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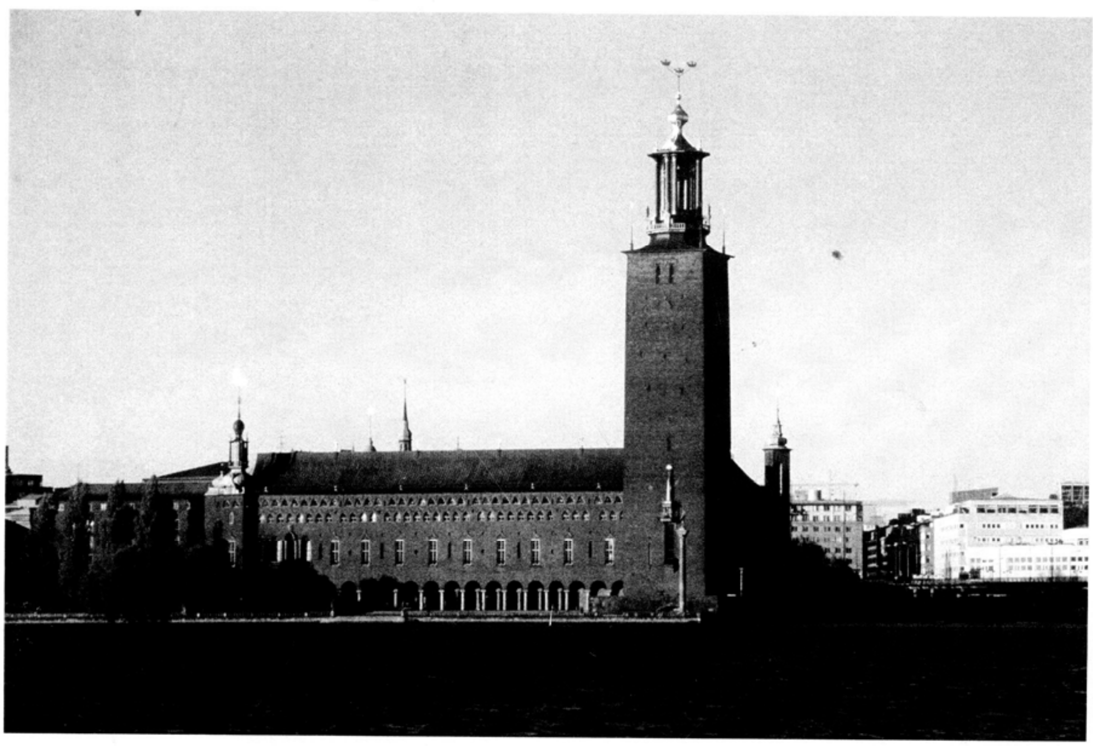

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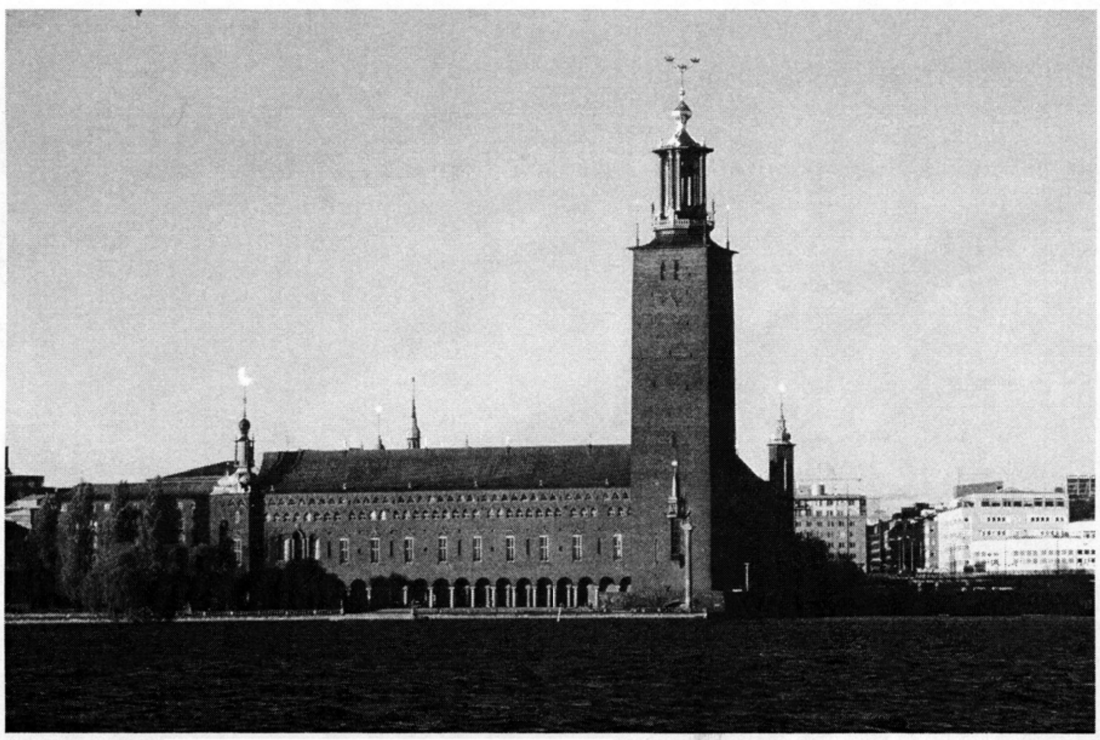

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Compact Feedback for MIMO-OFDM Systems over Frequency Selective Channels

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Abstract—**Transmit beamforming can improve the performance of multiple-input multiple-output (MIMO) antenna system. However, the feedback overhead of beamforming matrixes may significantly reduce the FDD system throughput. Efficient feedback techniques were reported in [2]-[10] recently. However, the complexities of them can be prohibitively high for systems with large numbers of transmit antennas (M_t) and spatial streams (M_s) since numerous large matrix codebooks of Grassman manifold packing G(M_t, M_s) [12] are required. In [14], it is shown that the beamforming matrix can be parameterized by independent unit vectors. This implies that quantizing the matrix jointly is equivalent to quantizing the vectors separately. First, we show that the distributions of the unit vectors are isotropic on unit spheres. Secondly, a low-complexity, scalable quantization scheme is derived. The key innovation lies in the quantization of the large dimension unit vectors. Each large dimension unit vector is partitioned into smaller dimension vectors and quantized by small size codebooks designed for low dimension unit vectors, and the partition itself is quantized by a 3-bit codebook. Finally, an interpolation scheme is proposed for beamforming matrixes downsampled in frequency domain, which significantly reduces overhead for MIMO-OFDM systems. The missing beamforming vectors are interpolated along the geodesic connecting two fed back vectors on a unit sphere. Simulations demonstrate that the overhead in 802.16d/e draft can be reduced by a factor of three using the proposed scheme.**

Keywords-MIMO;feedback;SVD;beamforming;interpolation

I. INTRODUCTION

The performance of MIMO systems can be improved by exploiting channel state information (CSI) at transmitter especially when the number of transmit antennas is greater than that of spatial streams. Transmit beamforming, adaptive bit loading, and antenna subset selection are common utilizations of the CSI. This paper focuses on the feedback for the optimal transmit beamforming, i.e. the SVD algorithm [1]. Since the CSI at transmitter may require feedback from the receiver, system throughput can be significantly reduced for time varying channels. Efficient feedback schemes were recently proposed in both academia and IEEE standard groups [2]-[10]. In [7] and [8], a quantization method of 2×2 beamforming matrix is proposed, which parameterizes the matrix by two angles and uniformly quantizes them. In [2] and [3], $M_t \times M_s$ beamforming matrix is quantized by a matrix codebook using various precoding criteria, where M_t and M_s are the numbers of transmit antennas and spatial streams respectively. The matrix codebook is optimized through Grassmannian subspace packing. The schemes in [2]-[10] except [4] require dedicated matrix codebooks for each $M_t \times M_s$ configuration and this may not be desired for large MIMO systems with many options of M_t and M_s. In [4], a general parameterization of any unitary beamforming matrix is derived, where the extracted parameters are independent angles and are obtained through a sequence of Givens rotations. The original matrix quantization of large size can be converted into the quantization of the parameters of small sizes.

For MIMO-OFDM system, two feedback techniques are reported in [9] and [5] respectively, which exploits the coherence between adjacent tones. Frequency domain downsampling and interpolation of the beamforming matrixes are proposed in [9]. Instead of downsampling, the difference between two beamforming matrixes on adjacent tones is quantized in [5] using a parametric codebook proposed in [6].

Following [14], we derive compact feedback schemes for MIMO-OFDM systems. First, we recursively parameterize and quantize the beamforming matrix column by column. The output parameters of each column are a unit vector, whose size reduces by one after each iteration. Secondly, we show that the distributions of the unit vectors are isotropic on unit spheres of various dimensions. Thirdly, since the direct quantization of the unit vectors of large dimensions requires large codebooks and thus incurs memory and computational complexities, a low-complexity quantization scheme is derived. Each large dimension unit vector is partitioned into smaller dimension vectors and quantized by small codebooks designed for smaller dimension unit vectors, and the partition itself is indexed by a 3-bit codebook with little performance degradation. Finally, an interpolation scheme is proposed for beamforming matrixes downsampled in frequency domain. The missing beamforming vectors are interpolated along the geodesic connecting two fed back vectors on a unit sphere. Simulations verify that the overhead in 802.16d/e draft can be reduced by a factor of three.

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II. System Overview

We consider a MIMO-OFDM system with M_t transmit and M_r receive antennas sending M_s spatial streams on N_f subcarriers. The MIMO channel with uncorrelated antennas for the f-th subcarrier is modeled by a matrix $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ [1], whose entry $\mathbf{H}_{i,j} \sim \mathrm{CN}(0,1)$ [2] for all i and j. The index of subcarrier is omitted in expression in section II and III for simplicity. The received signal vector for the f-th subcarrier is

$$\mathbf{r} = \mathbf{H}\hat{\mathbf{V}}\mathbf{d} + \mathbf{n}, \tag{1}$$

where $\mathbf{d}$ is a vector containing M_s data symbols one for each spatial stream; $\mathbf{n} \sim \mathrm{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ is the complex i.i.d. AWGN vector; $\hat{\mathbf{V}}$ is a M_t by M_s beamforming matrix. The transmit beamforming algorithm at the transmitter is the SVD algorithm [1]. The beamforming are conducted as follows. The transmitter first sounds the channel and the receiver computes singular value decomposition of the channel matrix $\mathbf{H}$ as $\mathbf{H} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^H$. The first M_s columns of the unitary matrix $\mathbf{V}$ are quantized and the quantization indexes are fed back to the transmitter. Finally, the transmitter reconstructs the beamforming matrix $\mathbf{V}$ from the feedback indexes and performs transmit beamforming.

A recursive quantization and its corresponding reconstruction of the beamforming matrix $\mathbf{V}$ are illustrated in **Figure** 1 and **Figure** 2, where $M_s = 3$ and $M_t = 4$.

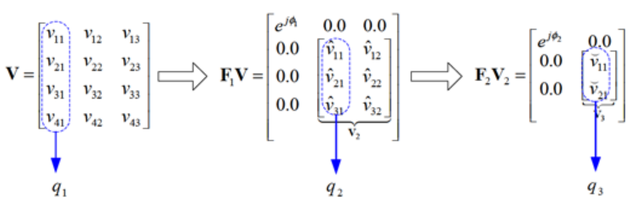

Figure 1. Recursive quantization of beamforming matrix.

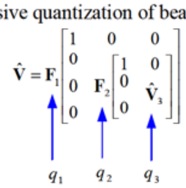

Figure 2. Recursive reconstruction of the beamforming matrix in Figure 1.

In **Figure 1**, the first column of $\mathbf{V}$, $\mathbf{v}_1$, is quantized by a vector codebook and the quantization index q_1 is fed back to the transmitter. A Householder reflection matrix is computed using the quantized vector, $\hat{\mathbf{v}}_1$, as

$$\mathbf{F}_1 = \mathbf{I} - \frac{2}{\|\mathbf{w}_1\|^2}\mathbf{w}_1\mathbf{w}_1^H, \tag{2}$$

where $\mathbf{w}_1 = \hat{\mathbf{v}}_1 e^{-j\varphi_1} - \mathbf{e}_1$, $\varphi_1 = \arg(\hat{\mathbf{v}}_1(1))$ and $\mathbf{e}_1 = [1,0,\ldots,0]^T$. Householder transformation on $\mathbf{V}$, i.e. $\mathbf{F}_1\mathbf{V}$, sets all the entries in the first column except the first to zero. Since $\mathbf{F}_1\mathbf{V}$ is unitary, the transformation also sets all entries in the first row to zero except the first. Due to quantization errors, these entries will not be exactly zero. The quantization and transformation comprise the first iteration. After the first iteration, the size of $\mathbf{V}$ reduces by one row and one column, and we repeat the iteration on the remaining matrix $\mathbf{V}_2$ that is obtained by striking out the first column and row of $\mathbf{F}_1\mathbf{V}$. The reconstruction of $\mathbf{V}$ at the transmitter using the fed back indexes q_i is shown in **Figure 2**. The reconstructed matrix $\hat{\mathbf{V}}$ can differ from $\hat{\mathbf{V}}$ by a global phase ϕ_i on each column. The quantization and feedback of these phases are not needed for narrow band systems, while they usually benefit interpolation of $\hat{\mathbf{V}}$ in frequency domain for MIMO-OFDM. An interpolation scheme that doesn't require these phases is depicted in section IV.

The original beamforming matrix $\mathbf{V}$ and the remaining matrixes $\mathbf{V}_i$ in iterations are isotropically distributed in the Stiefel Manifold V_{M_t-k, M_s-k} respectively, where k is the iteration steps. [14]. Using further results from [13], [2] and [4], we can show (see appendix I)

1) *The vectors for quantization, i.e. the first columns of* $\mathbf{V}$ *and* $\mathbf{V}_i$ *are isotropically distributed on the complex unit spheres.*

2) *The vector for quantization and its corresponding remaining matrix are independently distributed.*

3) *When channel correlation only exists in the receiver end, the above conclusion also apply.*

Instead of designing matrix codebooks, these imply that we can design codebooks for unit vectors with dimensions $2,\cdots,M_t$, whose codewords are uniformly distributed on unit spheres of dimensions $2,\cdots,M_t$. Furthermore, the sizes of the vector codebooks can be optimized jointly using criteria in [4]. These codebooks are sufficient to quantize beamforming matrix with up to M_t transmit antennas and various numbers of spatial streams. The substitution of matrix quantization with vector quantization reduces the complexities significantly; however, the quantization complexity of unit vectors of very large dimensions is still prohibitive. A low complexity scheme is proposed next.

III. Quantization of Large Dimension Unit Vector

Each large dimension unit vector is partitioned into two lower dimension vectors and quantized by small codebooks designed for low dimension unit vectors, and the partition itself is indexed by a small codebook, where two or three bits suffice. Any unit vector $\mathbf{u}$ can be written as

[1] $\mathbf{H} \in \mathbb{C}^{M \times N}$ denotes that $\mathbf{H}$ is an M by N complex matrix.

[2] $\mathbf{H}_{i,j} \sim \mathrm{CN}(0,1)$ denotes that entry on i-th row and j-th column of matrix $\mathbf{H}$ is a complex Gaussian random variable with zero mean and unit variance.

$$\mathbf{u} = \Big[\underbrace{u_1 \quad \cdots \quad u_{m_1}}_{\cos\theta \mathbf{u}_1} \quad \underbrace{u_{m_1+1} \quad \cdots \quad u_m}_{\sin\theta \mathbf{u}_2} \Big]^T, \tag{3}$$

where $0 \le \theta \le \pi/2$ and $\|\mathbf{u}_1\| = \|\mathbf{u}_2\| = 1$. It can be shown that unit vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ are isotropically distributed on complex unit spheres of dimensions m_1 and $m - m_1$ respectively if $\mathbf{u}$ is isotropically distributed. Therefore, $\mathbf{u}_1$ and $\mathbf{u}_2$ can be quantized using isotropic vector codebooks initially designed for small antenna arrays. Denote the quantized unit vectors of $\mathbf{u}_1$ and $\mathbf{u}_2$ as $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$. The global phase difference between $\mathbf{u}_i$ and $\hat{\mathbf{u}}_i$, for $i = 1,2$, is computed as

$$\varphi_i = \mathrm{phase}\left(\hat{\mathbf{u}}_i^H \mathbf{u}_i\right). \tag{4}$$

Define $\varphi = \varphi_2 - \varphi_1$. We design a 3-bit codebook for each pair of (m, m_1), which jointly quantizes (θ, φ). The analytical distribution of (θ, φ) is derived (see appendix II)

$$\rho_{m,m_1}(\theta) = \frac{2(\cos\theta)^{2m_1-1}(\sin\theta)^{2m-2m_1-1}}{\Gamma(m-m_1)\Gamma(m_1)/\Gamma(m)}$$

The θ and φ are independent and φ is uniform in $[0, 2\pi)$. Below we show the distribution for m=5, m_1=2.

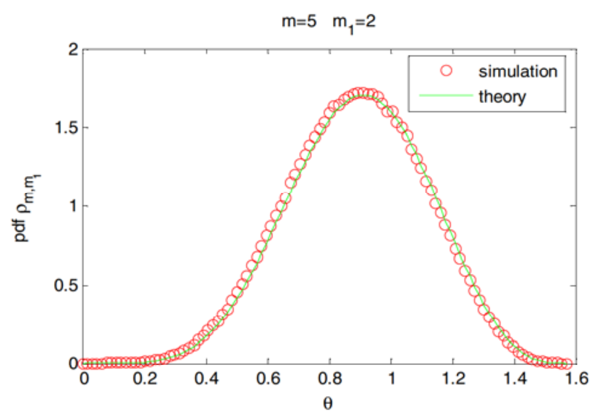

Figure 3 PDF of the θ distribution.

From the distribution, we find the optimal 3 bit codebook for the partition of a m vector into m_1 and $m - m_1$ sub-vectors, where (m, m_1) are (8,4), (7,3), (6,3) and (5,2). The reconstruction of $\mathbf{u}$ at the transmitter is

$$\hat{\mathbf{u}} = \begin{bmatrix} \cos\hat{\theta}\ \hat{\mathbf{u}}_1 \\ e^{j\hat{\varphi}} \sin\hat{\theta}\ \hat{\mathbf{u}}_2 \end{bmatrix}, \tag{5}$$

where the variable with hat notation is the quantized version of the original without hat.

For very large dimension, the partition can be carried out recursively. For example, a 12 dimension vector codebook can be constructed by 4 dimension vector codebook and (12,4) and (8,4) partition indexes.

IV. Interpolation of Beamforming Matrixes

The beamforming matrixes for adjacent OFDM subcarriers are usually correlated, the receiver can feed back every N_d subcarriers in order to reduce overhead, where $N_d > 1$. Since the global phases ϕ_is in **Figure 1** are not fed back, this creates a phase ambiguity during the interpolation at the transmitter. An example for a real 2×2 channel is illustrated in **Figure 4**, where $\mathbf{v}_1(f)$ and $\tilde{\mathbf{v}}_1(f)$, $f = 1,2,3$, are the correct and reconstructed beamforming vectors for the first eigenmode on subcarriers 1,2,3 respectively. We assume feedbacks are only conducted on subcarriers 1 and 3, i.e. $N_d = 2$, and interpolation is required for subcarrier 2. Because of the phase ambiguity $\tilde{\mathbf{v}}_1(3)$ is 180^o from $\mathbf{v}_1(3)$ and this won't affect the beamforming performance on subcarrier 3. However, the interpolated $\tilde{\mathbf{v}}_1(2)$ is 90^o from $\mathbf{v}_1(2)$ and it then directs little energy into the first eigenmode.

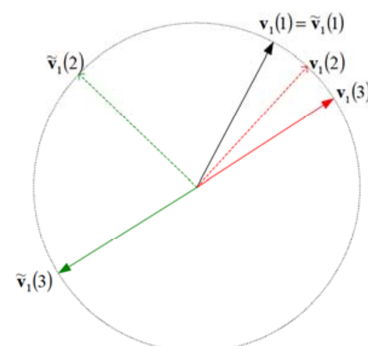

Figure 4. Illustration of interpolation error.

To remove the phase ambiguity, we use the reconstructed vector for the previous feedback subcarrier as reference to correct the global phase of the reconstructed vector of the current subcarrier as follows.

$$\hat{\mathbf{v}}_i(f) = \hat{\mathbf{v}}_i(f)\, \frac{\hat{\mathbf{v}}_i^H(f)\hat{\mathbf{v}}_i(f-2)}{\left\|\hat{\mathbf{v}}_i^H(f)\hat{\mathbf{v}}_i(f-2)\right\|} \tag{6}$$

where $\hat{\mathbf{v}}_i(f)$ can be either the *i*-th column of the beamforming matrix $\mathbf{V}$ or the reconstructed unit vector used in the Householder reconstruction in **Figure 2**. The phase corrected vectors of two adjacent feedback subcarriers are then interpolated along the geodesic connecting them on the unit sphere.

V. SIMULATIONS

We verify the proposed schemes by simulations using IEEE 802.16d modulations and ITU-R pedestrian channel model B. The packet size is 288 bytes, which is defined as a short test packet length in 802.16d standard. The OFDM subcarrier spacing is 11 kHz and bandwidth is 2.5 MHz. Packet error rate (PER) vs. signal to noise ratio (SNR) curves are plotted.

The performances of 4×2 with 2 spatial streams are shown in **Figure 5**. "16d draft" is the feedback scheme defined in 802.16d draft standard, which takes 40 bit per subcarrier. Its performance is within 0.2 dB to that with perfect channel information at the transmitter. "Grass-MSE" is the scheme in [2] with MSE criterion and a codebook downloaded from http://dynamo.ecn.purdue.edu/~djlove/grass.html, which takes 7 bits per subcarrier and within 0.9 dB to the ideal. "House" is for the proposed recursive scheme, where a 4 bit and 3 bit codebooks for 4 and 3 dimensions are used. "Grass-MSE" is better than the proposed scheme by 0.2 dB at cost of high complexities: "Grass-MSE" performs 2^7=128 searches to quantize the 4x2 beam forming matrix. "House" only searches 2^4+2^3=24 times. The performance loss is because "Grass-MSE" employs a better quantization criterion. This is verified by "House-MSE", which employs the same codebooks as "House" and the same MSE matrix selection criterion as "Grass-MSE". "House-MSE" outperforms "Grass-MSE" by 0.1 dB. The efficiency of the proposed interpolation scheme is verified by "House 13/9 b/t" and "House 13 b/t". The first one feeds back 13 bit per matrix and one matrix every 9 subcarrier, while the second one feeds back one matrix every subcarrier. There is almost no performance loss due to the downsampling, and both performances are very close to "16d draft", whose overhead is greater by more than 3 times than no downsampling and 28 times than downsampling. All the closed-loop 4×2 systems outperform 2×2 open loop system by more than 4 dB at PER 10%. This is supporting evidence for having more transmit antennae than the number of spatial streams.

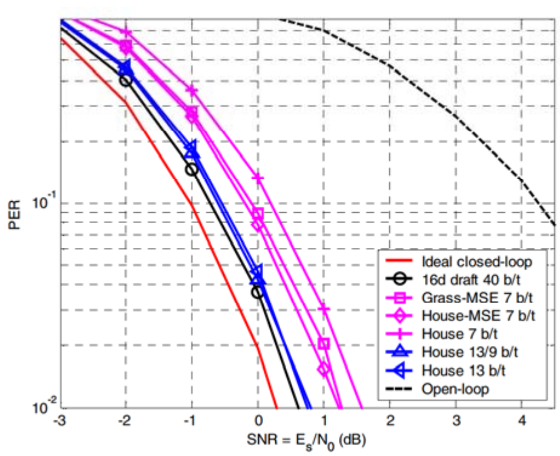

Figure 5. PER vs. SNR for 4×2 MIMO-OFDM with 2 spatial streams, BPSK, code rate ½.

The performances of 8×4 with 4 spatial streams are shown in **Figure 6**. For this configuration, the quantization complexity is prohibitive for schemes using matrix codebooks. We employ the partition technique in section III to reduce the quantization complexity of 8-, 7-, 6-, and 5-unit vectors. We start with a 7,6,5 bits codebook for 4-,3-,2-unit vector respectively. Larger dimension vector codebook construction parameters are listed in **Table 1**: For example, 8 dimension unit vector codebook is constructed by joining two 4 dimension vector codebooks of 7 bits each. The partition (8,4) is indexed by 3 bit. The total size of the 8 dimension vector codebook comes up to 7+7+3=17 bits.

m			codebook size	partition codebook	total
8	$m-m_1$	4	7	3	17
	m_1	4	7		
7	$m-m_1$	4	7	3	16
	m_1	3	6		
6	$m-m_1$	3	6	3	15
	m_1	3	6		
5	$m-m_1$	3	6	3	14
	m_1	2	5		
8x4 matrix codebook					62

Table 1 parameters of 8x4 matrix codebook

We then use the technique outline in session II to construct 62 bit 8x4 matrix codebook from 8-, 7-, 6- and 5-d unit vector codebook. The efficiency of the interpolation scheme is again verified by two "House" curves, whose performances are slightly better than that of "16d draft" and within 0.3 dB of the ideal. The overhead reduction is 23 and 2.6 times with and without downsampling respectively. All the closed-loop 8×4 systems outperform open loop 4×4 system by more than 10 dB at PER 10%. Quantization complexity of this 62 bit codebook is very low, only

$$2^7+2^7+2^7+2^6+2^6+2^6+2^6+2^5+4\times 2^3=704$$

searches are needed. This highlights the most valuable aspect of the systematic construction of large size codebooks. The memory space to store codebooks is also minimized.

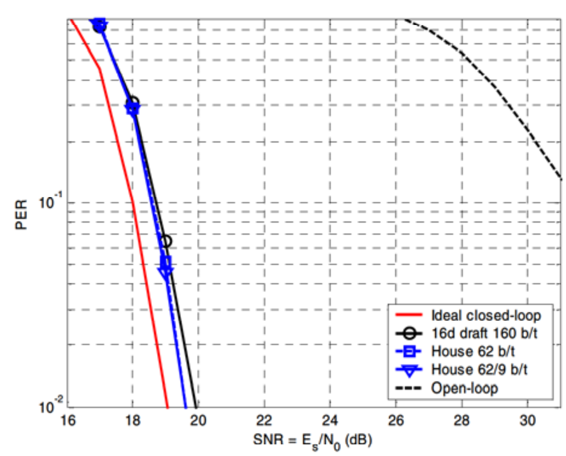

Figure 6. PER vs. SNR for 8×4 MIMO-OFDM with 4 spatial streams, 64 QAM, code rate ¾.

VI. CONCLUSIONS

We proposed a low-complexity, scalable approach for the quantization of beamforming matrix with flexible numbers of transmit antennas and spatial streams. Since the codeword searching is based on vector instead of matrix, the computational complexity of the proposed scheme is much lower than most of those schemes reported in literature. Furthermore, a low complexity method for the quantization of large vector is derived, which partitions a large vector into two small sub-vectors and quantizes them separately. Finally, a downsampling and interpolation scheme is also proposed for MIMO-OFDM systems, and no global phase is needed to be fed back. The feedback overhead is reduced with almost no performance degradation by exploiting the coherence between OFDM subcarriers.

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The authors would like to thank Ada Y. Poon and Minnie Ho for helpful suggestions and discussions.

APPENDIX I

The probability density of a matrix $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$, whose entry $\mathbf{H}_{i,j} \sim \mathrm{CN}(0,1)$ for all *i* and *j*, is

$$\exp(-\|H\|_F)\bigwedge_{i=1}^{M_r}\bigwedge_{j=1}^{M_t} dH_{i,j}$$

Where $\| \ \|_F$ denotes the Frobenius norm. Now let's consider HQ, where Q is an arbitrary $n \times n$ unitary matrix. The probability density of HQ is preserved since

$\|HQ\|_F = \|H\|_F$ and $\bigwedge_{i=1}^{M_r}\bigwedge_{j=1}^{M_t} d(HQ)_{i,j} = \bigwedge_{i=1}^{M_r}\bigwedge_{j=1}^{M_t} d(H)_{i,j}$.

Therefore, the SVD of $H = U\Sigma V^H$ and $HQ = U\Sigma(QV)^H$ shows that the probability density of V and QV are equal. So we demonstrate that the probability density is uniform. By applying Householder reflection (unitary matrix) recursively, we can show the uniformity of V_i as well.

With channel correlation at the receiver end only, we can write the channel matrix as $\sqrt{R}H$ [15]. The above proof applies too.

APPENDIX II

We start with the real m dimension volume integral

$$A = \int dx_1 dx_2 \ldots dx_m = \int f_m x^{m-1} dx \qquad \text{(A1)}$$

Where $f_m = 2\pi^{m/2}/\Gamma(m/2)$ and Γ is gamma function. We partition the m dimensional space into m_1 and $m-m_1$ dimensions, then

$$A = \int dy_1 \ldots dy_{m_1} \int dw_1 \ldots dw_{m-m_1}$$

$$= f_{m_1} f_{m-m_1} \int y^{m_1-1} w^{m-m_1-1} dy dw$$

If we rewrite $y = x\cos\theta$, $w = x\sin\theta$, and use $dydw = xdxd\theta$, we arrive at the following

$$A = f_{m_1} f_{m-m_1} \int x^{m-1} dx \int (\cos\theta)^{m_1-1}(\sin\theta)^{m-m_1-1} d\theta \qquad \text{(A2)}$$

Combined with equation A1, we get the density distribution of the θ

$$\rho^r_{m,m_1}(\theta) = \frac{2\Gamma(m/2)(\cos\theta)^{m_1-1}(\sin\theta)^{m-m_1-1}}{\Gamma((m-m_1)/2)\Gamma(m_1/2)} \qquad \text{(A3)}$$

In m dimension complex space, we have

$$\int dz_1 dz_2 \ldots dz_m = \int f(2m) z^{2m-1} dz$$

And by the same argument, we obtain the density distribution of the θ

$$\rho^c_{m,m_1}(\theta) = \frac{2\Gamma(m)(\cos\theta)^{2m_1-1}(\sin\theta)^{2m-2m_1-1}}{\Gamma(m-m_1)\Gamma(m_1)} \qquad \text{(A4)}$$

References

[1] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *AT&T Bell Labs Tech. Memo.*, 1995.

[2] D. J. Love and R. W. Heath Jr., "Limited feedback precoding for spatial multiplexing systems," in *Proc. IEEE Globecom 2003*, vol. 4, pp. 1857-1861, Dec. 2003.

[3] R. W. Heath Jr., J. C.hiang, B. Mondal, *et al.*, "11n partial proposal: Quantized precoding with feedback," *IEEE 802.11 TGn doc.* P802.11n-04/962, Aug. 2004.

[4] J. C. Roh and B. D. Rao, "An efficient feedback method for MIMO systems with slowly time-varing channels," in *Proc. IEEE WCNC 2004*, vol. 2, pp. 760–764, Mar. 2004.

[5] J. Zhang, H. Zhang, N. V. Waes, *et al.*, "Closed-loop MIMO precoding to improve MIMO link performance with limited feedback," *IEEE 802.16 TGe doc.* C802.16e-04/262r1, Aug. 2004.

[6] B. M. Hochwald, T. L. Marzetta, T. J. Richardson, *et al.*, "Systematic design of unitary space-time constellations," *IEEE Trans. Info. Th.*, vol. 46, pp. 1962-1973, Sept. 2000.

[7] E. N. Onggosanusi and A. G. Dabak, "A feedback-based adaptive multi-input multi-output signaling scheme," In *Proc. IEEE Asilomar Conf. 2002*, vol. 2, pp. 1694-1698, Nov. 2002.

[8] M. Ikram, E. N. Onggosanusi, V. Raghavan, *et al.*, "An enhanced closed-loop MIMO design for OFDM/OFDMA-PHY," *IEEE 802.16 TGe doc.* C802.16e-04/267, Aug. 2004.

[9] J. Choi and R. W. Heath Jr., "Interpolation based transmit beamforming for MIMO-OFDM with limited feedback," In *Proc. IEEE ICC 2004*, pp. 249-253, June, 2004.

[10] R. W. Heath Jr., "Precoding and interpolation for spatial multiplexing MIMO-OFDM with limited feedback," In *Proc. The 2004 Workshop on Smart Antennas in Wireless Comm.*, Stanford University, July 2004.

[11] *IEEE P802.16e/D5*, "Draft IEEE standard for local and metropolitan area networks, Part 16 ," Sept. 2004.

[12] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing line, planes, etc.: Packings in Grassmannian spaces," *Experimental Mathematics*, vol. 5, pp. 139-159, 1996.

[13] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *The Annals of Math. Stat.*, vol. 35, pp. 475-501, Jun. 1964.

[14] J. C. Roh and B. D. Rao, "Channel feedback quantization methods for MISO and MIMO systems" in *PIMRC 2004*, vol. 2, pp. 805-809

[15] A. Paulraj etc, "Introduction to space-time wireless communications", Cambridge Univ. 2003.