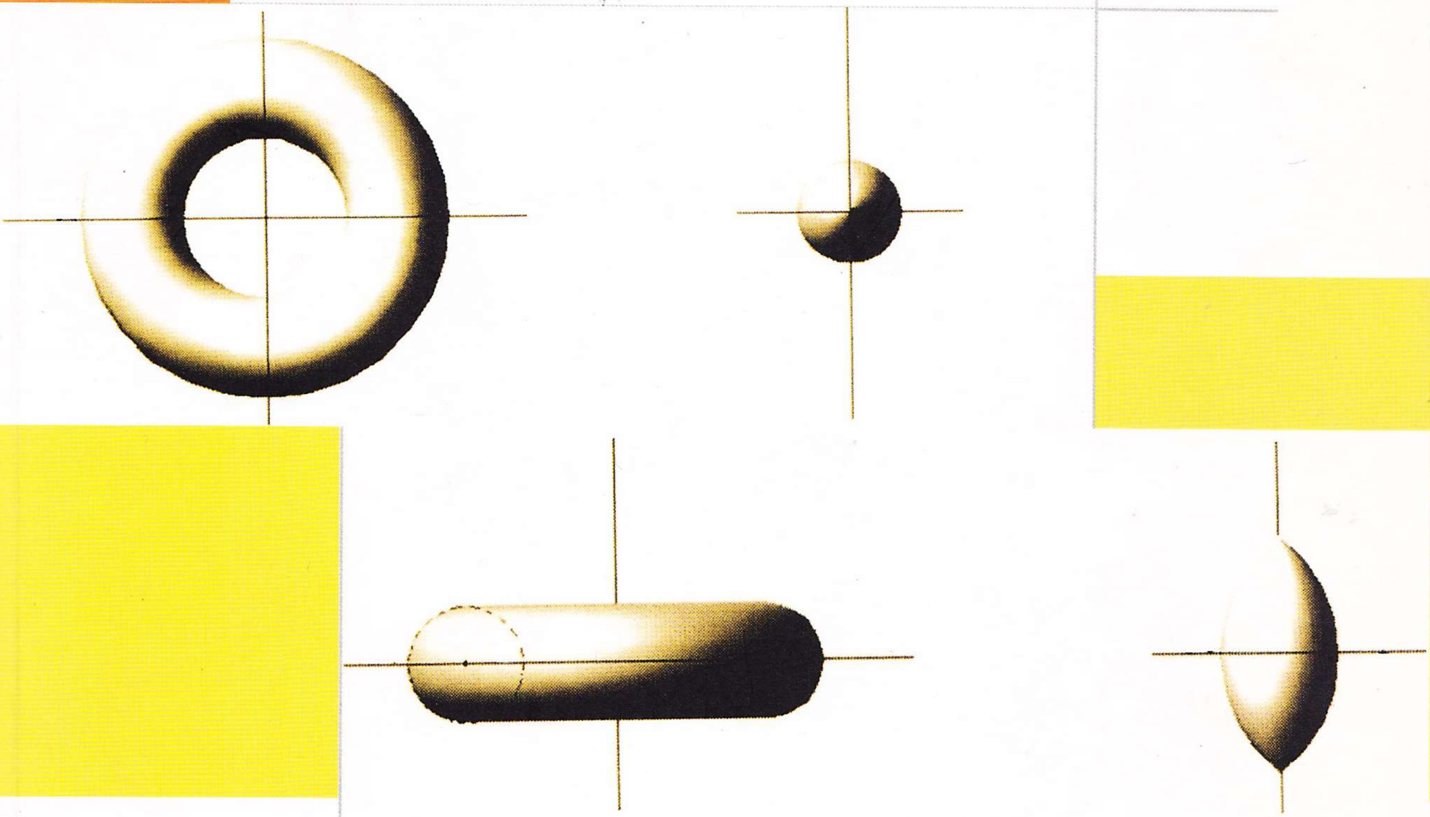

Introduction to

Geometrical Optics

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INTRODUCTION TO GEOMETRICAL OPTICS

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7.2 LENS NOMENCLATURE

The radii of curvature of the first and second surfaces of a lens are $\mathbf{r}_1$ and $\mathbf{r}_2$. Their corresponding centers of curvature are $\mathbf{C}_1$ and $\mathbf{C}_2$. A lens has a front vertex $\mathbf{A}_1$, and a rear vertex $\mathbf{A}_2$. Lens thickness is $\mathbf{d} = \mathbf{A}_1\mathbf{A}_2$. The radius of curvature is measured from the vertex **A** to the center **C**, i.e., $\mathbf{r}_1 = \mathbf{A}_1\mathbf{C}_1$; $\mathbf{r}_2 = \mathbf{A}_2\mathbf{C}_2$

According to our sign convention, a biconvex lens has a positive first surface radius, and a negative back surface radius. In an *equiconvex* lens, $\mathbf{r}_1 = -\mathbf{r}_2$. The *symmetrical* form of diverging lenses is called *equiconcave*. A sphere is a symmetrical lens with *concentric* surfaces, i.e., $\mathbf{C}_1$ and $\mathbf{C}_2$ coincide.

The *optical axis* is the straight line through the centers of curvature. The indices of refraction of the object space, the lens, and the image space media are $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$. Images formed by the lens may be found by using the refraction equation for a spherical refracting surface at each surface. At the first surface, the ray refracts from object space into the lens, i.e., from index $\mathbf{n}_1$ into $\mathbf{n}_1'$, where, $\mathbf{n}_1' = \mathbf{n}_2$. The ray then refracts from the lens into image space, i.e., from $\mathbf{n}_2$ into $\mathbf{n}_2'$, where, $\mathbf{n}_2' = \mathbf{n}_3$. See Figure 7.2.

When the lens is surrounded by a uniform medium the first and last indices are identical, i.e., $\mathbf{n}_1 = \mathbf{n}_3$. In this case we may simplify our nomenclature by calling the index of the surrounding medium **n**, and the index of the lens **n'**. A further simplification is made when the surrounding medium is air. Then $\mathbf{n}_1 = \mathbf{n}_3 = 1$, and the lens index is **n**. The following quantities are applicable to thin lenses:

Object distance:	AM	=	u	Image distance:	AM'	=	u'
Object height:	MQ	=	y	Image height	M'Q'	=	y'
First focal length	FA	=	f	Second focal length	F'A	=	f'

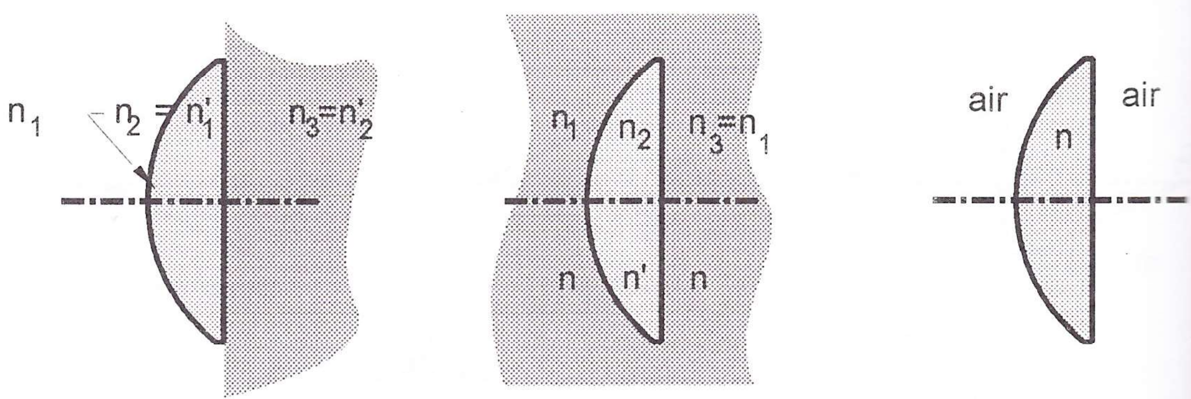

Figure 7.2 The nomenclature for refractive index varies with the media surrounding the lens.

7.3 THE OPTICAL CENTER OF A LENS

The *chief* ray from each off-axis object point, really or virtually, crosses the optical axis at the optical *center* **O** of the lens. Its path through a lens is shown in Figure 7.3. The incident chief ray heads toward the *first nodal* point **N**. After refracting into the lens the ray crosses the optical center, **O**. The ray refracts out of the lens as if leaving from the *second nodal* point **N'**, traveling in a direction parallel to the incident ray. By definition, rays that cross, or appear to cross the nodal points of a lens, will emerge undeviated. The position of the optical center of a lens, as a function of its bending and thickness, is given by

$$A_1O = \frac{r_1 d}{r_1 - r_2} \qquad 7.1$$

The path of a ray through the optical center is constructed by drawing parallel radii of curvature from the two centers of curvature. Within the lens, the path of the ray is found by connecting the points $\mathbf{I}_1$ and $\mathbf{I}_2$ at which the radii intersect the surfaces. If the ray path does not cross the axis, project it to the axis to find **O**. The direction of the incident and emergent rays depends on the indices of the lens and surrounding media. Figure 7.4 shows an arbitrarily chosen ray path to illustrate the positions of the nodal points. The incident ray crosses the axis at **N**, but the emergent ray must be projected back to the axis to find **N'**. Note how the points **O, N,** and **N'** of the meniscus lens are shifted, compared with their positions in the biconvex form.

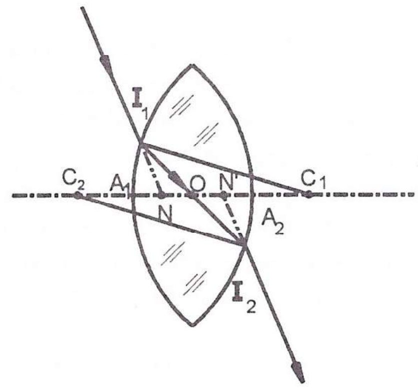

Figure 7.3 The optical center and nodal points of a biconvex lens.

EXAMPLE 1: Find the optical center of a lens with r_1 = +10 cm,
r_2 = +20 cm, and d = 2 cm. See Figure 7.4.

$\mathbf{A_1O}$ = 10(2)/10-20 = -2.00 cm.

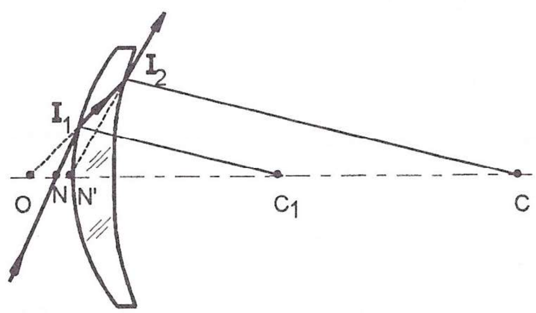

Figure 7.4 The optical center and nodal points of a positive meniscus lens.

7.4 THE THIN LENS

We may neglect the thickness of a lens if is very small compared with the radii of curvature. Consequently, d = 0, the vertices A_1 and A_2 coincide, and the lens vertex is simply denoted by **A**. The vertex **A** coincides with the optical center **0** and the nodal points **N** and **N'**. These assumptions enable us to find a simple thin lens equation.

7.5 THE THIN LENS EQUATION

The thin-lens equation for a lens in a uniform medium is obtained by applying the refraction equation for a spherical refracting surface at each of the two surfaces of the lens. Refraction at the first surface produces an image that serves as the object for the second surface. Although d = 0, the lens thickness is exaggerated in Figure 7.5 for the sake of clarity.

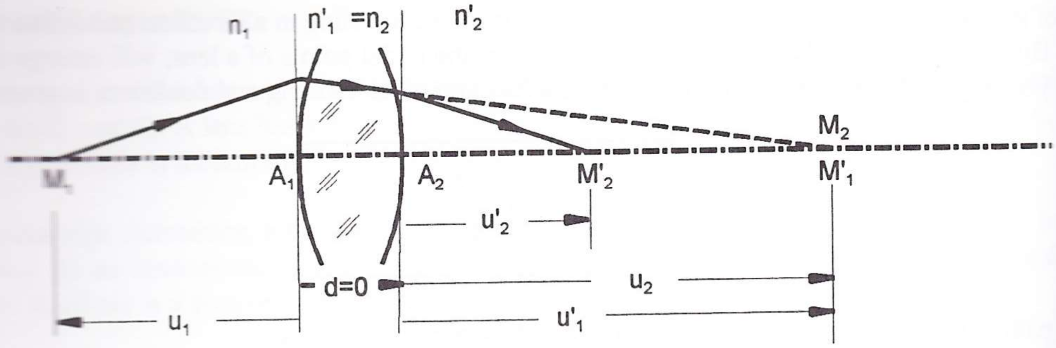

Figure 7.5 Refraction at each surface of a lens. M_1' is an intermediate image that becomes intermediate object M_2.

Point M_1 is the object to be imaged by the lens. It is at object distance $\mathbf{u}_1 = AM_1$. After refraction at the first lens surface, an image $\mathbf{M}_1'$ is produced at a distance $\mathbf{u}_1' = \mathbf{AM}_1'$. Although the image appears to be to the right of the lens, it belongs to the lens medium into which the rays have refracted. The image point M_1' serves as a virtual object for the second lens surface, consequently, it is renamed $\mathbf{M}_2$. Refraction at the second surface produces a final image $\mathbf{M}_2'$. The figure shows that $\mathbf{M}_1$ and $\mathbf{M}_1'$ are conjugate points, as are M_2 and M_2'. Consequently, $\mathbf{M}_1$ and $\mathbf{M}_2'$ are conjugate to each other.

The equation for refraction at the first surface of the lens is $$\frac{n_1'}{u_1'} = \frac{n_1}{u_1} + \frac{n_1' - n_1}{r_1} \qquad 7.2a$$

However, as noted in Section 7.2, $n_1' = n'$, and $n_1 = n$. Also, the image distance u_1' equals the object distance u_2. Thus, Eq. 7.2a becomes $$\frac{n'}{u_2} = \frac{n}{u_1} + \frac{n' - n}{r_1} \qquad 7.2b$$

The refraction equation at the second surface is $$\frac{n_2'}{u_2'} = \frac{n_2}{u_2} + \frac{n_2' - n_2}{r_2} \qquad 7.3a$$

This equation becomes $$\frac{n}{u_2'} = \frac{n'}{u_2} - \frac{n' - n}{r_2} \qquad 7.3b$$

Upon substituting Eq. 7.2b into Eq. 7.3b, and substituting $u_1 = u$ and $u_2' = u'$, we obtain

$$\frac{1}{u'} - \frac{1}{u} = \frac{n' - n}{n}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad 7.4$$

As defined for a spherical refracting surface, the image of an object at infinity ($u = \infty$) is termed the *second focal point* **F'** of a lens. An object at the *first focal point* **F** produces an image at ∞. That is, when the object distance AM = -FA or u = -f, the image distance u' = ∞. Substitution of these values of u and u' into Eq. 7.4 results in the *Lensmakers' Formula:*

$$\boxed{\frac{1}{f} = \frac{n' - n}{n}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \qquad 7.5$$

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