MOTHERSON EXHIBIT 1012

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Rapid Design through Virtual and Physical Prototyping

Index

Carnegie Mellon University

Introduction to Mechanisms

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4 Basic Kinematics of Constrained Rigid Bodies

4.1 Degrees of Freedom of a Rigid Body

4.1.1 Degrees of Freedom of a Rigid Body in a Plane

The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has. Figure 4-1 shows a rigid body in a plane. To determine the DOF of this body we must consider how many distinct ways the bar can be moved. In a two dimensional plane such as this computer screen, there are 3 DOF. The bar can be *translated* along the *x* axis, translated along the *y* axis, and *rotated* about its centroid.

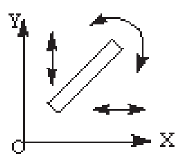

Figure 4-1 Degrees of freedom of a rigid body in a plane

4.1.2 Degrees of Freedom of a Rigid Body in Space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the *x*, *y* and *z* axes and three rotary motions around the *x*, *y* and *z* axes respectively.

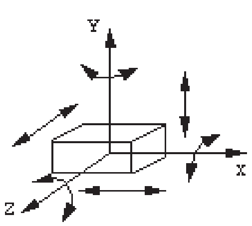

Figure 4-2 Degrees of freedom of a rigid body in space

Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with ***kinematic constraints***. *Kinematic constraints* are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The term kinematic pairs actually refers to *kinematic constraints* between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

4.2.1 Lower Pairs in Planar Mechanisms

There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.

A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.

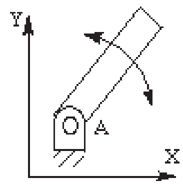

Figure 4-3 A planar revolute pair (R-pair)

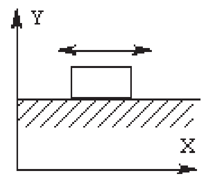

Figure 4-4 A planar prismatic pair (P-pair)

4.2.2 Lower Pairs in Spatial Mechanisms

There are six kinds of lower pairs under the category of spatial mechanisms. The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

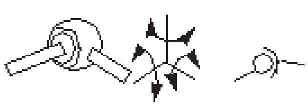

Figure 4-5 A spherical pair (S-pair)

A *spherical pair* keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to *rotate* relatively around *x*, *y* and *z* axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. **DOF = 3**.

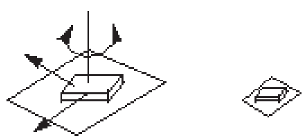

Figure 4-6 A planar pair (E-pair)

A *plane pair* keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table. Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane. Therefore, a plane

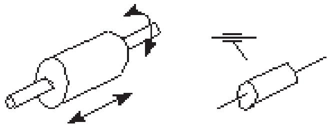

Figure 4-7 A cylindrical pair (C-pair)

A *cylindrical pair* keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis. Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. **DOF = 2.**

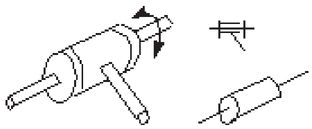

Figure 4-8 A revolute pair (R-pair)

A *revolute pair* keeps the axes of two rigid bodies together. Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis. Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

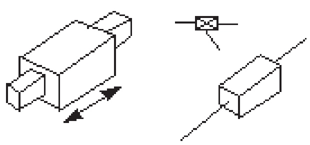

Figure 4-9 A prismatic pair (P-pair)

A *prismatic pair* keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis. Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**

Figure 4-10 A screw pair (H-pair)

The *screw pair* keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis. Therefore, a screw pair removes five degrees of freedom in spatial mechanism.

4.3 Constrained Rigid Bodies

Rigid bodies and kinematic constraints are the basic components of mechanisms. A constrained rigid body system can be a kinematic chain, a mechanism, a structure, or none of these. The influence of kinematic constraints in the motion of rigid bodies has two intrinsic aspects, which are the geometrical and physical aspects. In other words, we can analyze the motion of the constrained rigid bodies from their geometrical relationships or using Newton's Second Law.

A mechanism is a constrained rigid body system in which one of the bodies is the frame. The degrees of freedom are important when considering a constrained rigid body system that is a mechanism. It is less crucial when the system is a structure or when it does not have definite motion.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. We will discuss more on this topic for planar mechanisms in the next section.

4.4 Degrees of Freedom of Planar Mechanisms

4.4.1 Gruebler's Equation

The definition of the ***degrees of freedom*** of a mechanism is the number of independent relative motions among the rigid bodies. For example, Figure 4-11 shows several cases of a rigid body constrained by different kinds of pairs.

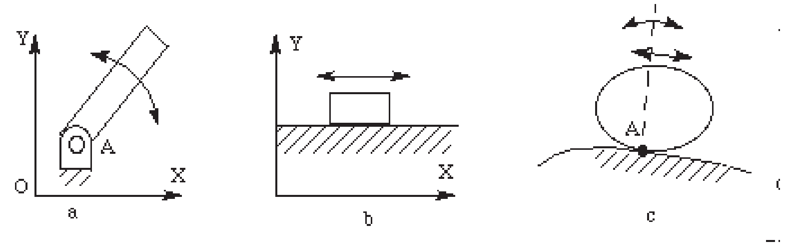

Figure 4-11 Rigid bodies constrained by different kinds of planar pairs

In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the *x* and *y* axes. The only way the rigid body can move is to rotate about the fixed point A.

In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the *x* axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the *y* axis.

In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.

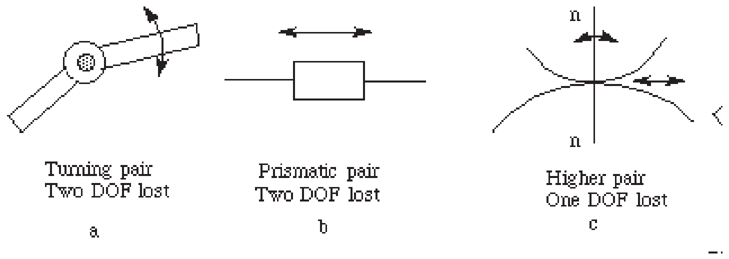

Figure 4-12 Kinematic Pairs in Planar Mechanisms

Therefore, we can write the following equation:

$$F = 3(n-1) - 2l - h$$

(4-1)

Where

F = total degrees of freedom in the mechanism

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