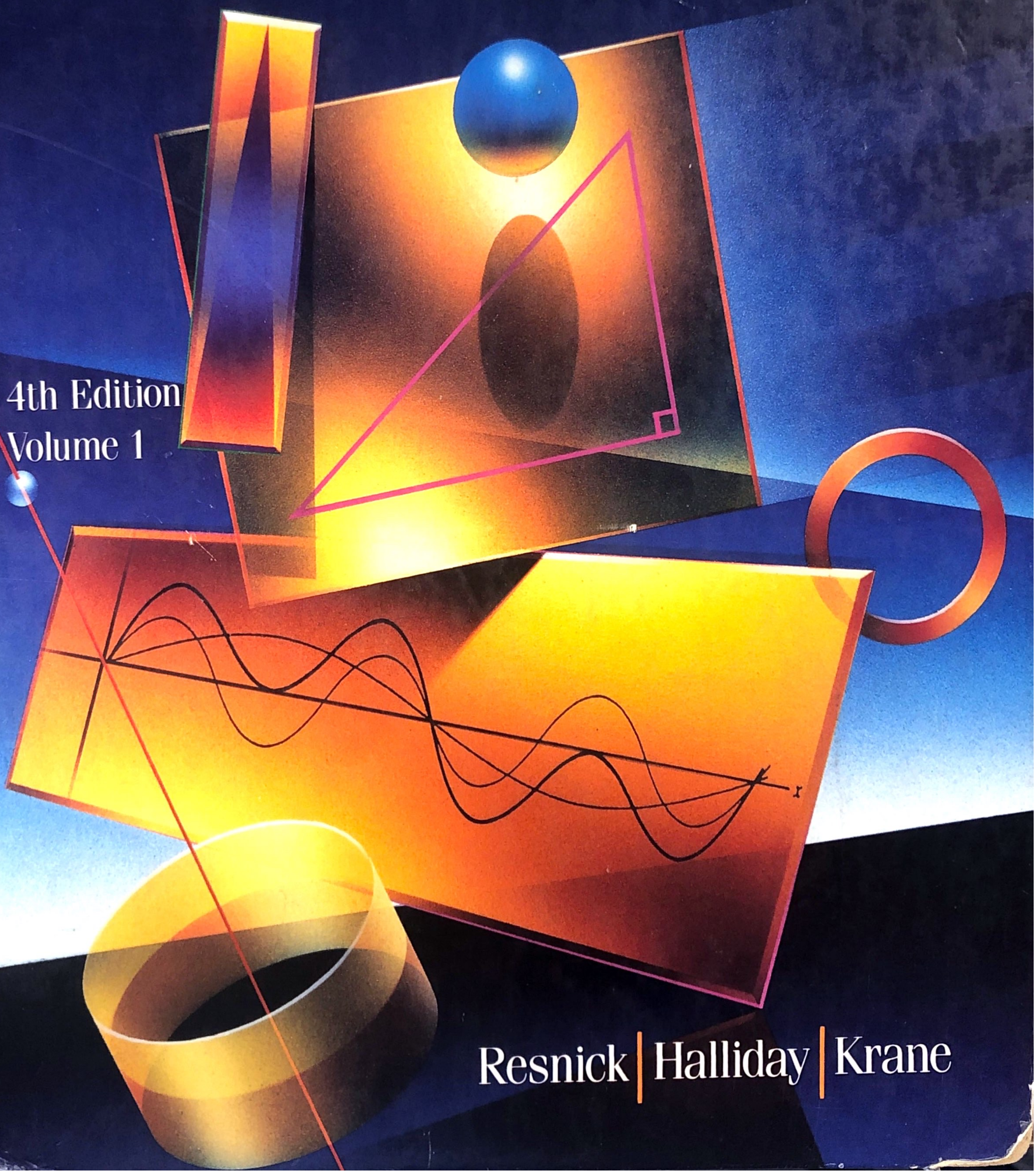
# PHYSICS



4th Edition
Volume 1

Resnick | Halliday | Krane

VOLUME ONE

# PHYSICS

FOURTH EDITION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**Library of Congress Cataloging-in-Publication Data**

Halliday, David, 1916–
Physics / David Halliday, Robert Resnick, Kenneth S. Krane. – – 4th ed.
p. cm.
Includes index.
ISBN 0-471-80458-4 (lib. bdg. : v. 1)
1. Physics. I. Resnick, Robert, 1923– . II. Krane, Kenneth S.
III. Title.
QC21.2.H355 1992 91-35885
530– –dc20 CIP

Printed and bound by Von Hoffmann Press, Inc.

10 9 8 7 6 5 4 3

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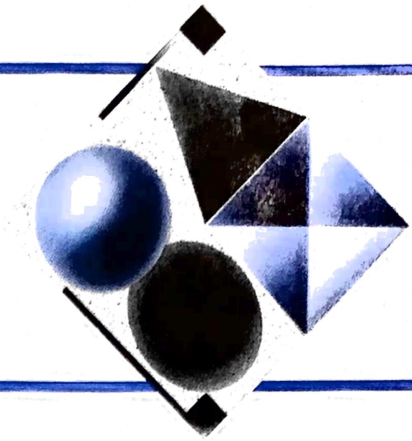
# CHAPTER 3

# VECTORS



*Many of the laws of physics involve not only algebraic relationships among quantities, but geometrical relationships as well. For example, picture a spinning top that rotates rapidly about its axis, while the axis of rotation itself rotates slowly about the vertical. This geometrical relationship is complicated to represent by algebraic equations. However, if we use* vectors *to represent the physical variables, a single equation is sufficient to explain the behavior. Vectors permit such economy of expression in a great variety of physical laws. Sometimes the vector form of a physical law permits us to see relationships or symmetries that would otherwise be obscured by a cumbersome algebraic equation.*

*In this chapter we explore some of the properties and uses of vectors and we introduce the mathematical operations that involve vectors. In the process you will learn that familiar symbols from arithmetic, such as +, −, and ×, have different meanings when applied to vectors.*

## 3-1 VECTORS AND SCALARS

A change of position of a particle is called a *displacement.* If a particle moves from position $A$ to position $B$ (Fig. 1*a*), we can represent its displacement by drawing a line from $A$ to $B$. The direction of displacement can be shown by putting an arrowhead at $B$ indicating that the displacement was *from A to B.* The path of the particle need not necessarily be a straight line from $A$ to $B$; the arrow represents only the net effect of the motion, not the actual motion.

In Fig. 1*b*, for example, we plot an actual path followed by a particle from $A$ to $B$. The path is not the same as the displacement $AB$. If we were to take snapshots of the particle when it was at $A$ and, later, when it was at some intermediate position $P$, we could obtain the displacement vector $AP$, representing the net effect of the motion during this interval, even though we would not know the actual path taken between these points. Furthermore, a displacement such as $A'B'$ (Fig. 1*a*), which is parallel to $AB$, similarly directed, and equal in length to $AB$, represents the same *change* in position as $AB$. We make no distinction between these two displacements. A displacement is therefore characterized by a *length* and a *direction.*

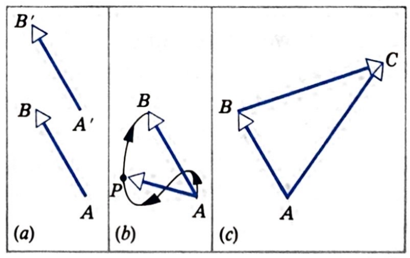



**Figure 1** Displacement vectors. (*a*) Vectors $AB$ and $A'B'$ are identical, since they have the same length and point in the same direction. (*b*) The actual *path* of the particle in moving from $A$ to $B$ may be the curve shown; the *displacement* is the vector $AB$. At the intermediate point $P$, the displacement is the vector $AP$. (*c*) After displacement $AB$, the particle undergoes another displacement $BC$. The net effect of the two displacements is the vector $AC$.

In a similar way, we can represent a subsequent displacement from $B$ to $C$ (Fig. 1*c*). The net effect of the two displacements is the same as a displacement from $A$ to $C$. We speak then of $AC$ as the *sum* or *resultant* of the displacements $AB$ and $BC$. Notice that this sum is not an algebraic sum and that a number alone cannot uniquely specify it.

Quantities that behave like displacements are called *vectors.* (The word *vector* means *carrier* in Latin. Biologists use the term *vector* to mean an insect, animal, or other agent that *carries* a cause of disease from one organism to another.) Vectors, then, are quantities that have both magnitude and direction and that follow certain rules of combination, which we describe below. The displacement vector is a convenient prototype. Some other physical quantities that are represented by vectors are force, velocity, acceleration, electric field, and magnetic field. Many of the laws of physics can be expressed in compact form by using vectors, and derivations involving these laws are often greatly simplified if we do so.

Quantities that can be specified completely by a number and unit and that therefore have magnitude only are called *scalars.* Some physical quantities that are scalars are mass, length, time, density, energy, and temperature. Scalars can be manipulated by the rules of ordinary algebra.

## 3-2 ADDING VECTORS: GRAPHICAL METHOD

To represent a vector on a diagram we draw an arrow. We choose the length of the arrow to be proportional to the magnitude of the vector (that is, we choose a scale), and we choose the direction of the arrow to be the direction of the vector, with the arrowhead giving the sense of the direction. For example, a displacement of 42 m in a northeast direction would be represented on a scale of 1 cm per 10 m by an arrow 4.2 cm long, drawn at an angle of 45° above a line pointing east with the arrowhead at the top right extreme (Fig. 2). A vector is usually represented in printing by a boldface symbol such as **d**. In handwriting we usually put an arrow above the symbol to denote a vector quantity, such as $\vec{d}$.

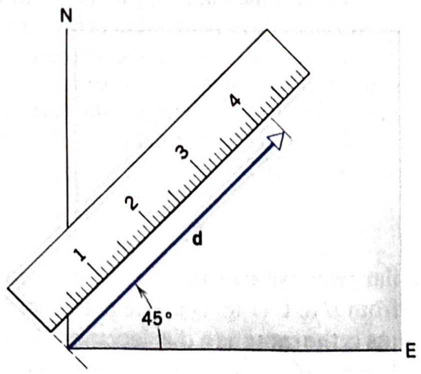



**Figure 2** The vector **d** represents a displacement of magnitude 42 m (on a scale in which 10 m = 1 cm) in a direction 45° north of east.

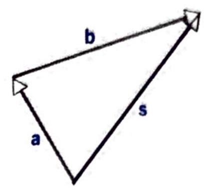



**Figure 3** The vector sum $\mathbf{a} + \mathbf{b} = \mathbf{s}$. Compare with Fig. 1*c*.

Often we are interested only in the magnitude (or length) of the vector and not in its direction. The magnitude of **d** is sometimes written as $|\mathbf{d}|$; more frequently we represent the magnitude alone by the italic letter symbol *d*. The boldface symbol is meant to signify both properties of the vector, magnitude and direction. When handwritten, the magnitude of the vector is represented by the symbol without the arrow.

Consider now Fig. 3 in which we have redrawn and relabeled the vectors of Fig. 1*c*. The relation among these vectors can be written

$$\mathbf{a} + \mathbf{b} = \mathbf{s}. \tag{1}$$

The rules to be followed in performing this vector addition graphically are these: (1) On a diagram drawn to scale lay out the vector **a** with its proper direction in the coordinate system. (2) Draw **b** to the same scale with its tail at the head of **a**, making sure that **b** has its own proper direction (generally different from the direction of **a**). (3) Draw a line from the tail of **a** to the head of **b** to construct the vector sum **s**. If the vectors were representing displacements, then **s** would be a displacement equivalent in length and direction to the successive displacements **a** and **b**. This procedure can be generalized to obtain the sum of any number of vectors.

Since vectors differ from ordinary numbers, we expect different rules for their manipulation. The symbol "+" in Eq. 1 has a meaning different from its meaning in arithmetic or scalar algebra. It tells us to carry out a different set of operations.

By careful inspection of Fig. 4 we can deduce two important properties of vector addition:

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a} \quad \text{(commutative law)} \tag{2}$$

and

$$\mathbf{d} + (\mathbf{e} + \mathbf{f}) = (\mathbf{d} + \mathbf{e}) + \mathbf{f} \quad \text{(associative law).} \tag{3}$$

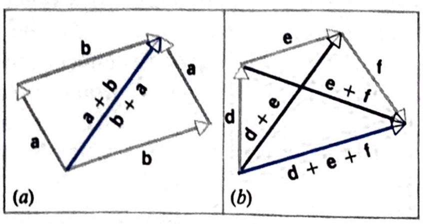



**Figure 4** (*a*) The commutative law for vector addition, which states that $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$. (*b*) The associative law, which states that $\mathbf{d} + (\mathbf{e} + \mathbf{f}) = (\mathbf{d} + \mathbf{e}) + \mathbf{f}$.

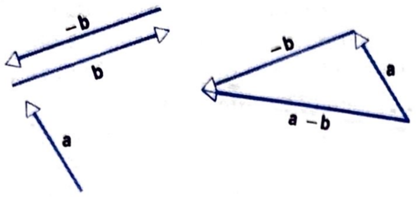



**Figure 5** The vector difference $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$.

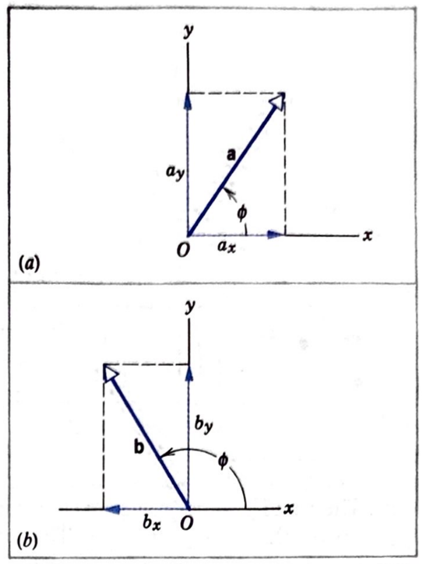



**Figure 6** (*a*) The vector **a** has component $a_x$ in the $x$ direction and component $a_y$ in the $y$ direction. (*b*) The vector **b** has a negative $x$ component.

These laws assert that it makes no difference in what order or in what grouping we add vectors; the sum is the same. In this respect, vector addition and scalar addition follow the same rules.

By inspection of Fig. 4*b*, you will see how the graphical method is used to find the sum of more than two vectors, in this case $\mathbf{d} + \mathbf{e} + \mathbf{f}$. Each succeeding vector is placed with its tail at the head of the previous one. The vector representing the sum is then drawn from the tail of the first vector to the head of the last one.

The operation of subtraction can be included in our vector algebra by defining the negative of a vector to be another vector of equal magnitude but opposite direction. Then

$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) \tag{4}$$

as shown in Fig. 5. Here $-\mathbf{b}$ means a vector with the same magnitude as **b** but pointing in the opposite direction. It follows from Eq. 4 that $\mathbf{a} - \mathbf{a} = \mathbf{a} + (-\mathbf{a}) = 0$.

Remember that, although we have used displacements to illustrate these operations, the rules apply to *all* vector quantities, such as velocities and forces.

## 3-3 COMPONENTS OF VECTORS

Even though we defined vector addition with the graphical method, it is not very useful for vectors in three dimensions. Often it is even inconvenient for the two-dimensional case. Another way of adding vectors is the analytical method, involving the resolution of a vector into components with respect to a particular coordinate system.

Figure 6*a* shows a vector **a** whose tail has been placed at the origin of a rectangular coordinate system. If we draw perpendicular lines from the head of **a** to the axes, the quantities $a_x$ and $a_y$ so formed are called the (Cartesian) *components* of the vector **a**. The process is called *resolving a vector into its components.* The vector **a** is completely and uniquely specified by these components; given $a_x$ and $a_y$, we could immediately reconstruct the vector **a**.

The components of a vector can be positive, negative, or zero. Figure 6*b* shows a vector **b** that has $b_x < 0$ and $b_y > 0$.

The components $a_x$ and $a_y$ in Fig. 6*a* are readily found from

$$\boxed{a_x = a\cos\phi \quad \text{and} \quad a_y = a\sin\phi,} \tag{5}$$

where $\phi$ is the angle that the vector **a** makes with the positive $x$ axis, measured counterclockwise from this axis. As shown in Fig. 6, the algebraic signs of the components of a vector depend on the quadrant in which the angle $\phi$ lies. For example, when $\phi$ is between 90° and 180°, as in Fig. 6*b*, the vector always has a negative $x$ component and a positive $y$ component. The components of a vector behave like scalar quantities because, in any particular coordinate system, only a number with an algebraic sign is needed to specify them.

Once a vector is resolved into its components, the components themselves can be used to specify the vector. Instead of the two numbers $a$ (magnitude of the vector) and $\phi$ (direction of the vector relative to the $x$ axis), we now have the two numbers $a_x$ and $a_y$. We can pass back and forth between the description of a vector in terms of its components ($a_x$ and $a_y$) and the equivalent description in terms of magnitude and direction ($a$ and $\phi$). To obtain $a$ and $\phi$ from $a_x$ and $a_y$, we note from Fig. 6*a* that

$$a = \sqrt{a_x^2 + a_y^2} \quad \vec{a} = \langle a_x, a_y \rangle \tag{6a}$$

and

$$\tan\phi = a_y/a_x. \tag{6b}$$

The quadrant in which $\phi$ lies is determined from the signs of $a_x$ and $a_y$.

In three dimensions the process works similarly: just draw perpendicular lines from the tip of the vector to the three coordinate axes $x$, $y$, and $z$. Figure 7 shows one way

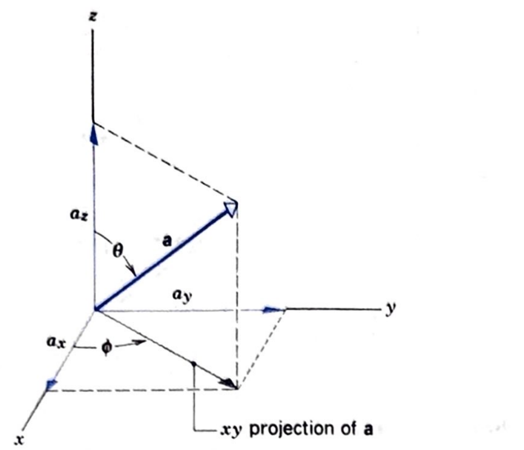



**Figure 7** A vector **a** in three dimensions with components $a_x$, $a_y$, and $a_z$. The $x$ and $y$ components are conveniently found by first drawing the $xy$ projection of **a**. The angle $\theta$ between **a** and the $z$ axis is called the *polar angle*. The angle $\phi$ in the $xy$ plane between the projection of **a** and the $x$ axis is called the *azimuthal angle*. The azimuthal angle $\phi$ has the same meaning here as it does in Fig. 6.

this is often drawn to make the components easier to recognize; the component (sometimes called a *projection*) of **a** in the $xy$ plane is first drawn, and then from its tip we can find the individual components $a_x$ and $a_y$. We would obtain exactly the same $x$ and $y$ components if we worked directly with the vector **a** instead of with its $xy$ projection, but the drawing would not be as clear. From the geometry of Fig. 7, we can deduce the components of the vector **a** to be

$$a_x = a \sin\theta \cos\phi, \quad a_y = a \sin\theta \sin\phi, \quad \text{and} \quad a_z = a\cos\theta. \tag{7}$$

When resolving a vector into components it is sometimes useful to introduce a vector of unit length in a given direction. Often it is convenient to draw unit vectors along the particular coordinate axes chosen. In the rectangular coordinate system the special symbols **i**, **j**, and **k** are usually used for unit vectors in the positive $x$, $y$, and $z$ directions, respectively (see Fig. 8). In handwritten notation, unit vectors are often indicated with a circumflex or "hat," such as $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Note that **i**, **j**, and **k** need not be located at the origin. Like all vectors, they can be translated anywhere in the coordinate space as long as their directions with respect to the coordinate axes are not changed.

In general, a vector **a** in a three-dimensional coordinate system can be written in terms of its components and the unit vectors as

$a = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}, \tag{8a}$$

or in two dimensions as

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j}. \tag{8b}$$

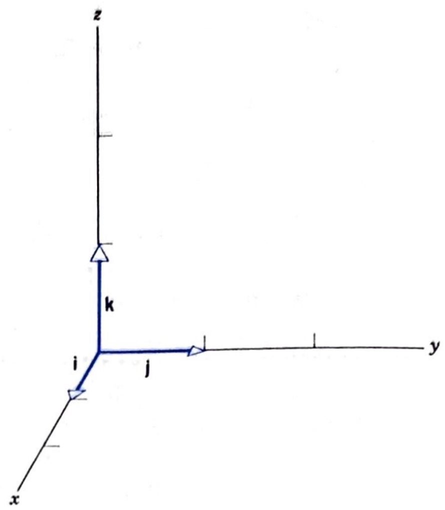



**Figure 8** The unit vectors **i**, **j**, and **k** are used to specify the positive $x$, $y$, and $z$ axes, respectively. Each vector is dimensionless and has a length of unity.

The vector relation Eq. 8*b* is equivalent to the scalar relations of Eq. 6. Each equation relates the vector (**a**, or $a$ and $\phi$) to its components ($a_x$ and $a_y$). Sometimes we call quantities such as $a_x\mathbf{i}$ and $a_y\mathbf{j}$ in Eq. 8*b* the *vector components* of **a**. Figure 9 shows the vectors **a** and **b** of Fig. 6 drawn in terms of their vector components. Many physical problems involving vectors can be simplified by replacing a vector by its vector components. That is, the

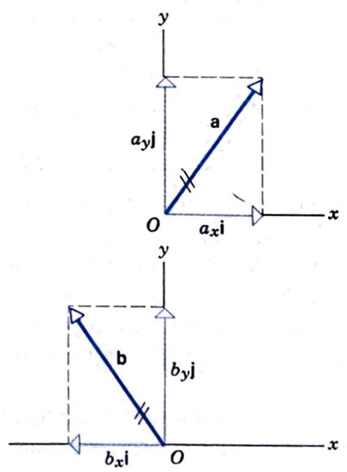



**Figure 9** The *vector* components of **a** and **b**. In any physical situation that involves vectors, we get the same outcome whether we use the vector itself, such as **a**, or its two vector components, $a_x\mathbf{i}$ and $a_y\mathbf{j}$. The effect of the single vector **a** is equivalent to the net effect of the two vectors $a_x\mathbf{i}$ and $a_y\mathbf{j}$. When we have replaced a vector with its vector components, it is helpful to draw a double line through the original vector, as shown; this helps us to remember not to consider the original vector any more.

action of a quantity represented as a vector can be replaced by the actions of its vector components. When necessary, we refer explicitly to *vector components,* while the word *component* alone continues to refer to the scalar quantities $a_x$ and $a_y$.

### Other Coordinate Systems *(Optional)*

Many other varieties of coordinate systems may be appropriate for analyzing certain physical situations. For example, the two-dimensional *xy* coordinate system may be changed in either of two ways: (1) by moving the origin to another location in the *xy* plane, which is called a *translation* of the coordinate system, or (2) by pivoting the *xy* axes about the fixed origin, which is a *rotation* of the coordinate system. In both of these operations we keep the vector fixed and move the coordinate axes. Figure 10 shows the effect of these two changes. In the case shown in Fig. 10*a*, the components are unchanged, but in the case shown in Fig. 10*b*, the components do change.

When the physical situation we are analyzing has certain symmetries, it may be advantageous to choose a different coordinate system for resolving a vector into its components. For instance, we might choose the radial and tangential directions of plane polar coordinates, shown in Fig. 11. In this case, we find the components on the coordinate axes just as we did in the ordinary *xyz* system: we draw a perpendicular from the tip of the vector to each coordinate axis.

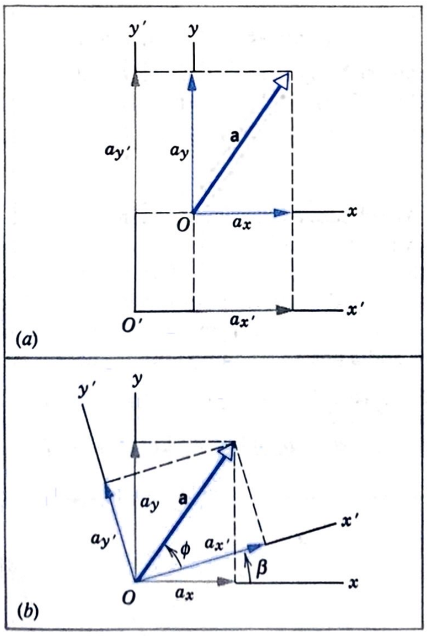



**Figure 10** (*a*) The origin *O* of the coordinate system of Fig. 6*a* has been moved or *translated* to the new position *O*′. The *x* and *y* components of **a** are identical to the *x*′ and *y*′ components. (*b*) The *x* and *y* axes have been rotated through the angle *β*. The *x*′ and *y*′ components are different from the *x* and *y* components (note that the *y*′ component is now smaller than the *x*′ component, while in Fig. 6*a* the *y* component was greater than the *x* component), but *the vector* **a** *is unchanged.* By what angle should we rotate the coordinate axes to make the *y*′ component zero?

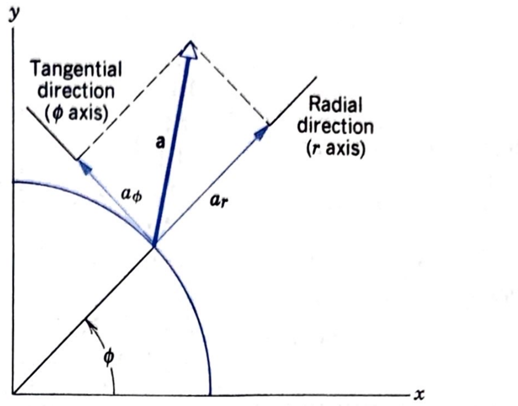



**Figure 11** The vector **a** is resolved into its radial and tangential components. These components will have important applications when we consider circular motion in Chapters 4 and 11.

The three-dimensional extensions of Fig. 11 (spherical polar or cylindrical polar coordinates) in many important cases are far superior to Cartesian coordinate systems for the analysis of physical problems. For example, the gravitational force exerted by the Earth on distant objects has the symmetry of a sphere, and thus its properties are most simply described in spherical polar coordinates. The magnetic force exerted by a long straight current-carrying wire has the symmetry of a cylinder and is therefore most simply described in cylindrical polar coordinates. ■

## 3-4 ADDING VECTORS: COMPONENT METHOD

Now that we have shown how to resolve vectors into their components, we can consider the addition of vectors by an analytic method.

Let **s** be the sum of the vectors **a** and **b**, or

$$\mathbf{s} = \mathbf{a} + \mathbf{b}. \qquad (9)$$

If two vectors, such as **s** and **a** + **b**, are to be equal, they must have the same magnitude and must point in the same direction. This can only happen if their corresponding components are equal. We stress this important conclusion:

*Two vectors are equal to each other only if their corresponding components are equal.*

For the vectors of Eq. 9, we can write

$$s_x\mathbf{i} + s_y\mathbf{j} = a_x\mathbf{i} + a_y\mathbf{j} + b_x\mathbf{i} + b_y\mathbf{j}$$
$$= (a_x + b_x)\mathbf{i} + (a_y + b_y)\mathbf{j}. \qquad (10)$$

Equating the *x* components on both sides of Eq. 10 gives

$$s_x = a_x + b_x, \qquad (11a)$$

and equating the $y$ components gives

$$s_y = a_y + b_y. \qquad (11b)$$

These two algebraic equations, taken together, are equivalent to the single vector relation of Eq. 9.

Instead of specifying the components of **s**, we can give its length and direction:

$$s = \sqrt{s_x^2 + s_y^2} = \sqrt{(a_x + b_x)^2 + (a_y + b_y)^2} \qquad (12a)$$

and

$$\tan \phi = \frac{s_y}{s_x} = \frac{a_y + b_y}{a_x + b_x}. = \frac{Y_1 + Y_2}{X_1 + X_2}. \qquad (12b)$$

Here is the rule for adding vectors by this method. (1) Resolve each vector into its components, keeping track of the algebraic sign of each component. (2) Add the components for each coordinate axis, taking the algebraic sign into account. (3) The sums so obtained are the components of the sum vector. Once we know the components of the sum vector, we can easily reconstruct that vector in space.

The advantage of the method of breaking up vectors into components, rather than adding directly with the use of suitable trigonometric relations, is that we always deal with right triangles and thus simplify the calculations.

In adding vectors by the component method, the choice of coordinate axes determines how simple the process will be. Sometimes the components of the vectors with respect to a particular set of axes are known at the start, so that the choice of axes is obvious. Other times a judicious choice of axes can greatly simplify the job of resolution of the vectors into components. For example, the axes can be oriented so that at least one of the vectors lies parallel to an axis; the components of that vector along the other axes will then be zero.

**Sample Problem 1** An airplane travels 209 km on a straight course making an angle of 22.5° east of due north. How far north and how far east did the plane travel from its starting point?

**Solution** We choose the positive $x$ direction to be east and the positive $y$ direction to be north. Next, we draw a displacement vector (Fig. 12) from the origin (starting point), making an angle of 22.5° with the $y$ axis (north) inclined along the positive $x$ direction (east). The length of the vector represents a magnitude of 209 km. If we call this vector **d**, then $d_x$ gives the distance traveled east of the starting point and $d_y$ gives the distance traveled north of the starting point. We have

$$\phi = 90.0° - 22.5° = 67.5°,$$

so that (see Eqs. 5)

$$d_x = d \cos \phi = (209 \text{ km}) (\cos 67.5°) = 80.0 \text{ km},$$

and

$$d_y = d \sin \phi = (209 \text{ km}) (\sin 67.5°) = 193 \text{ km}.$$

We have used Cartesian components in this sample problem, even though the Earth's surface is curved and therefore non-Cartesian. For example, a plane starting on the equator and flying northeast will eventually be due north of its starting point,

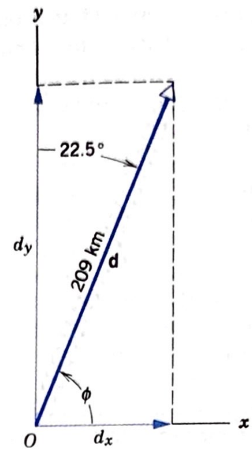



**Figure 12** Sample Problem 1.

which could never occur in a flat coordinate system. Similarly, two planes starting at different points on the equator and flying due north at the same speed along parallel paths will eventually collide at the north pole. This also would be impossible in a flat coordinate system. If we restrict our calculations to distances that are small with respect to the radius of the Earth (6400 km), we can safely use Cartesian coordinates for analyzing displacements on the Earth's surface.

**Sample Problem 2** An automobile travels due east on a level road for 32 km. It then turns due north at an intersection and travels 47 km before stopping. Find the resultant displacement of the car.

**Solution** We choose a coordinate system fixed with respect to the Earth, with the positive $x$ direction pointing east and the positive $y$ direction pointing north. The two successive displacements, **a** and **b**, are then drawn as shown in Fig. 13. The resultant displacement **s** is obtained from **s** = **a** + **b**. Since **b** has no $x$ component and **a** has no $y$ component, we obtain (see Eqs. 11)

$$s_x = a_x + b_x = 32 \text{ km} + 0 = 32 \text{ km},$$

$$s_y = a_y + b_y = 0 + 47 \text{ km} = 47 \text{ km}.$$

$S_x = X_1 + X_2$

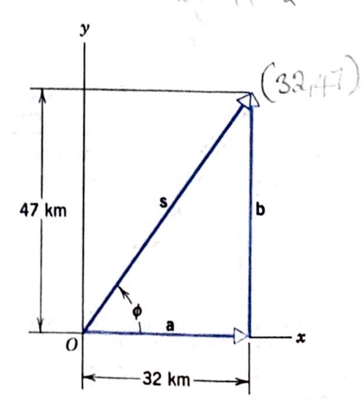
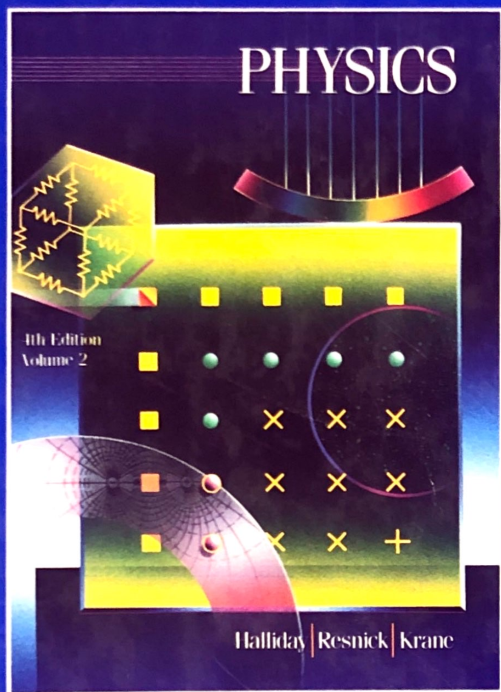
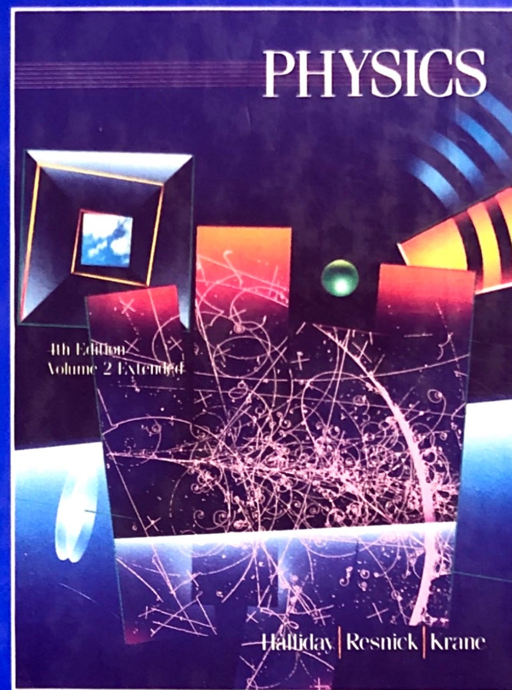



**Figure 13** Sample Problem 2.




ISBN 0-471-80458-4



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**New York | Chichester | Brisbane | Toronto | Singapore**