

SECOND EDITION

# Linear Algebra and Its Applications

Gilbert Strang

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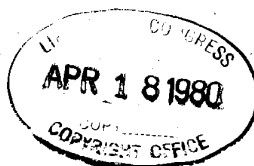
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