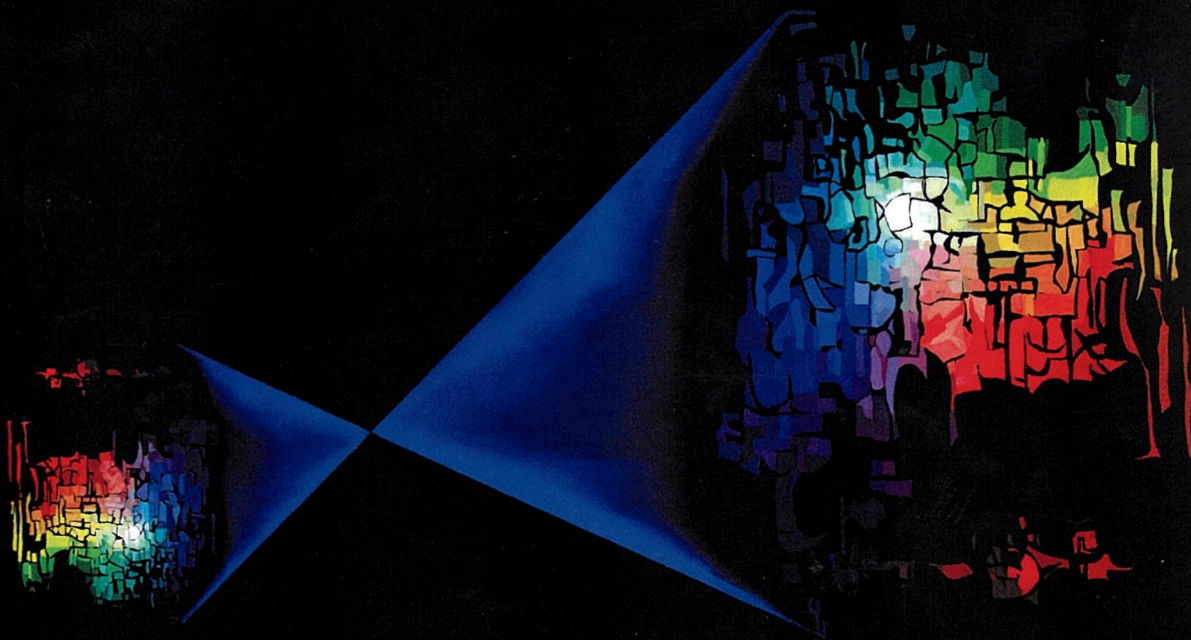


# Introduction to Aberrations in Optical Imaging Systems

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cosine of the angle  $\phi$  between these vectors. Table 5.1 summarizes the first four orders of aberrations using both vector and algebraic expressions. The fourth-order terms are often called the primary aberrations. The ten sixth-order terms can be divided into two groups. The first group (first six terms) can be considered as an improvement upon the primary aberrations by their increased field dependence, and the second group (last four terms) represents new wavefront deformation forms. Figure 5.1 shows the shape (aperture dependence only) of the zero, second, fourth, and the new wavefront shapes of the sixth-order aberrations.

In Table 5.1 the piston terms represent a uniform phase change across the aperture that does not degrade the image quality. Physically piston terms represent a time delay or advance in the time of arrival of the wavefront as it propagates from the object to the exit pupil. The second-order term magnification represents a change of magnification and the focus term represents a change in the axial location of the image. The coefficients for magnification and focus are set to zero given that Gaussian and Newtonian optics accurately predict the size and location of an image. However, a focus term is usually added to minimize aberrations or to select an observation plane other than the ideal image plane. In addition, the change of magnification and focus with respect to the wavelength are known as the transverse and longitudinal chromatic aberrations respectively.

### 5.5 Determination of the wavefront deformation

When rays of light do not pass through an ideal image point, the wavefront must be deformed. The wavefront deformation is measured with the aid of a reference sphere. The reference sphere for a given field point passes through the on-axis exit pupil point and its center coincides with the ideal image. As shown in Figure 5.3 the wavefront deformation multiplied by the index of refraction is the optical path between the wavefront and the reference sphere measured along the ray.

By convention the wavefront deformation is negative if the wavefront lags the reference sphere and positive if it leads the reference sphere. The units of the wavefront deformation are linear dimensions of millimeters, micrometers, etc. However, often the wavefront deformation is divided by the wavelength of light  $\lambda$ , and then the deformation is expressed in waves. The reference sphere is centered at point  $\vec{y}'_i \vec{H}$  in the image plane. Note that the tip of the aperture vector defines where the ray intersects the exit pupil plane. In this manner the aperture vector designates the same pupil point for all field points. This definition eventually makes easier the calculation of sixth-order coefficients that are coordinate-system dependent.

### 5.6 Parity of the aberrations

The aberrations can be classified as even or odd aberrations. For example, spherical aberration, astigmatism, field curvature, and the chromatic change of focus are even

aberrations. Coma, distortion, and the chromatic change of magnification are odd aberrations. The parity is found by observation of the algebraic power parity of the field and aperture vectors in the aberration coefficients. The odd aberrations have the important property that they cancel, or tend to cancel, in a system that has symmetry about the stop. That is, each half of the system contributes the same amount of aberration but with opposite algebraic sign. In contrast, in a symmetrical system the even aberrations from each half of the system add, rather than cancel.

### 5.7 Note on the choice of coordinates

The aberration theory developed in this book uses polar coordinates with the field vector  $\vec{H}$  serving as a reference to define the polar angle  $\phi$  and the aperture vector  $\vec{\rho}$ . Given the system's axial symmetry, inherently only three variables are necessary,  $|\vec{H}|$ ,  $|\vec{\rho}|$ , and  $\cos(\phi)$ , and eventually this leads to many simplifications. The other obvious choice is the use of Cartesian coordinates, which for historical reasons, previous works on wave aberration theory, and simplicity, are little used in the present treatment.

### 5.8 Summary

In this chapter we have introduced the aberration function as a polynomial depending on the field and aperture of the system. The terms in the aberration function represent aberrations as a wavefront deformation with respect to a reference sphere. The aberration coefficients provide the maximum amplitude of the deformation as an optical path. The aberration function provides a wealth of insight into the nature of an optical system and its aberrations. Symmetry considerations are important in developing the aberration function.

### Exercises

- 5.1. Using symmetry considerations, explain why the sine of the angle between the field and aperture vector does not appear in the aberration function.
- 5.2. Determine the aberration function up to fourth order of a system that has two orthogonal planes of symmetry. The intersection of these planes defines the optical axis. Use the unit vector  $\vec{i}$  to specify the direction of one of the planes of symmetry, and the field  $\vec{H}$  and aperture  $\vec{\rho}$  vectors.

### References

- [1] W. R. Hamilton, "Theory of systems of rays," *Trans. R. Irish Acad.* **15**(1828), 69–174.
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