported superconductivity at temperatures as high as 125 K in an oxide containing thallium. At this point one cannot rule out the possibility of room-temperature superconductivity, and the search for novel superconducting materials continues. It is an important search both for scientific reasons and because practical applications become more probable and widespread as the critical temperature is raised.

An important and useful application is superconducting magnets in which the magnetic field strengths are about ten times greater than those of the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. The idea of using superconducting power lines for transmitting power efficiently is also receiving some consideration. Modern superconducting electronic devices consisting of two thin-film superconductors separated by a thin insulator have been constructed. They include magnetometers (a magnetic-field measuring device) and various microwave devices.

Photograph of a small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is at 77 K. This superconductor has zero electric resistance at temperatures below 92 K and expels any applied magnetic field. *(Courtesy of IBM Research Laboratory)*

21.4 • A MODEL FOR ELECTRICAL CONDUCTION

The classical model of electrical conduction in metals leads to Ohm's law and shows that resistivity can be related to the motion of electrons in metals.

Consider a conductor as a regular array of atoms containing free electrons (sometimes called *conduction* electrons). Such electrons are free to move through the conductor (as we learned in our discussion of drift speed in Section 21.1) and are approximately equal in number to the atoms. In the absence of an electric field, the free electrons move in random directions with average speeds on the order of 10^6 m/s. The situation is similar to the motion of gas molecules confined in a vessel. In fact, some scientists refer to conduction electrons in a metal as an *electron gas*.

The conduction electrons are not totally free, because they are confined to the interior of the conductor and undergo frequent collisions with the array of atoms. The collisions are the predominant mechanism for the resistivity of a metal at normal temperatures. Note that there is no current through a conductor in the absence of an electric field, because the average velocity of the free electrons is zero. In

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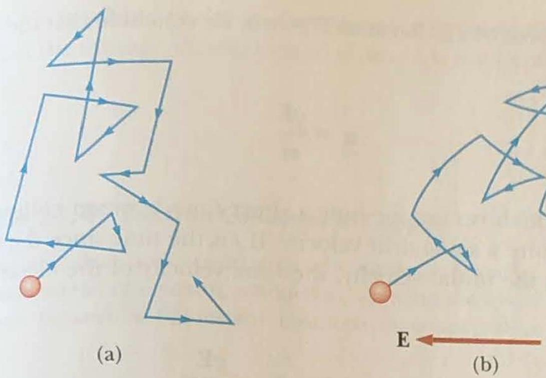

Figure 21.9 (a) A schematic diagram of the random motion of a charge carrier in a conductor in the absence of an electric field. The drift velocity is zero. (b) The motion of a charge carrier in a conductor in the presence of an electric field. Note that the random motion is modified by the field, and the charge carrier has a drift velocity.

other words, just as many electrons move in one direction as in the opposite direction, on the average, and so there is no net flow of charge.

The situation is modified when an electric field is applied to the metal. In addition to random thermal motion, the free electrons drift slowly in a direction opposite that of the electric field, with an average drift speed of v_d, which is much less (typically 10^{-4} m/s; see Example 21.1) than the average speed between collisions (typically 10^6 m/s). Figure 21.9 provides a crude depiction of the motion of free electrons in a conductor. In the absence of an electric field, there is no net displacement after many collisions (Fig. 21.9a). An electric field **E** modifies the random motion and causes the electrons to drift in a direction opposite that of **E** (Fig. 21.9b). The slight curvature in the paths in Figure 21.9b results from the acceleration of the electrons between collisions, caused by the applied field. One mechanical system somewhat analogous to this situation is a ball rolling down a slightly inclined plane through an array of closely spaced, fixed pegs (Fig. 21.10). The ball represents a conduction electron, the pegs represent defects in the crystal lattice, and the component of the gravitational force along the incline represents the electric force, $e\mathbf{E}$.

In our model, we shall assume that the excess energy acquired by the electrons in the electric field is lost to the conductor in the collision process. The energy given up to the atoms in the collisions increases the vibrational energy of the atoms, causing the conductor to warm up. The model also assumes that an electron's motion after a collision is independent of its motion *before* the collision.[3]

We are now in a position to obtain an expression for the drift velocity. When a mobile, charged particle of mass m and charge q is subjected to an electric field

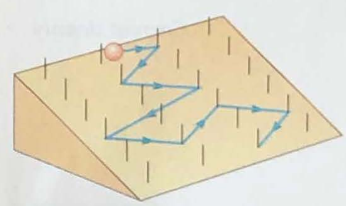

Figure 21.10 A mechanical system somewhat analogous to the motion of charge carriers in the presence of an electric field. The collisions of the ball with the pegs represent the resistance to the ball's motion down the incline.

[3]Because the collision process is random, each collision event is *independent* of what happened earlier. This is analogous to the random process of throwing a die. The probability of rolling a particular number on one throw is independent of the result of the previous throw. On the average, it would take six throws to come up with that number, starting at any arbitrary time.

$\mathbf{E}$, it experiences a force of $q\mathbf{E}$. Because $\mathbf{F} = m\mathbf{a}$, we conclude that the acceleration of the particle is

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \qquad [21.14]$$

This acceleration, which occurs for only a short time between collisions, enables the electrons to acquire a small drift velocity. If t is the time since the last collision (at $t = 0$), and $\mathbf{v}_0$ is the initial velocity, then the velocity of the electron after the time t is

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t = \mathbf{v}_0 + \frac{q\mathbf{E}}{m}t \qquad [21.15]$$

We now take the average value of $\mathbf{v}$ over all possible times t and all possible values of $\mathbf{v}_0$. If the initial velocities are assumed to be randomly distributed in space, we see that the average value of $\mathbf{v}_0$ is zero. The term $(qE/m)t$ is the velocity added by the field at the end of one trip between atoms. If the electron starts with zero velocity, the average value of the second term of Equation 21.15 is $(qE/m)\tau$, where τ is the *average time between collisions*. Because the average of $\mathbf{v}$ is equal to the drift velocity, we have

Drift velocity •

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m}\tau \qquad [21.16]$$

Substituting this result into Equation 21.6, we find that the magnitude of the current density is

Current density •

$$J = nqv_d = \frac{nq^2E}{m}\tau \qquad [21.17]$$

Comparing this expression with an alternative form of Equation 21.7,[4] $J = \sigma E$, we obtain the following relationships for the conductivity and resistivity:

Conductivity •

$$\sigma = \frac{nq^2\tau}{m} \qquad [21.18]$$

Resistivity •

$$\rho = \frac{1}{\sigma} = \frac{m}{nq^2\tau} \qquad [21.19]$$

According to this classical model, conductivity and resistivity do not depend on the electric field. This feature is characteristic of a conductor obeying Ohm's law. The model shows that the conductivity can be calculated from a knowledge of the density of the charge carriers, their charge and mass, and the average time between

[4]The relation $J = \sigma E$ can be derived as follows: The potential difference across a conductor of length ℓ is $\Delta V = E\ell$, and, from the definition of resistance, $\Delta V = IR$. Using these relations, together with Equations 21.5 and 21.10, we find that the magnitude of the current density is $J = I/A = \Delta V/RA = E\ell/RA = \sigma E$.

collisions, which is related to the average distance between collisions ℓ (the mean free path) and the average thermal speed $\bar{v}$ through the expression[5]

$$\tau = \frac{\ell}{\bar{v}} \qquad [21.20]$$

Example 21.4 Electron Collisions in Copper

(a) Using the data and results from Example 21.1 and the classical model of electron conduction, estimate the average time between collisions for electrons in copper at 20°C.

Solution From Equation 21.19 we see that

$$\tau = \frac{m}{nq^2\rho}$$

where $\rho = 1.7 \times 10^{-8}\ \Omega\cdot\text{m}$ for copper and the carrier density $n = 8.48 \times 10^{28}$ electrons/m^3 for the wire described in Example 21.1. Substitution of these values into the previous expression gives

$$\tau = \frac{(9.11 \times 10^{-31}\ \text{kg})}{(8.48 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.7 \times 10^{-8}\ \Omega\cdot\text{m})}$$

$$= 2.5 \times 10^{-14}\ \text{s}$$

(b) Assuming the average speed for free electrons in copper to be 1.6×10^6 m/s and using the result from part (a), calculate the mean free path for electrons in copper.

Solution

$$\ell = \bar{v}\tau = (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s}) = 4.0 \times 10^{-8}\ \text{m}$$

which is equivalent to 40 nm (compared with atomic spacings of about 0.2 nm). Thus, although the time between collisions is very short, the electrons travel about 200 atomic distances before colliding with an atom.

Although this classical model of conduction is consistent with Ohm's law, it is not satisfactory for explaining some important phenomena. For example, classical calculations for $\bar{v}$ using the ideal-gas model are about a factor of 10 smaller than the true values. Furthermore, according to Equations 21.19 and 21.20, the temperature variation of the resistivity is predicted to vary as $\bar{v}$, which, according to an ideal-gas model (Chap. 16, Eq. 16.15), is proportional to $\sqrt{T}$. This is in disagreement with the linear dependence of resistivity with temperature for pure metals (Fig. 21.5a). It is possible to account for such observations only by using a quantum mechanical model, which we shall describe briefly.

According to quantum mechanics, electrons have wave-like properties. If the array of atoms in a conductor is regularly spaced (that is, periodic), the wave-like character of the electrons makes it possible for them to move freely through the conductor, and a collision with an atom is unlikely. For an idealized conductor there would be no collisions, the mean free path would be infinite, and the resistivity would be zero. Electron waves are scattered only if the atomic arrangement is irregular (not periodic)—for example, as a result of structural defects or impurities. At low temperatures, the resistivity of metals is dominated by scattering caused by collisions between the electrons and impurities. At high temperatures, the resistivity is dominated by scattering caused by collisions between the electrons and the atoms of the conductor, which are continuously displaced as a result of thermal agitation. The thermal motion of the atoms makes the structure irregular (compared with an atomic array at rest), thereby reducing the electron's mean free path.

[5] Recall that the average speed is the average of the speeds that particles have as a consequence of the temperature of the system of particles (Chap. 16).

CONCEPTUAL PROBLEM 4

Why don't the free electrons in a metal fall to the bottom of the metal due to gravity? And charges in a conductor are supposed to reside on the surface—why don't the free electrons all go to the surface?

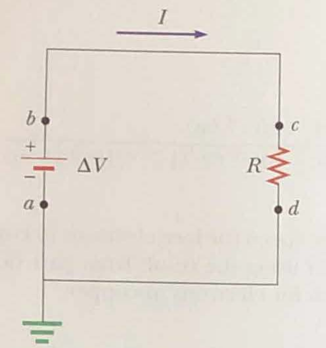

Figure 21.11 A circuit consisting of a battery of emf $\mathcal{E}$ and resistance R. Positive charge flows in the clockwise direction, from the negative to the positive terminal of the battery. Points a and d are grounded.

21.5 • ELECTRICAL ENERGY AND POWER

If a battery is used to establish an electric current in a conductor, there occurs a continuous transformation of chemical energy stored in the battery to kinetic energy of the charge carriers. This kinetic energy is quickly lost as a result of collisions between the charge carriers and the lattice ions, resulting in an increase in the temperature of the conductor. Thus, the chemical energy stored in the battery is continuously transformed into thermal energy.

In order to understand the process of energy transfer in a simple circuit, consider a battery the terminals of which are connected to a resistor R, as shown in Figure 21.11. (Remember that the positive terminal of the battery is always at the higher potential.) Now imagine following a positive quantity of charge ΔQ around the circuit from point a through the battery and resistor and back to a. Point a is a reference point that is grounded (the ground symbol is ⏚), and its potential is taken to be zero. As the charge moves from a to b through the battery the potential difference of which is ΔV, its electrical potential energy increases by the amount $\Delta Q\,\Delta V$, and the chemical potential energy in the battery decreases by the same amount. (Recall from Chap. 20 that $\Delta U = q\,\Delta V$.) However, as the charge moves from c to d through the resistor, it loses this electrical potential energy during collisions with atoms in the resistor, thereby producing thermal energy. Note that, if we neglect the resistance of the interconnecting wires, no loss in energy occurs for paths bc and da. When the charge returns to point a, it must have the same potential energy (zero) as it had at the start.[6]

The rate at which the charge ΔQ loses potential energy in going through the resistor is

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V = I\,\Delta V$$

where I is the current in the circuit. Of course, the charge regains this energy when it passes through the battery. Because the rate at which the charge loses energy equals the power P dissipated in the resistor, we have

$$P = I\,\Delta V \qquad [21.21]$$

In this case, the power is supplied to a resistor by a battery. However, Equation 21.21 can be used to determine the power transferred from a battery to *any* device carrying a current I and having a potential difference ΔV between its terminals.

Using Equation 21.21 and the fact that $\Delta V = IR$ for a resistor, we can express the power dissipated by the resistor in the alternative forms

This versatile circuit enables the experimenter to examine the properties of circuit elements such as capacitors and resistors and their effects on circuit behavior. *(Courtesy of Central Scientific Company)*

[6]Note that when the current reaches its steady-state value, there is *no* change with time in the kinetic energy associated with the current.

$$P = I^2R = \frac{(\Delta V)^2}{R} \qquad [21.22]$$

• *Power dissipated by a resistor*

The SI unit of power is the watt, introduced in Chapter 6. The power dissipated in a conductor of resistance R is also often referred to as an *I^2R loss.*

As we learned in Chapter 6, Section 6.5, the unit of energy the electric company uses to calculate energy consumption, the kilowatt-hour, is the energy consumed in 1 h at the constant rate of 1 kW. Because 1 W = 1 J/s, we have

$$1 \text{ kWh} = (10^3 \text{ W})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J} \qquad [21.23]$$

Thinking Physics 3

When is more power being delivered to a lightbulb—just after it is turned on and the glow of the filament is increasing or after it has been on for a few seconds and the glow is steady?

Reasoning Once the switch is closed, the line voltage is applied across the lightbulb. As the voltage is applied across the cold filament when first turned on, the resistance of the filament is low. Thus, the current is high, and a relatively large amount of power is delivered to the bulb. As the filament warms up, its resistance rises, and the current falls. As a result, the power delivered to the bulb falls. The large current spike at the beginning of operation is the reason that lightbulbs often fail just as they are turned on as noted in Conceptual Problem 3.

Thinking Physics 4

Two lightbulbs A and B are connected across the same potential difference, as in Figure 21.12. The resistance of A is twice that of B. Which lightbulb dissipates more power? Which carries the greater current?

Reasoning Because the voltage across each lightbulb is the same, and the power dissipated by a conductor is $P = (\Delta V)^2/R$, the conductor with the lower resistance will dissipate more power. In this case, the power dissipated by B is twice that of A and provides twice as much illumination. Furthermore, because $P = I\,\Delta V$, we see that the current carried by B is twice that of A.

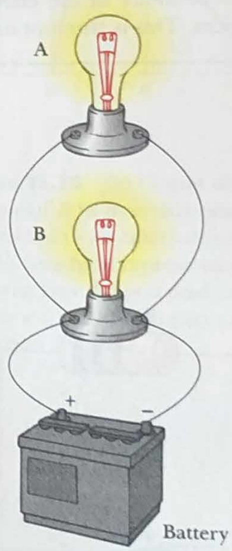

Figure 21.12 (Thinking Physics 4)

Example 21.5 Electrical Rating of a Lightbulb

A lightbulb is rated at 120 V/75 W, which means its operating voltage is 120 V and it has a power rating of 75.0 W. The bulb is powered by a 120-V direct-current power supply. Find the current in the bulb and its resistance.

Solution Because the power rating of the bulb is 75.0 W and the operating voltage is 120 V, we can use $P = I\,\Delta V$ to find the current:

$$I = \frac{P}{\Delta V} = \frac{75.0 \text{ W}}{120 \text{ V}} = 0.625 \text{ A}$$

Using $\Delta V = IR$, the resistance is calculated to be

$$R = \frac{\Delta V}{I} = \frac{120 \text{ V}}{0.625 \text{ A}} = 192\ \Omega$$

EXERCISE 5 What would the resistance be in a lamp rated at 120 V and 100 W? Answer 144 Ω

Example 21.6 The Cost of Operating a Lightbulb

How much does it cost to burn a 100-W lightbulb for 24 h if electricity costs eight cents per kilowatt hour?

Solution Because the energy consumed equals power × time, the amount of energy you must pay for, expressed in kWh, is

$$\text{Energy} = (0.10\ \text{kW})(24\ \text{h}) = 2.4\ \text{kWh}$$

If energy is purchased at 8¢ per kWh, the cost is

$$\text{Cost} = (2.4\ \text{kWh})(\$0.080/\text{kWh}) = \$0.19$$

That is, it costs 19¢ to operate the lightbulb for one day. This is a small amount, but when larger and more complex devices are being used, the costs go up rapidly.

Demands on our energy supplies have made it necessary to be aware of the energy requirements of our electric devices. This is true not only because they are becoming more expensive to operate but also because, with the dwindling of the coal and oil resources that ultimately supply us with electrical energy, increased awareness of conservation becomes necessary. On every electric appliance is a label that contains the information you need to calculate the power requirements of the appliance. The power consumption in watts is often stated directly, as on a lightbulb. In other cases, the amount of current used by the device and the voltage at which it operates are given. This information and Equation 21.2 are sufficient to calculate the operating cost of any electric device.

EXERCISE 6 If electricity costs 8¢ per kilowatt hour, what does it cost to operate an electric oven, which operates at 20.0 A and 220 V, for 5.00 h? Answer $1.76

EXERCISE 7 A 12-V battery is connected to a 60-Ω resistor. Neglecting the internal resistance of the battery, calculate the power dissipated in the resistor. Answer 2.4 W

21.6 • SOURCES OF emf

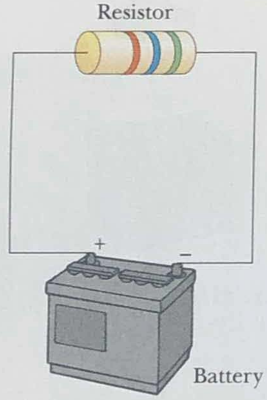

Figure 21.13 A circuit consisting of a resistor connected to the terminals of a battery.

The source that maintains the constant voltage in Figure 21.13 is called an "emf."[7] Sources of emf are any devices (such as batteries and generators) that increase the potential energy of charges circulating in circuits. One can think of a source of emf as a charge pump that forces electrons to move in a direction opposite the electrostatic field inside the source. The emf, $\mathcal{E}$, of a source describes the work done per unit charge, and hence the SI unit of emf is the volt.

Consider the circuit shown in Figure 21.13, consisting of a battery connected to a resistor. We shall assume that the connecting wires have no resistance. If we neglect the internal resistance of the battery, the potential difference across the battery (the terminal voltage) equals the emf of the battery. However, because a real battery always has some internal resistance, r, the terminal voltage is not equal to the emf. The circuit shown in Figure 21.13 can be described by the circuit diagram in Figure 21.14a. The battery within the dashed rectangle is represented by a source of emf, $\mathcal{E}$, in series with the internal resistance r. Now imagine a positive charge moving from a to b in Figure 21.14a. As the charge passes from the negative to the positive terminal within the battery, the potential of the charge increases by $\mathcal{E}$. However, as it moves through the resistance r, its potential decreases by an amount Ir, where I is the current in the circuit. Thus, the terminal voltage of the battery, $\Delta V = V_b - V_a$, is[8]

[7]The term was originally an abbreviation for *electromotive force*, but it is not a force, so the long form is discouraged.

[8]The terminal voltage in this case is less than the emf by the amount Ir. In some situations, the terminal voltage may *exceed* the emf by the amount Ir. This happens when the direction of the current is opposite that of the emf, as when a battery is charged with another source of emf.

$$\Delta V = \mathcal{E} - Ir \qquad [21.24]$$

Note from this expression that $\mathcal{E}$ is equivalent to the **open-circuit voltage—that is, the terminal voltage when the current is zero.** Figure 21.14b is a graphical representation of the changes in potential as the circuit is traversed clockwise. By inspecting Figure 21.14a, we see that the terminal voltage ΔV must also equal the potential difference across the external resistance R, often called the **load resistance;** that is, $\Delta V = IR$. Combining this with Equation 21.24, we see that

$$\mathcal{E} = IR + Ir \qquad [21.25]$$

Solving for the current gives

$$I = \frac{\mathcal{E}}{R + r}$$

This shows that the current in this simple circuit depends on both the resistance external to the battery and the internal resistance. If R is much greater than r, we can neglect r in our analysis. In many circuits we shall ignore this internal resistance.

If we multiply Equation 21.25 by the current I, we get

$$I\mathcal{E} = I^2R + I^2r$$

This equation tells us that the total power output of the source of emf, $I\mathcal{E}$, is equal to the power that is dissipated in the load resistance, I^2R, *plus* power that is dissipated in the internal resistance, I^2r. Again, if $r \ll R$, most of the power delivered by the battery is dissipated in the load resistance.

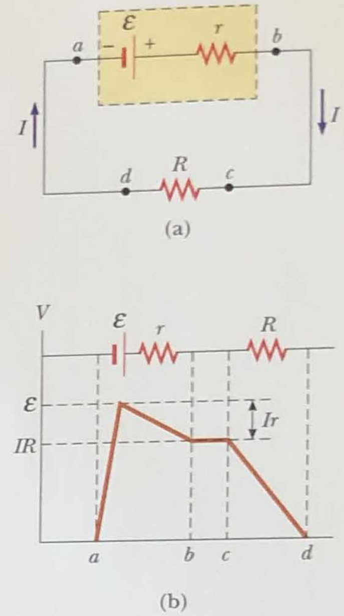

Figure 21.14 (a) Circuit diagram of an emf $\mathcal{E}$ of internal resistance r connected to an external resistor R. (b) Graphical representation showing how the potential changes as the series circuit in part (a) is traversed clockwise.

CONCEPTUAL PROBLEM 5

If the energy transferred to a dead battery during charging is E, is the total energy transferred out of the battery to an electrical load during use in which it completely discharges also E?

CONCEPTUAL PROBLEM 6

If you have your headlights on while you start your car, why do they dim while the car is starting?

CONCEPTUAL PROBLEM 7

Electrical devices are often rated with a voltage and a current—for example, 120 volts, 5 amperes. Batteries, however, are only rated with a voltage—for example, 1.5 volts. Why?

EXERCISE 8 A battery with an emf of 12 V and an internal resistance of 0.90 Ω is connected across a load resistor R. If the current in the circuit is 1.4 A, what is the value of R?
Answer 7.7 Ω

21.7 • RESISTORS IN SERIES AND IN PARALLEL

When two or more resistors are connected together so that they have only one common point per pair, they are said to be in *series*. Figure 21.15 shows two resistors connected in series. Note that the current is the same through the two resistors,

• ***For a series connection of resistors, the current is the same in all the resistors.***

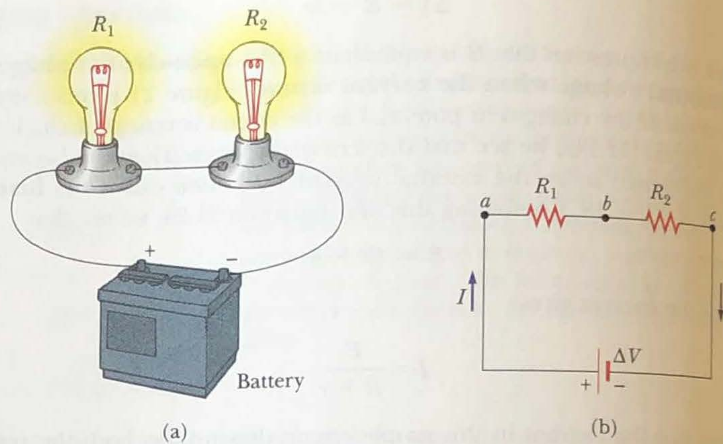

Figure 21.15 Series connection of two resistors, R_1 and R_2. The current is the same in each resistor.

because any charge that flows through R_1 must also flow through R_2. Because the potential difference between a and b in Figure 21.15b equals IR_1 and the potential difference between b and c equals IR_2, the potential difference between a and c is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Therefore, we can replace the two resistors in series with a single **equivalent resistance,** R_{eq}, the value of which is the *sum* of the individual resistances:

$$R_{eq} = R_1 + R_2 \qquad [21.26]$$

The resistance R_{eq} is equivalent to the series combination $R_1 + R_2$ in the sense that the circuit current is unchanged when R_{eq} replaces $R_1 + R_2$. The equivalent resistance of three or more resistors connected in series is simply

Equivalent resistance of several resistors in series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad [21.27]$$

Therefore, **the equivalent resistance of a series connection of resistors is always greater than any individual resistance and is the algebraic sum of the individual resistances.**

Note that if the filament of one lightbulb in Figure 21.15 were to break, or "burn out," the circuit would no longer be complete (an open-circuit condition would exist) and the second bulb would also go out. Some Christmas-tree-light sets (especially older ones) are connected in this way, and the tedious task of determining which bulb is burned out is familiar to many people.

In many circuits, fuses are used in series with other circuit elements for safety purposes. The conductor in the fuse is designed to melt and open the circuit at some maximum current, the value of which depends on the nature of the circuit. If a fuse were not used, excessive currents could damage circuit elements, overheat wires, and perhaps cause a fire. In modern home construction, circuit breakers are used in place of fuses. When the current in a circuit exceeds some value (typically 15 A), the circuit breaker acts as a switch and opens the circuit.

Now consider two resistors connected in *parallel,* as shown in Figure 21.16. In this case, **the potential differences across the resistors are equal.** However, the currents are generally not the same. When the current I reaches point a (called a *junction*) in Figure 21.16b, it splits into two parts, I_1 going through R_1 and I_2 going through R_2. If R_1 is greater than R_2, then I_1 is less than I_2. Clearly, because charge must be conserved, the current I that enters point a must equal the total current leaving point b:

$$I = I_1 + I_2$$

The potential drops across the resistors must be the *same,* and applying $I = \Delta V/R$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

From this result, we see that the equivalent resistance of two resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad [21.28]$$

This can be rearranged to become

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

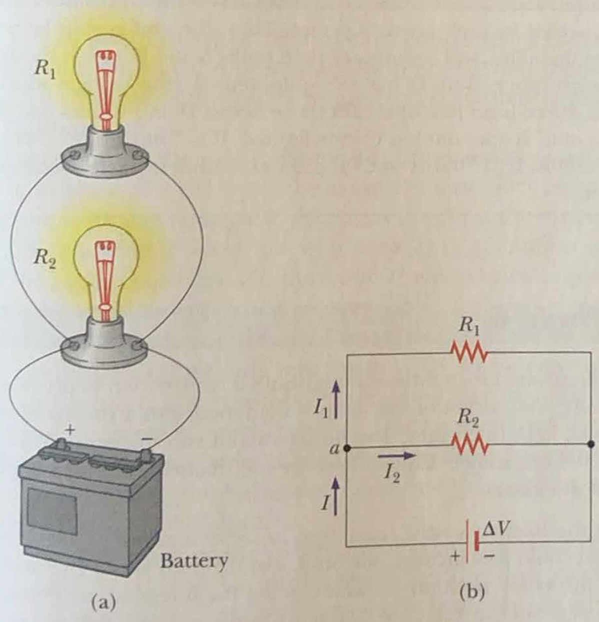

Figure 21.16 Parallel connection of two resistors, R_1 and R_2. The potential difference across each resistor is the same, and the equivalent resistance of the combination is given by $R_{eq} = R_1R_2/(R_1 + R_2)$.

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An extension of this analysis to three or more resistors in parallel yields the following general expression:

Equivalent resistance of several resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad [21.29]$$

From this expression it can be seen that **the equivalent resistance of two or more resistors connected in parallel is always less than the smallest resistance in the group and the inverse of the equivalent resistance is the algebraic sum of the inverses of the individual resistances.**

Household circuits are always wired so that the lightbulbs (or appliances, or whatever) are connected in parallel, as in Figure 21.16a. In this manner, each device operates independently of the others, so that if one is switched off, the others remain on. Equally important, each device operates on the same voltage.

Finally, it is interesting to note that parallel resistors combine in the same way that series capacitors combine, and vice versa.

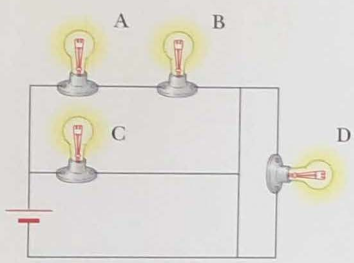


Figure 21.17 (Thinking Physics 5)

Thinking Physics 5

Predict the relative brightnesses of the four identical bulbs in Figure 21.17. What happens if bulb A "burns out," so that it cannot conduct current? What if C "burns out"? What if D "burns out"?

Reasoning Bulbs A and B are connected in series across the emf of the battery, whereas bulb C is connected by itself across this emf. Thus, the emf is split between bulbs A and B. As a result, bulb C will be brighter than bulbs A and B, which should be equally as bright as each other. Bulb D has an equipotential (the vertical wire) connected across it. Thus, there is no potential difference across D and it does not glow at all. If bulb A "burns out," B goes out but C stays lighted. If C "burns out," there is no effect on the other bulbs. If D "burns out," the event is undetectable, because D was not glowing anyway.

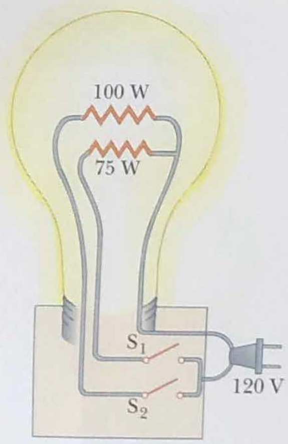


Figure 21.18 (Thinking Physics 6)

Thinking Physics 6

Figure 21.18 illustrates how a three-way lightbulb is constructed to provide three levels of light intensity. The socket of the lamp is equipped with a three-way switch for selecting different light intensities. The bulb contains two filaments. Why are the filaments connected in parallel? Explain how the two filaments are used to provide three different light intensities.

Reasoning If the filaments were connected in series and one of them were to burn out, no current could pass through the bulb, and the bulb would give no illumination, regardless of the switch position. However, when the filaments are connected in parallel and one of them (say the 75-W filament) burns out, the bulb will still operate in one of the switch positions as current passes through the other (100 W) filament. The three light intensities are made possible by selecting one of three values of filament resistance, using a single value of 120 V for the applied voltage. The 75-W filament offers one value of resistance, the 100-W filament offers a second value, and the third resistance is obtained by combining the two filaments in parallel. When switch 1 is closed and switch 2 is opened, current passes only through the 75-W filament. When switch 1 is open and switch 2 is closed, current passes only through the 100-W filament. When both switches are closed, current passes through both filaments, and a total illumination of 175 W is obtained.

CONCEPTUAL PROBLEM 8

Connecting batteries in series increases the emf. What advantage might there be in connecting them in parallel?

CONCEPTUAL PROBLEM 9

You have a large supply of lightbulbs and a battery. You start with one lightbulb connected to the battery and notice its brightness. You then add one lightbulb at a time, each new bulb being added in series to the previous bulbs. As you add the lightbulbs, what happens to the brightness of the bulbs? To the current through the bulbs? To the power transferred from the battery? To the lifetime of the battery? To the terminal voltage of the battery? Answer the same questions if the lightbulbs are added one by one in parallel with the first.

PROBLEM-SOLVING STRATEGY • Resistors

1. When two or more unequal resistors are connected in *series,* they carry the same current, but the potential differences across them are not the same. The resistors add directly to give the equivalent resistance of the series combination.
2. When two or more unequal resistors are connected in *parallel,* the potential differences across them are the same. Because the current is inversely proportional to the resistance, the currents through them are not the same. The equivalent resistance of a parallel combination of resistors is found through reciprocal addition, and the equivalent resistor is always *less* than the smallest individual resistor.
3. A complicated circuit consisting of resistors can often be reduced to a simple circuit containing only one resistor. To do so, examine the initial circuit and replace any resistors in series or any in parallel using the procedures outlined in Steps 1 and 2. Draw a sketch of the new circuit after these changes have been made. Examine the new circuit and replace any series or parallel combinations. Continue this process until a single equivalent resistance is found.
4. If the current through or the potential difference across a resistor in the complicated circuit is to be found, start with the final circuit found in Step 3 and gradually work your way back through the circuits, using $\Delta V = IR$ and the rules of Steps 1 and 2.

Example 21.7 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 21.19a. (a) Find the equivalent resistance between a and c.

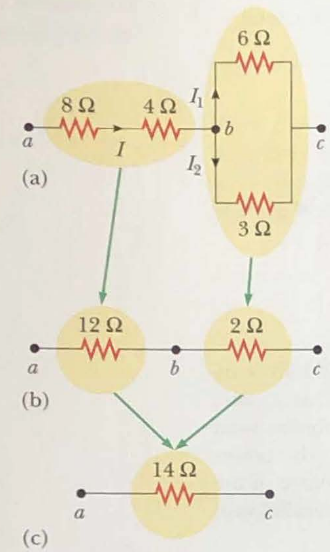


Figure 21.19 (Example 21.17) The resistances of the four resistors shown in (a) can be reduced in steps to an equivalent 14-Ω resistor.

Solution The circuit can be reduced in steps as shown in Figure 21.19. The 8.0-Ω and 4.0-Ω resistors are in series, and so the equivalent resistance between a and b is 12 Ω (Eq. 21.26). The 6.0-Ω and 3.0-Ω resistors are in parallel, and so from Equation 21.28 we find that the equivalent resistance from b to c is 2.0 Ω. Hence, the equivalent resistance from a to c is 14 Ω.

(b) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?

Solution The current I in the 8.0-Ω and 4.0-Ω resistors is the same because they are in series. Using Equation 21.7 and the results from part (a), we get

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14\ \Omega} = 3.0\ \text{A}$$

When this current enters the junction at b, it splits. Part of it passes through the 6.0-Ω resistor (I_1) and part goes through the 3.0-Ω resistor (I_2). Because the potential difference across these resistors, ΔV_{bc}, is the same (they are in parallel), we see that $6I_1 = 3I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this from the start by noting that the current through the 3.0-Ω resistor has to be twice the current through the 6.0-Ω resistor in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check, note that $\Delta V_{bc} = 6I_1 = 3I_2 = 6.0$ V and $\Delta V_{ab} = 12I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

Example 21.8 Three Resistors in Parallel

Three resistors are connected in parallel, as in Figure 21.20. A potential difference of 18 V is maintained between points a and b. (a) Find the current in each resistor.

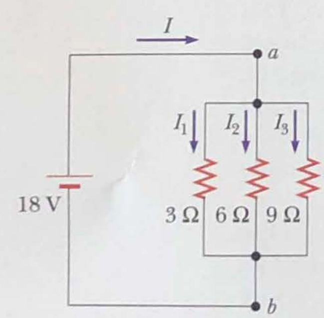


Figure 21.20 (Example 21.8) Three resistors connected in parallel. The voltage across each resistor is 18 V.

Solution The resistors are in parallel, and the potential difference across each is 18 V. Applying $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\ \text{V}}{6.0\ \Omega} = 3.0\ \text{A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}$$

(b) Calculate the power dissipated by each resistor and the total power dissipated by the three resistors.

Solution Applying $P = I^2R$ to each resistor gives

$$3.0\text{-}\Omega\text{:}\quad P_1 = I_1^{\,2}R_1 = (6.0\ \text{A})^2(3.0\ \Omega) = 110\ \text{W}$$

6.0-Ω: $P_2 = I_2^2 R_2 = (3.0\text{ A})^2(6.0\ \Omega) = 54\text{ W}$

9.0-Ω: $P_3 = I_3^2 R_3 = (2.0\text{ A})^2(9.0\ \Omega) = 36\text{ W}$

This shows that the smallest resistor dissipates the most power because it carries the most current. (Note that you can also use $P = (\Delta V)^2/R$ to find the power dissipated by each resistor.) Summing the three quantities gives a total power of 200 W.

(c) Calculate the equivalent resistance of the three resistors. We can use Equation 21.28 to find R_{eq}:

Solution

$$\frac{1}{R_{eq}} = \frac{1}{3.0} + \frac{1}{6.0} + \frac{1}{9.0}$$

$$R_{eq} = \frac{18}{11}\ \Omega = 1.6\ \Omega$$

EXERCISE 9 Use R_{eq} to calculate the total power dissipated in the circuit. Answer 200 W

21.8 • KIRCHHOFF'S RULES AND SIMPLE DC CIRCUITS

As indicated in the preceding section, we can analyze simple circuits using $\Delta V = IR$ and the rules for series and parallel combinations of resistors. However, there are many ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor. The procedure for analyzing such complex circuits is greatly simplified by the use of two simple rules called **Kirchhoff's rules:**

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction. (This rule is often referred to as the **junction rule.**)
2. The sum of the potential differences across each element around any closed circuit loop must be zero. (This rule is usually called the **loop rule.**)

The junction rule is a statement of **conservation of charge.** Whatever current enters a given point in a circuit must leave that point, because charge cannot build up or disappear at a point. If we apply this rule to the junction in Figure 21.21a, we get

$$I_1 = I_2 + I_3$$

Figure 21.21b represents a mechanical analog to this situation in which water flows through a branched pipe with no leaks. The flow rate into the pipe equals the total flow rate out of the two branches.

The second rule is equivalent to the law of **conservation of energy.** A charge that moves around any closed loop in a circuit (the charge starts and ends at the same point) must gain as much energy as it loses if a potential is defined for each point in the circuit. Its energy may decrease in the form of a potential drop, $-IR$, across a resistor or as a result of having the charge move in the reverse direction through an emf. In the latter case, electric potential energy is converted to chemical energy as the battery is charged. In a similar way, electrical energy may be converted to mechanical energy for operating a motor.

As an aid in applying the loop rule, the following points should be noted. They are summarized in Figure 21.22, where it is assumed that movement is from point a toward point b:

- If a resistor is traversed in the direction of the current, the change in potential across the resistor is $-IR$ (Fig. 21.22a).

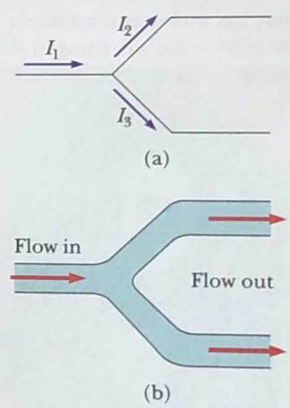


Figure 21.21 (a) A schematic diagram illustrating Kirchhoff's junction rule. Conservation of charge requires that whatever current enters a junction must leave that junction. Therefore, in this case, $I_1 = I_2 + I_3$. (b) A mechanical analog of the junction rule: The flow out must equal the flow in.

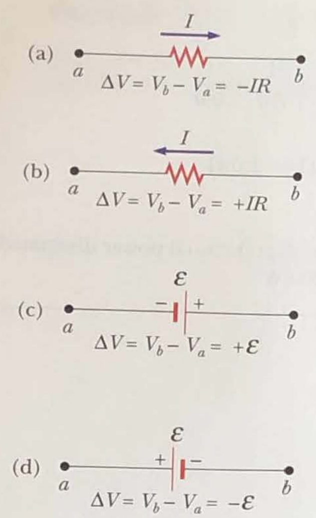


Figure 21.22 Rules for determining the potential changes across a resistor and a battery, assuming the battery has no internal resistance.

- If a resistor is traversed in the direction *opposite* the current, the change in potential across the resistor is $+IR$ (Figure 21.22b).
- If a source of emf is traversed in the direction of the emf (from − to + on the terminals), the change in potential is $+\mathcal{E}$ (Fig. 21.22c).
- If a source of emf is traversed in the direction opposite the emf (from + to − on the terminals), the change in potential is $-\mathcal{E}$ (Fig. 21.22d).

There are limitations on the use of the junction rule and the loop rule. You may use the junction rule as often as needed so long as each time you write an equation, you include in it a current that has not been used in a previous junction rule equation. In general, the number of times the junction rule must be used is one fewer than the number of junction points in the circuit. The loop rule can be used as often as needed so long as a new circuit element (a resistor or battery) or a new current appears in each new equation. In general, **the number of independent equations you need must equal the number of unknowns in order to solve a particular circuit problem.**

The following examples illustrate the use of Kirchhoff's rules in analyzing circuits. In all cases, it is assumed that the circuits have reached steady-state conditions—that is, the currents in the various branches are constant. If a capacitor is included as an element in one of the branches, **it acts as an open circuit: The current in the branch containing the capacitor is zero under steady-state conditions.**

Gustav Robert Kirchhoff (1824–1887). *(Courtesy of North Wind Picture Archives)*

PROBLEM-SOLVING STRATEGY AND HINTS • Kirchhoff's Rules

1. First, draw the circuit diagram and assign labels to all the known quantities and symbols to all the unknown quantities. You must assign *directions* to the currents in each part of the circuit. Do not be alarmed if you guess the direction of a current incorrectly; the result will have a negative value, but *its magnitude will be correct.* Although the assignment of current directions is arbitrary, you must adhere *rigorously* to the directions you assigned when you apply Kirchhoff's rules.
2. Apply the junction rule (Kirchhoff's first rule) to all but one of the junctions in the circuit; doing so provides independent equations relating the currents. (This step is easy!)
3. Now apply the loop rule (Kirchhoff's second rule) to as many loops in the circuit as are needed to solve for the unknowns. In order to apply this rule, you must correctly identify the change in potential as you cross each element in traversing the closed loop (either clockwise or counterclockwise). Watch out for signs!
4. Solve the equations simultaneously for the unknown quantities. Be careful in your algebraic steps, and check your numerical answers for consistency.

Example 21.9 Applying Kirchhoff's Rules

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 21.23.

Reasoning We choose the directions of the currents as in Figure 21.23. Applying Kirchhoff's first rule to junction *c* gives

$$(1) \quad I_1 + I_2 = I_3$$

There are three loops in the circuit, *abcda*, *befcb*, and *aefda* (the outer loop). Therefore, we need only two loop equations to determine the unknown currents. The third loop equation

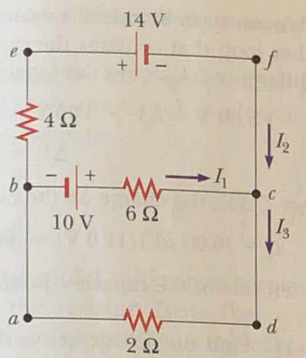


Figure 21.23 (Example 21.9) A circuit containing three loops.

would give no new information. Applying Kirchhoff's second rule to loops *abcda* and *befcb* and traversing these loops in the clockwise direction, we obtain the expressions

(2) Loop *abcda*: $10\text{ V} - (6\ \Omega)I_1 - (2\ \Omega)I_3 = 0$

(3) Loop *befcb*: $-(4\ \Omega)I_2 - 14\text{ V} + (6\ \Omega)I_1 - 10\text{ V} = 0$

Note that in loop *befcb*, a positive sign is obtained when traversing the 6-Ω resistor, because the direction of the path is opposite the direction of I_1. A third loop equation for *aefda* gives $-14 = 2I_3 + 4I_2$, which is just the sum of (2) and (3).

Solution Expressions (1), (2), and (3) represent three independent equations with three unknowns. We can solve the problem as follows: Substituting (1) into (2) gives

$$10 - 6I_1 - 2(I_1 + I_2) = 0$$

(4) $10 = 8I_1 + 2I_2$

Dividing each term in (3) by 2 and rearranging the equation gives

(5) $-12 = -3I_1 + 2I_2$

Subtracting (5) from (4) eliminates I_2, giving

$$22 = 11I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in (5) gives a value for I_2:

$$2I_2 = 3I_1 - 12 = 3(2) - 12 = -6$$

$$I_2 = -3\text{ A}$$

Finally, $I_3 = I_1 + I_2 = -1$ A. Hence, the currents have the values

$$I_1 = 2\text{ A} \qquad I_2 = -3\text{ A} \qquad I_3 = -1\text{ A}$$

The fact that I_2 and I_3 are both negative indicates only that we chose the wrong direction for these currents. However, the numerical values are correct.

EXERCISE 10 Find the potential difference between points *b* and *c*. Answer $V_b - V_c = 2$ V

Example 21.10 A Multiloop Circuit

(a) Under steady-state conditions, find the unknown currents in the multiloop circuit shown in Figure 21.24.

Reasoning First note that the capacitor represents an open circuit, and hence there is no current along path *ghab* under steady-state conditions. Therefore, $I_{fg} = I_{gb} = I_{bc} \equiv I_1$. Labeling the currents as shown in Figure 21.24 and applying Kirchhoff's first rule to junction *c*, we get

(1) $I_1 + I_2 = I_3$

Kirchhoff's second rule applied to loops *defcd* and *cfgbc* gives

(2) Loop *defcd*: $4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$

(3) Loop *cfgbc*: $(3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$

Solution From (1) we see that $I_1 = I_3 - I_2$, which when substituted into (3) gives

(4) $8.00\text{ V} - (5.00\ \Omega)I_3 + (8.00\ \Omega)I_2 = 0$

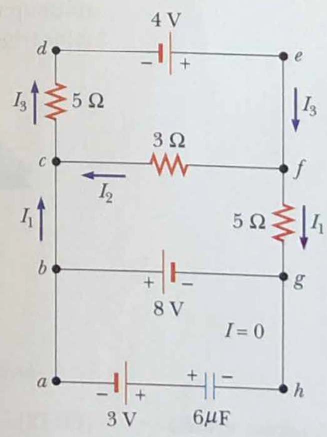


Figure 21.24 (Example 21.10) A multiloop circuit. Note that Kirchhoff's loop equation can be applied to *any* closed loop, including one containing the capacitor.

Subtracting (4) from (2), we eliminate I_3 and find

$$I_2 = -\frac{4.00}{11.0}\text{ A} = -0.364\text{ A}$$

Because I_2 is negative, we conclude that the direction of I_2 is from c to f through the 3.00-Ω resistor. Using this value of I_2 in (3) and (1) gives the following values for I_1 and I_3:

$$I_1 = 1.38\text{ A} \qquad I_3 = 1.02\text{ A}$$

Under steady-state conditions, the capacitor represents an *open* circuit, and so there is no current in the branch *ghab*.

(b) What is the charge on the capacitor?

Solution We can apply Kirchhoff's second rule to loop abgha (or any other loop that contains the capacitor) to find the potential difference ΔV_c across the capacitor:

$$-8.00\text{ V} + \Delta V_c - 3.00\text{ V} = 0$$

$$\Delta V_c = 11.0\text{ V}$$

Because $Q = C\,\Delta V_c$, the charge on the capacitor is

$$Q = (6.00\ \mu\text{F})(11.0\text{ V}) = 66.0\ \mu\text{C}$$

Why is the left side of the capacitor positively charged?

EXERCISE 11 Find the voltage across the capacitor by traversing any other loop. Answer 11.0 V

21.9 • *RC* CIRCUITS

So far we have discussed circuits with constant currents, or so-called *steady-state circuits*. We shall now consider circuits containing capacitors, in which the currents may vary in time.

Charging a Capacitor

Consider the series circuit shown in Figure 21.25. Let us assume that the capacitor is initially uncharged. There is no current when switch S is open (Fig. 21.25b). If the switch is closed at $t = 0$, charges begin to flow, setting up a current in the circuit, and the capacitor begins to charge (Fig. 21.25c). Note that during the charging, charges do not jump across the plates of the capacitor, because the gap between the plates represents an open circuit. Instead, electrons move from the top plate to the bottom plate only by moving through the resistor, switch, and battery until the capacitor is fully charged. The value of the maximum charge depends on the

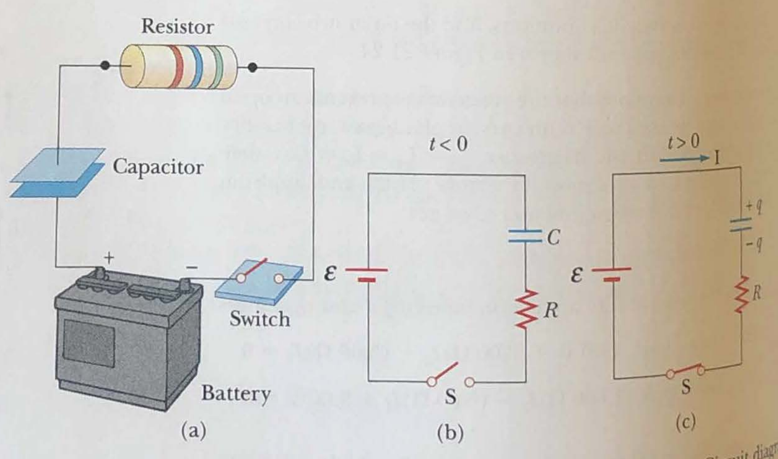


Figure 21.25 (a) A capacitor in series with a resistor, battery, and switch. (b) Circuit diagram representing this system before the switch is closed, $t < 0$. (c) Circuit diagram after the switch is closed, $t > 0$.

emf of the battery. Once the maximum charge is reached, the current in the circuit is zero.

To put this discussion on a quantitative basis, let us apply Kirchhoff's second rule to the circuit after the switch is closed. Choosing clockwise as our direction around the circuit, we get

$$\mathcal{E} - \frac{q}{C} - IR = 0 \qquad [21.30]$$

where q/C is the potential difference across the capacitor and IR is the potential difference across the resistor. Note that q and I are *instantaneous* values of the charge and current, respectively, as the capacitor is charged.

We can use Equation 21.30 to find the initial current in the circuit and the maximum charge on the capacitor. At $t = 0$, when the switch is closed, the charge on the capacitor is zero, and from Equation 21.30 we find that the initial current in the circuit I_0 is a maximum and equal to

$$I_0 = \frac{\mathcal{E}}{R} \quad \text{(current at } t = 0) \qquad [21.31]$$

• ***Maximum current***

At this time, **the potential difference is entirely across the resistor.** Later, when the capacitor is charged to its maximum value Q, charges cease to flow, the current in the circuit is zero, and **the potential difference is entirely across the capacitor.** Substituting $I = 0$ into Equation 21.30 yields the following expression for Q:

$$Q = C\mathcal{E} \quad \text{(maximum charge)} \qquad [21.32]$$

• ***Maximum charge on the capacitor***

To determine analytical expressions for the time dependence of the charge and current, we must solve Equation 21.30, a single equation containing two variables, q and I. In order to do this, let us substitute $I = dq/dt$ and rearrange the equation:

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

An expression for q may be found in the following way. Rearrange the equation by placing terms involving q on the left side and those involving t on the right side. Then integrate both sides:

$$\frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\,dt$$

$$\int_0^q \frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

From the definition of the natural logarithm, we can write this expression as

$$q(t) = C\mathcal{E}\,[1 - e^{-t/RC}] = Q[1 - e^{-t/RC}] \qquad [21.33]$$

• ***Charge versus time for a capacitor being charged through a resistor***

where e is the base of the natural logarithm and $Q = C\mathcal{E}$ is the *maximum* charge on the capacitor.

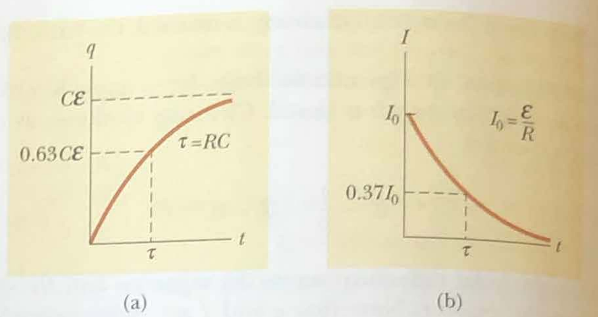


Figure 21.26 (a) Plot of capacitor charge versus time for the circuit shown in Figure 21.25. After one time constant, τ, the charge is 63.2% of the maximum value, $C\mathcal{E}$. The charge approaches its maximum value as t approaches infinity. (b) Plot of current versus time for the RC circuit shown in Figure 21.25. The current has its maximum value, $I_0 = \mathcal{E}/R$, at $t = 0$ and decays to zero exponentially as t approaches infinity. After one time constant, τ, the current decreases to 36.8% of its initial value.

An expression for the charging current may be found by differentiating Equation 21.33 with respect to time. Using $I = dq/dt$, we obtain

Current versus time •

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \qquad [21.34]$$

where $\mathcal{E}/R$ is the initial current in the circuit.

Plots of charge and current versus time are shown in Figure 21.26. Note that the charge is zero at $t = 0$ and approaches the maximum value of $C\mathcal{E}$ as $t \to \infty$ (Fig. 21.26a). Furthermore, the current has its maximum value of $I_0 = \mathcal{E}/R$ at $t = 0$ and decays exponentially to zero as $t \to \infty$ (Fig. 21.26b). The quantity RC, which appears in the exponential of Equations 21.33 and 21.34, is called the **time constant,** τ, of the circuit. It represents the time it takes the current to decrease to $1/e$ of its initial value; that is, in the time τ, $I = e^{-1} I_0 = 0.37I_0$. In a time of 2τ, $I = e^{-2}I_0 = 0.135I_0$, and so forth. Likewise, in a time τ the charge increases from zero to $C\mathcal{E}[1 - e^{-1}] = 0.63C\mathcal{E}$.

The following dimensional analysis shows that τ has units of time:

$$[\tau] = [RC] = \left[\frac{\Delta V}{I} \times \frac{Q}{\Delta V}\right] = \left[\frac{Q}{Q/t}\right] = \mathrm{T}$$

The energy decrease of the battery during the charging process is $Q\mathcal{E} = C\mathcal{E}^2$. After the capacitor is fully charged, the energy stored in it is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$, which is just half the energy decrease of the battery. It is left to an end-of-chapter problem to show that the remaining half of the energy supplied by the battery goes into thermal energy dissipated in the resistor (Problem 52).

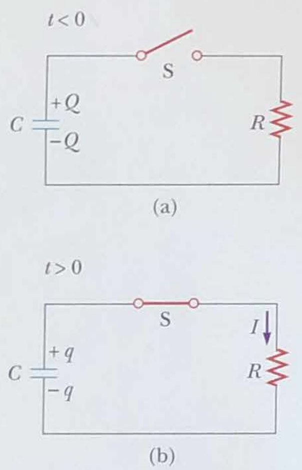


Figure 21.27 (a) A charged capacitor connected to a resistor and a switch, which is open at $t < 0$. (b) After the switch is closed, a nonsteady current is set up in the direction shown and the charge on the capacitor decreases exponentially with time.

Discharging a Capacitor

Now consider the circuit in Figure 21.27, consisting of a capacitor with an initial charge of Q, a resistor, and a switch. When the switch is open (Fig. 21.27a), there is a potential difference of Q/C across the capacitor and zero potential difference across the resistor, because $I = 0$. If the switch is closed at $t = 0$, the capacitor begins to discharge through the resistor. At some time during the discharge, the current in the circuit is I and the charge on the capacitor is q (Fig. 21.27b). From Kirchhoff's second rule, we see that the potential difference across the resistor, IR, must equal the potential difference across the capacitor, q/C:

$$IR = \frac{q}{C} \qquad [21.35]$$

However, **the current in the circuit must equal the rate of decrease of charge on the capacitor.** That is, $I = -dq/dt$, and so Equation 21.35 becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$, gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{-t/RC} \qquad [21.36]$$

• *Charge versus time for a discharging capacitor*

Differentiating Equation 21.36 with respect to time gives the current as a function of time:

$$I(t) = -\frac{dq}{dt} = I_0 e^{-t/RC} \qquad [21.37]$$

• *Current versus time for a discharging capacitor*

where the initial current is $I_0 = Q/RC$. Thus we see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

Thinking Physics 7

Many automobiles are equipped with windshield wipers that can be used intermittently during a light rainfall. How does the operation of this feature depend on the charging and discharging of a capacitor?

Reasoning The wipers are part of an RC circuit the time constant of which can be varied by selecting different values of R through a multipositioned switch. The brief time that the wipers remain on and the time they are off are determined by the value of the time constant of the circuit.

Example 21.11 Charging a Capacitor in an RC Circuit

An uncharged capacitor and a resistor are connected in series to a battery as in Figure 21.25. If $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as a function of time.

Solution The time constant of the circuit is $\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = 4.00$ s. The maximum charge on the capacitor is $Q = C\mathcal{E} = (5.00 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$. The maximum current in the circuit is $I_0 = \mathcal{E}/R = (12.0\ \text{V})/(8.00 \times 10^5\ \Omega) = 15.0\ \mu\text{A}$. Using these values and Equations 21.33 and 21.34, we find that

$$q(t) = 60.0[1 - e^{-t/4}]\ \mu\text{C}$$

$$I(t) = 15.0e^{-t/4}\ \mu\text{A}$$

EXERCISE 12 Calculate the charge on the capacitor and the current in the circuit after one time constant has elapsed. Answer 37.9 μC, 5.52 μA

Example 21.12 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor C being discharged through a resistor R as in Figure 21.27. (a) After how many time constants is the charge on the capacitor one fourth of its initial value?

Solution The charge on the capacitor varies with time according to Equation 21.36, $q(t) = Qe^{-t/RC}$. To find the time it takes the charge q to drop to one fourth of its initial value, we substitute $q(t) = Q/4$ into this expression and solve for t:

$$\tfrac{1}{4}Q = Qe^{-t/RC}$$

$$\tfrac{1}{4} = e^{-t/RC}$$

Taking logarithms of both sides, we find

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC \ln 4 = 1.39\ RC$$

(b) The energy stored in the capacitor decreases with time as it discharges. After how many time constants is this stored energy one fourth of its initial value?

Solution Using Equations 20.29 and 21.36, we can express the energy stored in the capacitor at any time t as

$$U = \frac{q^2}{2C} = \frac{Q^2}{2C}e^{-2t/RC} = U_0e^{-2t/RC}$$

where U_0 is the initial energy stored in the capacitor. As in part (a), we now set $U = U_0/4$ and solve for t:

$$\tfrac{1}{4}U_0 = U_0e^{-2t/RC}$$

$$\tfrac{1}{4} = e^{-2t/RC}$$

Again, taking logarithms of both sides and solving for t gives

$$t = \tfrac{1}{2}RC \ln 4 = 0.693\ RC$$

EXERCISE 13 After how many time constants is the current in the RC circuit one half of its initial value? Answer 0.693 RC

EXERCISE 14 An uncharged capacitor and a resistor are connected in series to a source of emf. If $\mathcal{E} = 9.0$ V, $C = 20\ \mu$F, and $R = 100\ \Omega$, find (a) the time constant of the circuit, (b) the maximum charge on the capacitor, and (c) the maximum current in the circuit. Answer (a) 2.0 ms (b) 180 μC (c) 90 mA

SUMMARY

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \qquad [21.2]$$

where dQ is the charge that passes through a cross-section of the conductor in the time dt. The SI unit of current is the ampere (A); 1 A = 1 C/s.

The current in a conductor is related to the motion of the charge carriers through the relationship

$$I = nqv_dA \qquad [21.4]$$

where n is the density of charge carriers, q is their charge, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The **current density** $\boldsymbol{J}$ in a conductor is defined as the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \qquad [21.5]$$

The **resistance** $\boldsymbol{R}$ of a conductor is defined as the ratio of the potential difference across the conductor to the current:

$$R \equiv \frac{\Delta V}{I} \qquad [21.7]$$

The SI units of resistance are volts per ampere, defined as ohms (Ω). That is, $1\ \Omega = 1\ \text{V/A}$.

If the resistance is independent of the applied voltage, the conductor obeys Ohm's law, and conductors that have a constant resistance over a wide range of voltages are said to be *ohmic*.

If a conductor has a uniform cross-sectional area of A and a length of ℓ, its resistance is

$$R = \rho\frac{\ell}{A} \qquad [21.9]$$

where ρ is called the **resistivity** of the conductor. The inverse of the resistivity is defined as the **conductivity,** $\boldsymbol{\sigma}$. That is, $\sigma = 1/\rho$.

The resistivity of a conductor varies with temperature in an approximately linear fashion; that is,

$$\rho = \rho_0[1 + \alpha(T - T_0)] \qquad [21.11]$$

where α is the temperature coefficient of resistivity and ρ_0 is the resistivity at some reference temperature T_0.

In a classical model of electronic conduction in a metal, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** $\mathbf{v}_d$, which is opposite the electric field:

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m}\tau \qquad [21.16]$$

where τ is the average time between collisions with the atoms of the metal. The resistivity of the material according to this model is

$$\rho = \frac{m}{nq^2\tau} \qquad [21.19]$$

where n is the number of free electrons per unit volume.

If a potential difference ΔV is maintained across a resistor, the **power,** or rate at which energy is supplied to the resistor, is

$$P = I\Delta V \qquad [21.21]$$

Because the potential difference across a resistor is $\Delta V = IR$, we can express the power dissipated in a resistor in the form

$$P = I^2R = \frac{(\Delta V)^2}{R} \qquad [21.22]$$

The electrical energy supplied to a resistor appears in the form of thermal energy in the resistor.

The **emf** of a battery is the voltage across its terminals when the current is zero. That is, the emf is equivalent to the open-circuit voltage of the battery.

The **equivalent resistance** of a set of resistors connected in **series** is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \quad [21.27]$$

The **equivalent resistance** of a set of resistors connected in *parallel* is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad [21.29]$$

Complex circuits involving more than one loop are conveniently analyzed using two simple rules called **Kirchhoff's rules:**

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
2. The sum of the potential differences across the elements around any closed-circuit loop must be *zero.*

The first rule is a statement of **conservation of charge;** the second rule is equivalent to a statement of **conservation of energy.**

When a resistor is traversed in the direction of the current, the change in potential, ΔV, across the resistor is $-IR$. If a resistor is traversed in the direction opposite the current, $\Delta V = +IR$.

If a source of emf is traversed in the direction of the emf (negative to positive) the change in potential is $+\mathcal{E}$. If it is traversed opposite the emf (positive to negative), the change in potential is $-\mathcal{E}$.

If a capacitor is charged with a battery of emf $\mathcal{E}$ through a resistance R, the charge on the capacitor and the current in the circuit vary in time according to the expressions

$$q(t) = Q[1 - e^{-t/RC}] \quad [21.33]$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \quad [21.34]$$

where $Q = C\mathcal{E}$ is the *maximum* charge on the capacitor. The product RC is called the **time constant** of the circuit.

If a charged capacitor is discharged through a resistance R, the charge and current decrease exponentially in time according to the expressions

$$q(t) = Qe^{-t/RC} \quad [21.36]$$

$$I(t) = I_0 e^{-t/RC} \quad [21.37]$$

where $I_0 = Q/RC$ is the initial current in the circuit and Q is the initial charge on the capacitor.

CONCEPTUAL QUESTIONS

1. In an analogy between automobile traffic flow and electrical current, what would correspond to the charge Q? What would correspond to the current I?
2. What factors affect the resistance of a conductor?
3. Two wires A and B of circular cross section are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross-sectional areas? How do their radii compare?
4. Use the atomic theory of matter to explain why the resistance of a material should increase as its temperature increases.
5. Explain how a current can persist in a superconductor without any applied voltage.
6. What would happen to the drift velocity of the electrons in a wire and to the current in the wire if the electrons could move freely without resistance through the wire?
7. If charges flow very slowly through a metal, why does it not

require several hours for a light to come on when you throw a switch?

8. If you were to design an electric heater using Nichrome wire as the heating element, what parameters of the wire could you vary to meet a specific power output, such as 1000 W?
9. Car batteries are often rated in ampere-hours. Does this designate the amount of current, power, energy, or charge that can be drawn from the battery?
10. How would you connect resistors so that the equivalent resistance is larger than the individual resistances? Give an example involving two or three resistors.
11. How would you connect resistors so that the equivalent resistance is smaller than the individual resistances? Give an example involving two or three resistors.
12. Why is it possible for a bird to sit on a high-voltage wire without being electrocuted?
13. A "short circuit" is a circuit containing a path of very low resistance in parallel with some other part of the circuit. Discuss the effect of a short circuit on the portion of the circuit it parallels. Use a lamp with a frayed line cord as an example.
14. A series circuit consists of three identical lamps connected to a battery, as in Figure Q21.14. When the switch S is closed, what happens (a) to the intensities of lamps A and B; (b) to the intensity of lamp C; (c) to the current in the circuit; and (d) to the voltage drop across the three lamps? (e) Does the power dissipated in the circuit increase, decrease, or remain the same?

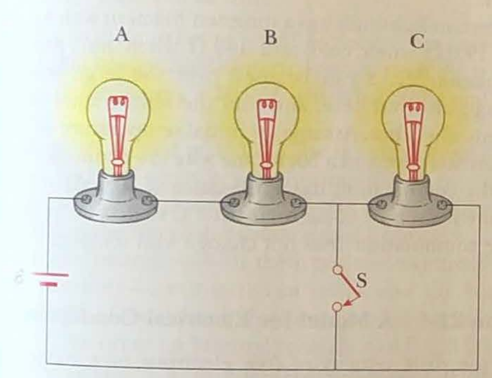


Figure Q21.14

15. Two lightbulbs both operate from 110 V, but one has a power rating of 25 W and the other of 100 W. Which bulb has the higher resistance? Which bulb carries the greater current?
16. If electrical power is transmitted over long distances, the resistance of the wires becomes significant. Why? Which mode of transmission would result in less energy loss—high current and low voltage or low current and high voltage? Discuss.
17. Two sets of Christmas tree lights are available. For set A, when one bulb is removed, the remaining bulbs remain illuminated. For set B, when one bulb is removed, the remaining bulbs do not operate. Explain the difference in wiring for the two sets.
18. Are the two headlights on a car wired in series or in parallel? How can you tell?
19. A ski resort consists of a few chair lifts and several interconnected downhill runs on the side of a mountain, with a lodge at the bottom. The lifts are analogous to batteries and the runs are analogous to resistors. Sketch how two runs can be in series. Sketch how three runs can be in parallel. Sketch a junction of one lift and two runs. One of the skiers is carrying an altimeter. State Kirchhoff's junction rule and Kirchhoff's loop rule for ski resorts.
20. In Figure Q21.20, describe what happens to the lightbulb after the switch is closed. Assume the capacitor has a large capacitance and is initially uncharged, and assume that the light illuminates when connected directly across the battery terminals.

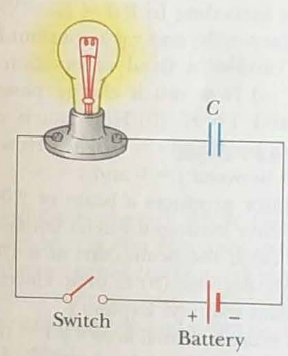


Figure Q21.20

21. Figure Q21.21 shows a series connection of three lamps, all rated at 120 V, with power ratings of 60 W, 75 W, and

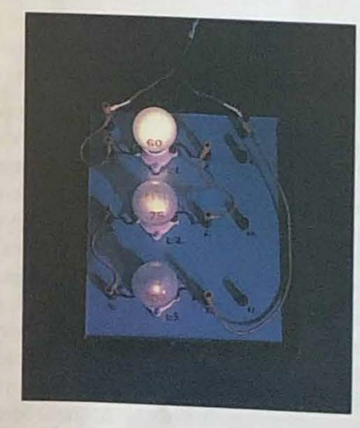


Figure Q21.21 *(Henry Leap and Jim Lehman)*

200 W. Why do the intensities of the lamps differ? Which lamp has the greatest resistance? How would their intensities differ if they were connected in parallel?

22. A student claims that a second lightbulb in series is less bright than the first, because the first bulb uses up some of the current. How would you respond to this statement?

PROBLEMS

Section 21.1 Electric Current

1. In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?
2. A teapot with a surface area of 700 cm^2 is to be silver plated. It is attached to the negative electrode of an electrolytic cell containing silver nitrate ($Ag^+NO_3^-$). If the cell is powered by a 12.0-V battery and has a resistance of 1.80 Ω, how long does it take to build up a 0.133-mm layer of silver on the teapot? (Density of silver $= 10.5 \times 10^3$ kg/m^3.)
3. Suppose that the current through a conductor decreases exponentially with time according to $I(t) = I_0 e^{-t/\tau}$, where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor. (a) How much charge passes this point between $t = 0$ and $t = \tau$? (b) How much charge passes this point between $t = 0$ and $t = 10\tau$? (c) How much charge passes this point between $t = 0$ and $t = \infty$?
4. A Van de Graaff generator produces a beam of 2.00-MeV deuterons, which are heavy hydrogen nuclei containing a proton and a neutron. (a) If the beam current is 10.0 μA, how far apart are the deuterons? (b) Is their electrostatic repulsion a factor of beam stability? Explain.
5. An aluminum wire has cross-sectional area 4.00×10^{-6} m^2 and carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is 2.70 g/cm^3. (Assume one electron is supplied by each atom.)

Section 21.2 Resistance and Ohm's Law

6. A lightbulb has a resistance of 240 Ω when operating at a voltage of 120 V. What is the current through the lightbulb?
7. A 0.900-V potential difference is maintained across a 1.50-m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire?
8. Suppose that you wish to fabricate a uniform wire out of 1.00 g of copper. If the wire is to have a resistance of $R =$ 0.500 Ω, and all of the copper is to be used, what will be (a) the length and (b) the diameter of this wire?
9. A 12.0-Ω metal wire is cut into three equal pieces that are then connected side by side to form a new wire the length of which is equal to one third the original length. What is the resistance of this new wire?
10. (a) Make an order-of-magnitude estimate of the resistance between the ends of a rubber band. (b) Make an order-of-magnitude estimate of the resistance between the "heads" and "tails" sides of a penny. In each case state what quantities you take as data and the values you measure or estimate for them. (c) *Don't* try this at home, but each would carry current of what order of magnitude if it were connected across a 120-V power supply?
11. While traveling through Death Valley on a day when the temperature is 58.0°C, Bill Hiker finds that a certain voltage applied to a copper wire produces a current of 1.00 A. Bill then travels to Antarctica and applies the same voltage to the same wire. What current does he register there if the temperature is −88.0°C? Assume no change in the wire's shape and size.
12. An aluminum rod has a resistance of 1.234 Ω at 20.0°C. Calculate the resistance of the rod at 120°C by accounting for the changes in both the resistivity and the dimensions of the rod.
13. A certain lightbulb has a tungsten filament with a resistance of 19.0 Ω when cold and 140 Ω when hot. Assume that Equation 21.13 can be used over the large temperature range involved here, and find the temperature of the filament when hot. Assume an initial temperature of 20.0°C.
14. A carbon wire and a Nichrome wire are connected in series. If the combination has a resistance of 10.0 kΩ at 0°C, what is the resistance of each wire at 0°C so that the resistance of the combination does not change with temperature?

Section 21.4 A Model for Electrical Conduction

15. If the drift velocity of free electrons in a copper wire is 7.84×10^{-4} m/s, calculate the electric field in the conductor.
16. If the current carried by a conductor is doubled, what happens to the (a) charge carrier density? (b) current density? (c) electron drift velocity? (d) average time between collisions?

Section 21.5 Electrical Energy and Power

17. A toaster is rated at 600 W when connected to a 120-V source. What current does the toaster carry, and what is its resistance?

18. In a hydroelectric installation, a turbine delivers 1500 hp to a generator, which in turn converts 80.0% of the mechanical energy into electrical energy. Under these conditions, what current will the generator deliver at a terminal potential difference of 2000 V?

19. What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0°C to 50.0°C in 10.0 min while operating at 110 V?

20. What is the required resistance of an immersion heater that will increase the temperature of a mass m of water from T_1 to T_2 in a time interval Δt while operating at a voltage ΔV?

21. Suppose that a voltage surge produces 140 V for a moment. By what percentage will the power output of a 120-V, 100-W lightbulb increase assuming its resistance does not change?

22. A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the electric field intensity in the wire, and (b) the power dissipated in it? (c) If the temperature is increased to 340°C and the voltage across the wire remains constant, what is the power dissipated?

23. Batteries are rated in terms of ampere hours (A·h), where a battery that can produce a current of 2.00 A for 3.00 h is rated at 6.00 A·h. (a) What is the total energy, in kilowatt hours, stored in a 12.0-V battery rated at 55.0 A·h? (b) At $0.0600 per kilowatt hour, what is the value of the electricity produced by this battery?

Section 21.6 Sources of emf

24. (a) What is the current in a 5.60-Ω resistor connected to a battery that has a 0.200-Ω internal resistance if the terminal voltage of the battery is 10.0 V? (b) What is the emf of the battery?

25. A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance of the battery?

26. Two 1.50-V batteries—with their positive terminals in the same direction—are inserted in series into the barrel of a flashlight. One battery has an internal resistance of 0.255 Ω, the other an internal resistance of 0.153 Ω. When the switch is closed, a current of 600 mA occurs in the lamp. (a) What is the lamp's resistance? (b) What fraction of the power dissipated is dissipated in the batteries?

Section 21.7 Resistors in Series and in Parallel

27. A television repairperson needs a 100-Ω resistor to repair a malfunctioning set. She is temporarily out of resistors of this value. All she has in her toolbox are a 500-Ω resistor and two 250-Ω resistors. How can the desired resistance be obtained from the resistors on hand?

28. (a) Find the equivalent resistance between points a and b in Figure P21.28. (b) If a potential difference of 34.0 V is applied between points a and b, calculate the current in each resistor.

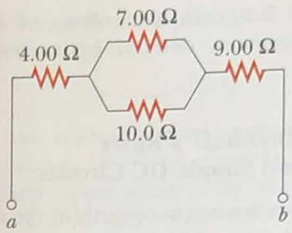


Figure P21.28

29. Consider the circuit shown in Figure P21.29. Find (a) the current in the 20.0-Ω resistor and (b) the potential difference between points a and b.

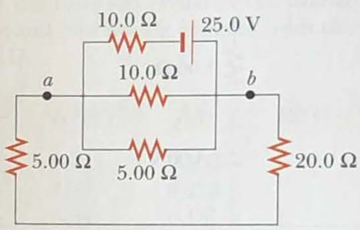


Figure P21.29

30. A lightbulb marked "75 W [at] 120 V" is screwed into a socket at one end of a long extension cord in which each of the two conductors has resistance 0.800 Ω. The other end of the extension cord is plugged into a 120-V outlet. Draw a circuit diagram and find the actual power of the bulb in this circuit.

31. Three 100-Ω resistors are connected, as shown in Figure P21.31. The maximum power that can safely be dissipated in any one resistor is 25.0 W. (a) What is the maximum voltage that can be applied to the terminals a and b? (b) For the voltage determined in part (a), what is the power dissipation in each resistor? What is the total power dissipation?

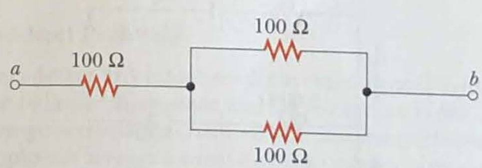


Figure P21.31

32. Four copper wires of equal length are connected in series. Their cross-sectional areas are 1.00 cm², 2.00 cm²,

3.00 cm², and 5.00 cm². If a voltage of 120 V is applied to the arrangement, determine the voltage across the 2.00-cm² wire.

Section 21.8 Kirchhoff's Rules and Simple DC Circuits

(*Note:* The currents are not necessarily in the directions shown for some circuits.)

33. Determine the current in each branch of the circuit in Figure P21.33.

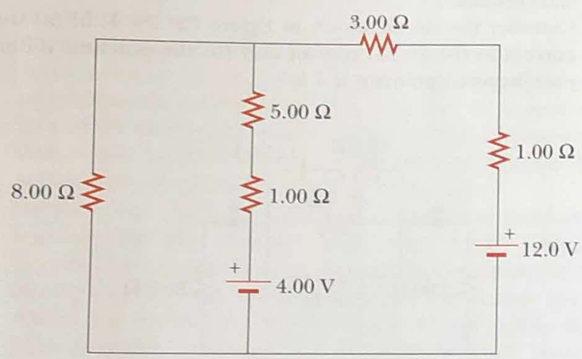


Figure P21.33

34. In Figure P21.33, show how to add just enough ammeters to measure every different current that is flowing. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.

35. The circuit considered in problem 33 and drawn in Figure P21.33 is connected for two minutes. (a) Find the energy converted by each battery. (b) Find the energy converted by each resistor. (c) Find the total energy converted by the circuit.

36. The ammeter in Figure 21.36 reads 2.00 A. Find I_1, I_2, and $\mathcal{E}$.

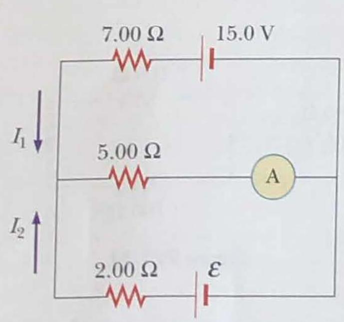


Figure P21.36

37. Using Kirchhoff's rules, (a) find the current in each resis[tor] in Figure P21.37. (b) Find the potential difference betwe[en] points *c* and *f*. Which point is at the higher potential?

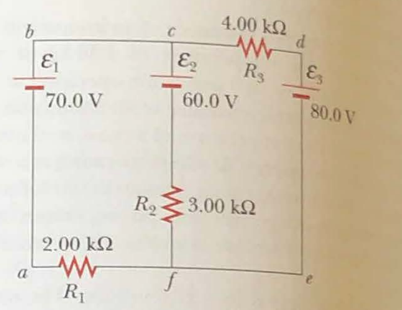


Figure P21.37

Section 21.9 *RC* Circuits

38. A 2.00×10^{-3} μF capacitor with an initial charge [of] 5.10 μC is discharged through a 1.30-kΩ resis[tor]. (a) Calculate the current through the resistor 9.00 μs af[ter] the resistor is connected across the terminals of the ca[pacitor]. (b) What charge remains on the capaci[tor] after 8.00 μs? (c) What is the maximum current in the re[sistor]?

39. Consider a series *RC* circuit (Fig. 21.25) for which R = 1.00 MΩ, C = 5.00 μF, and $\mathcal{E}$ = 30.0 V. Find (a) the tim[e] constant of the circuit and (b) the maximum charge on t[he] capacitor after the switch is closed. (c) If the switch is clos[ed] at t = 0, find the current in the resistor 10.0 s later.

40. In the circuit of Figure P21.40, the switch S has be[en] open for a long time. It is then suddenly closed. Determi[ne] the time constant (a) before the switch is closed a[nd] (b) after the switch is closed. (c) If the switch is closed [at] t = 0 s, determine the current through it as a function [of] time.

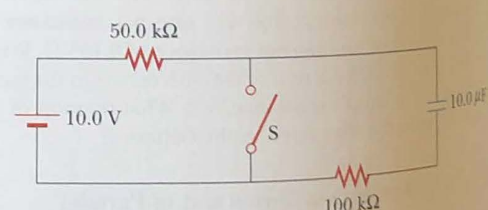


Figure P21.40

41. The circuit in Figure P21.41 has been connected for a lo[ng] time. (a) What is the voltage across the capacitor? (b) If t[he] battery is disconnected, how long does it take the capaci[tor] to discharge to one tenth of its initial voltage?

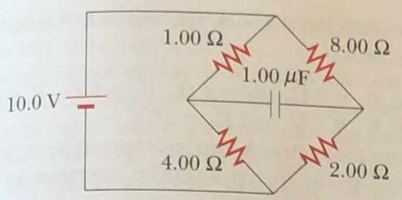


Figure P21.41

Additional Problems

42. One lightbulb is marked "25 W 120 V" and another "100 W 120 V" to mean that each converts that respective power when plugged into a constant 120-V potential difference. (a) Find the resistance of each. (b) In what time will 1.00 C pass through the dim bulb? How is this charge different on its exit versus its entry? (c) In what time will 1.00 J pass through the dim bulb? How is this energy different on its exit versus its entry? (d) Find the cost of running the dim bulb continuously for 30.0 days if the electric company sells its product at \$0.0700 per kWh. What *physical quantity* does the electric company sell? What is its price for one SI unit of this quantity?

43. A high-voltage transmission line of diameter 2.00 cm and length 200 km carries a steady current of 1000 A. If the conductor is a wire made of copper with a free charge density of 8.00×10^{28} electrons/m^3, how long does it take one electron to travel the full length of the cable?

44. A high-voltage transmission line carries 1000 A starting at 700 kV for a distance of 100 miles. If the resistance in the wire is 0.500 Ω/mi, what is the power loss due to resistive losses?

45. A copper cable is to be designed to carry a current of 300 A with a power loss of only 2.00 W/m. What is the required radius of the copper cable?

46. Four 1.50-V AA batteries in series are used to power a transistor radio. If the batteries can move a charge of 240 C before being depleted, how long will they last if the radio has a resistance of 200 Ω?

47. A battery has emf 9.20 V and internal resistance 1.20 Ω. (a) What resistance across the battery will dissipate heat energy from it at a rate of 12.8 W? (b) 21.2 W?

48. A 10.0-μF capacitor is charged by a 10.0-V battery through a resistance R. The capacitor reaches a potential difference of 4.00 V in a time 3.00 s after charging begins. Find R.

49. An electric heater is rated at 1500 W, a toaster at 750 W, and an electric grill at 1000 W. The three appliances are connected to a common 120-V circuit. (a) How much current does each draw? (b) Is a circuit fused at 25.0 A sufficient in this situation? Explain.

50. A more general definition of the temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$$

where ρ is the resistivity at temperature T. (a) Assuming that α is constant, show that

$$\rho = \rho_0 e^{\alpha(T-T_0)}$$

where ρ_0 is the resistivity at temperature T_0. (b) Using the series expansion ($e^x \approx 1 + x$; $x \ll 1$), show that the resistivity is given approximately by the expression $\rho = \rho_0[1 + \alpha(T - T_0)]$ for $\alpha(T - T_0) \ll 1$.

51. An experiment is conducted to measure the electrical resistivity of Nichrome in the form of wires with different lengths and cross-sectional areas. For one set of measurements, a student uses #30-gauge wire, which has a cross-sectional area of 7.3×10^{-8} m^2. The voltage across the wire and the current in the wire are measured with a voltmeter and ammeter, respectively. For each of the measurements given in the table below taken on wires of three different lengths, calculate the resistance of the wires and the corresponding values of the resistivity. What is the average value of the resistivity, and how does it compare with the value given in Table 21.1?

ℓ(m)	ΔV(V)	I(A)	R(Ω)	ρ(Ω · m)
0.54	5.22	0.500		
1.028	5.82	0.276		
1.543	5.94	0.187		

52. A battery is used to charge a capacitor through a resistor, as in Figure 21.25. Show that in the process of charging the capacitor, half of the energy supplied by the battery is dissipated as heat in the resistor and half is stored in the capacitor.

53. A straight cylindrical wire lying along the x axis has length ℓ and diameter d. It is made of a material described by Ohm's law with resistivity ρ. Assume that potential V_0 is maintained at $x = 0$, and $V = 0$ at $x = \ell$. In terms of ℓ, d, V_0, ρ, and physical constants, derive expressions for: (a) the electric field in the wire; (b) the resistance of the wire; (c) the electric current in the wire; and (d) the current density in the wire. Express vectors in vector notation. (e) Prove that $\mathbf{E} = \rho\mathbf{J}$.

Spreadsheet Problems

S1. Spreadsheet 21.1 calculates the average annual lighting cost per bulb for fluorescent and incandescent bulbs and the average yearly savings realized with fluorescent bulbs. It also graphs the average annual lighting cost per bulb versus the cost of electrical energy. (a) Suppose that a fluorescent bulb costs \$5, lasts for 5000 h, consumes 40 W of power, but provides the light intensity of a 100-W incandescent bulb. Assume that a 100-W incandescent bulb is on at all times and that energy costs 8.3 cents per kWh. How much does a consumer save each year by switching to fluorescent bulbs? (b) Check with your local electric company for their current rates, and find the cost of bulbs in your area. Would it pay you to switch to fluorescent bulbs? (c) Vary the parameters for bulbs of different wattages and reexamine the annual savings.

S2. The current-voltage characteristic curve for a semiconductor diode as a function of temperature T is given by

$$I = I_0(e^{e\Delta V/k_B T} - 1)$$

where e is the charge on the electron, k_B is Boltzmann's constant, ΔV is the applied voltage, and T is the absolute temperature. Set up a spreadsheet to calculate I and $R = \Delta V/I$ for $\Delta V = 0.40$ V to $\Delta V = 0.60$ V in increments of 0.05 V. Assume $I_0 = 1.0$ nA. Plot R versus ΔV for $T = 280$ K, 300 K, and 320 K.

S3. The application of Kirchhoff's rules to a dc circuit leads to a set of n linear equations in n unknowns. It is very tedious to solve these algebraically if $n > 3$. The purpose of this problem is to solve for the currents in a moderately complex circuit using matrix operations on a spreadsheet. You can solve equations very easily this way, and you can also readily explore the consequences of changing the values of the circuit parameters. (a) Consider the circuit in Figure S21.3. Assume the four unknown currents are in the directions shown.

- Apply Kirchhoff's rules to get four independent equations for the four unknown currents I_i, $i = 1, 2, 3$, and 4.
- Write these equations in matrix form **AI** = **B**, that is,

$$\sum_{j=1}^{4} A_{ij} I_j = B_i \qquad i = 1, 2, 3, 4$$

The solution is **I** = **A**$^{-1}$**B**, where **A**$^{-1}$ is the inverse m trix of **A**.

- Set $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 8\ \Omega$, $\mathcal{E}_1$ 3 V, $\mathcal{E}_2 = 9$ V, and $\mathcal{E}_3 = 12$ V.
- Enter the matrix **A** into your spreadsheet, one value p cell. Use the matrix inversion operation of the spre sheet to calculate **A**$^{-1}$.
- Find the currents by using the matrix multiplication o eration of the spreadsheet to calculate **I** = **A**$^{-1}$**B**.

(b) Change the sign of $\mathcal{E}_3$, and repeat the calculations part (a). This is equivalent to changing the polarity of $\mathcal{E}$ (c) Set $\mathcal{E}_1 = \mathcal{E}_2 = 0$ and repeat the calculations in part For these values, the circuit can be solved using sim series-parallel rules. Compare your results using both me ods. (d) Investigate any other cases of interest. For examp see how the currents change if you vary R_4.

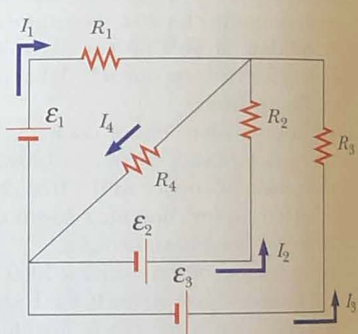


Figure S21.3

ANSWERS TO CONCEPTUAL PROBLEMS

1. A voltage is not something that "surges" through a completed circuit. A voltage is a potential difference that is applied *across* a device or a circuit. What goes *through* the circuit is *current.* Thus, it would be more correct to say, "1 ampere of electricity surges through the victim's body." Although this current would have disastrous results on the human body, a value of 1 (ampere) doesn't sound as exciting for a newspaper article as 10 000 (volts). Another possibility is to write, "10 000 volts of electricity were applied across the victim's body," which still doesn't sound quite as exciting!
2. The length of the line cord will double in this event. This would tend to increase the resistance of the line cord. But the doubling of the radius of the line cord results in the cross-sectional area increasing by a factor of 4. This would reduce the resistance more than the doubling of length in creases it. The net result is a decrease in resistance. The sa effect will occur for the lightbulb filament. The lowered sistance will result in more current flowing through the ament, causing it to glow more brightly.
3. The bulb filaments are cold when the lamp is first switch on, hence they have a lower resistance and draw more c rent than when they are hot. The increased current can o heat the filament and destroy it.
4. The gravitational force pulling the electrons to the botto of a piece of metal is much smaller than the electrical pulsion pushing the electrons apart. Thus, they stay dist uted throughout the metal. The concept of charges resid on the surface of a metal is true for a metal with an exc charge. The number of free electrons in a piece of meta

the same as the number of positive crystal lattice ions—the metal has zero net charge.

5. The total amount of energy delivered by the battery will be less than *E*. Recall that a battery can be considered to be an ideal, resistanceless battery in series with the internal resistance. When charging, the energy delivered to the battery includes the energy necessary to charge the ideal battery, plus the energy that goes into raising the temperature of the battery due to "joule heating" in the internal resistance. This latter energy is not available during the discharge of the battery. During discharge, part of the reduced available energy again transforms into internal energy in the internal resistance, further reducing the available energy below *E*.

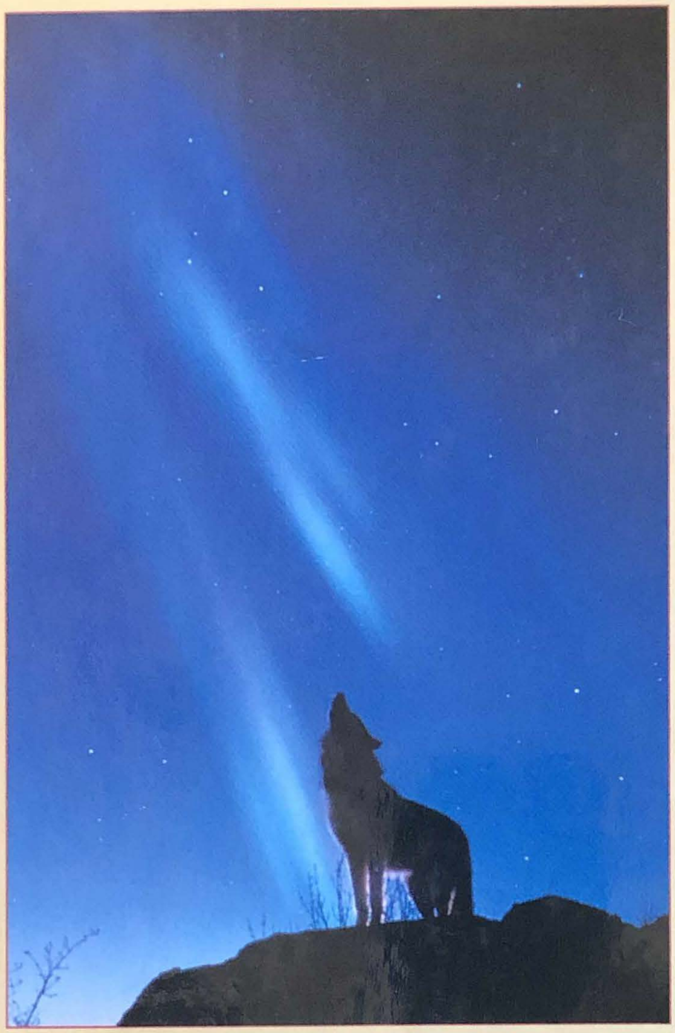
6. The starter in the automobile draws a relatively large current from the battery. This large current causes a significant voltage drop across the internal resistance of the battery. As a result, the terminal voltage of the battery is reduced, and the headlights dim accordingly.

7. An electrical device has a given resistance. Thus, when it is attached to a power source with a known potential difference, a definite current will be drawn. The device can be labeled with both the voltage and the current. Batteries, however, can be applied to a number of devices. Each device will have a different resistance, so the current from the battery will vary with the device. As a result, only the voltage of the battery can be specified.

8. Connecting batteries in parallel does not increase the emf. A high-current device connected to batteries in parallel can draw current from both batteries. Thus, connecting the batteries in parallel does increase the possible current output, and, therefore, the possible power output.

9. As you add more lightbulbs in *series*, the overall resistance of the circuit is increasing. Thus, the current through the bulbs will decrease. This decrease in current will result in a decrease in power transferred from the battery. As a result, the battery lifetime will increase. The whole string will be dimmer than the original lightbulb. As the current drops, the terminal voltage across the battery will become closer and closer to the battery emf.

   As you add more lightbulbs in *parallel*, the overall resistance of the circuit is decreasing. The current through each bulb remains nearly the same (until the battery starts to get hot). Each new bulb will be nearly as bright as the original lightbulb. The current leaving the battery will increase with the addition of each bulb. This increase in current will result in an increase in power transferred from the battery. As a result, the battery lifetime will decrease. As the current rises, the terminal voltage across the battery will drop further below the battery emf.



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