A New Method of Channel Feedback Quantization for High Data Rate MIMO Systems

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***Abstract*— In this work, we study a Multiple-Input Multiple-Output wireless system, where the channel state information is partially available at the transmitter through a feedback link. Based on Singular Value Decomposition, the MIMO channel is split into independent subchannels, which allows separate, and therefore, efficient decoding of the transmitted data signal. Effective feedback of the required spatial channel information entails efficient quantization/encoding of a Haar unitary matrix. The parameter reduction of an $n \times n$ unitary matrix to its $n^2 - n$ basic parameters is performed through Givens decomposition. We prove that Givens matrices of a Haar unitary matrix are statistically independent. Subsequently, we derive the probability distribution function (PDF) of the corresponding matrix elements. Based on these analyses, an efficient quantization scheme is proposed. The performance evaluation is provided for a scenario where the rates allocated to each independent channel are selected according to its corresponding gain. The results indicate a significant performance improvement compared to the performance of MIMO systems without feedback at the cost of a very low-rate feedback link.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) communication systems have received considerable attention in response to the increasing requirements of high spectral efficiency in wireless communications. In fact, the capacity of MIMO systems equipped with M_t transmit and M_r receive antennas scales up almost linearly with the minimum of M_t and M_r in flat Rayleigh fading environments [1] [2].

In recent years, researchers have examined the transmission strategies for MIMO systems, in which the transmitter and/or the receiver have full or partial knowledge of the Channel State Information (CSI). In [3], it has been shown that the achievable bit rate when perfect CSI is available both at the transmitter and the receiver is significantly higher than that when CSI is only available at the receiver. Due to practical restrictions such as an imperfect channel estimation and a limited feedback data rate, CSI is not perfect at the transmitter. However, unlike the single antenna systems, where exploiting CSI at the transmitter does not significantly enhance the capacity, in multiple antenna systems, the capacity is substantially improved through even partial CSI [4].

When CSI is available at the transmitter of a Multiple-Input Single-Output (MISO) system, beamforming can be used to exploit transmit diversity through spatial match filtering. In the context of MISO systems, several quantization schemes have been suggested to feed back instantaneous CSI to the transmitter. A simple and effective scheme has been suggested for a 3G wireless standard [5]. In [6] the authors have designed a codebook of beamformer vectors with the objective of minimizing the outage probability. Similar works, titled in single-substream precoding, have been reported in [7] [8], where the codebook design criterion is derived to maximize the received Signal to Noise Ratio (SNR).

For MIMO systems, the problem of quantizing CSI is more involved than for MISO systems. In [9] a precoder is combined with space-time encoder. The precoder is designed so as to reduce an upper bound on the worst pairwise codeword error probability conditioned on imperfect CSI at the transmitter. In [10], by assuming the availability of partial CSI at the transmitter of a MIMO system, a criterion has been presented to design a precoder based on the capacity maximization. However, [10] has not provided a practical approach to design such a precoder when the number of receive antennas is more than one. In this paper, we present a technique to address the need for a practical feedback scheme for a MIMO system (as opposed to a MISO). After we accomplished this work [11], we became aware of a similar work in quantizing the spatial information of the channel [12]. Specifically, the authors in [12] have come up with the same idea of using Givens rotations to reduce the number of parameters which need to be quantized. They use the time-dependency of the corresponding parameters of adjacent frames in slowly time-varying channels and employ a differential quantization for each parameter. However, in this work, we quantize the parameters of the channel's spatial information frame by frame. The quantization design and optimum bit allocation among the quantizers are accomplished based on the interference measure we define in Section III.

Consider the situation in which a MIMO channel is split into several independent subchannels by means of Singular Value Decomposition (SVD) based on the CSI at the transmitter and the receiver. This allows independent decoding of the subchannels and results in a low decoding complexity. In general, the optimum Maximum Likelihood (ML) decoding in a MIMO system without feedback is equivalent to a lattice decoding problem, which incurs significant complexity. Lower complexity decoding algorithms can be devised by the proper design of a transmit strategy, e.g., the Bell Labs Layered

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Space Time system [1]. However, this is achieved at the cost of degraded performance [13] [14]. This indicates that, in addition to the gain in the SNR performance, a reduction in the decoding complexity is another important advantage of a closed loop MIMO system based on the SVD.

In this work, the modulation format is selected to match the subchannel SNR on each subchannel. In this scheme, the spatial information of the channel and the constellation index of each subchannel is needed at the transmitter. We develop an algorithm to quantize the spatial information of the channel, based on minimizing the interference between the subchannels. The rate allocation strategy is determined at the receiver and fed back to the transmitter by using an efficient low rate approach.

The system model is described in Section II. In Section III, the parametrization and statistics of the right singular matrix of a Gaussian matrix is discussed. The feedback design is developed according to these properties, and the decoding strategy at the receiver. In Section IV, feedback scheme for transferring the rate information of each subchannel is discussed. In Section V, the simulation results are presented. Section VI concludes the paper.

II. System Model

We consider an independent and identically distributed block fading channel model. For a multiple transmit antenna system with M_t transmit and M_r receive antennas, the model leads to the following complex baseband representation of the received signal:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$ is the $M_t \times 1$ vector of the transmitted symbols, $\mathbf{H}$ is the $M_r \times M_t$ channel matrix, $\mathbf{n}$ is the $M_r \times 1$ zero mean Gaussian noise vector with the autocorrelation $\sigma^2\mathbf{I}$ where $\mathbf{I}$ is the identity matrix, and $\mathbf{y}$ is the received signal. The power constraint of the transmitted signal is defined as $E(\mathbf{x}\mathbf{x}^*) = \mathcal{E}\mathbf{I}$, where E represents the expectation and $(.)^*$ is the hermitian of $(.)$. The elements of the channel matrix $\mathbf{H}$ are circularly symmetric complex Gaussian distributed with zero mean and unit variance.

The SVD of matrix $\mathbf{H}$ is defined as [15]

$$\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{U}^*, \tag{2}$$

where $\mathbf{V}$ and $\mathbf{U}$ are the unitary matrices, and $\mathbf{\Lambda}$ is a diagonal matrix. If $\mathbf{U}$ is available at the transmitter and the transmitted signal is prefiltered by $\mathbf{U}$, then the received signal is given by

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{U}\mathbf{x} + \mathbf{n} \\ &= \mathbf{V}\mathbf{\Lambda}\mathbf{x} + \mathbf{n}. \end{aligned} \tag{3}$$

The receiver filters the received vector $\mathbf{y}$ by $\mathbf{V}^*$,

$$\mathbf{r} = \mathbf{V}^*\mathbf{y} = \mathbf{\Lambda}\mathbf{x} + \mathbf{n}. \tag{4}$$

Therefore, a MIMO channel with M_t transmit antennas and M_r receive antennas is transformed to ‘rank $\mathbf{H}$’ parallel subchannels. This transformation substantially reduces the decoding complexity. In the transition from (3) to (4), we take advantage of the fact that the elements of $\mathbf{n}$ are statistically independent, and rotating $\mathbf{n}$ by the unitary matrix $\mathbf{V}^*$ does not change the distribution of the noise.

As it can be seen in (4), the subchannels provide different gains corresponding to $\mathbf{\Lambda}$. We consider a case in which data is transmitted and received separately in each subchannel with different rates and with equal energy. It can be shown that the use of equal energy maximizes the rate under the assumption of continuous approximation for a cubical shaping region (subject to a constraint on total energy). This method involves the allocation of an appropriate data rate to each subchannel, while a certain target error rate on each subchannel is met. By this assumption, the transmitter requires the rate information of each subchannel, in addition to the right singular matrix of the channel.

III. Feedback Design:Channel Singular Matrix Quantization

In the scenario described above, the transmitter needs to know the right singular matrix of the channel. We assume that a noiseless feedback link from the receiver to the transmitter is available. By the SVD of $\mathbf{H}$ at the receiver, the unitary matrix $\mathbf{U}$ is computed, quantized and sent to the transmitter.

If we assume the quantization error $\mathbf{\Delta U}$ for $\mathbf{U}$, the received signal is

$$\mathbf{r} = \mathbf{\Lambda}\mathbf{x} + \mathbf{\Lambda}\mathbf{U}^*\mathbf{\Delta U}\mathbf{x} + \mathbf{n}. \tag{5}$$

The quantization scheme is based on minimizing the interference between the parallel subchannels, since the receiver strategy is to detect the data in each subchannel independently. The variance of the interference signal is expressed as follows:

$$\begin{aligned} E(\|\mathbf{\Lambda}\mathbf{U}^*\mathbf{\Delta U}\mathbf{x}\|^2) &= E\mathrm{Tr}(\mathbf{\Lambda}\mathbf{U}^*\mathbf{\Delta U}\mathbf{x}\mathbf{x}^*\mathbf{\Delta U}^*\mathbf{U}\mathbf{\Lambda}) \\ &= \lambda E\mathrm{Tr}(\mathbf{\Delta U}\mathbf{\Delta U}^*\mathbf{x}\mathbf{x}^*) \\ &= \lambda\mathcal{E}E\mathrm{Tr}(\|\mathbf{\Delta U}\|^2), \end{aligned} \tag{6}$$

where $E(\mathbf{\Lambda}^2) = \lambda\mathbf{I}$, $E(\mathbf{x}\mathbf{x}^*) = \mathcal{E}\mathbf{I}$ and Tr denotes the trace function. In (6), we use the property that the singular values of a Gaussian matrix are independent from the corresponding singular vectors [16], and also the equality $\mathrm{Tr}(\mathbf{AB}) = \mathrm{Tr}(\mathbf{BA})$. As a result, minimizing the mean of the interference power leads to the minimization of the Frobenius norm of $\mathbf{\Delta U}$. In order to minimize the interference, the unitary matrix $\mathbf{U}$ should be quantized, based on minimizing the expression in (6). In the following, we examine the statistical properties of the underlying unitary matrices.

A. Statistics of Singular Matrices of a Random Gaussian Matrix

In most analytic studies of MIMO systems, the channel between the transmitter and the receiver is assumed to be Rayleigh fading. This indicates that the entries of the channel matrix are statistically independent and identically distributed, and have a complex Gaussian distribution with a zero mean. We are interested in the probability distribution of the singular matrices[1] of the mentioned channel matrix in the space of

[1]The probability distribution of a matrix is the joint PDF of its elements.

$M(n)$, namely the group of $n \times n$ unitary matrices. It is known that such a random unitary matrix takes its values uniformly from $M(n)$ in the sense of the following property [17].

Theorem 1 *Let us assume that* $\boldsymbol{U}$ *is a singular matrix of a random Gaussian matrix. For all* $\boldsymbol{V} \in M(n)$*, the distribution of* $\boldsymbol{U}$ *and* $\boldsymbol{VU}$ *are the same.*

Such a distribution is called the Haar distribution and the corresponding unitary matrices are called Haar unitary matrices [17]. We refer to this property as the right invariance property.

A complex $n \times n$ matrix can be described by $2n^2$ real parameters. However, the definition of a unitary matrix implies there is a dependency between these parameters. The number of equations describing this dependency for an $n \times n$ unitary matrix is $n + 2\binom{n}{2}$ (as the norm of each column is unit and every two columns are orthogonal to each other). Therefore, a unitary matrix $\mathbf{U}$ has $n^2 = 2n^2 - (n + 2\binom{n}{2})$ independent parameters. Here, for the purpose of matrix decomposition using SVD, n out of n^2 parameters are also redundant, since SVD can be performed such that the diagonal elements of $\mathbf{U}$ in (2) are set to be real. Several different approaches such as the Cayley transform, Householder reflection, and Givens rotations can be used to parameterize a complex $n \times n$ unitary matrix $\mathbf{U}$ in its $n^2 - n$ real parameters [15].

In this work, we consider the matrix decomposition using Givens matrices. Besides their ability to decompose the unitary matrix to the minimum number of parameters, the resulting parameters are statistically independent (Theorem 2). The independence property facilitates the quantization procedure.

A complex unitary matrix $\mathbf{U}$ can be decomposed in terms of the products of Givens matrices [15], i.e.,

$$\mathbf{U} = \prod_{k=1}^{n-1} \prod_{i=k+1}^{n} \mathbf{G}(k, i), \tag{7}$$

where each $\mathbf{G}(k, i)$ is an $n \times n$ unitary matrix with two parameters, c, and, s. Parameter c is in the position (k, k) and (i, i), s is in (k, i) and $-s^*$ is in (i, k), $k < i$. The other diagonal elements of the matrix $\mathbf{G}(k, i)$ are 1 and the remaining elements are zero. Since $\mathbf{G}(k, i)$ is a unitary matrix, then $|c|^2 + |s|^2 = 1$. In this work, we can assume that c is real since the SVD operation allows $\mathbf{U}$ to be multiplied by an arbitrary diagonal unitary matrix

In the following, the statistical properties of Givens matrices corresponding to a Haar unitary matrix $\mathbf{U}$ is derived. This will be later used to determine the quantization strategy. The key point of the codebook design for a Haar unitary matrix is the following result[2].

Theorem 2 *Let us assume that* $\boldsymbol{U}$ *is an* $n \times n$ *unitary matrix with a Haar distribution which is decomposed into Givens matrices as in (7). The set of Givens matrices* $\{\boldsymbol{G}(k, i)\}$ *for* $1 \le k < i \le n$ *are statistically independent of each other. Moreover, the PDF of the elements of* $\boldsymbol{G}(k, i)$ *is*

$$p_{k,i}(c, \angle s) = p_{k,i}(c)p(\angle s) = \frac{i-k}{\pi} c^{2(i-k)-1},$$
$$0 \le c \le 1, \quad \angle s \in [-\pi, \pi]. \tag{8}$$

The proof is omitted because of the limited space. See [11] for the details.

B. Quantization of Unitary Matrices

Based on the criterion presented for the quantizer design in (6), the distortion measure of the quantizer for matrix $\mathbf{U}$ is defined as follows:

$$D(U) = \frac{1}{2} E \mathrm{Tr}(\|\mathbf{U} - \widehat{\mathbf{U}}\|^2). \tag{9}$$

Substituting (7) in (9), we derive the first order approximation of $D(U)$ as follows:

$$D(U) \simeq \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} D(G(k, i)), \tag{10}$$

where $D(G)$ is defined as follows:

$$D(G) = \frac{1}{2} E \mathrm{Tr}(\|\mathbf{G} - \widehat{\mathbf{G}}\|^2), \tag{11}$$

and $\widehat{\mathbf{G}}$ is the quantized version of $\mathbf{G}$. In the following, we use

$$\mathbf{G} = \begin{pmatrix} c & s \\ -s^* & c \end{pmatrix} \tag{12}$$

to refer to the non-trivial part of a Givens matrix.

1) Method A: The basic parameters of the Givens matrix, named c and $\theta = \angle s$, are quantized as $\widehat{c}$ and $\widehat{\theta}$, independently. The transmitter uses $\widehat{c}$ and $\widehat{\theta}$ to construct $\widehat{\mathbf{G}}$ as follows:

$$\widehat{\mathbf{G}} = \begin{pmatrix} \widehat{c} & |\widehat{s}| e^{j\widehat{\theta}} \\ -|\widehat{s}| e^{-j\widehat{\theta}} & \widehat{c} \end{pmatrix}, \tag{13}$$

where $|\widehat{s}| = \sqrt{1 - \widehat{c}^2}$. According to the construction scheme in (13) , $\widehat{\mathbf{G}}$ is also unitary. It can be easily demonstrated that the first order approximation of $D(G)$ is

$$D(G) \simeq E\left(\frac{(c - \widehat{c})^2}{1 - c^2}\right) + E(1 - c^2)E(\theta - \widehat{\theta})^2. \tag{14}$$

We apply (8) to simplify the following expression,

$$E(1 - c_{k,i}^2) = \frac{1}{2(i-k)+1}. \tag{15}$$

By applying (10) and (14), and (15), we write,

$$D(U) \simeq \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} E\left(\frac{(c_{k,i} - \widehat{c}_{k,i})^2}{1 - c_{k,i}^2}\right) + \frac{1}{2(i-k)+1} E(\theta_{k,i} - \widehat{\theta}_{k,i})^2. \tag{16}$$

We design Linde-Buzo-Gray (LBG) quantizers for different $c_{k,i}$ and $\theta_{k,i}$ to minimize

$$E\left(\frac{(c_{k,i} - \widehat{c}_{k,i})^2}{1 - c_{k,i}^2}\right),$$

[2]As we mentioned earlier, after we accomplished this work, we became aware of [12] which independently proves a similar result.

and,

$$E(\theta_{k,i} - \widehat{\theta}_{k,i})^2,$$

respectively.

We utilize dynamic programming to find the optimum allocation of bits among the quantizers. We use a trellis diagram with $B+1$ states and $n^2 - n$ stages to allocate B bits to the quantizers of the independent parameters $c_{k,i}$ and $\theta_{k,i}$, $1 \leq i < k \leq n$ of the $n \times n$ unitary matrix. The lth state in jth stage corresponds to the distortion caused by the jth parameter using $l-1$ bits. In the trellis diagram, each branch represents the difference between the number of bits corresponding to the two ending states on the branch. The search through the trellis determines the path with minimum overall distortion and the corresponding number of bits for each parameter.

2) Method B: In this method, we quantize each Givens matrix as a unit and define a new parameterization for this purpose. The non-trivial part of a Givens matrix can be shown as follows:

$$\mathbf{G} = \begin{pmatrix} \cos(\eta) & e^{j\theta}\sin(\eta) \\ -e^{-j\theta}\sin(\eta) & \cos(\eta) \end{pmatrix}, \qquad (17)$$

where $0 \leq \theta \leq 2\pi$ and $0 \leq \eta \leq \pi$. The distortion measure for $\mathbf{G}$, relative to a reference matrix with parameters η_0 and θ_0, is

$$D_0(G) = 1 - E(\cos(\eta)\cos(\eta_0) + \sin(\eta)\sin(\eta_0)\cos(\theta - \theta_0)). \qquad (18)$$

We use the LBG algorithm to determine the regions and centroids of the two-dimensional quantizers corresponding to various (η, θ). The distortion function is

$$D = \sum_{m=1}^{M} \int_{R_m} D_m(G) p(\eta,\theta) d\eta d\theta, \qquad (19)$$

where R_m is the mth quantization region and M is the number of quantization partitions. The centroid (η_m, θ_m) is determined iteratively by minimizing the distortion function in the region R_m,

$$\theta_m = \tan^{-1}(\frac{\varsigma_m}{\gamma_m}), \qquad (20)$$

$$\eta_m = \tan^{-1}\left(\frac{\sqrt{\varsigma_m^2 + \gamma_m^2}}{\int_{R_m} \cos^{l+1}(\eta)\sin(\eta) d\eta d\theta}\right), \qquad (21)$$

where

$$\gamma_m = \int_{R_m} \cos^l(\eta)\sin^2(\eta)\cos(\theta) d\eta d\theta, \qquad (22)$$

and,

$$\varsigma_m = \int_{R_m} \cos^l(\eta)\sin^2(\eta)\sin(\theta) d\eta d\theta, \qquad (23)$$

and $l = 2(i-k) - 1$, in the case of quantizing $\mathbf{G}(k,i)$ in (7). By applying the above algorithm, we design codebooks of the matrices for different rates. In this method, a trellis diagram with the same structure as method A trellis diagram is used for optimum bit allocation. The trellis diagram contains $\frac{n^2-n}{2}$ stages, each corresponds to a Givens component of an $n \times n$ unitary matrix, and $B+1$ states (B is the number of bits). The lth state in jth stage corresponds to the distortion caused by the jth Givens matrix using $l-1$ bits.

IV. Feedback Design: Encoding of Rate Allocation Information

Besides the quantized right singular matrix of the channel that is fed back to the transmitter, information pertaining the rate that will be allocated to each subchannel is also fed back. This indicates a set of M_t indices from a set of N_R predetermined rates, e.g., the different modulation schemes. Obviously, the total rate is bounded, and since we can perform the SVD of the channel matrix so that the singular values become ordered, the M_t indices correspond to an ordered set of increasing positive integers (rates). To encode this information, we can use a trellis diagram with N_R states and M_t stages. The states correspond to the set of possible rates in an increasing fashion, and there is a branch from each state to another state in the next stage, only if the entering state is located at the same or at a lower level position. Each path in the trellis then corresponds to a set of subchannel rates, whose index is chosen by the receiver and fed back to the transmitter. The trellis structure exploits the ordering property of the rates, and therefore, allows their efficient coding at a rate of [18]

$$R_{rate} = \left\lceil \log_2 \binom{M_t + N_R - 1}{N_R - 1} \right\rceil. \qquad (24)$$

The complexity of this algorithm is very low and is, in fact, proportional to the number of states. Similar structures have been used to address the points of a block-based trellis quantizer in [18], or a pyramid vector quantizer in [19].

V. Performance Evaluation

In this section, we present the performance results of the system, described in Section II. We assume that the precoding is performed by the quantized version of the right singular matrix of the channel by applying the quantization methods presented in Section III-B. For the different subchannels, we use different modulation schemes. The process of selecting the appropriate modulation scheme for each subchannel is accomplished at the receiver. We restrict the system to transmit data with the power $\frac{\mathcal{E}}{M_t}$ on each transmit antenna. It means that the power is equally distributed among data symbols, since we use an orthonormal precoder. Therefore the rate is maximized based on continuous approximation concept. At the receiver, the channel state information and the instantaneous quantization noise power is assumed to be available. For each subchannel, the probability of error is computed for different modulation schemes. The receiver selects a modulation scheme for each subchannel that achieves the target Bit Error Rate (BER) of the system and sends the indices of the corresponding modulation schemes to the transmitter through the feedback channel that was described in Section IV. The received SNR at the kth subchannel is,

$$SNR_k = \frac{\mathcal{E}\lambda_k^2}{M_t(\sigma^2 + \widehat{\sigma}_k^2)}, \qquad (25)$$

where $\widehat{\sigma}_k^2$ is the corresponding quantization noise variance of the kth subchannel. We consider a set of QAM modulation formats. At the receiver, the rate r_k of kth subchannel is computed as follows,

$$\max_{P(SNR_k) \leq P_b} r_k, \tag{26}$$

where P_b is the target BER of the system and $P(SNR)$, the BER function of the modulation scheme with rate r, is [20]

$$P(SNR) \approx \frac{4}{r} Q\left(\sqrt{\frac{3r\,SNR}{2^r - 1}}\right), \tag{27}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{x^2}{2}} dx$.

The SVD and Givens decomposition are performed at the receiver. The number of computations required by the SVD and Givens decomposition for an $n \times n$ matrix are $21n^3$ and $3n^2(n-1)$ flops, respectively [15].

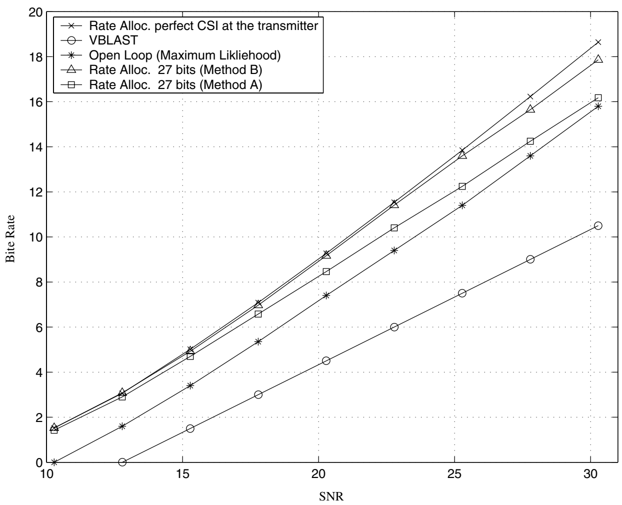

Fig. 1. The average bit rate for different schemes where $M_t = 3$ and $M_r = 3$. The target BER$= 5 \times 10^{-3}$.

Figure 1 shows the average bit rate versus SNR for different MIMO systems with $M_t = 3$ and $M_r = 3$ at the target BER$=$ 5×10^{-3}. We use 8 bits for the feedback of the right singular matrix. The modulation schemes we use are QAMs with bit rates between 1 and 7, inclusively, and then $N_R = 8$. We use 6 bits for the feedback of the rate allocation vector in each transmission block (the number of bits is derived by applying (24)). The two quantization methods presented in Section III-B are compared. Method B outperforms method A at the cost of complexity. The average bit rate of a 3×3 MIMO system with ML decoding is depicted. It can be seen that the performance gain, compared to the gain of the ML decoding of the open loop system is noticeable. For example, at the bit rate$=$ 10 the system has a 3 dB improvement in comparison to the optimum open loop system. We also compare the performance of this system with that of a V-BLAST system which is proposed as a solution to overcome the complexity problem. Figure 1 displays a significant improvement in comparison to the V-BLAST at the price of the feedback. The performance of the system, if perfect channel information is available at the transmitter, is also depicted.

VI. Conclusion

In this work, we have presented efficient methods for the channel information quantization in a high data rate MIMO system. We have developed efficient algorithms for the quantization of the underlying unitary matrices. Also, we have presented a low rate indexing of rate allocation information. The simulation results show a significant improvement compared to MIMO systems without feedback at the cost of a very low-rate feedback link.

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