Dec. 12, 1967 V. E. BENES 3,358,269

SQUARE SWITCH DISTRIBUTION NETWORK EMPLOYING
A MINIMAL NUMBER OF CROSSPOINTS

Filed April 10, 1964

3 Sheets-Sheet 1

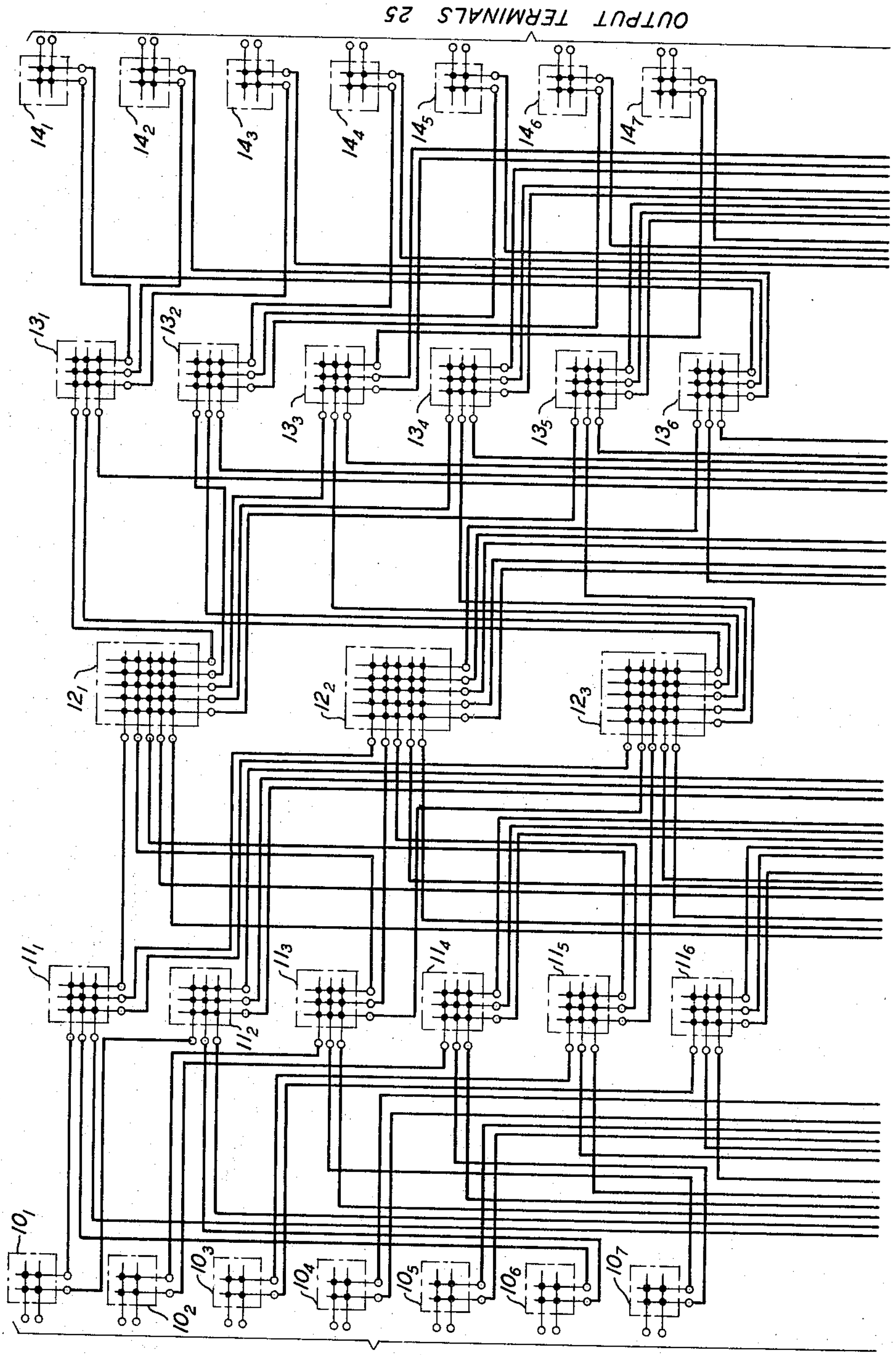

FIG. 1(A)

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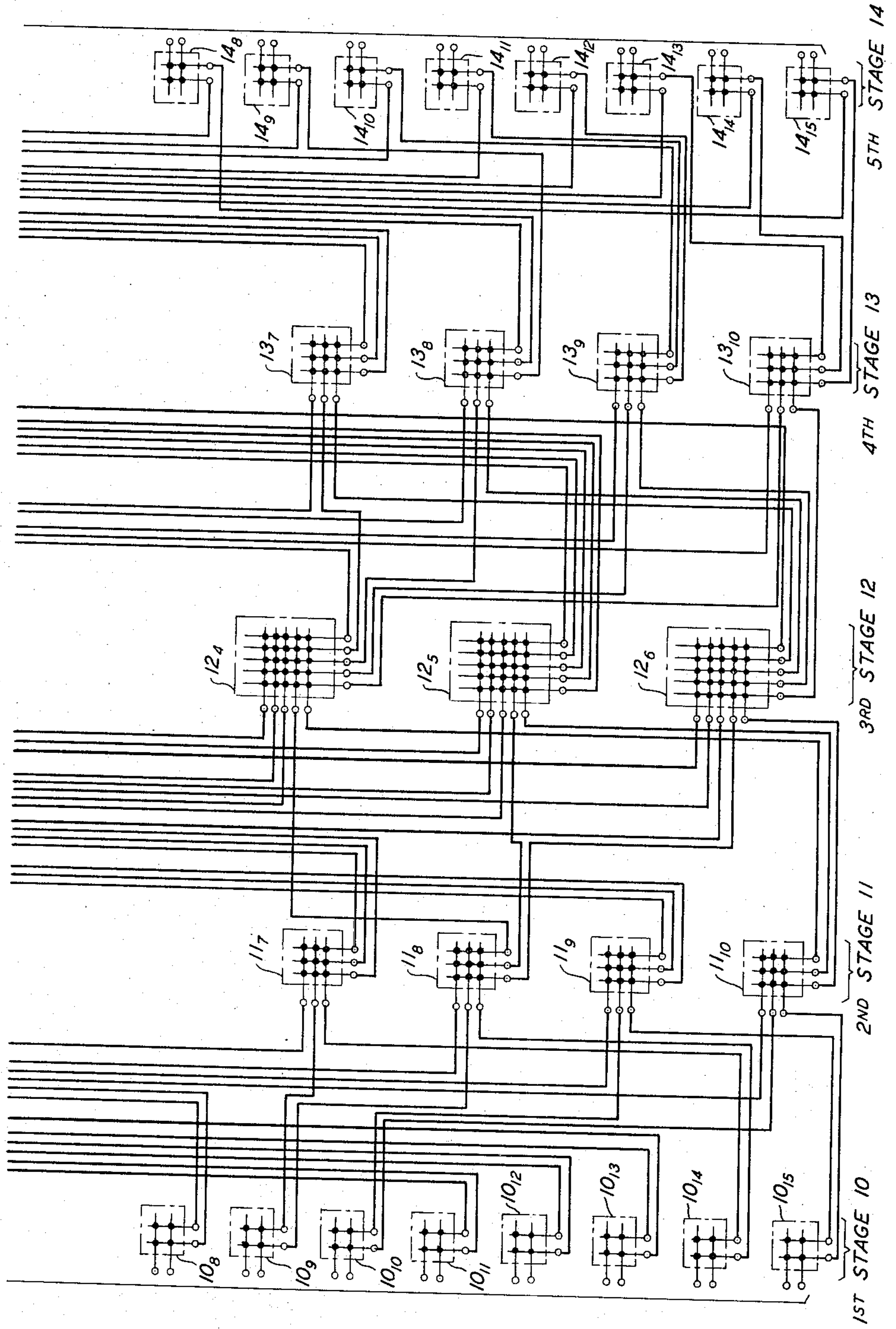
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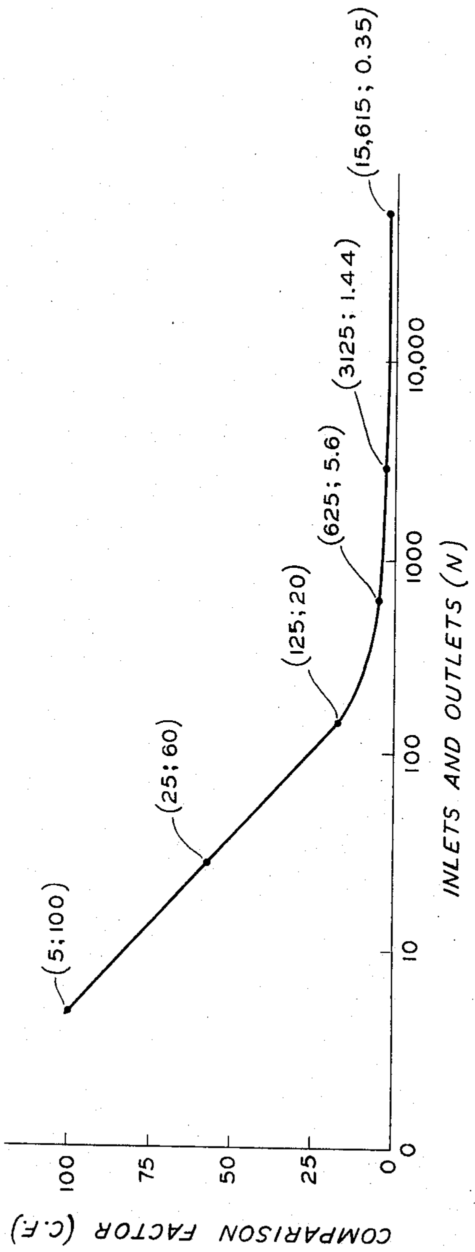
14_8
14_9
14_{10}
14_{11}
14_{12}
14_{13}
14_{14}
14_{15}
4TH STAGE 13
13_7
13_8
13_9
13_{10}
3RD STAGE 12
12_4
12_5
12_6
2ND STAGE 11
11_7
11_8
11_9
11_{10}
1ST STAGE 10
10_8
10_9
10_{10}
10_{11}
10_{12}
10_{13}
10_{14}
10_{15}

FIG. 1(B)

FIG. 2


(5;100)
(25;60)
(125;20)
(625;5.6)
(3125;1.44)
(15,615;0.35)
0
10
100
1000
10,000
INLETS AND OUTLETS (N)
0
25
50
75
100
COMPARISON FACTOR (C.F.)

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SQUARE SWITCH DISTRIBUTION NETWORK EMPLOYING A MINIMAL NUMBER OF CROSSPOINTS

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9 Claims. (Cl. 340—166)

ABSTRACT OF THE DISCLOSURE

A rearrangeable symmetrical distribution network is disclosed for connecting any one of N input terminals to any one of N output terminals where N is any positive integer such that $2^{\alpha_2} \cdot 3^{\alpha_3} \ldots k^{\alpha_k}$ is the prime decomposition of N into a product of prime numbers raised to integer exponents. The network includes

$$2\left(\sum_{j=2}^{k} \alpha_j\right)-1$$

interconnected stages if the largest prime factor of N exceeds three or if it equals three and N is odd, or

$$2\left(\sum_{j=2}^{k} \alpha_j\right)-3$$

interconnected stages if the largest prime factor of N equals two or if it equals three and N is even. Each stage includes a plurality of square switches of like size. The number of inputs and outputs of each stage equals N.

This invention relates to switching circuits and, more specifically, to a distribution switching netword employing a minimal number of crosspoint contact pairs.

Pursuant to relatively recent innovations in the electronic and magnetic arts, much effort is currently being directed towards the development of economic telephone systems employing electronic switching principles. Such arrangements are inherently capable of faster operative speeds and increased flexibility than were prior telephone communication embodiments.

Experience with electronic telephone switching systems has indicated that the crosspoint switches found in signal distribution switching arrays included therein comprise a greater percentage of the over-all system cost than these circuit combinations did in older, electro-mechanical switching arrangements. Accordingly, electronic telephone systems require distribution networks comprising a minimal number of crosspoints. In addition, it is further desirable that such connecting networks be rearrangeable, i.e., allow each idle inlet to be connected to each idle outlet by rearranging the existing connection pattern. An illustrative rearrangeable network, along with common control equipment associated therewith, is disclosed in a copending application by M. C. Paull, Ser. No. 154,477, filed Nov. 24, 1961 (now patent 3,129,407 issued Apr. 14, 1964). However, the above-described combination of features in a distribution connector, viz., rearrangeability and a crosspoint minimum, are in general conflicting, and have therefore not been embodied in prior art switching structures.

It is thus an object of the present invention to provide an improved distribution switching network.

More specifically, an object of the present invention is the provision of an economical, rearrangeable distribution network which includes a minimum number of crosspoint switches.

It is another object of the present invention to provide a distribution connector which may easily and inexpensively be fabricated from a plurality of similar circuit combinations.

These and other objects of the present invention are realized in a specific, illustrative, rearrangeable symmetrical distribution connector which includes a minimum number of crosspoints for each input and output terminal connected thereto. The composite distribution network includes an odd number of stages each comprising a plurality of square switches of a like size.

Corresponding to N input and N output terminals, where N is any positive integer whose prime decomposition $2^{\alpha_3} \cdot 3^{\alpha_3} \ldots k^{\alpha_k}$ includes a prime factor greater than three or includes a prime factor equal to three and N is odd, the switching structure includes

$$2\left(\sum_{j=2}^{k} \alpha_j\right)-1$$

stages. Where the largest prime factor of the prime decomposition of N is equal to three and N is even or where the largest prime factor is equal to two, the switching structure includes

$$2\left(\sum_{j=2}^{k} \alpha_j\right)-3$$

stages. Each stage includes a plurality of square switches of like size. In the first situation mentioned above, the number of square switches included in each stage is derived by dividing N by a corresponding prime factor. In the second situation the number of square switches included in each stage except the middle stage is also derived by dividing N by a corresponding prime factor. The middle stage in this latter case includes either four or six square switches depending on the value of N.

The total number of crosspoints included in the instant connector is proportional to N log N, which compares favorably with the N^2 factor characterizing prior art square matrix switches.

It is thus a feature of the present invention that a distribution switch include a plurality of switching stages each comprising a plurality of like size, square switches.

It is another feature of the present invention that a distribution network comprise a like number of inlets and outlets, an odd number of symmetrically-connected switching stages each employing like-sized square switches, wherein the network comprises a minimum number of crosspoint contact pairs.

It is still another feature of the present invention that a symmetrical distribution network employing square switches comprise N input and N output terminals, and S switching stages serially connected between the input and output terminals, where

$$S=\left(2\sum_{j=2}^{k} \alpha_j\right)-1$$

when the largest prime factor of the prime decomposition of N is greater than three or is equal to three and N is odd, and

$$S=2\left(\sum_{j=2}^{k} \alpha_j\right)-3$$

when the largest prime factor of the prime decomposition of N is equal to two or is equal to three and N is even.

A complete understanding of the present invention and of the above and other features, advantages and variations thereof may be gained from a consideration of the following detailed description of an illustrative embodiment thereof presented hereinbelow in conjunction with the accompanying drawing, in which:

FIGS. 1(A) and 1(B) are schematic diagrams respectively illustrating the upper and lower halves of a distribution network which embodies the principles of the present invention; and

FIG. 2 is a graph comparing the number of crosspoint switches employed in the instant class of distribution arrangements with the number of corresponding elements required in prior art square matrix embodiments.

As indicated above, it is considered important in the switching art to provide a rearrangeable distribution network which connects each of N input terminals, or inlets, to each of N output terminals, or outlets, and which employs a relatively small number of crosspoint switches. Accordingly, one aspect of the present invention is the provision of a distribution network which employs an absolute minimum number of crosspoints for the class of rearrangeable distribution connectors which comprise an odd number of serially-connected symmetrically-arranged switching stages each employing square switches. Each square switch, in turn, comprises a matrix array including a like number of inlets and outlets which are interconnected via a plurality of crosspoint switches. The crosspoint switches may advantageously comprise electronic devices, such as PNPN rectifiers or transistors, or electromechanical elements such as relays.

The over-all network is symmetrical to facilitate the fabrication thereof and also to simplify the associated common control circuit which may advantageously be of the type disclosed in the aforementioned Paull application. Further, the serially-connected switching stages include square switches to further facilitate the fabrication of the present distribution network.

General network structure

A distribution network constructed according to the principles of the present invention is exactly defined when the number of switching stages, the composition of each stage, and the interstage linkage patterns are specified. The former two criteria are dependent upon the particular value of N, corresponding to the N input and N output terminals to be interconnected by the composite distribution network. The precise nature of this dependence is given hereinbelow.

As is well known, every integer N has associated therewith a corresponding prime decomposition into a product of prime numbers each raised to an integer exponent. That is, for every integer N, there exists a sequence of prime factors 2, 3, 5, . . . k such that

$$N=2^{\alpha_2}\cdot 3^{\alpha_3}\cdot 5^{\alpha_5} \ldots k^{\alpha_k} \qquad (1)$$

where the α exponents may take on any integer value including zero. For example, the integer 126 has the prime decomposition.

$$126=2^1\cdot 3^2\cdot 5^0\cdot 7^1 \qquad (2)$$

Also, it is well known that there is only one, unique prime decomposition for any particular integer.

With the above in mind, a distribution network embodying N inlets and N outlets is derived as follows. The case where the number of switching stages S is given by

$$S=\left(2\sum_{j=2}^{k}\alpha_j\right)-1 \qquad (3)$$

will be considered. In this case, the central switching stage includes N/k square switches each comprising k inlets and outlets. The remaining switching stages are symmetrically distributed about the aforementioned central stage and comprise N/f square switches each having f inlets and outlets, where f is the corresponding prime factor included in the prime decomposition of N. Each prime factor f gives rise to 2 α_f switching stages, where α_f is the corresponding exponent of the prime factor f, with these switching stages being symmetrically connected on both sides of the central switching stage.

Adjacent ones of the serially-connected switching stages are interconnected in a repeating, overlapping manner. That is, a junctor connects the first outlet of the first (or top-most) switch includes in a left-hand switching stage with the first inlet of the first switch in the right-adjacent stage. The second outlet on the first switch of the left stage is joined with the first inlet of the second switch of the adjacent switching stage. This connection pattern continues until each switch of the right stage has one junctor connected thereto. At this point, the process starts over again with the first switch, continuing cyclically until all the junctors emanating from the left stage have been assigned.

It is noted that the above relationships define a complete distribution structure for any value of N. The structure embodying the above-described class of distribution networks, and the particular method of fabrication thereof, will become more clear from the discussion hereinafter of a particular, illustrative connector shown in FIG. 1, which succeeds a mathematical proof of crosspoint minimization.

Proof of minimization

The following proof demonstrates that the herein-presented class of distribution networks in fact employs an absolute minimum number of crosspoint switches for the above-described connector requirements, viz., and odd number of symmetrically-connected switching stages each including square switches of a like size.

SECTION 1.—PRELIMINARIES

The symbol C_N, $N \geqq 2$, is used to denote the class of all connecting networks ν with the following properties:

(1) ν is two-sided, with N terminals on each side;
(2) ν is built of an odd number s of stages δ_k, $k=$v, . . . , of square switches, each stage having N inlets and N outlets;
(3) ν is symmetric in the sense that $\delta_k=\delta_{s-k+1}$ for $k=1, \ldots, \frac{1}{2}(s-1)$; and
(4) Employing the notation
$s=s(\nu)$=number of stages of ν, $n_k=n_k(\nu)$=switch size in the k^{th} stage of ν, where ν has N/n_k identical switches in the k^{th} stage.

The defining conditions of C_N imply that

$$n_k=n_{s-k+1} \text{ for } k=1, \ldots, \frac{s-1}{2} \qquad (4)$$

and that

$$\prod_{k=1}^{\frac{1}{2}(s+1)} n_k=N \qquad (5)$$

It is assumed throughout that $n_k(\nu) \geqq 2$ for all ν and all $k=1, \ldots, s(\nu)$.

The cost per terminal (on a side) $c(\nu)$ of a network $\nu \epsilon C_N$ is defined to be the total number of crosspoints of ν divided by the number N of terminals on a side. Since there are

$$\frac{N}{n_k} n_k x n_k$$

switches in stage k, the total number of crosspoints is (using the symmetry condition)

$$\sum_{k=1}^{S}\frac{N}{n_k}\cdot n_k^2=N\sum_{k=1}^{S} n_k=N\left(n_{1/2(s+1)}+2\sum_{k=0}^{\frac{1}{2}(S-1)} n_k\right) \qquad (6)$$

and so

$$c(\nu)=n_{1/2(s+1)}+2\sum_{k=1}^{\frac{1}{2}(S-1)} n_k \qquad (7)$$

A network ν is called optimal if

$$c(\nu)=\min\{c(\mu):\mu \epsilon C_N\} \qquad (8)$$

It is clear that the cost per terminal of a network $\nu \epsilon C_N$ depends only on the switch sizes, and not at all on the

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