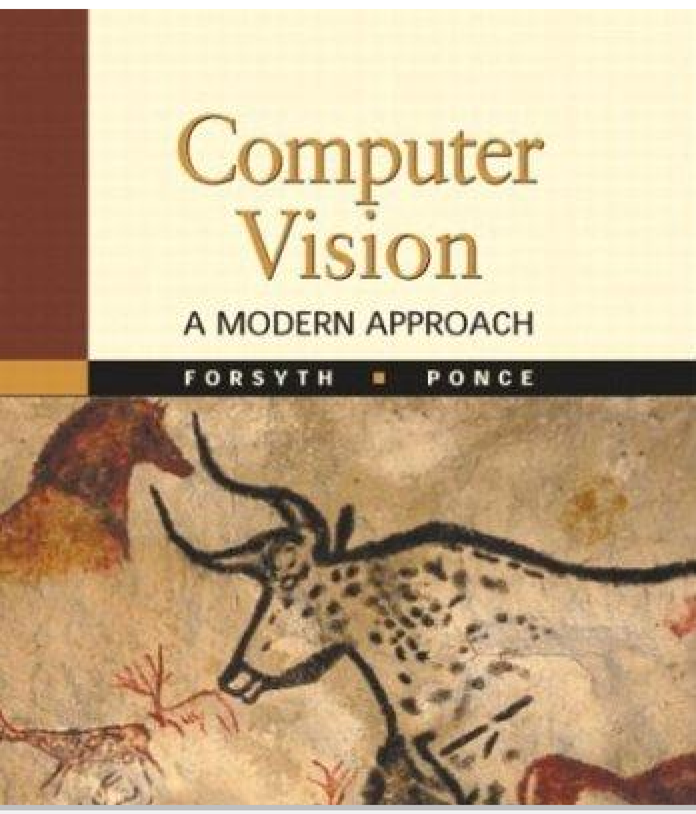
Computer
Vision
A MODERN APPROACH
FORSYTH PONCE

in the algorithms for establishing stereo correspondences presented in Sections 12.2 and 12.3.

12.1.2 Image Rectification

The calculations associated with stereo algorithms are often considerably simplified when the images of interest have been rectified, i.e., replaced by two projectively equivalent pictures with a common image plane parallel to the baseline joining the two optical centers (Figure 12.5). The rectification process can be implemented by projecting the original pictures onto the new image plane. With an apropriate choice of coordinate system, the rectified epipolar lines are scanlines of the new images, and they are also parallel to the baseline.

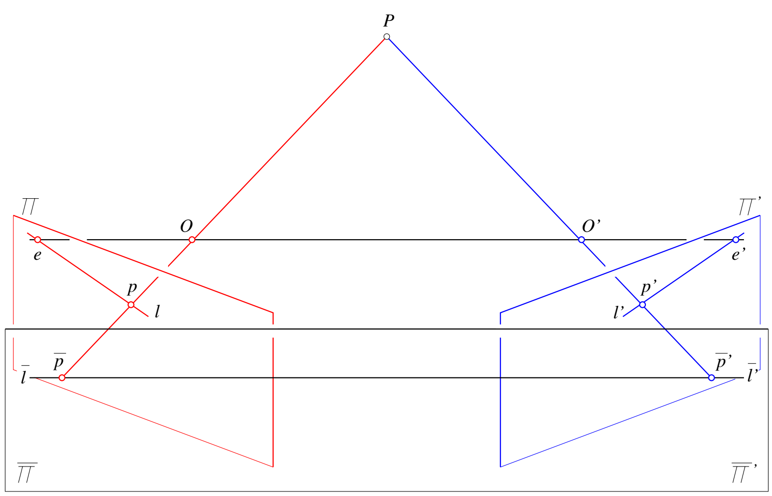

Figure 12.5. A rectified stereo pair: the two image planes Π and Π' are reprojected onto a common plane $\bar{\Pi} = \bar{\Pi}'$ parallel to the baseline. The epipolar lines l and l' associated with the points p and p' in the two pictures map onto a common scanline $\bar{l} = \bar{l}'$ also parallel to the baseline and passing through the reprojected points $\bar{p}$ and $\bar{p}'$. The rectified images are easily constructed by considering each input image as a polyhedral mesh and using texture mapping to render the projection of this mesh into the plane $\bar{\Pi} = \bar{\Pi}'$.

As noted in [?], there are two degrees of freedom involved in the choice of the rectified image plane: (1) the distance between this plane and the baseline, which is essentially irrelevant since modifying it will only change the scale of the rectified pictures, an effect easily balanced by an inverse scaling of the image coordinate axes, and (2) the direction of the rectified plane normal in the plane perpendicular to the baseline. Natural choices include picking a plane parallel to the line where the two original retinas intersect, and minimizing the distortion associated with the reprojection process.

In the case of rectified images, the notion of disparity introduced informally earlier takes a precise meaning: given two points p and p' located on the same scanline of the left and right images, with coordinates (u, v) and (u', v), the disparity is defined as the difference $d = u' - u$. Let us assume from now on normalized image coordinates. If B denotes the distance between the optical centers, also called baseline in this context, it is easy to show that the depth of P in the (normalized) coordinate system attached to the first camera is $z = -B/d$ (Figure 12.6).

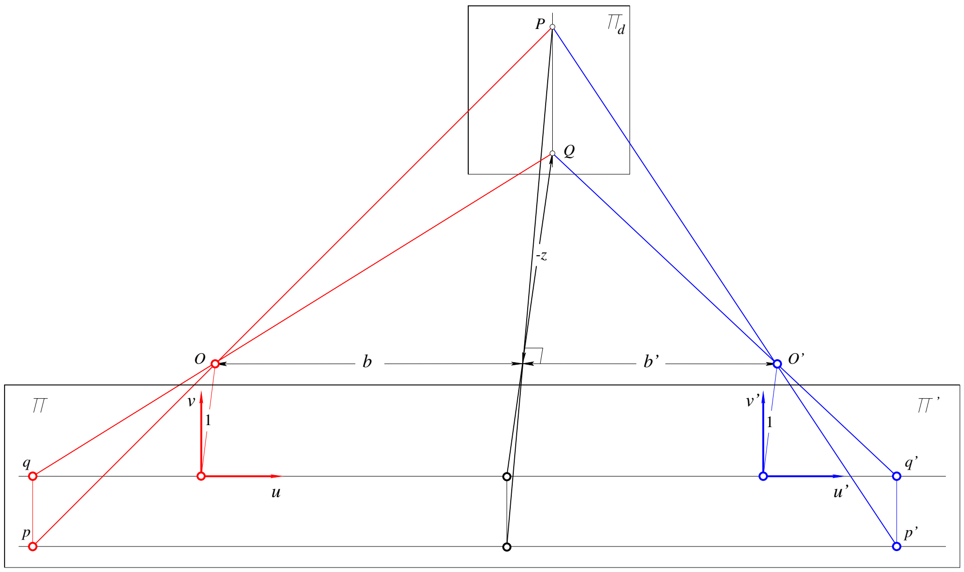

Figure 12.6. Triangulation for rectified images: the rays associated with two points p and p' on the same scanline are by construction guaranteed to intersect in some point P. As shown in the text, the depth of P relative to the coordinate system attached to the left camera is inversely proportional to the disparity $d = u' - u$. In particular, the preimage of all pairs of image points with constant disparity d is a *frontoparallel* plane Π_d (i.e., a plane parallel to the camera retinas).

To show this, let us consider first the points q and q' with coordinates $(u, 0)$ and $(u', 0)$, and the corresponding scene point Q. Let b and b' denote the respective distances between the orthogonal projection of Q onto the baseline and the two optical centers O and O'. The triangles qQq' and OQO' are similar, and it follows immediately that $b = zu$ and $b' = -zu'$. Thus $B = -zd$, which proves the result for q and q'. The general case involving p and p' with $v \neq 0$ follows immediately from the fact that the line PQ is parallel to the two lines pq and $p'q'$ and therefore also parallel to the rectified image plane. In particuliar, the coordinate vector of the point P in the frame attached to the first camera is $\boldsymbol{P} = -(B/d)\boldsymbol{p}$, where $\boldsymbol{p} = (u, v, 1)^T$ is the vector of normalized image coordinates of p. This provides yet another reconstruction method for rectified stereo pairs.