FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS

Venkat Konda

CROSS REFERENCE TO RELATED APPLICATIONS

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0037US entitled "FULLY CONNECTED GENERALIZED MULTI-STAGE NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0038US entitled "FULLY CONNECTED GENERALIZED BUTTERFLY FAT TREE NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0039US entitled "FULLY CONNECTED GENERALIZED REARRANGEABLY NONBLOCKING MULTI-LINK MULTI-STAGE NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0040US entitled "FULLY CONNECTED GENERALIZED MULTI-LINK BUTTERFLY FAT TREE NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0042US entitled "FULLY CONNECTED GENERALIZED STRICTLY NONBLOCKING MULTI-LINK MULTI-STAGE NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

This application is related to and incorporates by reference in its entirety the U.S. Provisional Patent Application Docket No. M-0045US entitled "VLSI LAYOUTS OF FULLY CONNECTED GENERALIZED NETWORKS" by Venkat Konda assigned to the same assignee as the current application, filed concurrently.

BRIEF DESCRIPTION OF DRAWINGS

FIG. 1A is a diagram 100A of an exemplary symmetrical folded multi-stage network $V_{fold}(N,d,s)$ having inverse Benes connection topology of five stages with N = 8, d = 2 and s=2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1A1 is a diagram 100A1 of an exemplary symmetrical folded multi-stage network $V_{fold}(N,d,2)$ having Omega connection topology of five stages with N = 8, d = 2 and s=2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1A2 is a diagram 100A2 of an exemplary symmetrical folded multi-stage network $V_{fold}(N,d,2)$ having nearest neighbor connection topology of five stages with N = 8, d = 2 and s=2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1B is a diagram 100B of a general symmetrical folded multi-stage network $V_{fold}(N,d,2)$ with $(2\times\log_d N)-1$ stages strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections in accordance with the invention.

FIG. 1C is a diagram 100C of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1,N_2,d,2)$ having inverse Benes connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1C1 is a diagram 100C1 of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ having Omega connection topology of five stages with N_1 = 8, N_2 = p* N_1 = 24 where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1C2 is a diagram 100C2 of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ having nearest neighbor connection topology of five stages with N_1 = 8, N_2 = p* N_1 = 24 where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1D is a diagram 100D of a general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ with N_2 = p* N_1 and with $(2 \times \log_d N) - 1$ stages strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections in accordance with the invention.

FIG. 1E is a diagram 100E of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ having inverse Benes connection topology of five stages with N_2 = 8, N_1 = p* N_2 = 24, where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1E1 is a diagram 100E1 of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ having Omega connection topology of five stages with N_2 = 8, N_1 = p* N_2 = 24, where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1E2 is a diagram 100E2 of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ having nearest neighbor connection topology of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, and d = 2 with exemplary multicast connections, strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections, in accordance with the invention.

FIG. 1F is a diagram 100F of a general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 2)$ with $N_1 = p^* N_2$ and with $(2 \times \log_d N) - 1$ stages strictly nonblocking network for unicast connections and rearrangeably nonblocking network for arbitrary fan-out multicast connections in accordance with the invention.

FIG. 2A is a diagram 200A of an exemplary symmetrical folded multi-stage network $V_{fold}(N, d, s)$ having inverse Benes connection topology of five stages with N = 8, d = 2 and s=1 with exemplary unicast connections rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 2B is a diagram 200B of a general symmetrical folded multi-stage network $V_{fold}(N, d, 1)$ with $(2 \times \log_d N) - 1$ stages rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 2C is a diagram 200C of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 1)$ having inverse Benes connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, and d = 2 with exemplary unicast connections rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 2D is a diagram 200D of a general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 1)$ with $N_2 = p^* N_1$ and with $(2 \times \log_d N) - 1$ stages rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 2E is a diagram 200E of an exemplary asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 1)$ having inverse Benes connection topology of five stages with

$N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, and d = 2 with exemplary unicast connections rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 2F is a diagram 200F of a general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, 1)$ with $N_1 = p^* N_2$ and with $(2 \times \log_d N) - 1$ stages rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 3A is a diagram 300A of an exemplary symmetrical multi-stage network $V(N, d, s)$ having inverse Benes connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3B is a diagram 300B of an exemplary symmetrical multi-stage network $V(N, d, s)$ (having a connection topology built using back-to-back Omega Networks) of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3C is a diagram 300C of an exemplary symmetrical multi-stage network $V(N, d, s)$ having an exemplary connection topology of five stages with N = 8, d = 2 and s=1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3D is a diagram 300D of an exemplary symmetrical multi-stage network $V(N, d, s)$ having an exemplary connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3E is a diagram 300E of an exemplary symmetrical multi-stage network $V(N, d, s)$ (having a connection topology called flip network and also known as inverse shuffle exchange network) of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3F is a diagram 300F of an exemplary symmetrical multi-stage network $V(N,d,s)$ having Baseline connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3G is a diagram 300G of an exemplary symmetrical multi-stage network $V(N,d,s)$ having an exemplary connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3H is a diagram 300H of an exemplary symmetrical multi-stage network $V(N,d,s)$ having an exemplary connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3I is a diagram 300I of an exemplary symmetrical multi-stage network $V(N,d,s)$ (having a connection topology built using back-to-back Banyan Networks or back-to-back Delta Networks or equivalently back-to-back Butterfly networks) of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3J is a diagram 300J of an exemplary symmetrical multi-stage network $V(N,d,s)$ having an exemplary connection topology of five stages with N = 8, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3K is a diagram 300K of a general symmetrical multi-stage network $V(N,d,s)$ with $(2\times\log_d N)-1$ stages with s = 1, rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 3A1 is a diagram 300A1 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ having inverse Benes connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3B1 is a diagram 300B1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ (having a connection topology built using back-to-back Omega Networks) of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3C1 is a diagram 300C1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_1 = 8$, N2 $= p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3D1 is a diagram 300D1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3E1 is a diagram 300E1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ (having a connection topology called flip network and also known as inverse shuffle exchange network) of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3F1 is a diagram 300F1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having Baseline connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3G1 is a diagram 300G1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_1 = 8$, $N_2 = p^* N_1 = 24$ where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3H1 is a diagram 300H1 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_1 = 8$, N_2

= p* N_1 = 24 where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3I1 is a diagram 300I1 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ (having a connection topology built using back-to-back Banyan Networks or back-to-back Delta Networks or equivalently back-to-back Butterfly networks) of five stages with N_1 = 8, N_2 = p* N_1 = 24 where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3J1 is a diagram 300J1 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ having an exemplary connection topology of five stages with N_1 = 8, N_2 = p* N_1 = 24 where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3K1 is a diagram 300K1 of a general asymmetrical multi-stage network $V(N_1,N_2,d,s)$ with $(2\times\log_d N)-1$ stages with N_1 = p* N_2 and s = 1, rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 3B2 is a diagram 300B2 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ (having a connection topology built using back-to-back Omega Networks) of five stages with N_2 = 8, N_1 = p* N_2 = 24, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3A2 is a diagram 300A2 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ having inverse Benes connection topology of five stages with N_2 = 8, N_1 = p* N_2 = 24, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3C2 is a diagram 300C2 of an exemplary asymmetrical multi-stage network $V(N_1,N_2,d,s)$ having an exemplary connection topology of five stages with N_2 = 8, N_1 = p* N_2 = 24, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3D2 is a diagram 300D2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3E2 is a diagram 300E2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ (having a connection topology called flip network and also known as inverse shuffle exchange network) of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3F2 is a diagram 300F2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having Baseline connection topology of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3G2 is a diagram 300G2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3H2 is a diagram 300H2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections, in accordance with the invention.

FIG. 3I2 is a diagram 300I2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ (having a connection topology built using back-to-back Banyan Networks or back-to-back Delta Networks or equivalently back-to-back Butterfly networks) of five stages with $N_2 = 8$, $N_1 = p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3J2 is a diagram 300J2 of an exemplary asymmetrical multi-stage network $V(N_1, N_2, d, s)$ having an exemplary connection topology of five stages with $N_2 = 8$, N_1

$= p^* N_2 = 24$, where p = 3, d = 2 and s = 1, rearrangeably nonblocking network for unicast connections.

FIG. 3K2 is a diagram 300K2 of a general asymmetrical multi-stage network $V(N_1, N_2, d, s)$ with $(2 \times \log_d N) - 1$ stages with $N_2 = p^* N_1$ and s = 1, rearrangeably nonblocking network for unicast connections in accordance with the invention.

FIG. 4A is high-level flowchart of a scheduling method according to the invention, used to set up the multicast connections in all the networks disclosed in this invention.

DETAILED DESCRIPTION OF THE INVENTION

The present invention is concerned with the design and operation of large scale crosspoint reduction using arbitrarily large folded multi-stage switching networks for broadcast, unicast and multicast connections. Particularly folded multi-stage networks with stages more than three and radices greater than or equal to two offer large scale crosspoint reduction when configured with optimal links as disclosed in this invention.

When a transmitting device simultaneously sends information to more than one receiving device, the one-to-many connection required between the transmitting device and the receiving devices is called a multicast connection. A set of multicast connections is referred to as a multicast assignment. When a transmitting device sends information to one receiving device, the one-to-one connection required between the transmitting device and the receiving device is called unicast connection. When a transmitting device simultaneously sends information to all the available receiving devices, the one-to-all connection required between the transmitting device and the receiving devices is called a broadcast connection.

In general, a multicast connection is meant to be one-to-many connection, which includes unicast and broadcast connections. A multicast assignment in a switching network is nonblocking if any of the available inlet links can always be connected to any of the available outlet links.

In certain folded multi-stage networks of the type described herein, any connection request of arbitrary fan-out, i.e. from an inlet link to an outlet link or to a set of outlet links of the network, can be satisfied without blocking if necessary by rearranging some of the previous connection requests. In certain other folded multi-stage networks of the type described herein, any connection request of arbitrary fan-out, i.e. from an inlet link to an outlet link or to a set of outlet links of the network, can be satisfied without blocking with never needing to rearrange any of the previous connection requests.

In certain folded multi-stage networks of the type described herein, any connection request of unicast from an inlet link to an outlet link of the network, can be satisfied without blocking if necessary by rearranging some of the previous connection requests. In certain other folded multi-stage networks of the type described herein, any connection request of unicast from an inlet link to an outlet link of the network, can be satisfied without blocking with never needing to rearrange any of the previous connection requests.

The folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ disclosed, in the current invention, is topologically exactly the same as the multi-stage network $V(N_1, N_2, d, s)$, disclosed in U.S. Provisional Patent Application Docket No. M-0037US that is incorporated by reference above, excepting that in the illustrations folded network $V_{fold}(N_1, N_2, d, s)$ is shown as it is folded at middle stage $130 + 10 * (Log_d N_2 - 2)$. This is true for all the embodiments presented in the current invention.

Nonblocking configurations for other types of networks with numerous connection topologies and scheduling methods are disclosed as follows:

1) Strictly and rearrangeably nonblocking for arbitrary fan-out multicast and unicast for generalized multi-stage networks $V(N_1, N_2, d, s)$ with numerous connection topologies and the scheduling methods are described in detail in U.S. Provisional Patent Application, Attorney Docket No. M-0037 US that is incorporated by reference above.

2) Strictly and rearrangeably nonblocking for arbitrary fan-out multicast and unicast for generalized butterfly fat tree networks $V_{bft}(N_1, N_2, d, s)$ with numerous connection topologies and the scheduling methods are described in detail in U.S. Provisional Patent Application, Attorney Docket No. M-0038 US that is incorporated by reference above.

3) Rearrangeably nonblocking for arbitrary fan-out multicast and unicast, and strictly nonblocking for unicast for generalized multi-link multi-stage networks $V_{mlink}(N_1, N_2, d, s)$ and generalized folded multi-link multi-stage networks $V_{fold-mlink}(N_1, N_2, d, s)$ with numerous connection topologies and the scheduling methods are described in detail in U.S. Provisional Patent Application, Attorney Docket No. M-0039 US that is incorporated by reference above.

4) Strictly and rearrangeably nonblocking for arbitrary fan-out multicast and unicast for generalized multi-link butterfly fat tree networks $V_{mlink-bft}(N_1, N_2, d, s)$ with numerous connection topologies and the scheduling methods are described in detail in U.S. Provisional Patent Application, Attorney Docket No. M-0040 US that is incorporated by reference above.

5) Strictly nonblocking for arbitrary fan-out multicast for generalized multi-link multi-stage networks $V_{mlink}(N_1, N_2, d, s)$ and generalized folded multi-link multi-stage networks $V_{fold-mlink}(N_1, N_2, d, s)$ with numerous connection topologies and the scheduling methods are described in detail in U.S. Provisional Patent Application, Attorney Docket No. M-0042 US that is incorporated by reference above.

6) VLSI layouts of generalized multi-stage networks $V(N_1, N_2, d, s)$, generalized folded multi-stage networks $V_{fold}(N_1, N_2, d, s)$, generalized butterfly fat tree networks $V_{bft}(N_1, N_2, d, s)$, generalized multi-link multi-stage networks $V_{mlink}(N_1, N_2, d, s)$, generalized folded multi-link multi-stage networks $V_{fold-mlink}(N_1, N_2, d, s)$, generalized multi-link butterfly fat tree networks $V_{mlink-bft}(N_1, N_2, d, s)$, and generalized hypercube networks $V_{hcube}(N_1, N_2, d, s)$ for s = 1,2,3 or any number in general, are described in

detail in U.S. Provisional Patent Application, Attorney Docket No. M-0045 US that is incorporated by reference above.

Symmetric folded RNB Embodiments:

Referring to FIG. 1A, in one embodiment, an exemplary symmetrical folded multi-stage network 100A with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N}{d}$, where N is the total number of inlet links or outlet links. The number of middle switches in each middle stage is denoted by $2\times\frac{N}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d*2d$ and each output switch OS1-OS4 can be denoted in general with the notation $2d*d$. Likewise, the size of each switch in any of the middle stages can be denoted as $d*d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. A symmetric folded multi-stage network can be represented with the notation $V_{fold}(N,d,s)$, where N represents the total number of inlet links of all input switches (for example the links IL1-IL8), d represents the inlet links of each input switch or outlet links of each output switch, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch. Although it is not necessary that there be the same number of inlet links IL1-IL8 as there are outlet links OL1-OL8, in a symmetrical network they are the same.

Each of the $\frac{N}{d}$ input switches IS1 – IS4 are connected to exactly $2\times d$ switches in middle stage 130 through $2\times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2\times\frac{N}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,5) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $2\times\frac{N}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $2\times\frac{N}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N}{d}$ output switches OS1 – OS4 are connected from exactly $2\times d$ switches in middle stage 150 through $2\times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,2), MS(3,5) and MS(3,6) through the links ML(4,1), ML(4,3), ML(4,9) and ML(4,11) respectively).

Finally the connection topology of the network 100A shown in FIG. 1A is known to be back to back inverse Benes connection topology.

Referring to FIG. 1A1, in another embodiment of network $V_{fold}(N,d,s)$, an exemplary symmetrical folded multi-stage network 100A1 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N}{d}$, where N is the total number of inlet links or outlet links. The number of middle switches in each middle stage is denoted by $2\times\frac{N}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d*2d$ and each output switch OS1-OS4 can be denoted in general with the notation $2d*d$. Likewise, the size of each switch in any of the middle stages can be denoted as $d*d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The symmetric folded multi-stage network of FIG. 1A1 is also the network of the type $V_{fold}(N,d,s)$, where N represents the total number of inlet links of all input switches (for example the links IL1-IL8), d represents the inlet links of each input switch or outlet links of each output switch, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch. Although it is not necessary that there be the same number of inlet links IL1-IL8 as there are outlet links OL1-OL8, in a symmetrical network they are the same.

Each of the $\frac{N}{d}$ input switches IS1 – IS4 are connected to exactly $2\times d$ switches in middle stage 130 through $2\times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2\times\frac{N}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,9) are connected to the middle switch MS(1,1) from input switch IS1 and IS3 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2\times\frac{N}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,5) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2\times\frac{N}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,5) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N}{d}$ output switches OS1 – OS4 are connected from exactly $2\times d$ switches in middle stage 150 through $2\times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,3), MS(3,5) and MS(3,7) through the links ML(4,1), ML(4,5), ML(4,9) and ML(4,13) respectively).

Finally the connection topology of the network 100A1 shown in FIG. 1A1 is known to be back to back Omega connection topology.

Referring to FIG. 1A2, in another embodiment of network $V_{fold}(N,d,s)$, an exemplary symmetrical folded multi-stage network 100A2 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N}{d}$, where N is the total number of inlet links or outlet links. The number of middle switches in each middle stage is denoted by $2\times\frac{N}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d*2d$ and each output switch OS1-OS4 can be denoted in general with the notation $2d*d$. Likewise, the size of each switch in any of the middle stages can be denoted as $d*d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The symmetric folded multi-stage network of FIG. 1A2 is also the network of the type $V_{fold}(N,d,s)$, where N represents the total number of inlet links of all input switches (for example the links IL1-IL8), d represents the inlet links of each input switch or outlet links of each output switch, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch. Although it is not necessary that there be the same number of inlet links IL1-IL8 as there are outlet links OL1-OL8, in a symmetrical network they are the same.

Each of the $\frac{N}{d}$ input switches IS1 – IS4 are connected to exactly $2 \times d$ switches in middle stage 130 through $2 \times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2 \times \frac{N}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,14) are connected to the middle switch MS(1,1) from input switch IS1 and IS4 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2 \times \frac{N}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,8) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,4) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2 \times \frac{N}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,8) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,4) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N}{d}$ output switches OS1 – OS4 are connected from exactly $2 \times d$ switches in middle stage 150 through $2 \times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,4), MS(3,5) and MS(3,8) through the links ML(4,1), ML(4,2), ML(4,3) and ML(4,4) respectively).

Finally the connection topology of the network 100A2 shown in FIG. 1A2 is hereinafter called nearest neighbor connection topology.

In the three embodiments of FIG. 1A, FIG. 1A1 and FIG. 1A2 the connection topology is different. That is the way the links ML(1,1) - ML(1,16), ML(2,1) - ML(2,16), ML(3,1) - ML(3,16), and ML(4,1) - ML(4,16) are connected between the respective stages is different. Even though only three embodiments are illustrated, in general, the network $V_{fold}(N,d,s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N,d,s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N,d,s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N,d,s)$ can be built. The embodiments of FIG. 1A, FIG. 1A1, and FIG. 1A2 are only three examples of network $V_{fold}(N,d,s)$.

In the three embodiments of FIG. 1A, FIG. 1A1 and FIG. 1A2, each of the links ML(1,1) – ML(1,16), ML(2,1) – ML(2,16), ML(3,1) – ML(3,16) and ML(4,1) – ML(4,16) are either available for use by a new connection or not available if currently used by an existing connection. The input switches IS1-IS4 are also referred to as the network input ports. The input stage 110 is often referred to as the first stage. The output switches OS1-OS4 are also referred to as the network output ports. The output stage 120 is often referred to as the last stage. The middle stage switches MS(1,1) – MS(1,8), MS(2,1) – MS(2,8), and MS(3,1) – MS(3,8) are referred to as middle switches or middle ports.

In the example illustrated in FIG. 1A (or in FIG1A1, or in FIG. 1A2), a fan-out of four is possible to satisfy a multicast connection request if input switch is IS2, but only two switches in middle stage 130 will be used. Similarly, although a fan-out of three is possible for a multicast connection request if the input switch is IS1, again only a fan-out of two is used. The specific middle switches that are chosen in middle stage 130 when selecting a fan-out of two is irrelevant so long as at most two middle switches are selected to ensure that the connection request is satisfied. In essence, limiting the fan-out from input switch to no more than two middle switches permits the network 100A (or 100A1, or 100A2), to be operated in rearrangeably nonblocking manner in accordance with the invention.

The connection request of the type described above can be unicast connection request, a multicast connection request or a broadcast connection request, depending on the example. In case of a unicast connection request, a fan-out of one is used, i.e. a single middle stage switch in middle stage 130 is used to satisfy the request. Moreover, although in the above-described embodiment a limit of two has been placed on the fan-out into the middle stage switches in middle stage 130, the limit can be greater depending on the number of middle stage switches in a network (while maintaining the rearrangeably nonblocking nature of operation of the network for multicast connections). However any arbitrary fan-out may be used within any of the middle stage switches and the output stage switches to satisfy the connection request.

Generalized Symmetric folded RNB Embodiments:

Network 100B of FIG. 1B is an example of general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ with $(2\times\log_d N)-1$ stages. The general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ can be operated in rearrangeably nonblocking manner for multicast when $s=2$ according to the current invention. Also the general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ can be operated in strictly nonblocking manner for unicast if $s=2$ according to the current invention. (And in the example of FIG. 1B, $s = 2$). The general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ with $(2\times\log_d N)-1$ stages has d inlet links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links IL1-IL(d) to the input switch IS1) and $2\times d$ outgoing links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links ML(1,1) - ML(1,2d) to the input switch IS1). There are d outlet links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example the links OL1-OL(d) to the output switch OS1) and $2\times d$ incoming links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example $ML(2\times Log_d N-2,1)$ - $ML(2\times Log_d N-2,2\times d)$ to the output switch OS1).

Each of the $\frac{N}{d}$ input switches IS1 – IS(N/d) are connected to exactly $2\times d$ switches in middle stage 130 through $2\times d$ links (for example input switch IS1 is connected to middle switches MS(1,1) - MS(1,d) through the links ML(1,1) - ML(1,d) and to middle switches MS(1,N/d+1) – MS(1,{N/d}+d) through the links ML(1,d+1) – ML(1,2d) respectively.

Each of the $2\times\frac{N}{d}$ middle switches MS(1,1) – MS(1,2N/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $2\times\frac{N}{d}$ middle switches $MS(Log_d N-1,1)$ - $MS(Log_d N-1,2\times\frac{N}{d})$ in the middle stage $130+10*(Log_d N-2)$ are connected from exactly d switches in middle stage $130+10*(Log_d N-3)$ through d links and also are connected to exactly d switches in middle stage $130+10*(Log_d N-1)$ through d links.

Similarly each of the $2\times\frac{N}{d}$ middle switches $MS(2\times Log_d N-3,1)$ - $MS(2\times Log_d N-3,2\times\frac{N}{d})$ in the middle stage $130+10*(2*Log_d N-4)$ are connected from exactly d switches in middle stage $130+10*(2*Log_d N-5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N}{d}$ output switches OS1 – OS(N/d) are connected from exactly $2\times d$ switches in middle stage $130+10*(2*Log_d N-4)$ through $2\times d$ links.

As described before, again the connection topology of a general $V_{fold}(N,d,s)$ may be any one of the connection topologies. For example the connection topology of the network $V_{fold}(N,d,s)$ may be back to back inverse Benes networks, back to back Omega networks, back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the general $V_{fold}(N,d,s)$ network is, when no connections are setup from any input link if any output link should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N,d,s)$ can be built. The embodiments of FIG. 1A, FIG. 1A1, and FIG. 1A2 are three examples of network $V_{fold}(N,d,s)$.

The general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ can be operated in rearrangeably nonblocking manner for multicast when $s=2$ according to the current invention. Also the general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ can be operated in strictly nonblocking manner for unicast if $s=2$ according to the current invention.

Every switch in the folded multi-stage networks discussed herein has multicast capability. In a $V_{fold}(N,d,s)$ network, if a network inlet link is to be connected to more than one outlet link on the same output switch, then it is only necessary for the corresponding input switch to have one path to that output switch. This follows because that path can be multicast within the output switch to as many outlet links as necessary.

Multicast assignments can therefore be described in terms of connections between input switches and output switches. An existing connection or a new connection from an input switch to r' output switches is said to have fan-out r'. If all multicast assignments of a first type, wherein any inlet link of an input switch is to be connected in an output switch to at most one outlet link are realizable, then multicast assignments of a second type, wherein any inlet link of each input switch is to be connected to more than one outlet link in the same output switch, can also be realized. For this reason, the following discussion is limited to general multicast connections of the first type (with fan-out r', $1 \le r' \le \frac{N}{d}$) although the same discussion is applicable to the second type.

To characterize a multicast assignment, for each inlet link $i \in \left\{1,2,...,\frac{N}{d}\right\}$, let $I_i = O$, where $O \subset \left\{1,2,...,\frac{N}{d}\right\}$, denote the subset of output switches to which inlet link i is to be connected in the multicast assignment. For example, the network of Fig. 1A shows an exemplary five-stage network, namely $V_{fold}(8,2,2)$, with the following multicast assignment $I_1 = \{2,3\}$and all other $I_j = \phi$ for j = [2-8]. It should be noted that the connection I_1 fans out in the first stage switch IS1 into middle switches MS(1,1) and MS(1,5) in middle stage 130, and fans out in middle switches MS(1,1) and MS(1,5) only once into middle switches MS(2,1) and MS(2,5) respectively in middle stage 140.

The connection I_1 also fans out in middle switches MS(2,1) and MS(2,5) only once into middle switches MS(3,1) and MS(3,7) respectively in middle stage 150. The connection I_1 also fans out in middle switches MS(3,1) and MS(3,7) only once into output switches OS2 and OS3 in output stage 120. Finally the connection I_1 fans out once in the output stage switch OS2 into outlet link OL3 and in the output stage switch OS3 twice into the outlet links OL5 and OL6. In accordance with the invention, each connection can fan out in the input stage switch into at most two middle stage switches in middle stage 130.

Asymmetric folded RNB ($N_2 > N_1$) Embodiments:

Referring to FIG. 1C, in one embodiment, an exemplary asymmetrical folded multi-stage network 100C with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, eight by six switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by four switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches of size two by two in each of middle stage 130 and middle stage 140, and eight switches of size two by four in middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_1}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_1}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * 2d$ and each output switch OS1-OS4 can be denoted in general with the notation $(d+d_2)*d$, where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$. The size of each switch in any of the middle stages excepting the last middle stage can be denoted as $d*d$. The size of each switch in the last middle stage can be denoted as $d*\frac{(d+d_2)}{2}$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric folded multi-stage network can be represented with the notation $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL8), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL24), d represents the inlet links of each input switch where $N_2 > N_1$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

Each of the $\frac{N_1}{d}$ input switches IS1 – IS4 are connected to exactly $2 \times d$ switches in middle stage 130 through $2 \times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2 \times \frac{N_1}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,5) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $2 \times \frac{N_1}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $2\times\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly $\frac{d+d_2}{2}$ output switches in output stage 120 through $\frac{d+d_2}{2}$ links (for example the links ML(4,1), ML(4,2), ML(4,3) and ML(4,4) are connected to output switches OS1, OS2, OS3, and OS4 respectively from middle switches MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly $d+d_2$ switches in middle stage 150 through $d+d_2$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,2), MS(3,3), MS(3,4), MS(3,5), MS(3,6), MS(3,7), and MS(3,8) through the links ML(4,1), ML(4,5), ML(4,9), ML(4,13), ML(4,17), ML(4,21), ML(4,25) and ML(4,29) respectively).

Finally the connection topology of the network 100C shown in FIG. 1C is known to be back to back inverse Benes connection topology.

Referring to FIG. 1C1, in another embodiment of network $V_{fold}(N_1, N_2, d, s)$, an exemplary asymmetrical folded multi-stage network 100C1 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, eight by six switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by four switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches of size two by two in each of middle stage 130 and middle stage 140, and eight switches of size two by four in middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_1}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_1}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * 2d$ and each output switch OS1-OS4 can be denoted in general with the notation $(d + d_2) * d$, where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$. The size of each switch in any of the middle stages excepting the last middle stage can be denoted as $d * d$. The size of each switch in the last middle stage can be denoted as $d * \frac{(d + d_2)}{2}$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The asymmetric folded multi-stage network of FIG. 1C1 is also the network of the type $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL8), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL24), d represents the inlet links of each input switch where $N_2 > N_1$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

Each of the $\frac{N_1}{d}$ input switches IS1 – IS4 are connected to exactly $2 \times d$ switches in middle stage 130 through $2 \times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2 \times \frac{N_1}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,9) are connected to the middle switch MS(1,1) from input switch IS1 and IS3 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2 \times \frac{N_1}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,5) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2 \times \frac{N_1}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,5) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly $\frac{d + d_2}{2}$ output switches in output stage 120 through $\frac{d + d_2}{2}$ links (for example the links ML(4,1), ML(4,2), ML(4,3) and ML(4,4) are connected to output switches OS1, OS2, OS3, and OS4 respectively from middle switches MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly $d + d_2$ switches in middle stage 150 through $d + d_2$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,2), MS(3,3), MS(3,4), MS(3,5), MS(3,6), MS(3,7), and MS(3,8) through the links ML(4,1), ML(4,5), ML(4,9), ML(4,13), ML(4,17), ML(4,21), ML(4,25) and ML(4,29) respectively).

Finally the connection topology of the network 100C1 shown in FIG. 1C1 is known to be back to back Omega connection topology.

Referring to FIG. 1C2, in another embodiment of network $V_{fold}(N_1, N_2, d, s)$, an exemplary asymmetrical folded multi-stage network 100C2 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by four switches IS1-IS4 and output stage 120 consists of four, eight by six switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, two by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by four switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size two by four, the switches in output stage 120 are of size eight by six, and there are eight switches of size two by two in each of middle stage 130 and middle stage 140, and eight switches of size two by four in middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_1}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_1}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * 2d$ and each output switch OS1-OS4 can be denoted in general with the notation $(d + d_2) * d$, where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$. The size of each switch in any of the middle stages excepting the last middle stage can be denoted as $d * d$. The size of each switch in the last middle stage can be denoted as $d * \frac{(d + d_2)}{2}$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The asymmetric folded multi-stage network of FIG. 1C2 is also the network of the type $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL8), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL24), d represents the inlet links of each input switch where $N_2 > N_1$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

Each of the $\frac{N_1}{d}$ input switches IS1 – IS4 are connected to exactly $2 \times d$ switches in middle stage 130 through $2 \times d$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,5) and MS(1,6) through the links ML(1,1), ML(1,2), ML(1,3) and ML(1,4) respectively).

Each of the $2 \times \frac{N_1}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,14) are connected to the middle switch MS(1,1) from input switch IS1 and IS4 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2\times\frac{N_1}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,8) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,4) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2\times\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,8) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,4) respectively) and also are connected to exactly $\frac{d+d_2}{2}$ output switches in output stage 120 through $\frac{d+d_2}{2}$ links (for example the links ML(4,1), ML(4,2), ML(4,3) and ML(4,4) are connected to output switches OS1, OS2, OS3, and OS4 respectively from middle switches MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly $d+d_2$ switches in middle stage 150 through $d+d_2$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,2), MS(3,3), MS(3,4), MS(3,5), MS(3,6), MS(3,7), and MS(3,8) through the links ML(4,1), ML(4,5), ML(4,9), ML(4,13), ML(4,17), ML(4,21), ML(4,25) and ML(4,29) respectively).

Finally the connection topology of the network 100C2 shown in FIG. 1C2 is hereinafter called nearest neighbor connection topology.

In the three embodiments of FIG. 1C, FIG. 1C1 and FIG. 1C2 the connection topology is different. That is the way the links ML(1,1) - ML(1,16), ML(2,1) - ML(2,16), ML(3,1) - ML(3,16), and ML(4,1) - ML(4,16) are connected between the respective stages is different. Even though only three embodiments are illustrated, in general, the network $V_{fold}(N_1, N_2, d, s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiments of FIG. 1C, FIG. 1C1, and FIG. 1C2 are only three examples of network $V_{fold}(N_1, N_2, d, s)$.

In the three embodiments of FIG. 1C, FIG. 1C1 and FIG. 1C2, each of the links ML(1,1) – ML(1,32), ML(2,1) – ML(2,16), ML(3,1) – ML(3,16) and ML(4,1) – ML(4,16) are either available for use by a new connection or not available if currently used by an existing connection. The input switches IS1-IS4 are also referred to as the network input ports. The input stage 110 is often referred to as the first stage. The output switches OS1-OS4 are also referred to as the network output ports. The output stage 120 is often referred to as the last stage. The middle stage switches MS(1,1) – MS(1,8), MS(2,1) – MS(2,8), and MS(3,1) – MS(3,8) are referred to as middle switches or middle ports.

In the example illustrated in FIG. 1C (or in FIG1C1, or in FIG. 1C2), a fan-out of four is possible to satisfy a multicast connection request if input switch is IS2, but only two switches in middle stage 130 will be used. Similarly, although a fan-out of three is possible for a multicast connection request if the input switch is IS1, again only a fan-out of two is used. The specific middle switches that are chosen in middle stage 130 when selecting a fan-out of two is irrelevant so long as at most two middle switches are selected to ensure that the connection request is satisfied. In essence, limiting the fan-out from input switch to no more than two middle switches permits the network 100C (or 100C1, or 100C2), to be operated in rearrangeably nonblocking manner in accordance with the invention.

The connection request of the type described above can be unicast connection request, a multicast connection request or a broadcast connection request, depending on the example. In case of a unicast connection request, a fan-out of one is used, i.e. a single middle stage switch in middle stage 130 is used to satisfy the request. Moreover, although in the above-described embodiment a limit of two has been placed on the fan-out into the middle stage switches in middle stage 130, the limit can be greater depending on the number of middle stage switches in a network (while maintaining the rearrangeably nonblocking nature of operation of the network for multicast connections). However any arbitrary fan-out may be used within any of the middle stage switches and the output stage switches to satisfy the connection request.

Generalized Asymmetric folded RNB ($N_2 > N_1$) Embodiments:

Network 100D of FIG. 1D is an example of general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2 \times \log_d N_1) - 1$ stages where $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. In network 100D of FIG. 1D, $N_1 = N$ and $N_2 = p * N$. The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 2$ according to the current invention. Also the general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in strictly nonblocking manner for unicast if $s = 2$ according to the current invention. (And in the example of FIG. 1D, $s = 2$). The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2 \times \log_d N_1) - 1$ stages has d inlet links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links IL1-IL(d) to the input switch IS1) and $2 \times d$ outgoing links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links ML(1,1) - ML(1,2d) to the input switch IS1). There are d_2 (where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$) outlet links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example the links OL1-OL(p*d) to the output switch OS1) and $d + d_2$ (= $d + p \times d$)

incoming links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example $ML(2 \times Log_d N_1 - 2,1)$ - $ML(2 \times Log_d N_1 - 2, d + d_2)$ to the output switch OS1).

Each of the $\frac{N_1}{d}$ input switches IS1 – IS(N_1/d) are connected to exactly $2 \times d$ switches in middle stage 130 through $2 \times d$ links (for example in one embodiment the input switch IS1 is connected to middle switches MS(1,1) - MS(1,d) through the links ML(1,1) - ML(1,d) and to middle switches MS(1,N_1/d+1) – MS(1,{ N_1/d}+d) through the links ML(1,d+1) – ML(1,2d) respectively.

Each of the $2 \times \frac{N_1}{d}$ middle switches MS(1,1) – MS(1,2 N_1/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $2 \times \frac{N_1}{d}$ middle switches $MS(Log_d N_1 - 1,1)$ - $MS(Log_d N_1 - 1, 2 \times \frac{N_1}{d})$ in the middle stage $130 + 10 * (Log_d N_1 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 1)$ through d links.

Similarly each of the $2 \times \frac{N_1}{d}$ middle switches $MS(2 \times Log_d N_1 - 3,1)$ - $MS(2 \times Log_d N_1 - 3, 2 \times \frac{N_1}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_1 - 4)$ are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N_1 - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N_1}{d}$ output switches OS1 – OS(N_1/d) are connected from exactly $d+d_2$ switches in middle stage $130+10*(2*Log_d N_1 - 4)$ through $d+d_2$ links.

As described before, again the connection topology of a general $V_{fold}(N_1, N_2, d, s)$ may be any one of the connection topologies. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back inverse Benes networks, back to back Omega networks, back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the general $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link if any output link should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiments of FIG. 1C, FIG. 1C1, and FIG. 1C2 are three examples of network $V_{fold}(N_1, N_2, d, s)$ for s = 2 and $N_2 > N_1$.

The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 2$ according to the current invention. Also the general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in strictly nonblocking manner for unicast if $s = 2$ according to the current invention.

For example, the network of Fig. 1C shows an exemplary five-stage network, namely $V_{fold}(8,24,2,2)$, with the following multicast assignment $I_1 = \{2,3\}$ and all other $I_j = \phi$ for j = [2-8]. It should be noted that the connection I_1 fans out in the first stage switch IS1 into middle switches MS(1,1) and MS(1,5) in middle stage 130, and fans out in middle switches MS(1,1) and MS(1,5) only once into middle switches MS(2,1) and MS(2,5) respectively in middle stage 140.

The connection I_1 also fans out in middle switches MS(2,1) and MS(2,5) only once into middle switches MS(3,1) and MS(3,7) respectively in middle stage 150. The connection I_1 also fans out in middle switches MS(3,1) and MS(3,7) only once into output switches OS2 and OS3 in output stage 120. Finally the connection I_1 fans out once in the output stage switch OS2 into outlet link OL7 and in the output stage switch OS3 twice into the outlet links OL13 and OL16. In accordance with the invention, each connection can fan out in the input stage switch into at most two middle stage switches in middle stage 130.

Asymmetric folded RNB ($N_1 > N_2$) Embodiments:

Referring to FIG. 1E, in one embodiment, an exemplary asymmetrical folded multi-stage network 100E with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, six by eight switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, four by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches of size four by two in middle stage 130, and eight switches of size two by two in middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_2}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_1 > N_2$ and

$N_1 = p * N_2$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_2}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * (d + d_1)$ and each output switch OS1-OS4 can be denoted in general with the notation $(2 \times d * d)$, where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$. The size of each switch in any of the middle stages excepting the first middle stage can be denoted as $d * d$. The size of each switch in the first middle stage can be denoted as $\frac{(d + d_1)}{2} * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric folded multi-stage network can be represented with the notation $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL24), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL8), d represents the inlet links of each input switch where $N_1 > N_2$, and s is the ratio of number of incoming links to each output switch to the outlet links of each output switch.

Each of the $\frac{N_2}{d}$ input switches IS1 – IS4 are connected to exactly $d + d_1$ switches in middle stage 130 through $d + d_1$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,3), MS(1,4), MS(1,5), MS(1,6), MS(1,7), and MS(1,8) through the links ML(1,1), ML(1,2), ML(1,3), ML(1,4), ML(1,5), ML(1,6), ML(1,7), and ML(1,8) respectively).

Each of the $2 \times \frac{N_2}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly $\frac{(d + d_1)}{2}$ input switches through $\frac{(d + d_1)}{2}$ links (for example the links ML(1,1), ML(1,9), ML(1,17) and ML(1,25) are connected to the middle switch MS(1,1) from input switch IS1, IS2, IS3, and IS4 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $2\times\frac{N_2}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $2\times\frac{N_2}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N_2}{d}$ output switches OS1 – OS4 are connected from exactly $2\times d$ switches in middle stage 150 through $2\times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,2), MS(3,5), and MS(3,6) through the links ML(4,1), ML(4,3), ML(4,9), and ML(4,11) respectively).

Finally the connection topology of the network 100E shown in FIG. 1E is known to be back to back inverse Benes connection topology.

Referring to FIG. 1E1, in another embodiment of network $V_{fold}(N_1,N_2,d,s)$, an exemplary asymmetrical folded multi-stage network 100E1 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, six by eight switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, four by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches of size four by two in middle stage 130, and eight switches of size two by two in middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_2}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_2}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * (d + d_1)$ and each output switch OS1-OS4 can be denoted in general with the notation $(2 \times d * d)$, where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$. The size of each switch in any of the middle stages excepting the first middle stage can be denoted as $d * d$. The size of each switch in the first middle stage can be denoted as $\frac{(d + d_1)}{2} * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The asymmetric folded multi-stage network of FIG. 1E1 is also the network of the type $V_{fold}(N_1, N_2, d, s)$, where N_1

represents the total number of inlet links of all input switches (for example the links IL1-IL24), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL8), d represents the inlet links of each input switch where $N_1 > N_2$, and s is the ratio of number of incoming links to each output switch to the outlet links of each output switch.

Each of the $\frac{N_2}{d}$ input switches IS1 – IS4 are connected to exactly $d + d_1$ switches in middle stage 130 through $d + d_1$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,3), MS(1,4), MS(1,5), MS(1,6), MS(1,7), and MS(1,8) through the links ML(1,1), ML(1,2), ML(1,3), ML(1,4), ML(1,5), ML(1,6), ML(1,7), and ML(1,8) respectively).

Each of the $2 \times \frac{N_2}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly $\frac{(d + d_1)}{2}$ input switches through $\frac{(d + d_1)}{2}$ links (for example the links ML(1,1), ML(1,9), ML(1,17) and ML(1,25) are connected to the middle switch MS(1,1) from input switch IS1, IS2, IS3, and IS4 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2 \times \frac{N_2}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,5) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2\times\frac{N_2}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,5) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N_2}{d}$ output switches OS1 – OS4 are connected from exactly $2\times d$ switches in middle stage 150 through $2\times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,3), MS(3,5), and MS(3,7) through the links ML(4,1), ML(4,5), ML(4,9), and ML(4,13) respectively).

Finally the connection topology of the network 100E1 shown in FIG. 1E1 is known to be back to back Omega connection topology.

Referring to FIG. 1E2, in another embodiment of network $V_{fold}(N_1, N_2, d, s)$, an exemplary asymmetrical folded multi-stage network 100E2 with five stages of thirty two switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, six by eight switches IS1-IS4 and output stage 120 consists of four, four by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of eight, four by two switches MS(1,1) - MS(1,8), middle stage 140 consists of eight, two by two switches MS(2,1) - MS(2,8), and middle stage 150 consists of eight, two by two switches MS(3,1) - MS(3,8).

Such a network can be operated in strictly non-blocking manner for unicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches in each of middle stage 130, middle stage 140 and middle stage 150. Such a network can be operated in rearrangeably non-blocking manner for multicast connections, because the switches in the input stage 110 are of size six by eight, the switches in output stage 120 are of size four by two, and there are eight switches of size four by two in middle stage 130, and eight switches of size two by two in middle stage 140 and middle stage 150.

In one embodiment of this network each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_2}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. The number of middle switches in each middle stage is denoted by $2 \times \frac{N_2}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * (d + d_1)$ and each output switch OS1-OS4 can be denoted in general with the notation $(2 \times d * d)$, where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$. The size of each switch in any of the middle stages excepting the first middle stage can be denoted as $d * d$. The size of each switch in the first middle stage can be denoted as $\frac{(d + d_1)}{2} * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. The asymmetric folded multi-stage network of FIG. 1E1 is also the network of the type $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL24), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL8), d represents the inlet links of each input switch where $N_1 > N_2$, and s is the ratio of number of incoming links to each output switch to the outlet links of each output switch.

Each of the $\frac{N_2}{d}$ input switches IS1 – IS4 are connected to exactly $d + d_1$ switches in middle stage 130 through $d + d_1$ links (for example input switch IS1 is connected to middle switches MS(1,1), MS(1,2), MS(1,3), MS(1,4), MS(1,5), MS(1,6), MS(1,7), and MS(1,8) through the links ML(1,1), ML(1,2), ML(1,3), ML(1,4), ML(1,5), ML(1,6), ML(1,7), and ML(1,8) respectively).

Each of the $2\times\frac{N_2}{d}$ middle switches MS(1,1) – MS(1,8) in the middle stage 130 are connected from exactly $\frac{(d+d_1)}{2}$ input switches through $\frac{(d+d_1)}{2}$ links (for example the links ML(1,1), ML(1,9), ML(1,17) and ML(1,25) are connected to the middle switch MS(1,1) from input switch IS1, IS2, IS3, and IS4 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,2) respectively).

Similarly each of the $2\times\frac{N_2}{d}$ middle switches MS(2,1) – MS(2,8) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,8) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,4) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,2) respectively).

Similarly each of the $2\times\frac{N_2}{d}$ middle switches MS(3,1) – MS(3,8) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,8) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,4) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switches MS(3,1)).

Each of the $\frac{N_2}{d}$ output switches OS1 – OS4 are connected from exactly $2\times d$ switches in middle stage 150 through $2\times d$ links (for example output switch OS1 is connected from middle switches MS(3,1), MS(3,4), MS(3,5), and MS(3,8) through the links ML(4,1), ML(4,8), ML(4,9), and ML(4,16) respectively).

Finally the connection topology of the network 100E2 shown in FIG. 1E2 is hereinafter called nearest neighbor connection topology.

In the three embodiments of FIG. 1E, FIG. 1E1 and FIG. 1E2 the connection topology is different. That is the way the links ML(1,1) - ML(1,32), ML(2,1) - ML(2,16), ML(3,1) - ML(3,16), and ML(4,1) - ML(4,16) are connected between the respective stages is different. Even though only three embodiments are illustrated, in general, the network $V_{fold}(N_1, N_2, d, s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiments of FIG. 1E, FIG. 1E1, and FIG. 1E2 are only three examples of network $V_{fold}(N_1, N_2, d, s)$.

In the three embodiments of FIG. 1E, FIG. 1E1 and FIG. 1E2, each of the links ML(1,1) – ML(1,32), ML(2,1) – ML(2,16), ML(3,1) – ML(3,16) and ML(4,1) – ML(4,16) are either available for use by a new connection or not available if currently used by an existing connection. The input switches IS1-IS4 are also referred to as the network input ports. The input stage 110 is often referred to as the first stage. The output switches OS1-OS4 are also referred to as the network output ports. The output stage 120 is often referred to as the last stage. The middle stage switches MS(1,1) – MS(1,8), MS(2,1) – MS(2,8), and MS(3,1) – MS(3,8) are referred to as middle switches or middle ports.

In the example illustrated in FIG. 1E (or in FIG1E1, or in FIG. 1E2), a fan-out of four is possible to satisfy a multicast connection request if input switch is IS2, but only two switches in middle stage 130 will be used. Similarly, although a fan-out of three is possible for a multicast connection request if the input switch is IS1, again only a fan-out of two is used. The specific middle switches that are chosen in middle stage 130 when selecting a fan-out of two is irrelevant so long as at most two middle switches are selected to ensure that the connection request is satisfied. In essence, limiting the fan-out from input switch to no more than two middle switches permits the network 100E (or 100E1, or 100E2), to be operated in rearrangeably nonblocking manner in accordance with the invention.

The connection request of the type described above can be unicast connection request, a multicast connection request or a broadcast connection request, depending on the example. In case of a unicast connection request, a fan-out of one is used, i.e. a single middle stage switch in middle stage 130 is used to satisfy the request. Moreover, although in the above-described embodiment a limit of two has been placed on the fan-out into the middle stage switches in middle stage 130, the limit can be greater depending on the number of middle stage switches in a network (while maintaining the rearrangeably nonblocking nature of operation of the network for multicast connections). However any arbitrary fan-out may be used within any of the middle stage switches and the output stage switches to satisfy the connection request.

Generalized Asymmetric folded RNB ($N_1 > N_2$) Embodiments:

Network 100F of FIG. 1F is an example of general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2 \times \log_d N_2) - 1$ stages where $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. In network 100D of FIG. 1F, $N_2 = N$ and $N_1 = p * N$. The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 2$ according to the current invention. Also the general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in strictly nonblocking manner for unicast if $s = 2$ according to the current invention. (And in the example of FIG. 1F, $s = 2$). The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2 \times \log_d N_2) - 1$ stages has d_1 (where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$ inlet links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links IL1-IL(p*d) to the input switch IS1) and $d + d_1$ (= $d + p \times d$) outgoing links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links ML(1,1) - ML(1,(d+p*d)) to the input switch IS1). There are d outlet links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example the links OL1-OL(d) to the output switch OS1) and $2 \times d$ incoming links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example $ML(2 \times Log_d N_2 - 2,1)$ - $ML(2 \times Log_d N_2 - 2,2 \times d)$ to the output switch OS1).

Each of the $\frac{N_2}{d}$ input switches IS1 – IS(N_2/d) are connected to exactly $d + d_1$ switches in middle stage 130 through $d + d_1$ links (for example in one embodiment the input switch IS1 is connected to middle switches MS(1,1) - MS(1, (d+d_1)/2) through the links ML(1,1) - ML(1,(d+d_1)/2) and to middle switches MS(1,N_1/d+1) – MS(1,{ N_1/d}+(d+d_1)/2) through the links ML(1, ((d+d_1)/2)+1) – ML(1, (d+d_1)) respectively.

Each of the $2 \times \frac{N_2}{d}$ middle switches MS(1,1) – MS(1,2*N_2/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $2 \times \frac{N_2}{d}$ middle switches $MS(Log_d N_2 - 1,1)$ - $MS(Log_d N_2 - 1,2 \times \frac{N_2}{d})$ in the middle stage $130 + 10 * (Log_d N_2 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 1)$ through d links.

Similarly each of the $2 \times \frac{N_2}{d}$ middle switches $MS(2 \times Log_d N_2 - 3,1)$ - $MS(2 \times Log_d N_2 - 3,2 \times \frac{N_2}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_2 - 4)$ are connected from exactly d switches in middle stage $130+10*(2*Log_d N_2 - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N_2}{d}$ output switches OS1 – OS(N_2/d) are connected from exactly $2\times d$ switches in middle stage $130+10*(2*Log_d N_2 - 4)$ through $2\times d$ links.

As described before, again the connection topology of a general $V_{fold}(N_1, N_2, d, s)$ may be any one of the connection topologies. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back inverse Benes networks, back to back Omega networks, back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the general $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link if any output link should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiments of FIG. 1E, FIG. 1E1, and FIG. 1E2 are three examples of network $V_{fold}(N_1, N_2, d, s)$ for s = 2 and $N_1 > N_2$.

The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 2$ according to the current invention. Also the general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in strictly nonblocking manner for unicast if $s = 2$ according to the current invention.

For example, the network of Fig. 1E shows an exemplary five-stage network, namely $V_{fold}(24,8,2,2)$, with the following multicast assignment $I_1 = \{2,3\}$ and all other $I_j = \phi$ for j = [2-8]. It should be noted that the connection I_1 fans out in the first stage switch IS1 into middle switches MS(1,1) and MS(1,5) in middle stage 130, and fans out in middle switches MS(1,1) and MS(1,5) only once into middle switches MS(2,1) and MS(2,5) respectively in middle stage 140.

The connection I_1 also fans out in middle switches MS(2,1) and MS(2,5) only once into middle switches MS(3,1) and MS(3,7) respectively in middle stage 150. The connection I_1 also fans out in middle switches MS(3,1) and MS(3,7) only once into output switches OS2 and OS3 in output stage 120. Finally the connection I_1 fans out once in the output stage switch OS2 into outlet link OL3 and in the output stage switch OS3 twice into the outlet links OL5 and OL6. In accordance with the invention, each connection can fan out in the input stage switch into at most two middle stage switches in middle stage 130.

SNB Embodiments:

The folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ disclosed, in the current invention, is topologically exactly the same as the multi-stage network $V_{fold}(N_1, N_2, d, s)$, disclosed in U.S. Provisional Patent Application Docket No. M-0037US that is incorporated by reference above, excepting that in the illustrations folded network $V_{fold}(N_1, N_2, d, s)$ is shown as it is folded at middle stage $130 + 10 * (Log_d N_2 - 2)$.

The general symmetrical folded multi-stage network $V_{fold}(N, d, s)$ can also be operated in strictly nonblocking manner for multicast when $s = 3$ according to the current invention. Similarly the general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can also be operated in strictly nonblocking manner for multicast when $s = 3$ according to the current invention.

Symmetric folded RNB Unicast Embodiments:

Referring to FIG. 2A, an exemplary symmetrical folded multi-stage network 200A respectively with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by two switches IS1-IS4 and output stage 120 consists of four, two by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, two by two switches

MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by two switches MS(3,1) - MS(3,4).

Such a network can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size two by two, the switches in output stage 120 are of size two by two, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

The connection topology of the network 200A shown in FIG. 2A is known to be back to back inverse Benes connection topology. In other embodiments the connection topology is different. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different.

Even though only one embodiment is illustrated, in general, the network $V_{fold}(N,d,s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N,d,s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N,d,s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N,d,s)$ can be built. The embodiment of FIG. 2A is only one example of network $V_{fold}(N,d,s)$.

The network 200A of FIG. 2A is also rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N}{d}$, where N is the total number of inlet links or outlet links. The number of middle switches in each middle stage is denoted by $\frac{N}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d*d$ and each output switch OS1-OS4 can be denoted in general with the notation $d*d$. Likewise, the size of each switch in any of the middle stages can be denoted as $d * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. A symmetric folded multi-stage network can be represented with the notation $V_{fold}(N, d, s)$, where N represents the total number of inlet links of all input switches (for example the links IL1-IL8), d represents the inlet links of each input switch or outlet links of each output switch, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch. Although it is not necessary that there be the same number of inlet links IL1-IL8 as there are outlet links OL1-OL8, in a symmetrical network they are the same.

In network 200A of FIG. 2A, each of the $\frac{N}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d links (for example input switch IS1 is connected to middle switches MS(1,1) and MS(1,2) through the links ML(1,1) and ML(1,2) respectively).

Each of the $\frac{N}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,4) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switch MS(3,1)).

Each of the $\frac{N}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d links (for example output switch OS1 is connected from middle switches MS(3,1) and MS(3,2) through the links ML(4,1) and ML(4,4) respectively).

Generalized Symmetric folded RNB Unicast Embodiments:

Network 200B of FIG. 2B is an example of general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ with $(2\times\log_d N)-1$ stages. The general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ can be operated in rearrangeably nonblocking manner for unicast when $s=1$ according to the current invention (and in the example of FIG. 2B, $s=1$). The general symmetrical folded multi-stage network $V_{fold}(N,d,s)$ with $(2\times\log_d N)-1$ stages has d inlet links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links IL1-IL(d) to the input switch IS1) and d outgoing links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links ML(1,1) - ML(1,d) to the input switch IS1). There are d outlet links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example OL1-OL(d) to the output switch OS1) and d incoming links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example $ML(2\times Log_d N-2,1)$ - $ML(2\times Log_d N-2,d)$ to the output switch OS1).

Each of the $\frac{N}{d}$ input switches IS1 – IS(N/d) are connected to exactly d switches in middle stage 130 through d links.

Each of the $\frac{N}{d}$ middle switches MS(1,1) – MS(1,N/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N}{d}$ middle switches $MS(Log_d N - 1{,}1)$ - $MS(Log_d N - 1, \frac{N}{d})$ in the middle stage $130 + 10 * (Log_d N - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N - 1)$ through d links.

Similarly each of the $\frac{N}{d}$ middle switches $MS(2 \times Log_d N - 3{,}1)$ - $MS(2 \times Log_d N - 3, \frac{N}{d})$ in the middle stage $130 + 10 * (2 * Log_d N - 4)$ are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N}{d}$ output switches OS1 – OS(N/d) are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N - 4)$ through d links.

The general symmetrical folded multi-stage network $V_{fold}(N, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 1$ according to the current invention.

Asymmetric folded RNB ($N_2 > N_1$) Unicast Embodiments:

Referring to FIG. 2C, an exemplary symmetrical folded multi-stage network 200C respectively with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by two switches IS1-IS4 and output stage 120 consists of four, six by six switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, two by two switches MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by six switches MS(3,1) - MS(3,4).

Such networks can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size two by two, the switches in output stage 120 are of size six by six, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

The connection topology of the network 200C shown in FIG. 2C is known to be back to back inverse Benes connection topology. The connection topology of the networks 200C is different in the other embodiments. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different.

Even though only one embodiment is illustrated, in general, the network $V_{fold}(N_1, N_2, d, s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiment of FIG. 2C is only one example of network $V_{fold}(N_1, N_2, d, s)$.

The networks 200C of FIG. 2C is also rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_1}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. The number of middle switches in each middle stage is denoted by $\frac{N_1}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * d$ and each output switch OS1-OS4 can be denoted in general with the notation $d_2 * d_2$, where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$. The size of each switch in any of the middle stages excepting the last middle stage can be denoted as $d * d$. The size of each switch in the last middle stage can be denoted as $d * d_2$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric folded multi-stage network can be represented with the notation $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL8), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL24), d represents the inlet links of each input switch where $N_2 > N_1$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

In network 200C of FIG. 2C, each of the $\frac{N_1}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d links (for example input switch IS1 is connected to middle switches MS(1,1) and MS(1,2) through the links ML(1,1) and ML(1,2) respectively).

Each of the $\frac{N_1}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,4) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d_2 links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 from middle switch MS(3,1); the links ML(4,3) and ML(4,4) are connected to output switches OS2 from middle switch MS(3,1); the link ML(4,5) is connected to output switches OS3 from middle switch MS(3,1); and the link ML(4,6) is connected to output switches OS4 from middle switch MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d_2 links (for example output switch OS1 is connected from middle switch MS(3,1) through the links ML(4,1) and ML(4,2); output switch OS1 is connected from middle switch MS(3,2) through the links ML(4,7) and ML(4,8); output switch OS1 is connected from middle switch MS(3,3) through the link ML(4,13); and output switch OS1 is connected from middle switch MS(3,4) through the links ML(4,19)).

Generalized Asymmetric folded RNB ($N_2 > N_1$) Unicast Embodiments:

Network 200D of FIG. 2D is an example of general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2\times\log_d N)-1$ stages where $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. In network 200D of FIG. 2D, $N_1 = N$ and $N_2 = p * N$. The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for unicast when $s = 1$ according to the current invention (and in the example of FIG. 2D, $s = 1$). The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2\times\log_d N)-1$ stages has d inlet links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links IL1-IL(d) to the input switch IS1) and d outgoing links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links ML(1,1) - ML(1,d) to the input switch IS1). There are d_2 (where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$) outlet links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example the links OL1-OL(p*d) to the output switch OS1) and d_2 (= $p \times d$) incoming links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example $ML(2\times Log_d N_1 - 2,1)$ - $ML(2\times Log_d N_1 - 2, d_2)$ to the output switch OS1).

Each of the $\frac{N_1}{d}$ input switches IS1 – IS(N_1/d) are connected to exactly d switches in middle stage 130 through d links.

Each of the $\frac{N_1}{d}$ middle switches MS(1,1) – MS(1,N_1/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N_1}{d}$ middle switches $MS(Log_d N_1 - 1,1)$ - $MS(Log_d N_1 - 1, \frac{N_1}{d})$ in the middle stage $130 + 10 * (Log_d N_1 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 1)$ through d links.

Similarly each of the $\frac{N_1}{d}$ middle switches $MS(2 \times Log_d N_1 - 3,1)$ - $MS(2 \times Log_d N_1 - 3, \frac{N_1}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_1 - 4)$ are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N_1 - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d_2 links.

Each of the $\frac{N_1}{d}$ output switches OS1 – OS(N_1/d) are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N - 4)$ through d_2 links.

The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 1$ according to the current invention.

Asymmetric folded RNB ($N_1 > N_2$) Unicast Embodiments:

Referring to FIG. 2E, an exemplary symmetrical folded multi-stage network 200E with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, six by six switches IS1-IS4 and output stage 120 consists of four, two by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, six by two switches MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by two switches MS(3,1) - MS(3,4).

Such a network can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size six by six, the switches in output stage 120 are of size two by two, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

The connection topology of the network 200E shown in FIG. 2E is known to be back to back inverse Benes connection topology. The connection topology of the networks 200E is different in the other embodiments. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different.

Even though only one embodiment is illustrated, in general, the network $V_{fold}(N_1, N_2, d, s)$, comprise any arbitrary type of connection topology. For example the connection topology of the network $V_{fold}(N_1, N_2, d, s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V_{fold}(N_1, N_2, d, s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V_{fold}(N_1, N_2, d, s)$ can be built. The embodiment of FIG. 2E is only one example of network $V_{fold}(N_1, N_2, d, s)$.

The network 200E is rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_2}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. The number of middle switches in each middle stage is denoted by $\frac{N_2}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d_1 * d_1$ and each output switch OS1-OS4 can be denoted in general with the notation $(d * d)$, where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$. The size of each switch in any of the middle stages excepting the first middle stage can be denoted as $d * d$. The size of each switch in the first middle stage can be denoted as $d_1 * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric folded multi-stage network can be represented with the notation $V_{fold}(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL24), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL8), d represents the inlet links of each output switch where $N_1 > N_2$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

In network 200E of FIG. 2E, each of the $\frac{N_2}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d_1 links (for example input switch IS1 is connected to middle switch MS(1,1) through the links ML(1,1) and ML(1,2); input switch IS1 is connected to middle switch MS(1,2) through the links ML(1,3) and ML(1,4); input switch IS1 is connected to middle switch MS(1,3) through the link ML(1,5); and input switch IS1 is connected to middle switch MS(1,4) through the links ML(1,6)).

Each of the $\frac{N_2}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d_1 input switches through d links (for example the links ML(1,1) and ML(1,2) are connected to the middle switch MS(1,1) from input switch IS1; the links ML(1,7) and ML(1,8) are connected to the middle switch MS(1,1) from input switch IS2; the link ML(1,13) is connected to the middle switch MS(1,1) from input switch IS3; and the link ML(1,19) is connected to the middle switch MS(1,1) from input switch IS4), and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d_2 links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switch MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d_2 links (for example output switch OS1 is connected from middle switches MS(3,1) and MS(3,2) through the links ML(4,1) and ML(4,4) respectively).

Generalized Asymmetric folded RNB ($N_1 > N_2$) Unicast Embodiments:

Network 200F of FIG. 2F is an example of general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2\times\log_d N)-1$ stages where $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. In network 200F of FIG. 2F, $N_2 = N$ and $N_1 = p * N$.. The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for unicast when $s = 1$ according to the current invention (and in the example of FIG. 2F, $s = 1$). The general asymmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ with $(2\times\log_d N)-1$ stages has d_1 (where

$d_1 = N_1 \times \frac{d}{N_2} = p \times d$ inlet links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links IL1-IL(p*d) to the input switch IS1) and d_1 (= $p \times d$) outgoing links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links ML(1,1) - ML(1,(d+p*d)) to the input switch IS1). There are d outlet links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example the links OL1-OL(d) to the output switch OS1) and d incoming links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example $ML(2 \times Log_d N_2 - 2,1)$ - $ML(2 \times Log_d N_2 - 2, d)$ to the output switch OS1).

Each of the $\frac{N_2}{d}$ input switches IS1 – IS(N_2/d) are connected to exactly d switches in middle stage 130 through d_1 links.

Each of the $\frac{N_2}{d}$ middle switches MS(1,1) – MS(1,N_2/d) in the middle stage 130 are connected from exactly d input switches through d_1 links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N_2}{d}$ middle switches $MS(Log_d N_2 - 1,1)$ - $MS(Log_d N_2 - 1, \frac{N_2}{d})$ in the middle stage $130 + 10 * (Log_d N_2 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 1)$ through d links.

Similarly each of the $\frac{N_2}{d}$ middle switches $MS(2 \times Log_d N_2 - 3,1)$ - $MS(2 \times Log_d N_2 - 3, \frac{N_2}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_2 - 4)$ are connected from exactly d switches in middle stage $130+10*(2*Log_d N_2 -5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N_2}{d}$ output switches OS1 – OS(N_2/d) are connected from exactly d switches in middle stage $130+10*(2*Log_d N_2 -4)$ through d links.

The general symmetrical folded multi-stage network $V_{fold}(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s=1$ according to the current invention.

Symmetric RNB Unicast Embodiments:

Referring to FIG. 3A, FIG. 3B, FIG. 3C, FIG. 3D, FIG. 3E, FIG. 3F, FIG. 3G, FIG. 300H, FIG. 300I and FIG. 3J with exemplary symmetrical multi-stage networks 300A, 300B, 300C, 300D, 300E, 300F, 300G, 300H, 300I, and 300J respectively with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by two switches IS1-IS4 and output stage 120 consists of four, two by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, two by two switches MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by two switches MS(3,1) - MS(3,4).

Such networks can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size two by two, the switches in output stage 120 are of size two by two, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

In all the ten embodiments of FIG. 3A to FIG. 3J the connection topology is different. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different. For example, the connection topology of the network 300A shown in FIG. 3A is known to be back to back inverse Benes connection topology; the connection topology of the network 300B shown in FIG. 3B is known to be back to back Omega connection topology; and the connection topology of the network 300C shown in FIG. 3C is hereinafter called nearest neighbor connection topology.

Even though only ten embodiments are illustrated, in general, the network $V(N,d,s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V(N,d,s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V(N,d,s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V(N,d,s)$ can be built. The ten embodiments of FIG. 3A to FIG. 3J are only three examples of network $V(N,d,s)$.

The networks 300A - 300J of FIG. 3A - FIG. 3J are also rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N}{d}$, where N is the total number of inlet links or outlet links. The number of middle switches in each middle stage is denoted by $\frac{N}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d*d$ and each output switch OS1-OS4 can be denoted in general with the notation $d*d$. Likewise, the size of each switch in any of the middle stages can be denoted as $d*d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. A symmetric multi-stage network can be represented with the notation $V(N,d,s)$, where N represents the total number of inlet links of all input switches (for example the links IL1-IL8), d represents the inlet links of each input switch or outlet links of each output switch, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch. Although it is not necessary that there be the same number of inlet links IL1-IL8 as there are outlet links OL1-OL8, in a symmetrical network they are the same.

In network 300A of FIG. 3A, each of the $\frac{N}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d links (for example input switch IS1 is connected to middle switches MS(1,1) and MS(1,2) through the links ML(1,1) and ML(1,2) respectively).

Each of the $\frac{N}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,4) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switch MS(3,1)).

Each of the $\frac{N}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d links (for example output switch OS1 is connected from middle switches MS(3,1) and MS(3,2) through the links ML(4,1) and ML(4,4) respectively).

Generalized Symmetric RNB Unicast Embodiments:

Network 300K of FIG. 3K is an example of general symmetrical multi-stage network $V(N,d,s)$ with $(2\times\log_d N)-1$ stages. The general symmetrical multi-stage network $V(N,d,s)$ can be operated in rearrangeably nonblocking manner for unicast when $s=1$ according to the current invention (and in the example of FIG. 3K, $s=1$). The general symmetrical multi-stage network $V(N,d,s)$ with $(2\times\log_d N)-1$ stages has d inlet links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links IL1-IL(d) to the input switch IS1) and d outgoing links for each of $\frac{N}{d}$ input switches IS1-IS(N/d) (for example the links ML(1,1) - ML(1,d) to the input switch IS1). There are d outlet links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example OL1-OL(d) to the output switch OS1) and d incoming links for each of $\frac{N}{d}$ output switches OS1-OS(N/d) (for example $ML(2\times Log_d N-2,1)$ - $ML(2\times Log_d N-2,d)$ to the output switch OS1).

Each of the $\frac{N}{d}$ input switches IS1 – IS(N/d) are connected to exactly d switches in middle stage 130 through d links.

Each of the $\frac{N}{d}$ middle switches MS(1,1) – MS(1,N/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N}{d}$ middle switches $MS(Log_d N-1,1)$ - $MS(Log_d N-1,\frac{N}{d})$ in the middle stage $130+10*(Log_d N-2)$ are connected from exactly d switches in middle stage $130+10*(Log_d N-3)$ through d links and also are connected to exactly d switches in middle stage $130+10*(Log_d N-1)$ through d links.

Similarly each of the $\frac{N}{d}$ middle switches $MS(2\times Log_d N-3,1)$ - $MS(2\times Log_d N-3,\frac{N}{d})$ in the middle stage $130+10*(2*Log_d N-4)$ are connected from exactly d switches in middle stage $130+10*(2*Log_d N-5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N}{d}$ output switches OS1 – OS(N/d) are connected from exactly d switches in middle stage $130+10*(2*Log_d N-4)$ through d links.

The general symmetrical multi-stage network $V(N,d,s)$ can be operated in rearrangeably nonblocking manner for multicast when $s=1$ according to the current invention.

Asymmetric RNB ($N_2 > N_1$) Unicast Embodiments:

Referring to FIG. 3A1, FIG. 3B1, FIG. 3C1, FIG. 3D1, FIG. 3E1, FIG. 3F1, FIG. 3G1, FIG. 300H1, FIG. 300I1 and FIG. 3J1 with exemplary symmetrical multi-stage networks 300A1, 300B1, 300C1, 300D1, 300E1, 300F1, 300G1, 300H1, 300I1, and 300J1 respectively with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, two by two switches IS1-IS4 and output stage 120 consists of four, six by six switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, two by two switches MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by six switches MS(3,1) - MS(3,4).

Such networks can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size two by two, the switches in output stage 120 are of size six by six, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

In all the ten embodiments of FIG. 3A1 to FIG. 3J1 the connection topology is different. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different. For example, the connection topology of the network 300A1 shown in FIG. 3A1 is known to be back to back inverse Benes connection topology; the connection topology of the network 300B1 shown in FIG. 3B1 is known to be back to back Omega connection topology; and the connection topology of the network 300C1 shown in FIG. 3C1 is hereinafter called nearest neighbor connection topology.

Even though only ten embodiments are illustrated, in general, the network $V(N_1, N_2, d, s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V(N_1, N_2, d, s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V(N_1, N_2, d, s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V(N_1, N_2, d, s)$ can be built. The ten embodiments of FIG. 3A1 to FIG. 3J1 are only three examples of network $V(N_1, N_2, d, s)$.

The networks 300A1 - 300J1 of FIG. 3A1 - FIG. 3J1 are also rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_1}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. The number of middle switches in each middle stage is denoted by $\frac{N_1}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d * d$ and each output switch OS1-OS4 can be denoted in general with the notation $d_2 * d_2$, where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$. The size of each switch in any of the middle stages excepting the last middle stage can be denoted as $d * d$. The size of each switch in the last middle stage can be denoted as $d * d_2$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric multi-stage network can be represented with the notation $V(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL8), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL24), d represents the inlet links of each input switch where $N_2 > N_1$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

In network 300A1 of FIG. 3A1, each of the $\frac{N_1}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d links (for example input switch IS1 is connected to middle switches MS(1,1) and MS(1,2) through the links ML(1,1) and ML(1,2) respectively).

Each of the $\frac{N_1}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d input switches through d links (for example the links ML(1,1) and ML(1,4) are connected to the middle switch MS(1,1) from input switch IS1 and IS2 respectively) and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d

switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d_2 links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 from middle switch MS(3,1); the links ML(4,3) and ML(4,4) are connected to output switches OS2 from middle switch MS(3,1); the link ML(4,5) is connected to output switches OS3 from middle switch MS(3,1); and the link ML(4,6) is connected to output switches OS4 from middle switch MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d_2 links (for example output switch OS1 is connected from middle switch MS(3,1) through the links ML(4,1) and ML(4,2); output switch OS1 is connected from middle switch MS(3,2) through the links ML(4,7) and ML(4,8); output switch OS1 is connected from middle switch MS(3,3) through the link ML(4,13); and output switch OS1 is connected from middle switch MS(3,4) through the links ML(4,19)).

Generalized Asymmetric RNB ($N_2 > N_1$) Unicast Embodiments:

Network 300K1 of FIG. 3K1 is an example of general asymmetrical multi-stage network $V(N_1, N_2, d, s)$ with $(2 \times \log_d N) - 1$ stages where $N_2 > N_1$ and $N_2 = p * N_1$ where p > 1. In network 100K1 of FIG. 1K1, $N_1 = N$ and $N_2 = p * N$. The general symmetrical multi-stage network $V(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for unicast when $s = 1$ according to the current invention (and in the example of FIG. 3K1, $s = 1$). The general asymmetrical multi-stage network

$V(N_1, N_2, d, s)$ with $(2\times\log_d N)-1$ stages has d inlet links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links IL1-IL(d) to the input switch IS1) and d outgoing links for each of $\frac{N_1}{d}$ input switches IS1-IS(N_1/d) (for example the links ML(1,1) - ML(1,d) to the input switch IS1). There are d_2 (where $d_2 = N_2 \times \frac{d}{N_1} = p \times d$) outlet links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example the links OL1-OL(p*d) to the output switch OS1) and d_2 (= $p \times d$) incoming links for each of $\frac{N_1}{d}$ output switches OS1-OS(N_1/d) (for example $ML(2\times Log_d N_1 - 2,1)$ - $ML(2\times Log_d N_1 - 2, d_2)$ to the output switch OS1).

Each of the $\frac{N_1}{d}$ input switches IS1 – IS(N_1/d) are connected to exactly d switches in middle stage 130 through d links.

Each of the $\frac{N_1}{d}$ middle switches MS(1,1) – MS(1,N_1/d) in the middle stage 130 are connected from exactly d input switches through d links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N_1}{d}$ middle switches $MS(Log_d N_1 - 1,1)$ - $MS(Log_d N_1 - 1, \frac{N_1}{d})$ in the middle stage $130 + 10 * (Log_d N_1 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_1 - 1)$ through d links.

Similarly each of the $\frac{N_1}{d}$ middle switches $MS(2 \times Log_d N_1 - 3,1)$ - $MS(2 \times Log_d N_1 - 3, \frac{N_1}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_1 - 4)$ are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N_1 - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d_2 links.

Each of the $\frac{N_1}{d}$ output switches OS1 – OS(N_1/d) are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N - 4)$ through d_2 links.

The general symmetrical multi-stage network $V(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 1$ according to the current invention.

Asymmetric RNB ($N_1 > N_2$) Unicast Embodiments:

Referring to FIG. 3A2, FIG. 3B2, FIG. 3C2, FIG. 3D2, FIG. 3E2, FIG. 3F2, FIG. 3G2, FIG. 300H2, FIG. 300I2 and FIG. 3J2 with exemplary symmetrical multi-stage networks 300A2, 300B2, 300C2, 300D2, 300E2, 300F2, 300G2, 300H2, 300I2, and 300J2respectively with five stages of twenty switches for satisfying communication requests, such as setting up a telephone call or a data call, or a connection between configurable logic blocks, between an input stage 110 and output stage 120 via middle stages 130, 140, and 150 is shown where input stage 110 consists of four, six by six switches IS1-IS4 and output stage 120 consists of four, two by two switches OS1-OS4. And all the middle stages namely middle stage 130 consists of four, six by two switches MS(1,1) - MS(1,4), middle stage 140 consists of four, two by two switches MS(2,1) - MS(2,4), and middle stage 150 consists of four, two by two switches MS(3,1) - MS(3,4).

Such networks can be operated in rearrangeably nonblocking manner for unicast connections, because the switches in the input stage 110 are of size six by six, the switches in output stage 120 are of size two by two, and there are four switches in each of middle stage 130, middle stage 140 and middle stage 150.

In all the ten embodiments of FIG. 3A2 to FIG. 3J2 he connection topology is different. That is the way the links ML(1,1) - ML(1,8), ML(2,1) - ML(2,8), ML(3,1) - ML(3,8), and ML(4,1) - ML(4,8) are connected between the respective stages is different. For example, the connection topology of the network 300A2 shown in FIG. 3A2 is known to be back to back inverse Benes connection topology; the connection topology of the network 300B2 shown in FIG. 3B2 is known to be back to back Omega connection topology; and the connection topology of the network 300C2 shown in FIG. 3C2 is hereinafter called nearest neighbor connection topology.

Even though only ten embodiments are illustrated, in general, the network $V(N_1,N_2,d,s)$ can comprise any arbitrary type of connection topology. For example the connection topology of the network $V(N_1,N_2,d,s)$ may be back to back Benes networks, Delta Networks and many more combinations. The applicant notes that the fundamental property of a valid connection topology of the $V(N_1,N_2,d,s)$ network is, when no connections are setup from any input link all the output links should be reachable. Based on this property numerous embodiments of the network $V(N_1,N_2,d,s)$ can be built. The ten embodiments of FIG. 3A2 to FIG. 3J2 are only three examples of network $V(N_1,N_2,d,s)$.

The networks 300A2 - 300J2 of FIG. 3A2 - FIG. 3J2 are also rearrangeably nonblocking for unicast according to the current invention. In one embodiment of these networks each of the input switches IS1-IS4 and output switches OS1-OS4 are crossbar switches. The number of switches of input stage 110 and of output stage 120 can be denoted in general with the variable $\frac{N_2}{d}$, where N_1 is the total number of inlet links or and N_2 is the total number of outlet links and $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. The number of middle switches in each middle stage is denoted by $\frac{N_2}{d}$. The size of each input switch IS1-IS4 can be denoted in general with the notation $d_1 * d_1$ and each output switch OS1-OS4 can be denoted in general with the notation $(d * d)$, where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$. The size of each switch in any of the middle stages excepting the first middle stage can be denoted as $d * d$. The size of each switch in the first middle stage can be denoted as $d_1 * d$. A switch as used herein can be either a crossbar switch, or a network of switches each of which in turn may be a crossbar switch or a network of switches. An asymmetric multi-stage network can be represented with the notation $V(N_1, N_2, d, s)$, where N_1 represents the total number of inlet links of all input switches (for example the links IL1-IL24), N_2 represents the total number of outlet links of all output switches (for example the links OL1-OL8), d represents the inlet links of each output switch where $N_1 > N_2$, and s is the ratio of number of outgoing links from each input switch to the inlet links of each input switch.

In network 300A2 of FIG. 3A2, each of the $\frac{N_2}{d}$ input switches IS1 – IS4 are connected to exactly d switches in middle stage 130 through d_1 links (for example input switch IS1 is connected to middle switch MS(1,1) through the links ML(1,1) and ML(1,2); input switch IS1 is connected to middle switch MS(1,2) through the links ML(1,3) and ML(1,4); input switch IS1 is connected to middle switch MS(1,3) through the link ML(1,5); and input switch IS1 is connected to middle switch MS(1,4) through the links ML(1,6)).

Each of the $\frac{N_2}{d}$ middle switches MS(1,1) – MS(1,4) in the middle stage 130 are connected from exactly d_1 input switches through d links (for example the links ML(1,1) and ML(1,2) are connected to the middle switch MS(1,1) from input switch IS1; the links ML(1,7) and ML(1,8) are connected to the middle switch MS(1,1) from input switch IS2; the link ML(1,13) is connected to the middle switch MS(1,1) from input switch IS3; and the link ML(1,19) is connected to the middle switch MS(1,1) from input switch IS4), and also are connected to exactly d switches in middle stage 140 through d links (for example the links ML(2,1) and ML(2,2) are connected from middle switch MS(1,1) to middle switch MS(2,1) and MS(2,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(2,1) – MS(2,4) in the middle stage 140 are connected from exactly d switches in middle stage 130 through d links (for example the links ML(2,1) and ML(2,6) are connected to the middle switch MS(2,1) from middle switches MS(1,1) and MS(1,3) respectively) and also are connected to exactly d switches in middle stage 150 through d links (for example the links ML(3,1) and ML(3,2) are connected from middle switch MS(2,1) to middle switch MS(3,1) and MS(3,3) respectively).

Similarly each of the $\frac{N_1}{d}$ middle switches MS(3,1) – MS(3,4) in the middle stage 150 are connected from exactly d switches in middle stage 140 through d links (for example the links ML(3,1) and ML(3,6) are connected to the middle switch MS(3,1) from middle switches MS(2,1) and MS(2,3) respectively) and also are connected to exactly d output switches in output stage 120 through d_2 links (for example the links ML(4,1) and ML(4,2) are connected to output switches OS1 and OS2 respectively from middle switch MS(3,1)).

Each of the $\frac{N_1}{d}$ output switches OS1 – OS4 are connected from exactly d switches in middle stage 150 through d_2 links (for example output switch OS1 is connected from middle switches MS(3,1) and MS(3,2) through the links ML(4,1) and ML(4,4) respectively).

Generalized Asymmetric RNB ($N_1 > N_2$) Unicast Embodiments:

Network 300K2 of FIG. 3K2 is an example of general asymmetrical multi-stage network $V(N_1, N_2, d, s)$ with $(2 \times \log_d N) - 1$ stages where $N_1 > N_2$ and $N_1 = p * N_2$ where p > 1. In network 100K2 of FIG. 1K2, $N_2 = N$ and $N_1 = p * N$.. The general symmetrical multi-stage network $V(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for unicast when $s = 1$ according to the current invention (and in the example of FIG. 3K2, $s = 1$). The general asymmetrical multi-stage network $V(N_1, N_2, d, s)$ with $(2 \times \log_d N) - 1$ stages has d_1 (where $d_1 = N_1 \times \frac{d}{N_2} = p \times d$ inlet links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links IL1-IL(p*d) to the input switch IS1) and d_1 (= $p \times d$) outgoing links for each of $\frac{N_2}{d}$ input switches IS1-IS(N_2/d) (for example the links ML(1,1) - ML(1,(d+p*d)) to the input switch IS1). There are d outlet links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example the links OL1-OL(d) to the output switch OS1) and d incoming links for each of $\frac{N_2}{d}$ output switches OS1-OS(N_2/d) (for example $ML(2 \times Log_d N_2 - 2,1)$ - $ML(2 \times Log_d N_2 - 2, d)$ to the output switch OS1).

Each of the $\frac{N_2}{d}$ input switches IS1 – IS(N_2/d) are connected to exactly d switches in middle stage 130 through d_1 links.

Each of the $\frac{N_2}{d}$ middle switches MS(1,1) – MS(1,N_2/d) in the middle stage 130 are connected from exactly d input switches through d_1 links and also are connected to exactly d switches in middle stage 140 through d links.

Similarly each of the $\frac{N_2}{d}$ middle switches $MS(Log_d N_2 - 1,1)$ - $MS(Log_d N_2 - 1, \frac{N_2}{d})$ in the middle stage $130 + 10 * (Log_d N_2 - 2)$ are connected from exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 3)$ through d links and also are connected to exactly d switches in middle stage $130 + 10 * (Log_d N_2 - 1)$ through d links.

Similarly each of the $\frac{N_2}{d}$ middle switches $MS(2 \times Log_d N_2 - 3,1)$ - $MS(2 \times Log_d N_2 - 3, \frac{N_2}{d})$ in the middle stage $130 + 10 * (2 * Log_d N_2 - 4)$ are connected from exactly d switches in middle stage $130 + 10 * (2 * Log_d N_2 - 5)$ through d links and also are connected to exactly d output switches in output stage 120 through d links.

Each of the $\frac{N_2}{d}$ output switches OS1 – OS(N2/d) are connected from exactly d switches in middle stage $130+10*(2*Log_d N_2 - 4)$ through d links.

The general symmetrical multi-stage network $V(N_1, N_2, d, s)$ can be operated in rearrangeably nonblocking manner for multicast when $s = 1$ according to the current invention.

Scheduling Method Embodiments:

FIG. 4A shows a high-level flowchart of a scheduling method 1000, in one embodiment executed to setup multicast and unicast connections in network 100A of FIG. 1A (or any of the networks $V_{fold}(N_1, N_2, d, s)$ and $V(N_1, N_2, d, s)$ disclosed in this invention) . According to this embodiment, a multicast connection request is received in act 1010. Then the control goes to act 1020.

In act 1020, based on the inlet link and input switch of the multicast connection received in act 1010, from each available outgoing middle link of the input switch of the multicast connection, by traveling forward from middle stage 130 to middle stage $130+10*(Log_d N - 2)$, the lists of all reachable middle switches in each middle stage are derived recursively. That is, first, by following each available outgoing middle link of the input switch all the reachable middle switches in middle stage 130 are derived. Next, starting from the selected middle switches in middle stage 130 traveling through all of their available out going middle links to middle stage 140 all the available middle switches in middle stage 140 are derived. This process is repeated recursively until all the reachable middle switches, starting from the outgoing middle link of input switch, in middle stage $130+10*(Log_d N - 2)$ are derived. This process is repeated for each available outgoing middle link from the input switch of the multicast connection and separate reachable lists are derived in each middle stage from middle stage 130 to middle stage $130+10*(Log_d N - 2)$ for all the available outgoing middle links from the input switch. Then the control goes to act 1030.

In act 1030, based on the destinations of the multicast connection received in act 1010, from the output switch of each destination, by traveling backward from output stage 120 to middle stage $130+10*(Log_d N-2)$, the lists of all middle switches in each middle stage from which each destination output switch (and hence the destination outlet links) is reachable, are derived recursively. That is, first, by following each available incoming middle link of the output switch of each destination link of the multicast connection, all the middle switches in middle stage $130+10*(2*Log_d N-4)$ from which the output switch is reachable, are derived. Next, starting from the selected middle switches in middle stage $130+10*(2*Log_d N-4)$ traveling backward through all of their available incoming middle links from middle stage $130+10*(2*Log_d N-5)$ all the available middle switches in middle stage $130+10*(2*Log_d N-5)$ from which the output switch is reachable, are derived. This process is repeated recursively until all the middle switches in middle stage $130+10*(Log_d N-2)$ from which the output switch is reachable, are derived. This process is repeated for each output switch of each destination link of the multicast connection and separate lists in each middle stage from middle stage $130+10*(2*Log_d N-4)$ to middle stage $130+10*(Log_d N-2)$ for all the output switches of each destination link of the connection are derived. Then the control goes to act 1040.

In act 1040, using the lists generated in acts 1020 and 1030, particularly list of middle switches derived in middle stage $130+10*(Log_d N-2)$ corresponding to each outgoing link of the input switch of the multicast connection, and the list of middle switches derived in middle stage $130+10*(Log_d N-2)$ corresponding to each output switch of the destination links, the list of all the reachable destination links from each outgoing link of the input switch are derived. Specifically if a middle switch in middle stage $130+10*(Log_d N-2)$ is reachable from an outgoing link of the input switch, say "x", and also from the same middle switch in middle stage $130+10*(Log_d N-2)$ if the output switch of a destination link, say "y", is reachable then using the outgoing link of the input switch x, destination link y is reachable. Accordingly, the list of all the reachable destination links from each outgoing link of the input switch is derived. The control then goes to act 1050.

In act 1050, among all the outgoing links of the input switch, it is checked if all the destinations are reachable using only one outgoing link of the input switch. If one outgoing link is available through which all the destinations of the multicast connection are reachable (i.e., act 1050 results in “yes”), the control goes to act 1070. And in act 1070, the multicast connection is setup by traversing from the selected only one outgoing middle link of the input switch in act 1050, to all the destinations. Then the control transfers to act 1090.

If act 1050 results “no”, that is one outgoing link is not available through which all the destinations of the multicast connection are reachable, then the control goes to act 1060. In act 1060, it is checked if all destination links of the multicast connection are reachable using two outgoing middle links from the input switch. According to the current invention, it is always possible to find at most two outgoing middle links from the input switch through which all the destinations of a multicast connection are reachable. So act 1060 always results in “yes”, and then the control transfers to act 1080. In act 1080, the multicast connection is setup by traversing from the selected only two outgoing middle links of the input switch in act 1060, to all the destinations. Then the control transfers to act 1090.

In act 1090, all the middle links between any two stages of the network used to setup the connection in either act 1070 or act 1080 are marked unavailable so that these middle links will be made unavailable to other multicast connections. The control then returns to act 1010, so that acts 1010, 1020, 1030, 1040, 1050, 1060, 1070, 1080, and 1090 are executed in a loop, for each connection request until the connections are set up.

In the example illustrated in FIG. 1A, four outgoing middle links are available to satisfy a multicast connection request if input switch is IS2, but only at most two outgoing middle links of the input switch will be used in accordance with this method. Similarly, although three outgoing middle links is available for a multicast connection request if the input switch is IS1, again only at most two outgoing middle links is used. The specific outgoing middle links of the input switch that are chosen when selecting two outgoing middle links of the input switch is irrelevant to the method of FIG. 4A so long as at most two outgoing middle links of the input switch are selected to ensure that the connection request is satisfied, i.e. the destination switches identified by the connection request can be reached from the outgoing middle links of the input switch that are selected. In essence, limiting the outgoing middle links of the input switch to no more than two permits the network $V_{fold}(N_1, N_2, d, s)$ to be operated in nonblocking manner in accordance with the invention.

According to the current invention, using the method 1040 of FIG. 4A, the network $V_{fold}(N_1, N_2, d, s)$ is operated in rearrangeably nonblocking for unicast connections when $s \geq 1$, is operated in strictly nonblocking for unicast connections when $s \geq 2$, is operated in rearrangeably nonblocking for multicast connections when $s \geq 2$, and is operated in strictly nonblocking for multicast connections when $s \geq 3$. In addition according to the current invention, using the method 1040 of FIG. 4A, the network $V(N_1, N_2, d, s)$ is also operated in rearrangeably nonblocking for unicast connections when $s \geq 1$.

The connection request of the type described above in reference to method 1000 of FIG. 4A can be unicast connection request, a multicast connection request or a broadcast connection request, depending on the example. In case of a unicast connection request, only one outgoing middle link of the input switch is used to satisfy the request. Moreover, in method 1000 described above in reference to FIG. 4A any number of middle links may be used between any two stages excepting between the input stage and middle stage 130, and also any arbitrary fan-out may be used within each output stage switch, to satisfy the connection request.

As noted above method 1000 of FIG. 4A can be used to setup multicast connections, unicast connections, or broadcast connection of all the networks $V_{fold}(N, d, s)$, $V_{fold}(N_1, N_2, d, s)$, $V(N, d, s)$, and $V(N_1, N_2, d, s)$ disclosed in this invention.

Numerous modifications and adaptations of the embodiments, implementations, and examples described herein will be apparent to the skilled artisan in view of the disclosure.

Electronic Patent Application Fee Transmittal

Application Number:	
Filing Date:	
Title of Invention:	FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS
First Named Inventor/Applicant Name:	Venkat Konda
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Provisional Filing Fees

Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Basic Filing:				
Provisional Application filing fee	2005	1	100	100
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Miscellaneous-Filing:				
Petition:				
Patent-Appeals-and-Interference:				
Post-Allowance-and-Post-Issuance:				
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Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Miscellaneous:				
		Total in USD ($)		**100**

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Title of Invention:	FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS
First Named Inventor/Applicant Name:	Venkat Konda
Customer Number:	38139
Filer:	Venkar Konda
Filer Authorized By:	
Attorney Docket Number:	M-0041US
Receipt Date:	25-MAY-2007
Filing Date:	
Time Stamp:	23:22:05
Application Type:	Provisional

Payment information:

Submitted with Payment	yes
Payment was successfully received in RAM	$100
RAM confirmation Number	2881
Deposit Account	

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Warnings:					
Information:					
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New Applications Under 35 U.S.C. 111
If a new application is being filed and the application includes the necessary components for a filing date (see 37 CFR 1.53(b)-(d) and MPEP 506), a Filing Receipt (37 CFR 1.54) will be issued in due course and the date shown on this Acknowledgement Receipt will establish the filing date of the application.

National Stage of an International Application under 35 U.S.C. 371
If a timely submission to enter the national stage of an international application is compliant with the conditions of 35 U.S.C. 371 and other applicable requirements a Form PCT/DO/EO/903 indicating acceptance of the application as a national stage submission under 35 U.S.C. 371 will be issued in addition to the Filing Receipt, in due course.

New International Application Filed with the USPTO as a Receiving Office
If a new international application is being filed and the international application includes the necessary components for an international filing date (see PCT Article 11 and MPEP 1810), a Notification of the International Application Number and of the International Filing Date (Form PCT/RO/105) will be issued in due course, subject to prescriptions concerning national security, and the date shown on this Acknowledgement Receipt will establish the international filing date of the application.

FIG. 1A

100A

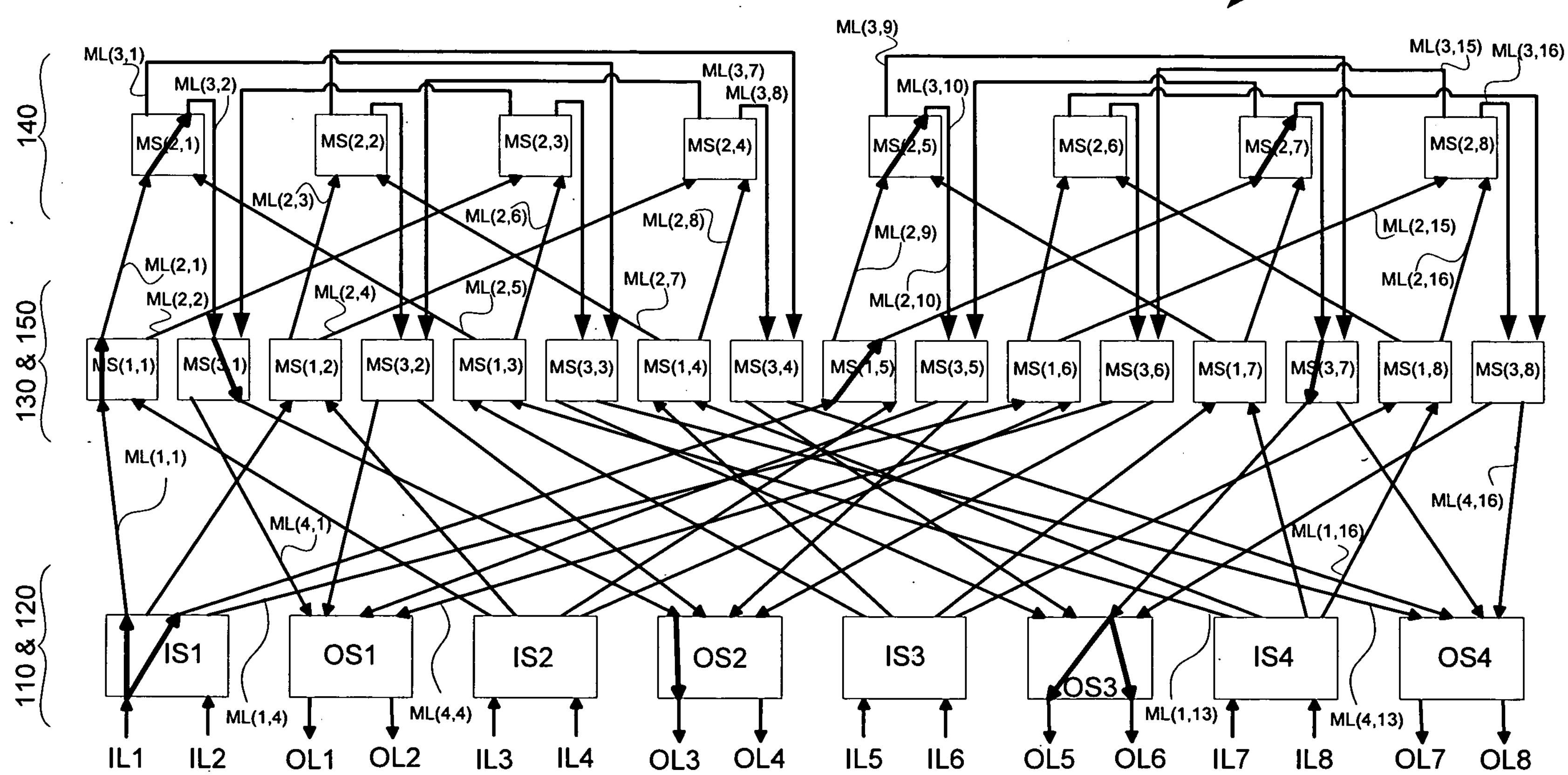
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FIG. 1A1

100A1

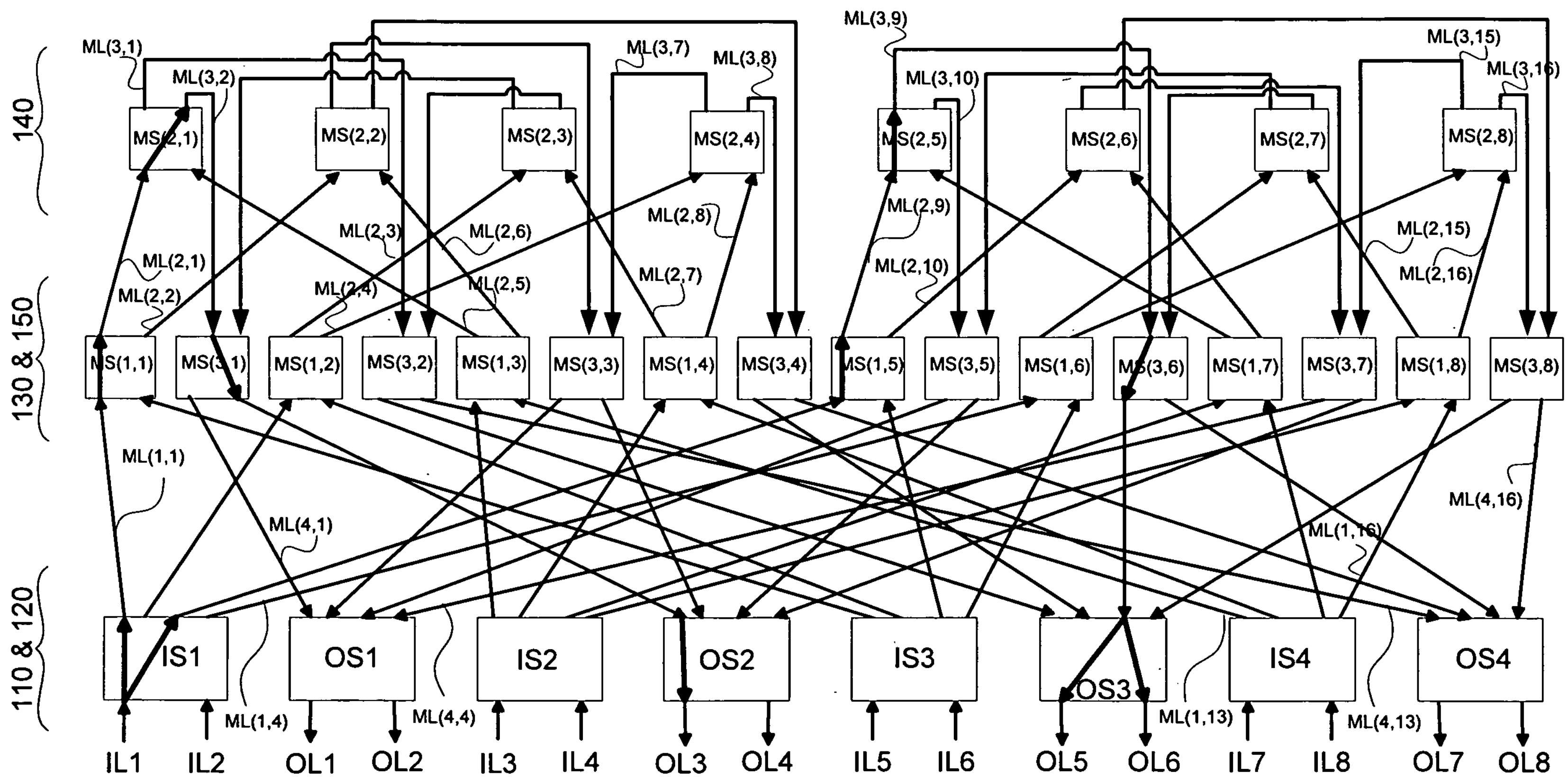

FIG. 1A2

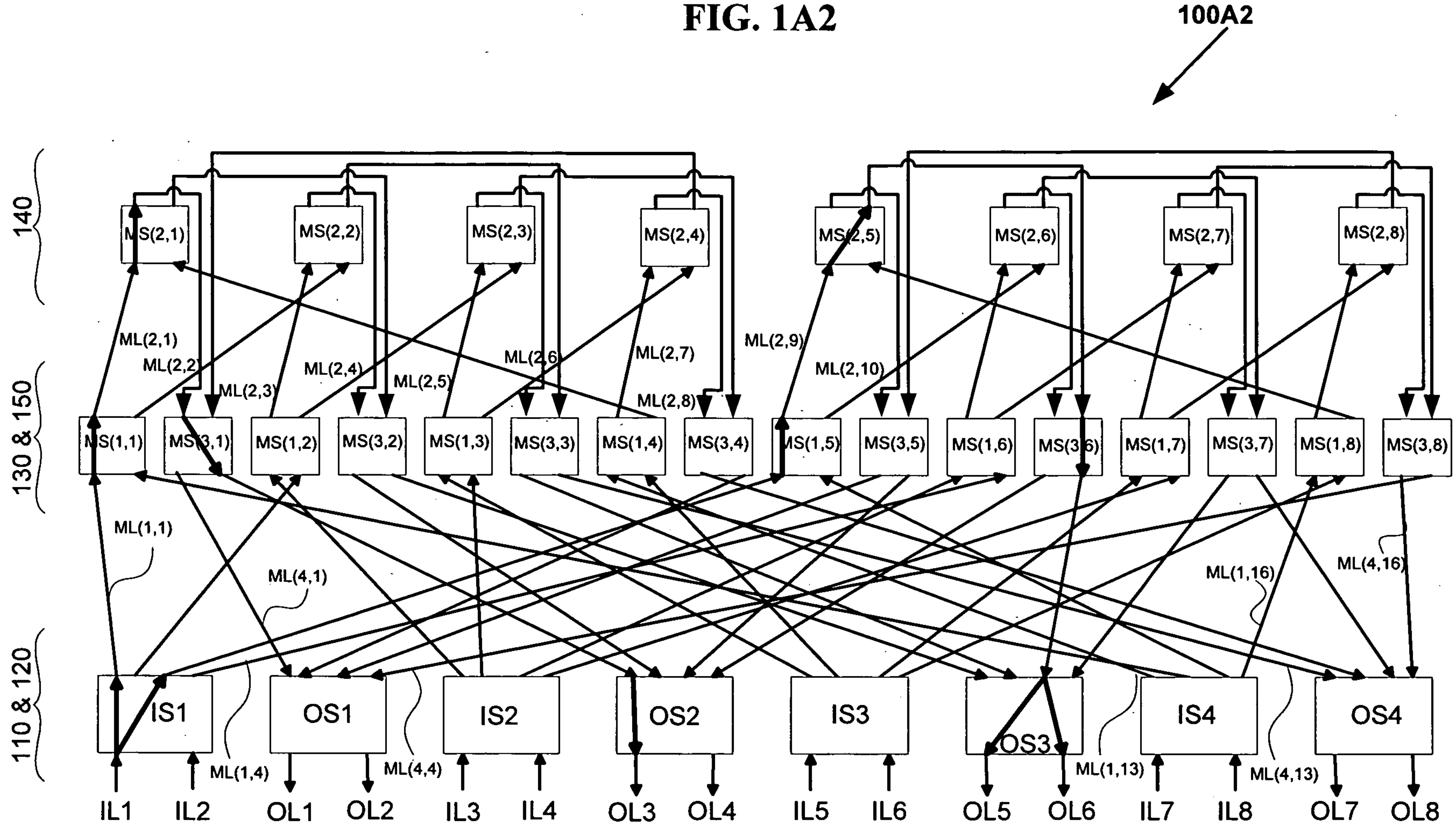
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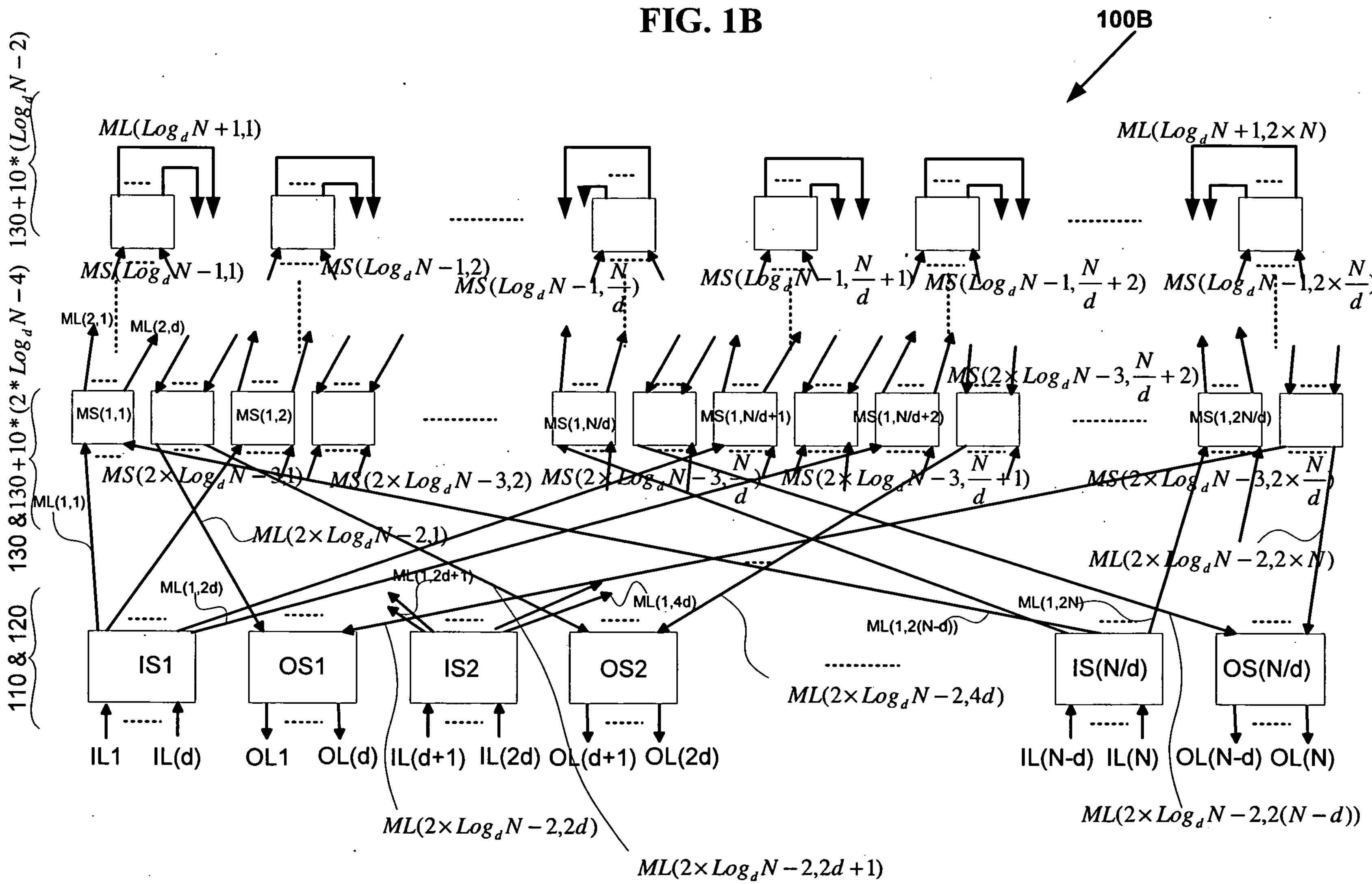
FIG. 1B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$MS(Log_d N-1,\frac{N}{d}+2)$
$MS(Log_d N-1,2\times\frac{N}{d})$
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MS(1,2)
MS(1,N/d)
MS(1,N/d+1)
MS(1,N/d+2)
MS(1,2N/d)
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$MS(2\times Log_d N-3,2)$
$MS(2\times Log_d N-3,\frac{N}{d})$
$MS(2\times Log_d N-3,\frac{N}{d}+1)$
$MS(2\times Log_d N-3,2\times\frac{N}{d})$
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$ML(2\times Log_d N-2,2\times N)$
ML(1,2d)
ML(1,2d+1)
ML(1,4d)
ML(1,2N)
ML(1,2(N-d))
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
$ML(2\times Log_d N-2,4d)$
IL1
IL(d)
OL1
OL(d)
IL(d+1)
IL(2d)
OL(d+1)
OL(2d)
IL(N-d)
IL(N)
OL(N-d)
OL(N)
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$ML(2\times Log_d N-2,2(N-d))$
$ML(2\times Log_d N-2,2d+1)$
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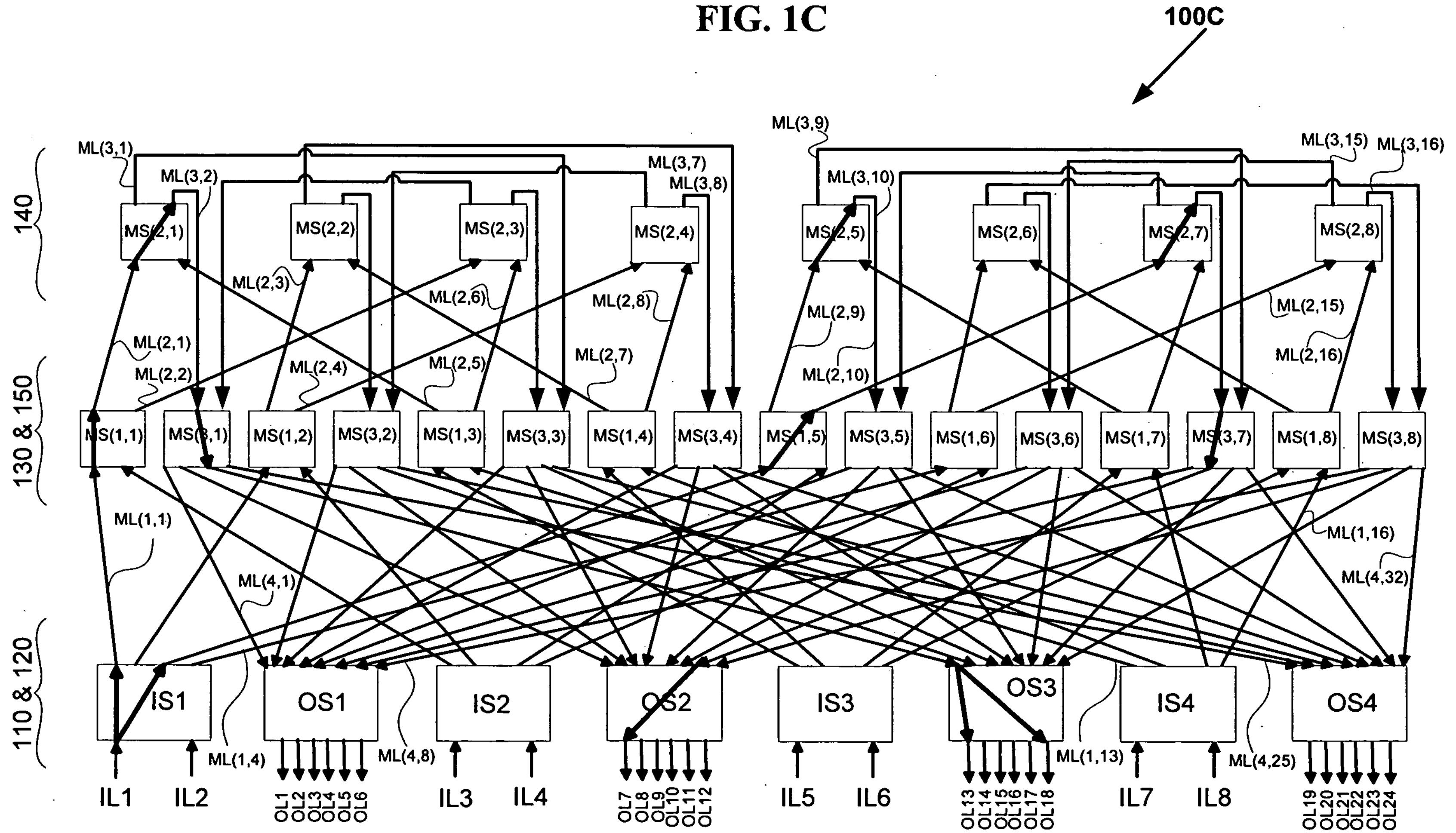
$130\ \&\ 130+10*(2*Log_d N-4)$
110 & 120

FIG. 1C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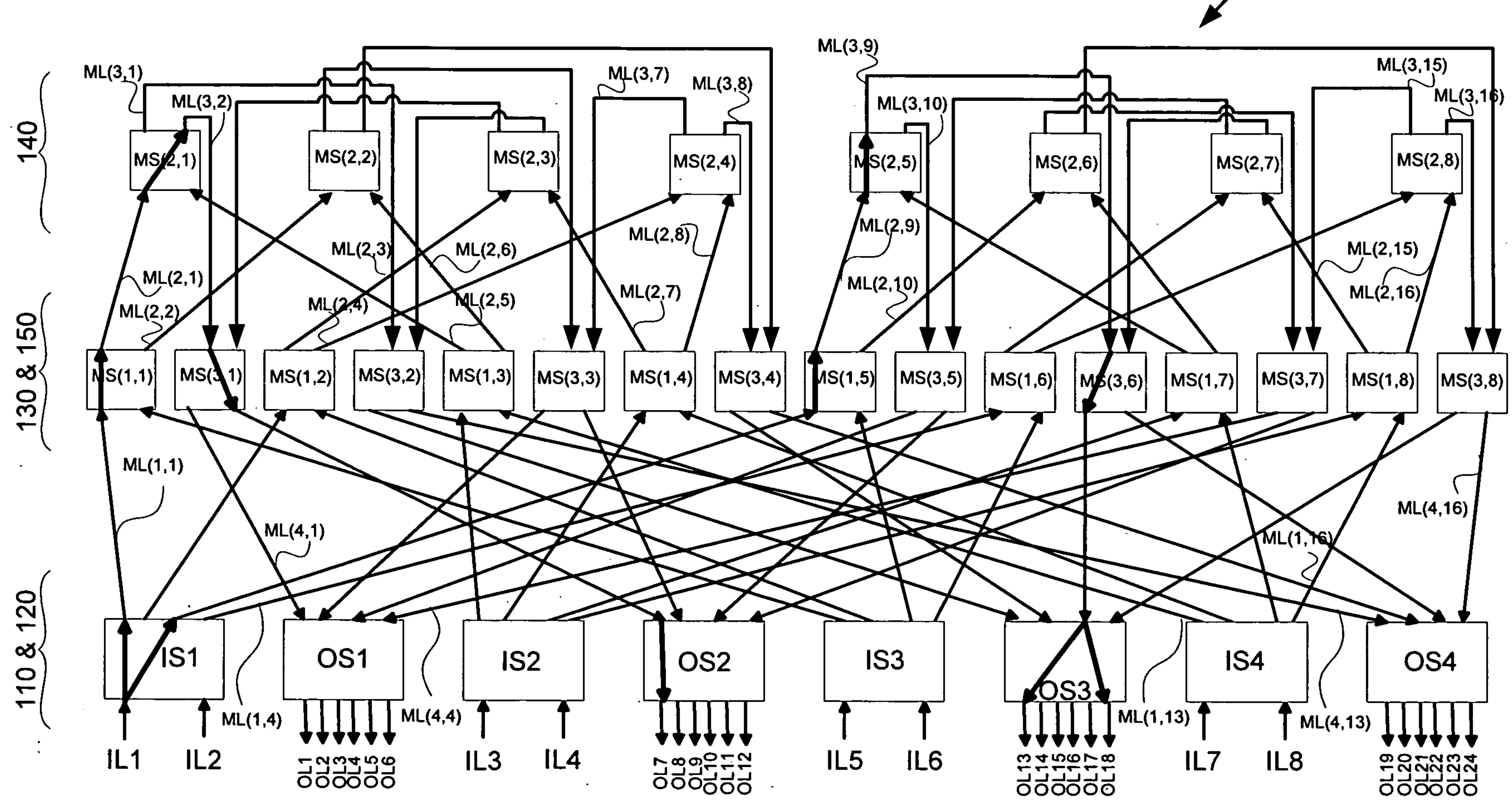
OL24

FIG. 1C1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OL21
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OL23
OL24
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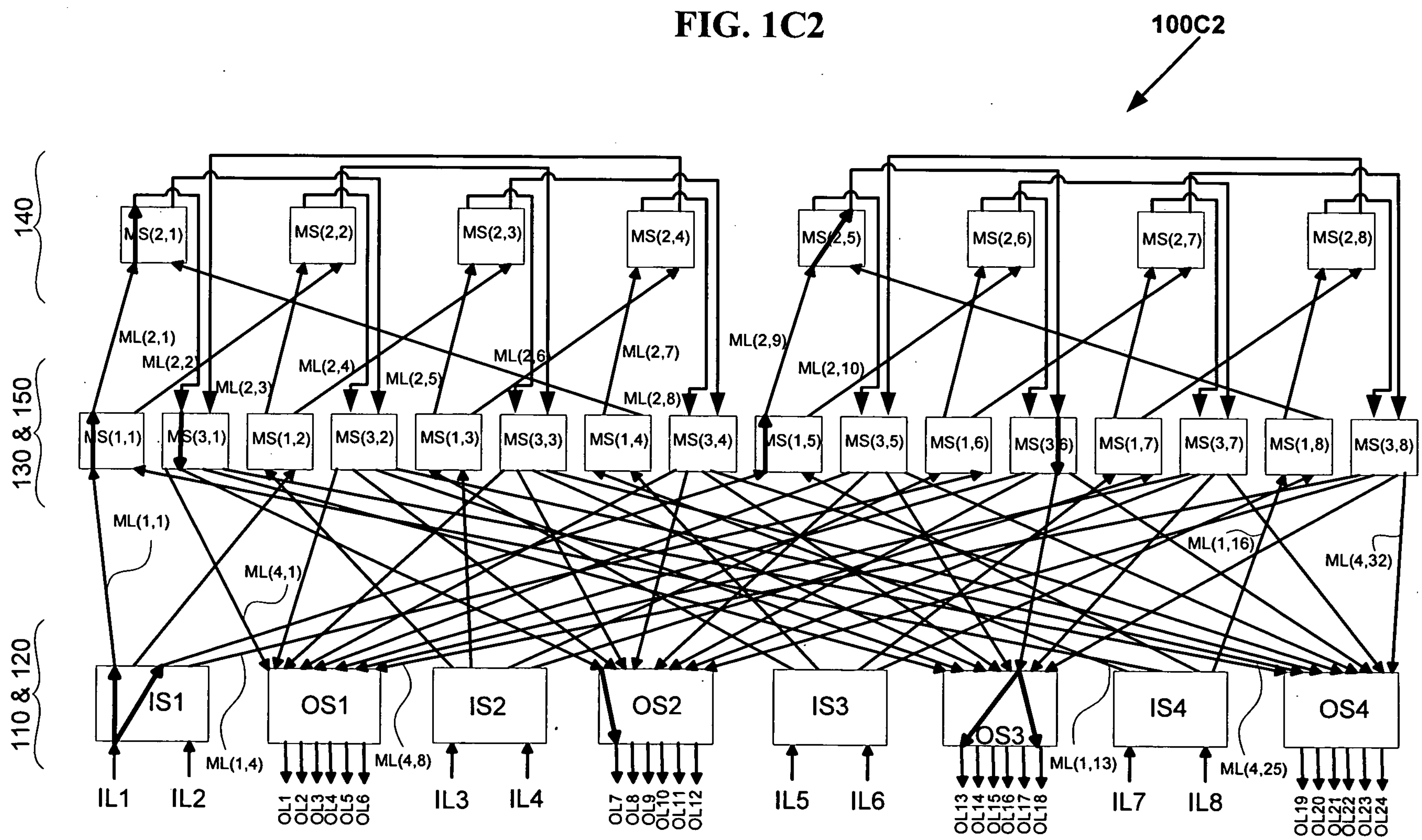
FIG. 1C2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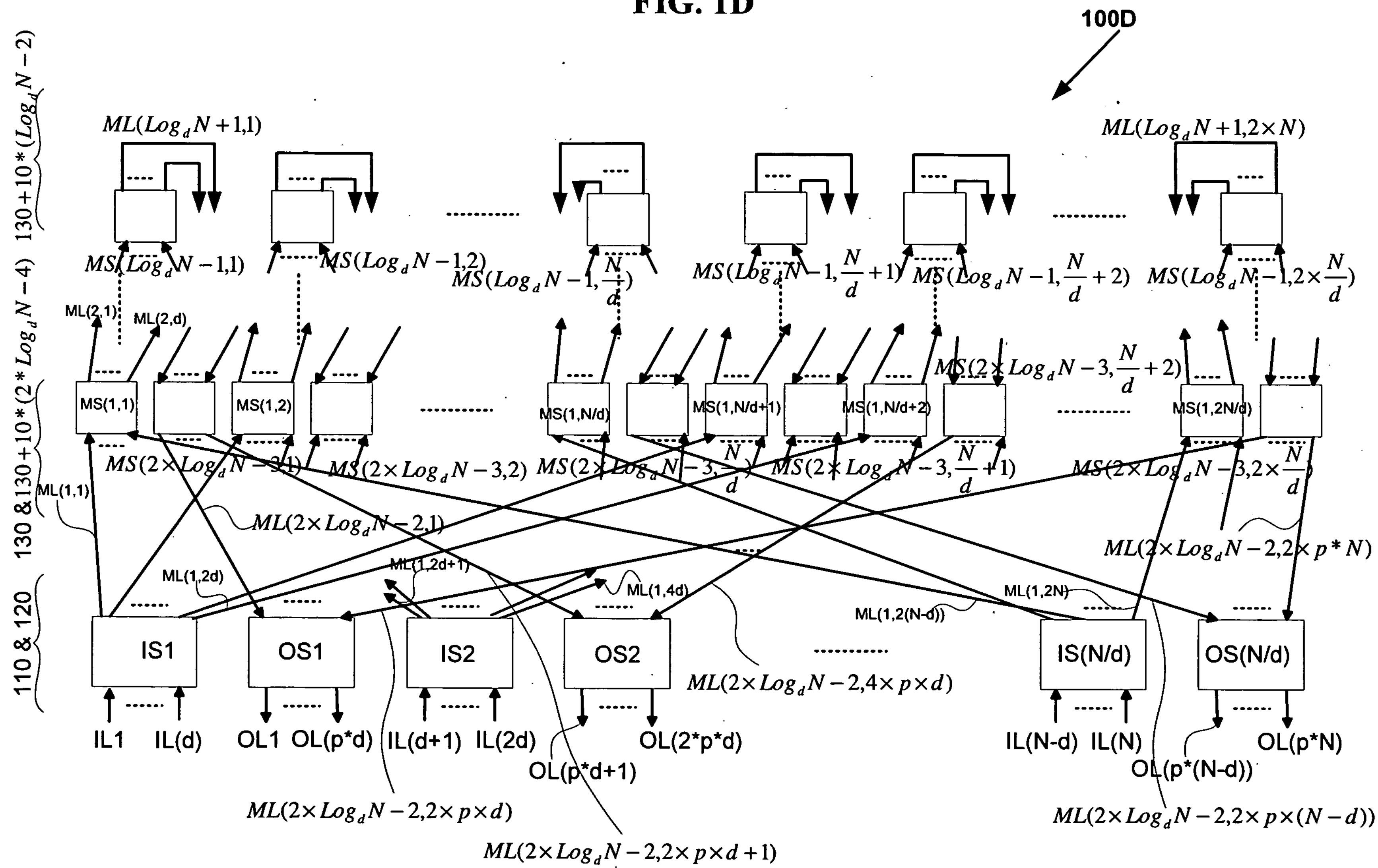
OL23
OL24

FIG. 1D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$MS(Log_d N-1,\frac{N}{d}+1)$
$MS(Log_d N-1,\frac{N}{d}+2)$
$MS(Log_d N-1,2\times\frac{N}{d})$
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MS(1,2)
MS(1,N/d)
MS(1,N/d+1)
MS(1,N/d+2)
MS(1,2N/d)
$MS(2\times Log_d N-3,\frac{N}{d}+2)$
$MS(2\times Log_d N-3,1)$
$MS(2\times Log_d N-3,2)$
$MS(2\times Log_d N-3,\frac{N}{d})$
$MS(2\times Log_d N-3,\frac{N}{d}+1)$
$MS(2\times Log_d N-3,2\times\frac{N}{d})$
ML(1,1)
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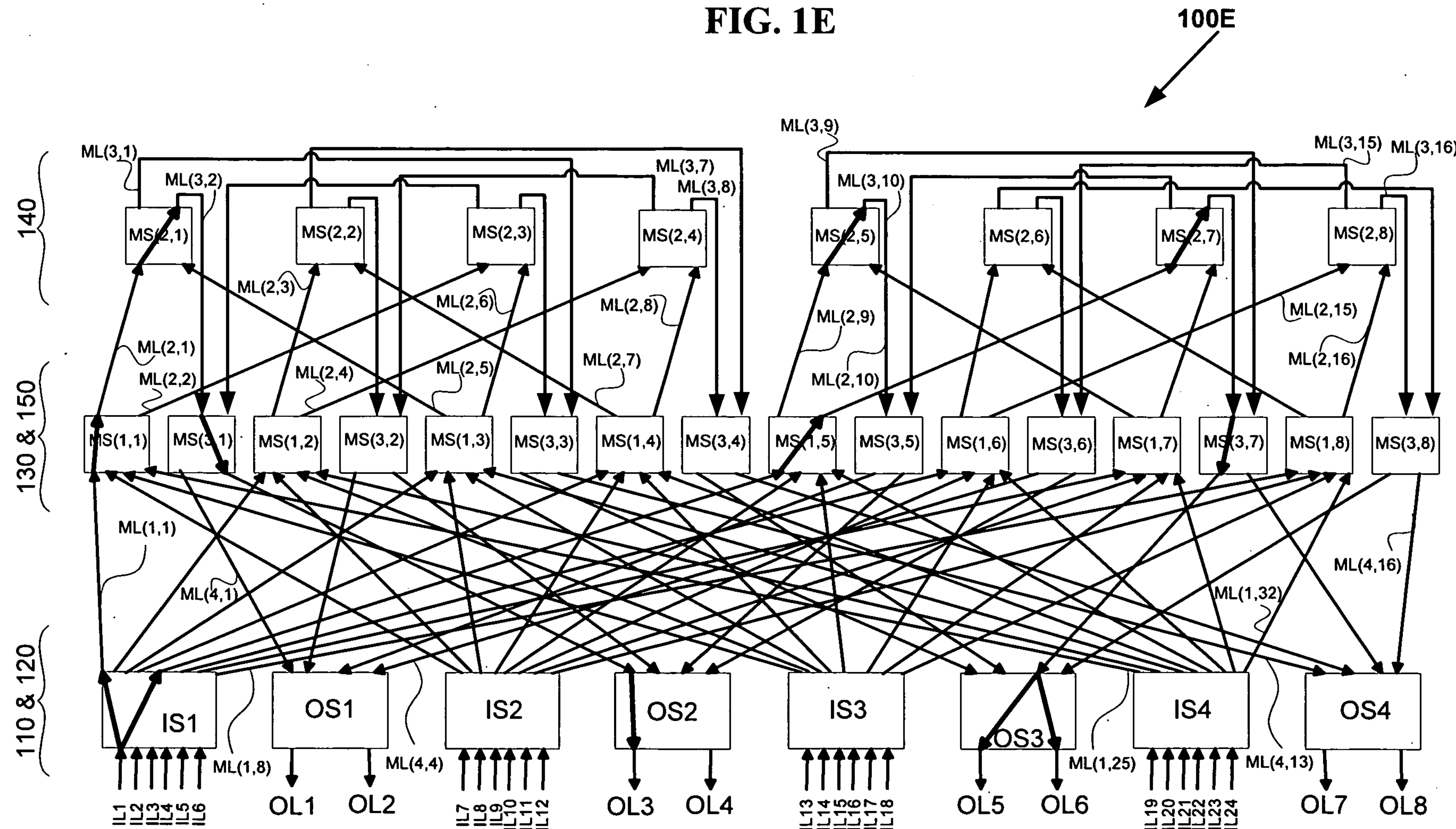
$ML(2\times Log_d N-2,2\times p*N)$
ML(1,2d)
ML(1,2d+1)
ML(1,4d)
ML(1,2N)
ML(1,2(N-d))
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
IL1
IL(d)
OL1
OL(p*d)
IL(d+1)
IL(2d)
OL(p*d+1)
OL(2*p*d)
IL(N-d)
IL(N)
OL(p*(N-d))
OL(p*N)
$ML(2\times Log_d N-2,4\times p\times d)$
$ML(2\times Log_d N-2,2\times p\times d)$
$ML(2\times Log_d N-2,2\times p\times d+1)$
$ML(2\times Log_d N-2,2\times p\times (N-d))$
110 & 120
130 & 130+10*(2*$Log_d N-4$)
130+10*($Log_d N-2$)

FIG. 1E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OL8

FIG. 1E1

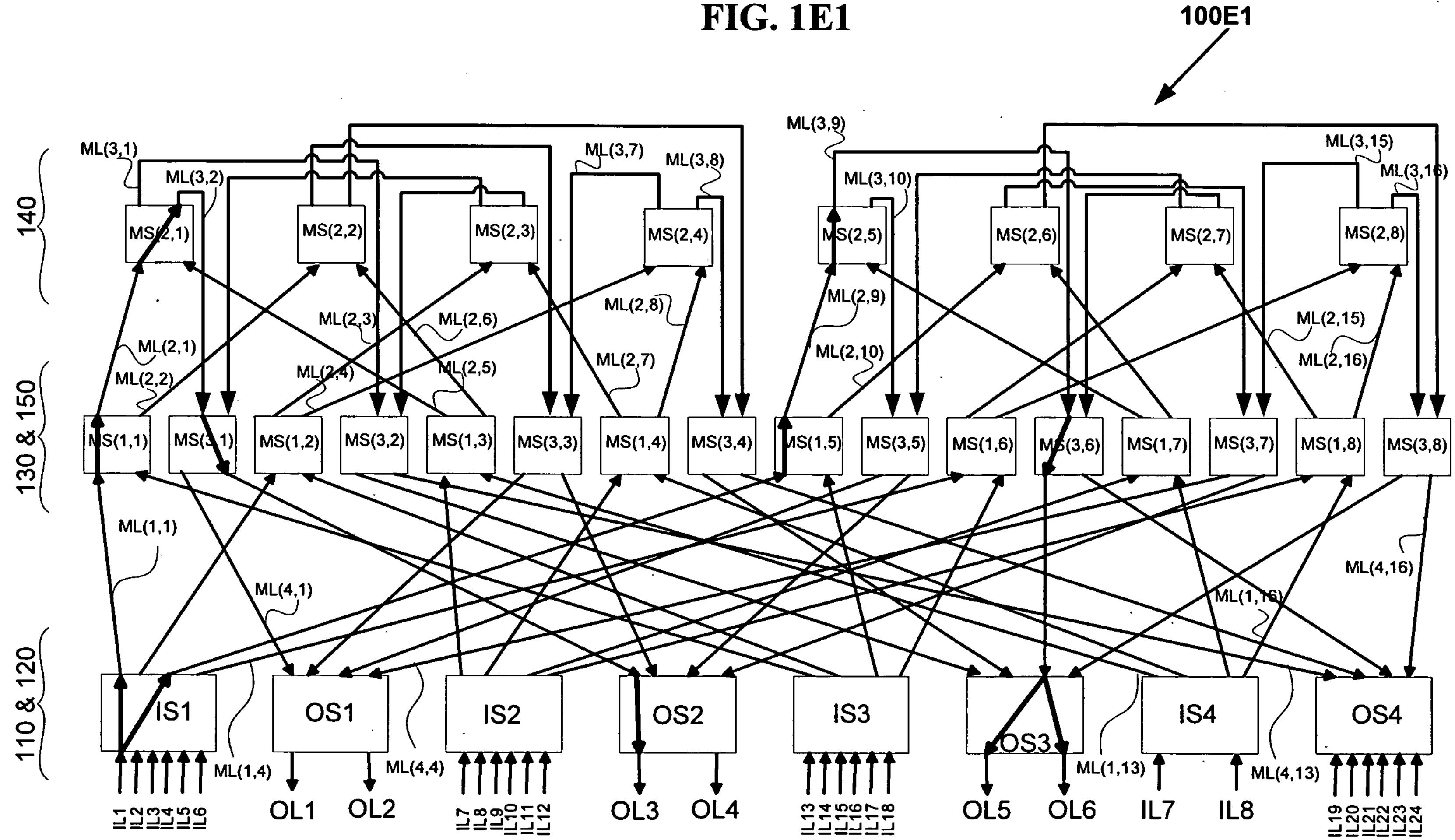
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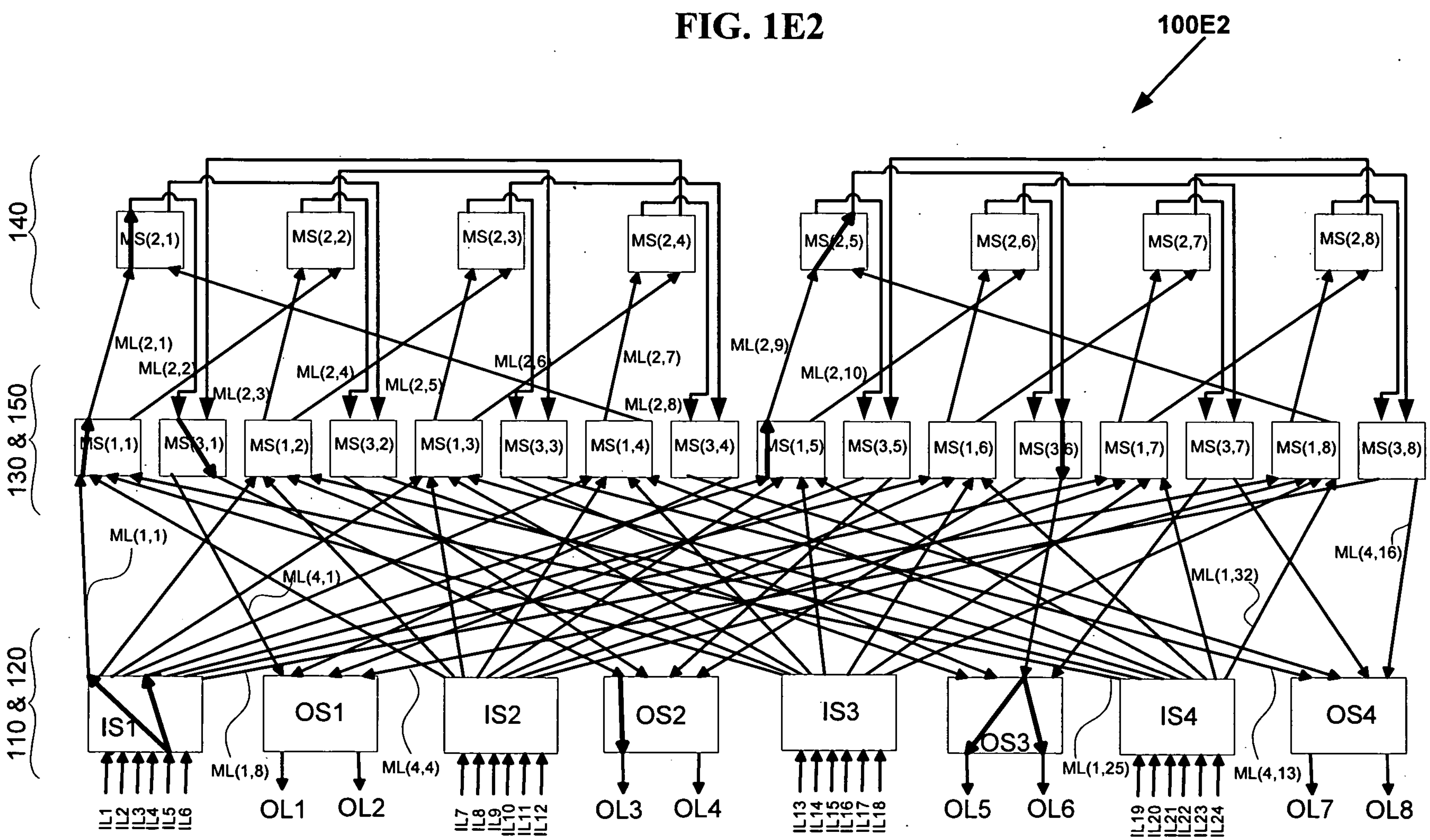
FIG. 1E2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OS2
IS3
OS3
IS4
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OL5
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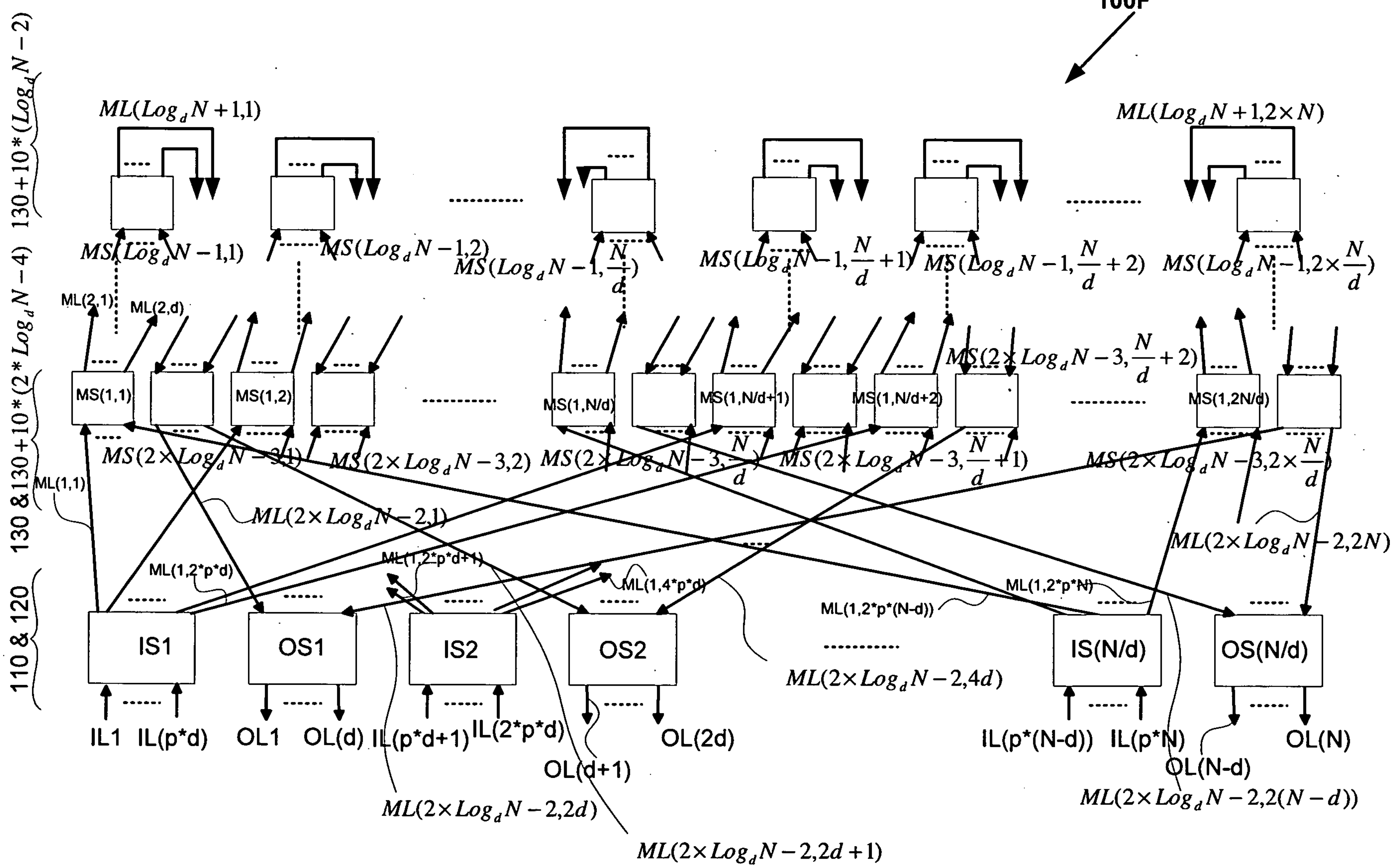
IL19 IL20 IL21 IL22 IL23 IL24
OL7
OL8

FIG. 1F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$MS(Log_d N-1,\frac{N}{d}+1)$
$MS(Log_d N-1,\frac{N}{d}+2)$
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MS(1,N/d+2)
MS(1,2N/d)
$MS(2\times Log_d N-3,\frac{N}{d}+2)$
$MS(2\times Log_d N-3,1)$
$MS(2\times Log_d N-3,2)$
$MS(2\times Log_d N-3,\frac{N}{d})$
$MS(2\times Log_d N-3,\frac{N}{d}+1)$
$MS(2\times Log_d N-3,2\times\frac{N}{d})$
ML(1,1)
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$ML(2\times Log_d N-2,2N)$
ML(1,2*p*d)
ML(1,2*p*d+1)
ML(1,4*p*d)
ML(1,2*p*N)
ML(1,2*p*(N-d))
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
$ML(2\times Log_d N-2,4d)$
IL1
IL(p*d)
OL1
OL(d)
IL(p*d+1)
IL(2*p*d)
OL(2d)
OL(d+1)
IL(p*(N-d))
IL(p*N)
OL(N)
OL(N-d)
$ML(2\times Log_d N-2,2(N-d))$
$ML(2\times Log_d N-2,2d)$
$ML(2\times Log_d N-2,2d+1)$
$130+10*(Log_d N-2)$
$130 \& 130+10*(2*Log_d N-4)$
110 & 120

FIG. 2A

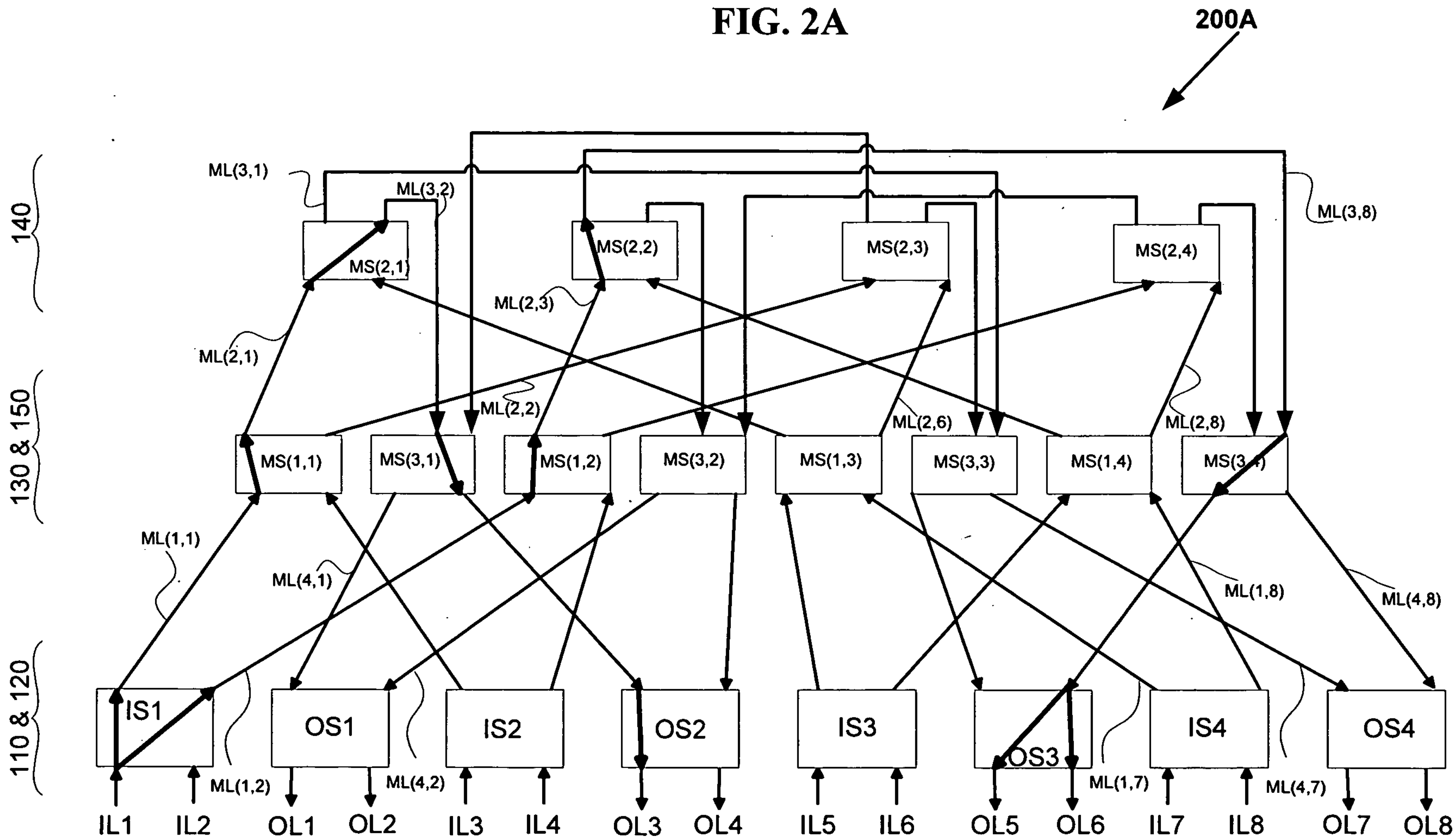
200A
140
130 & 150
110 & 120
ML(3,1)
ML(3,2)
ML(3,8)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
ML(2,3)
ML(2,1)
ML(2,2)
ML(2,6)
ML(2,8)
MS(1,1)
MS(3,1)
MS(1,2)
MS(3,2)
MS(1,3)
MS(3,3)
MS(1,4)
MS(3,4)
ML(1,1)
ML(4,1)
ML(1,8)
ML(4,8)
IS1
OS1
IS2
OS2
IS3
OS3
IS4
OS4
ML(1,2)
ML(4,2)
ML(1,7)
ML(4,7)
IL1
IL2
OL1
OL2
IL3
IL4
OL3
OL4
IL5
IL6
OL5
OL6
IL7
IL8
OL7
OL8

FIG. 2B

200B

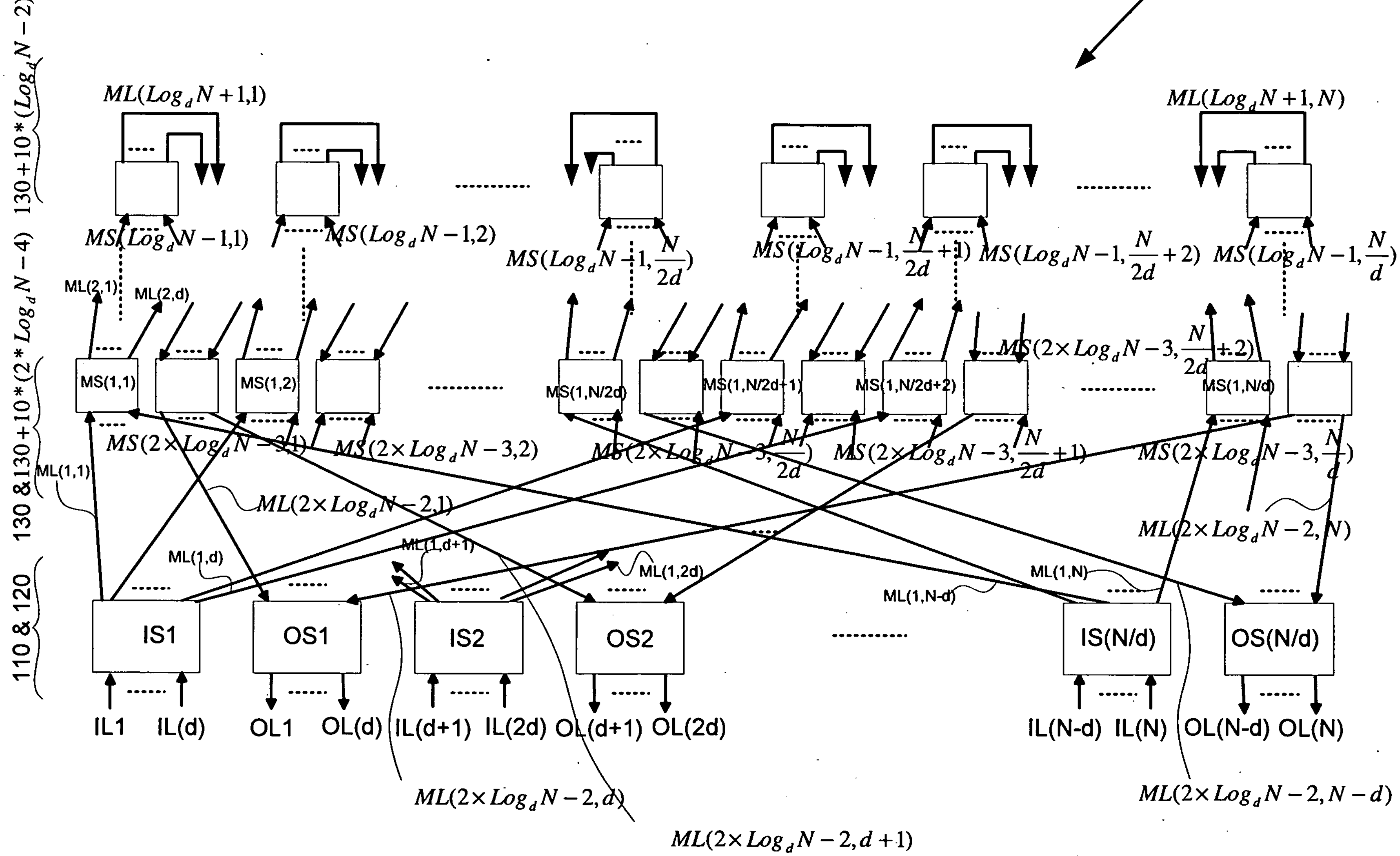
$ML(Log_d N+1,1)$
$ML(Log_d N+1,N)$
$MS(Log_d N-1,1)$
$MS(Log_d N-1,2)$
$MS(Log_d N-1,\frac{N}{2d})$
$MS(Log_d N-1,\frac{N}{2d}+1)$
$MS(Log_d N-1,\frac{N}{2d}+2)$
$MS(Log_d N-1,\frac{N}{d})$
ML(2,1)
ML(2,d)
MS(1,1)
MS(1,2)
MS(1,N/2d)
MS(1,N/2d+1)
MS(1,N/2d+2)
MS(1,N/d)
$MS(2\times Log_d N-3,\frac{N}{2d}+2)$
$MS(2\times Log_d N-3,1)$
$MS(2\times Log_d N-3,2)$
$MS(2\times Log_d N-3,\frac{N}{2d})$
$MS(2\times Log_d N-3,\frac{N}{2d}+1)$
$MS(2\times Log_d N-3,\frac{N}{d})$
ML(1,1)
$ML(2\times Log_d N-2,1)$
$ML(2\times Log_d N-2,N)$
ML(1,d)
ML(1,d+1)
ML(1,2d)
ML(1,N)
ML(1,N-d)
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
IL1
IL(d)
OL1
OL(d)
IL(d+1)
IL(2d)
OL(d+1)
OL(2d)
IL(N-d)
IL(N)
OL(N-d)
OL(N)
$ML(2\times Log_d N-2,d)$
$ML(2\times Log_d N-2,N-d)$
$ML(2\times Log_d N-2,d+1)$
$130+10*(Log_d N-2)$
$130 \& 130+10*(2*Log_d N-4)$
110 & 120

FIG. 2C

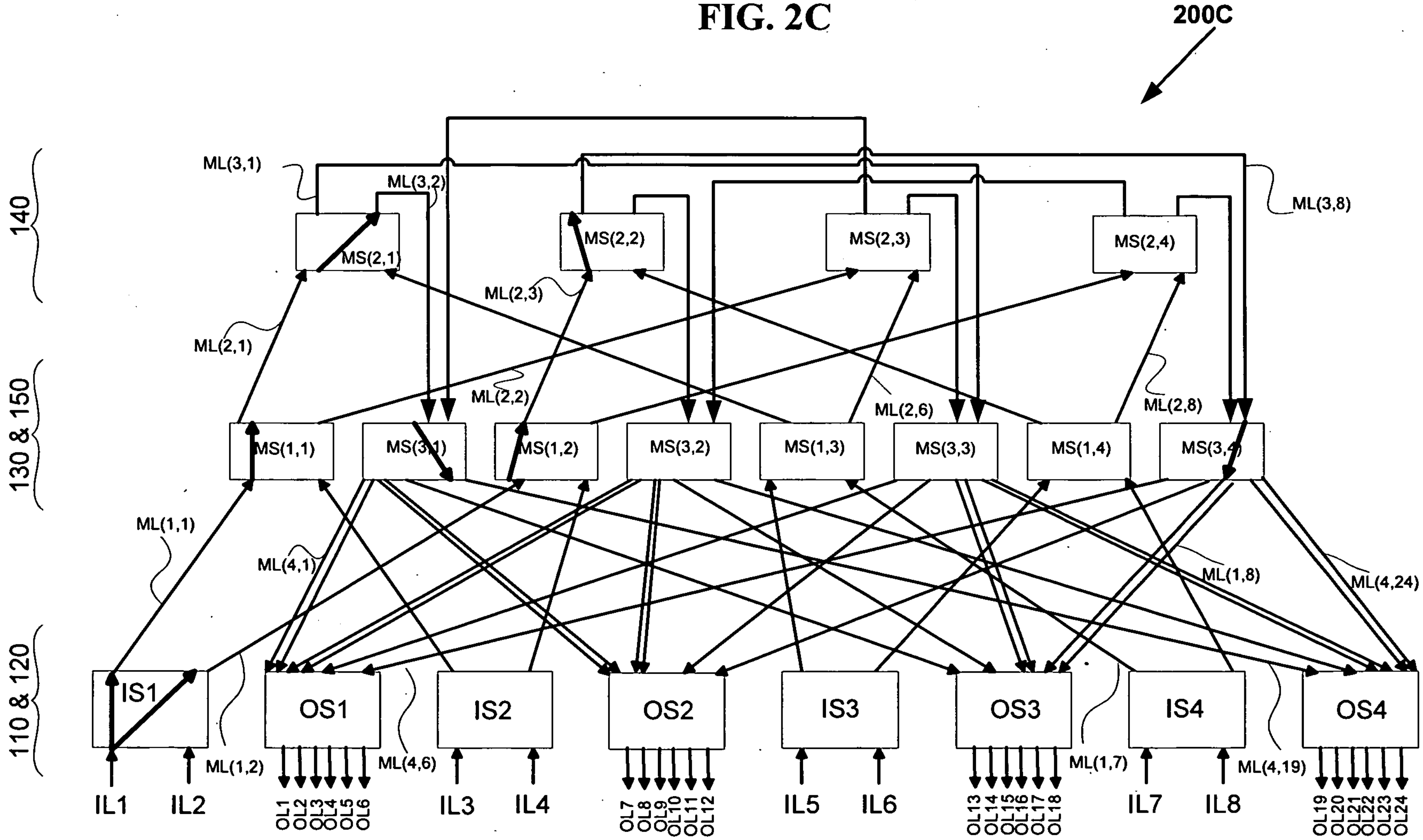
200C
140
130 & 150
110 & 120
ML(3,1)
ML(3,2)
ML(3,8)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
ML(2,3)
ML(2,1)
ML(2,2)
ML(2,6)
ML(2,8)
MS(1,1)
MS(3,1)
MS(1,2)
MS(3,2)
MS(1,3)
MS(3,3)
MS(1,4)
MS(3,4)
ML(1,1)
ML(4,1)
ML(1,8)
ML(4,24)
IS1
OS1
IS2
OS2
IS3
OS3
IS4
OS4
ML(1,2)
ML(4,6)
ML(1,7)
ML(4,19)
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
OL1 OL2 OL3 OL4 OL5 OL6
OL7 OL8 OL9 OL10 OL11 OL12
OL13 OL14 OL15 OL16 OL17 OL18
OL19 OL20 OL21 OL22 OL23 OL24

FIG. 2D

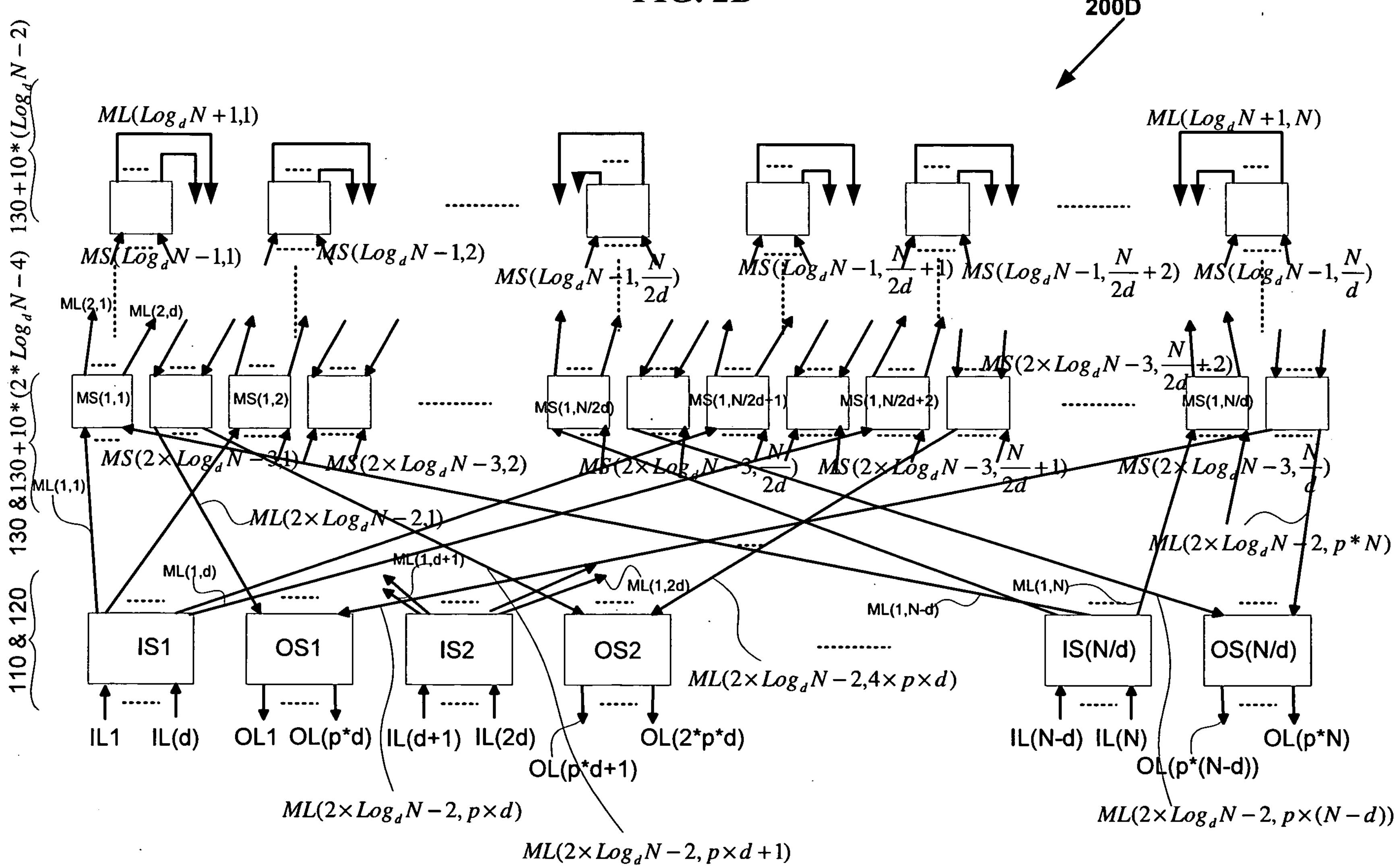
200D
$ML(Log_d N+1,1)$
$ML(Log_d N+1,N)$
$MS(Log_d N-1,1)$
$MS(Log_d N-1,2)$
$MS(Log_d N-1,\frac{N}{2d})$
$MS(Log_d N-1,\frac{N}{2d}+1)$
$MS(Log_d N-1,\frac{N}{2d}+2)$
$MS(Log_d N-1,\frac{N}{d})$
ML(2,1)
ML(2,d)
MS(1,1)
MS(1,2)
MS(1,N/2d)
MS(1,N/2d+1)
MS(1,N/2d+2)
MS(1,N/d)
$MS(2\times Log_d N-3,1)$
$MS(2\times Log_d N-3,2)$
$MS(2\times Log_d N-3,\frac{N}{2d})$
$MS(2\times Log_d N-3,\frac{N}{2d}+1)$
$MS(2\times Log_d N-3,\frac{N}{2d}+2)$
$MS(2\times Log_d N-3,\frac{N}{d})$
ML(1,1)
$ML(2\times Log_d N-2,1)$
$ML(2\times Log_d N-2,p*N)$
ML(1,d)
ML(1,d+1)
ML(1,2d)
ML(1,N-d)
ML(1,N)
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
$ML(2\times Log_d N-2,4\times p\times d)$
IL1
IL(d)
OL1
OL(p*d)
IL(d+1)
IL(2d)
OL(p*d+1)
OL(2*p*d)
IL(N-d)
IL(N)
OL(p*(N-d))
OL(p*N)
$ML(2\times Log_d N-2,p\times d)$
$ML(2\times Log_d N-2,p\times d+1)$
$ML(2\times Log_d N-2,p\times (N-d))$
110 & 120
130 & 130+10*(2*$Log_d N-4$)
130+10*($Log_d N-2$)

FIG. 2E

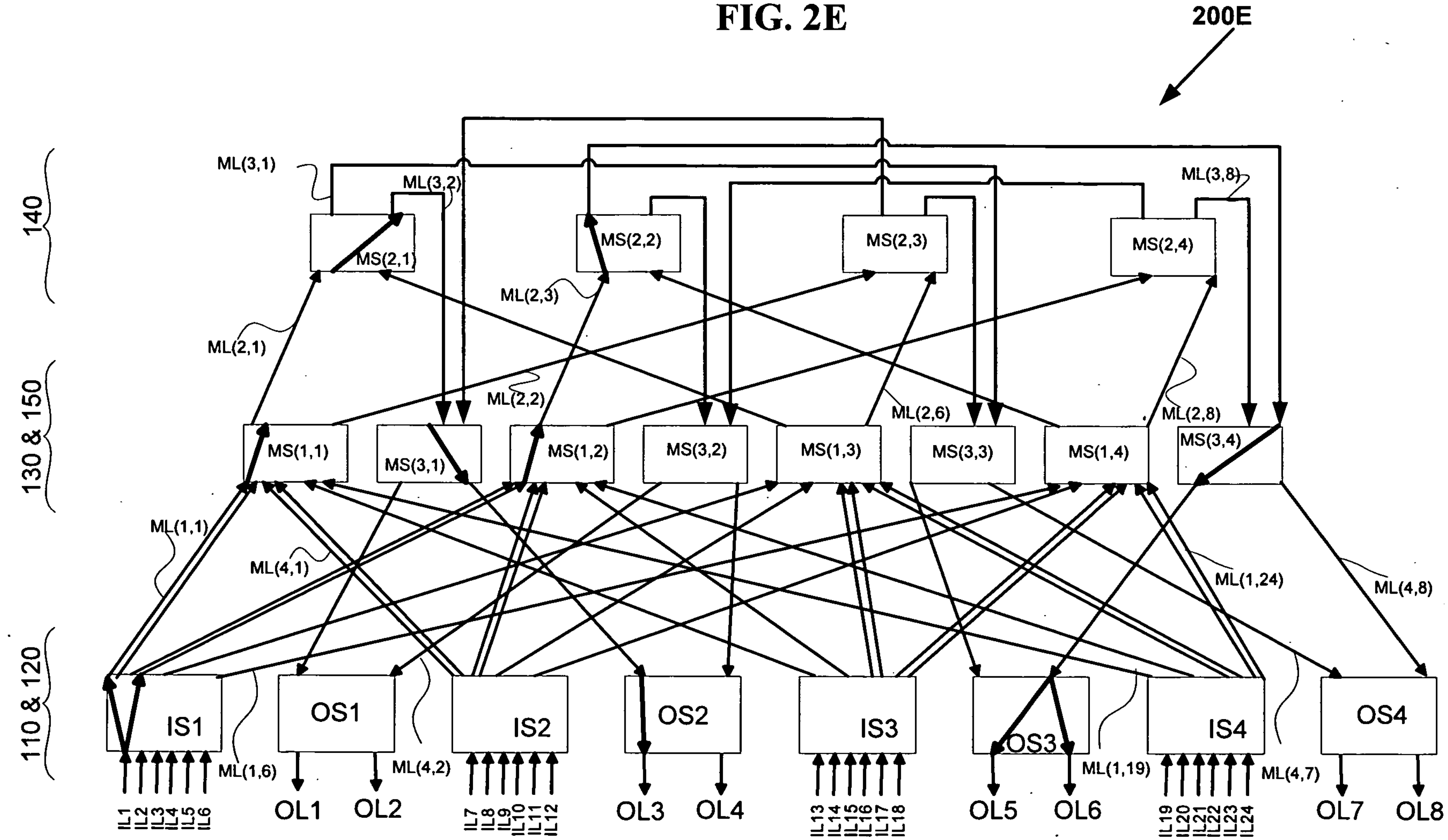
200E
140
130 & 150
110 & 120
ML(3,1)
ML(3,2)
ML(3,8)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
ML(2,3)
ML(2,1)
ML(2,2)
ML(2,6)
ML(2,8)
MS(1,1)
MS(3,1)
MS(1,2)
MS(3,2)
MS(1,3)
MS(3,3)
MS(1,4)
MS(3,4)
ML(1,1)
ML(4,1)
ML(1,24)
ML(4,8)
IS1
OS1
IS2
OS2
IS3
OS3
IS4
OS4
ML(1,6)
ML(4,2)
ML(1,19)
ML(4,7)
IL1 IL2 IL3 IL4 IL5 IL6
OL1 OL2
IL7 IL8 IL9 IL10 IL11 IL12
OL3 OL4
IL13 IL14 IL15 IL16 IL17 IL18
OL5 OL6
IL19 IL20 IL21 IL22 IL23 IL24
OL7 OL8

FIG. 2F

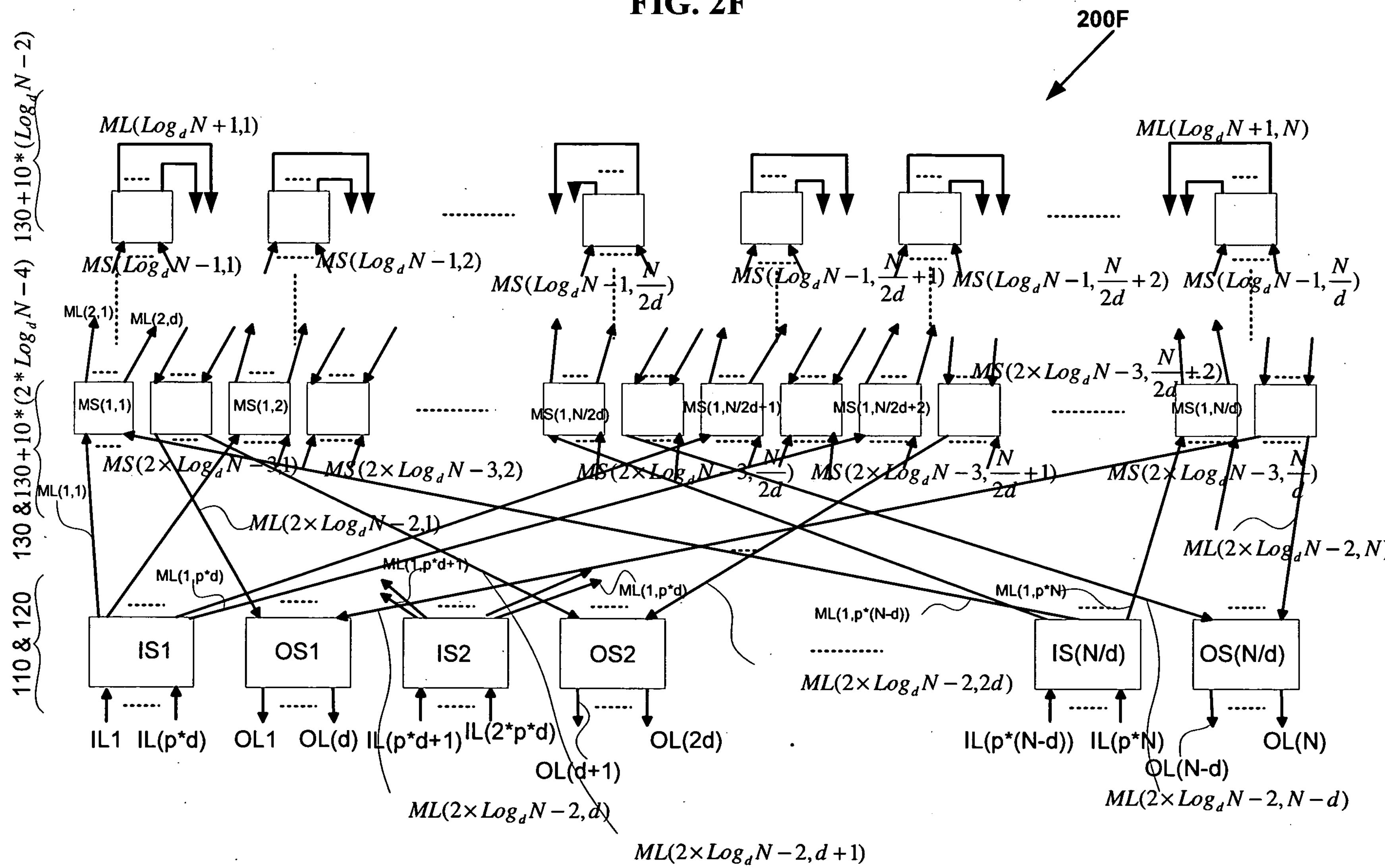
200F
$ML(Log_dN+1,1)$
$ML(Log_dN+1,N)$
$MS(Log_dN-1,1)$
$MS(Log_dN-1,2)$
$MS(Log_dN-1,\frac{N}{2d})$
$MS(Log_dN-1,\frac{N}{2d}+1)$
$MS(Log_dN-1,\frac{N}{2d}+2)$
$MS(Log_dN-1,\frac{N}{d})$
ML(2,1)
ML(2,d)
MS(1,1)
MS(1,2)
MS(1,N/2d)
MS(1,N/2d+1)
MS(1,N/2d+2)
MS(1,N/d)
$MS(2\times Log_dN-3,\frac{N}{2d}+2)$
$MS(2\times Log_dN-3,1)$
$MS(2\times Log_dN-3,2)$
$MS(2\times Log_dN-3,\frac{N}{2d})$
$MS(2\times Log_dN-3,\frac{N}{2d}+1)$
$MS(2\times Log_dN-3,\frac{N}{d})$
ML(1,1)
$ML(2\times Log_dN-2,1)$
$ML(2\times Log_dN-2,N)$
ML(1,p*d)
ML(1,p*d+1)
ML(1,p*d)
ML(1,p*N)
ML(1,p*(N-d))
IS1
OS1
IS2
OS2
IS(N/d)
OS(N/d)
$ML(2\times Log_dN-2,2d)$
IL1
IL(p*d)
OL1
OL(d)
IL(p*d+1)
IL(2*p*d)
OL(2d)
OL(d+1)
IL(p*(N-d))
IL(p*N)
OL(N)
OL(N-d)
$ML(2\times Log_dN-2,d)$
$ML(2\times Log_dN-2,d+1)$
$ML(2\times Log_dN-2,N-d)$
110 & 120
130 & 130+10*(2*Log_dN-4)
130+10*(Log_dN-2)

FIG. 3A

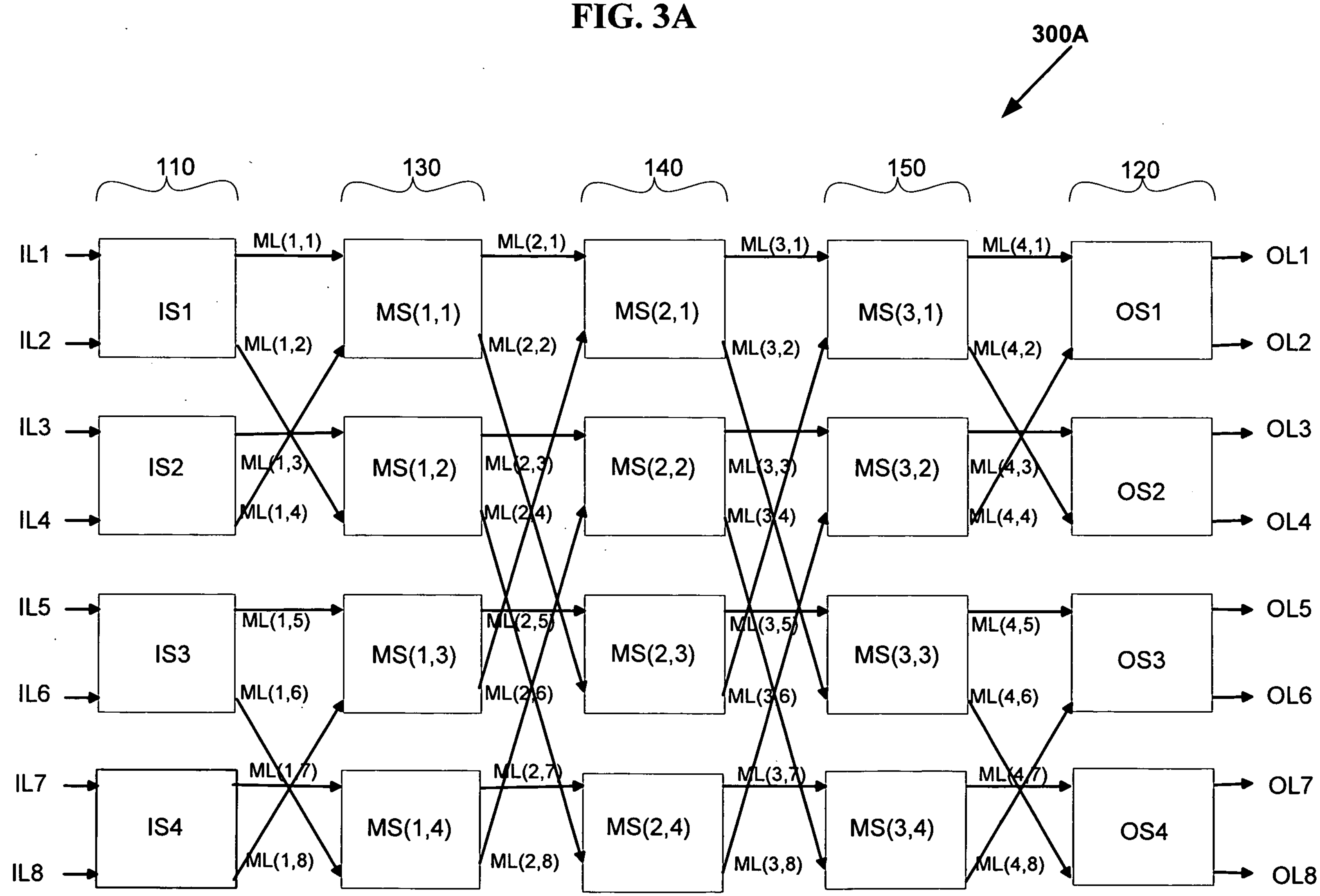
300A
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML(2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)

FIG. 3B

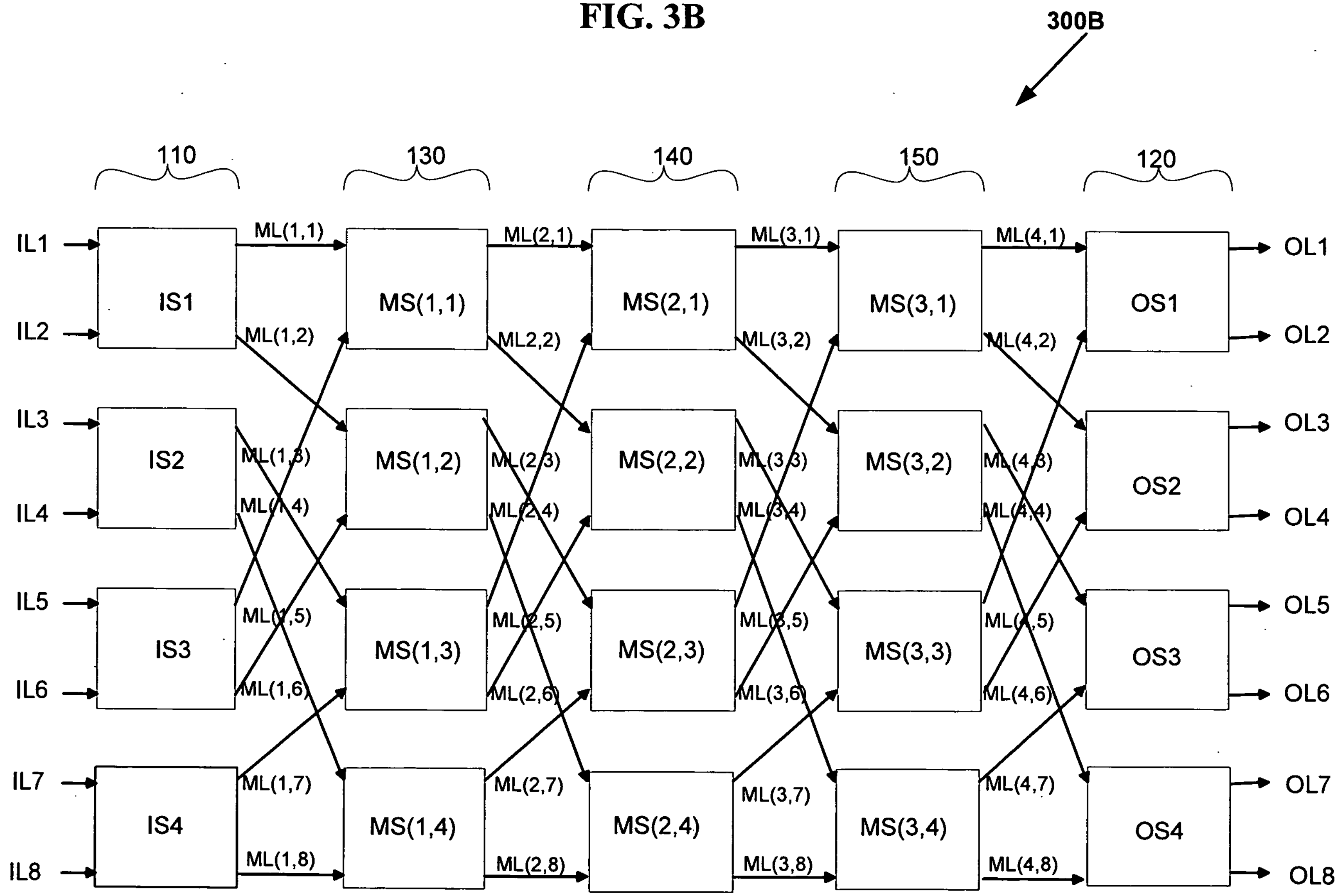
300B
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)

FIG. 3C

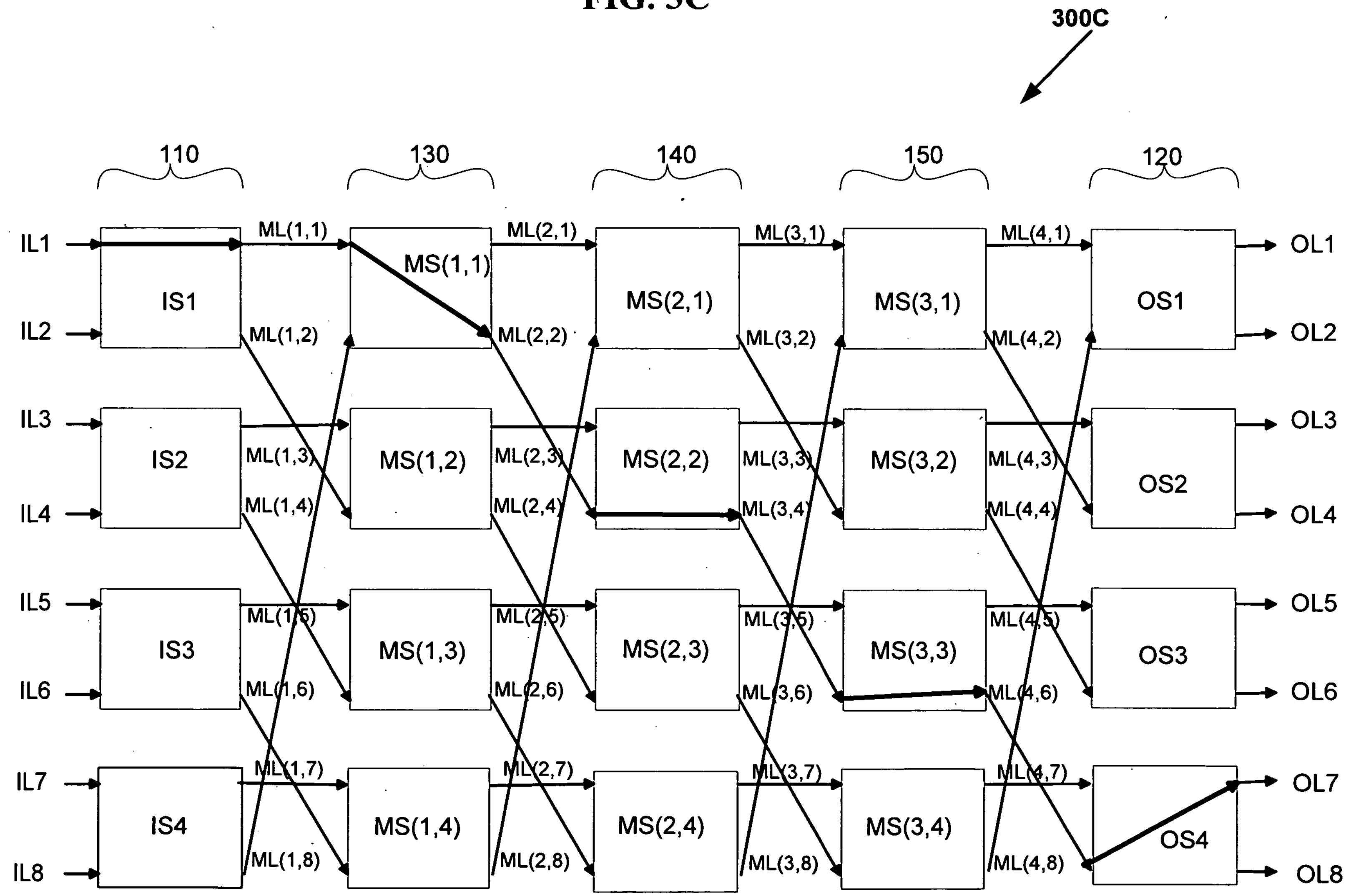
300C
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML(2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3D

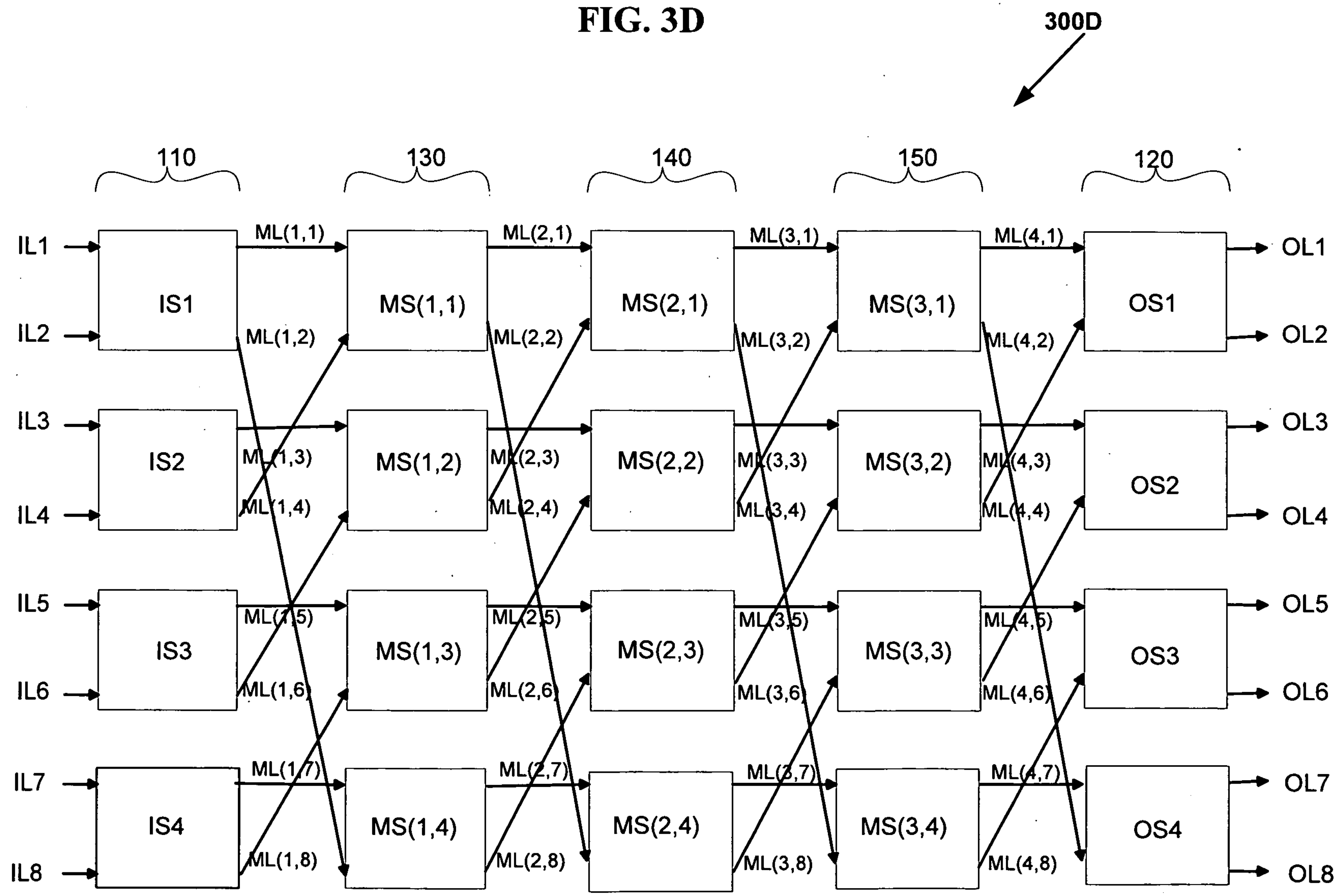
300D
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML(2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3E

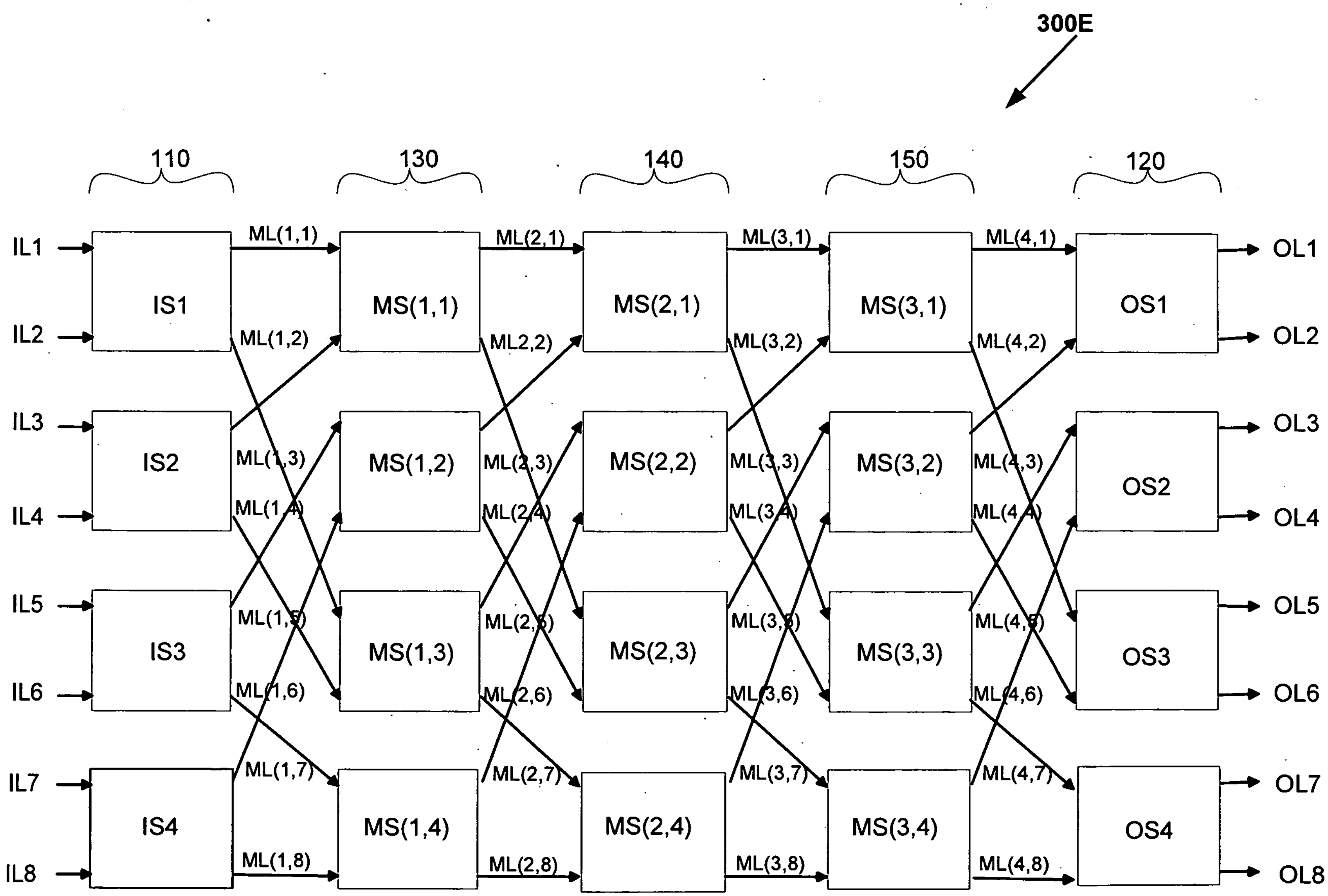
300E
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)

FIG. 3F

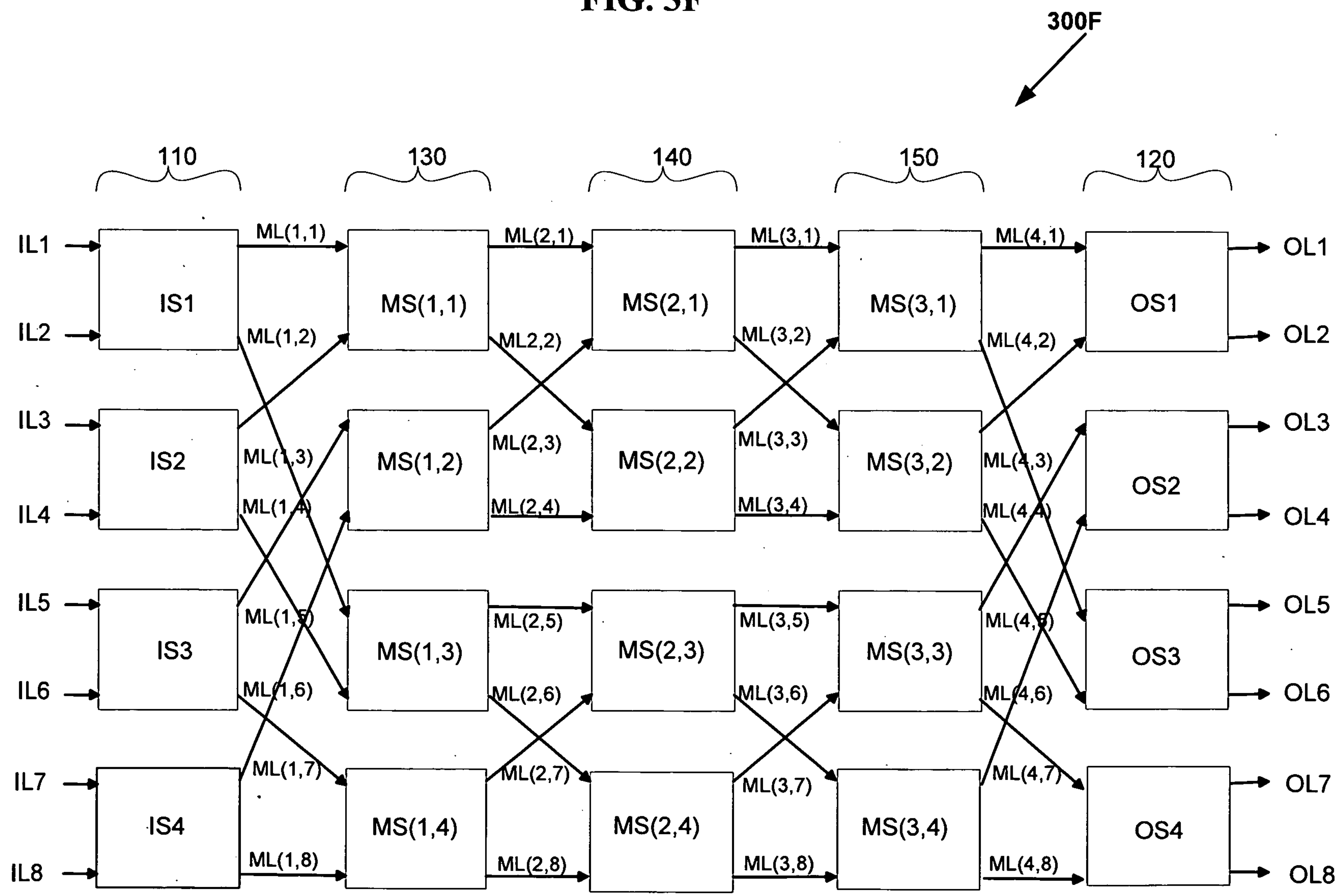
300F
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3G

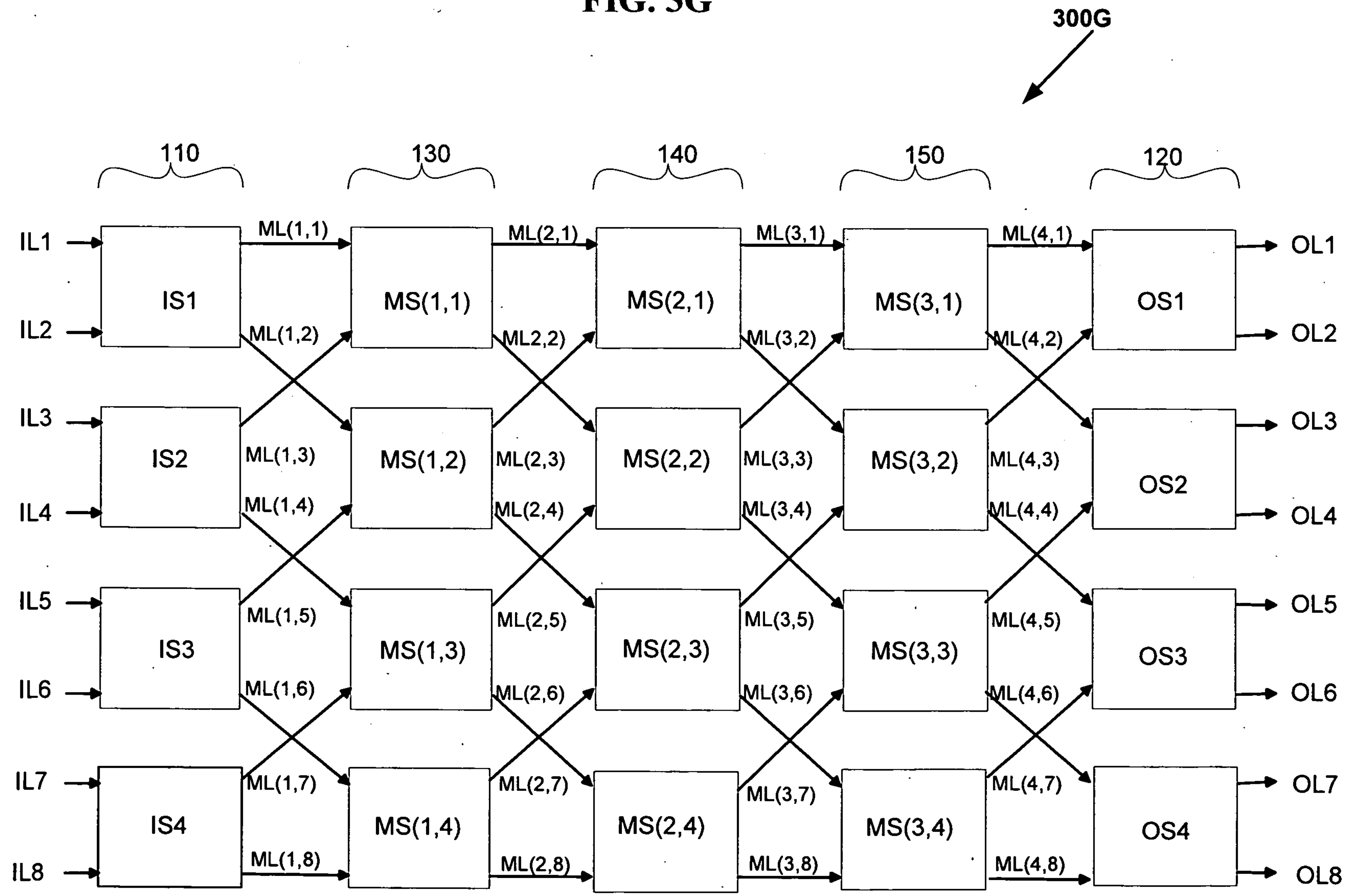
300G
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8
ML(1,1)
ML(1,2)
ML(1,3)
ML(1,4)
ML(1,5)
ML(1,6)
ML(1,7)
ML(1,8)
ML(2,1)
ML2,2)
ML(2,3)
ML(2,4)
ML(2,5)
ML(2,6)
ML(2,7)
ML(2,8)
ML(3,1)
ML(3,2)
ML(3,3)
ML(3,4)
ML(3,5)
ML(3,6)
ML(3,7)
ML(3,8)
ML(4,1)
ML(4,2)
ML(4,3)
ML(4,4)
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)

FIG. 3H

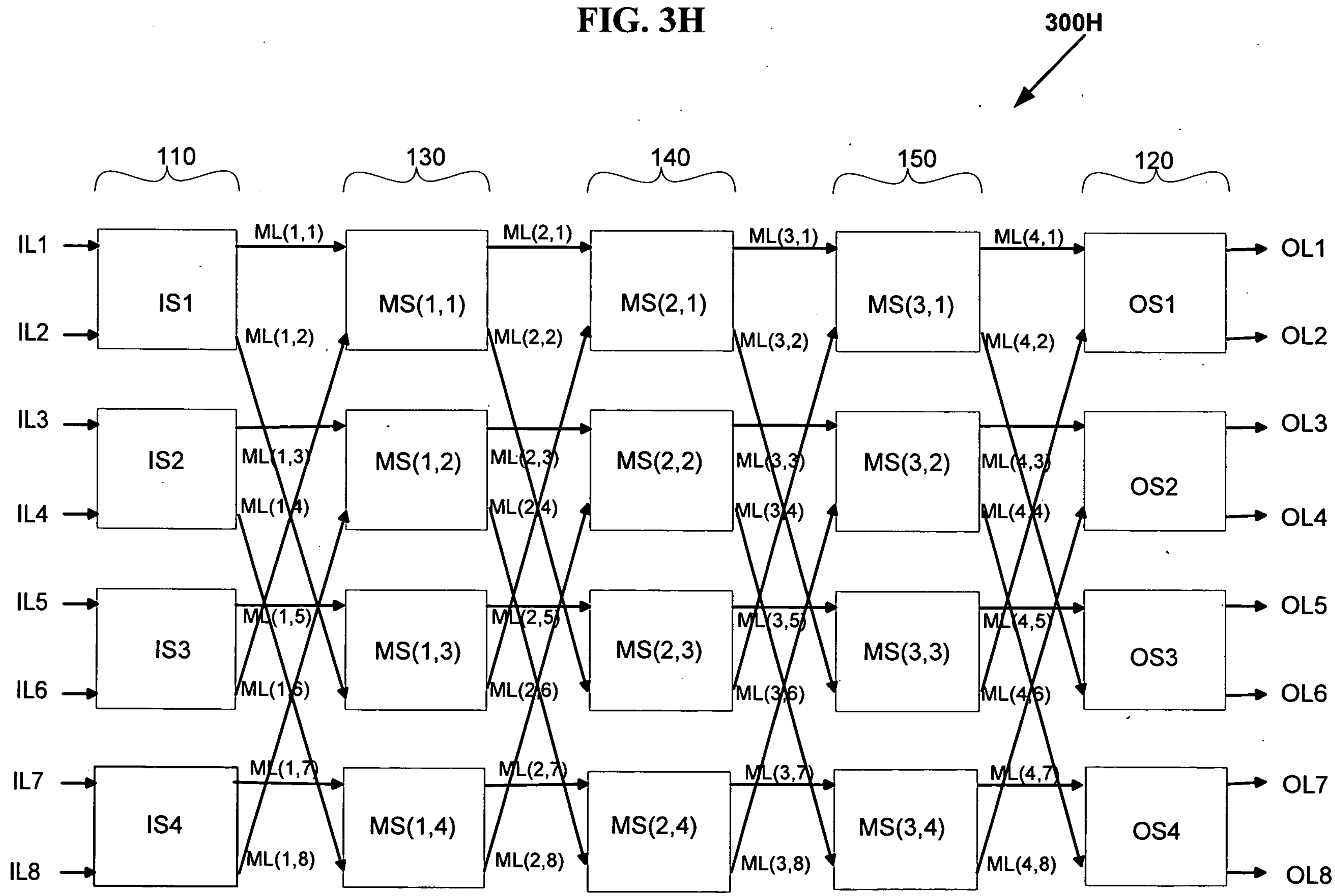
300H
110
130
140
150
120
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IS1
IS2
IS3
IS4
MS(1,1)
MS(1,2)
MS(1,3)
MS(1,4)
MS(2,1)
MS(2,2)
MS(2,3)
MS(2,4)
MS(3,1)
MS(3,2)
MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8
ML(1,1)
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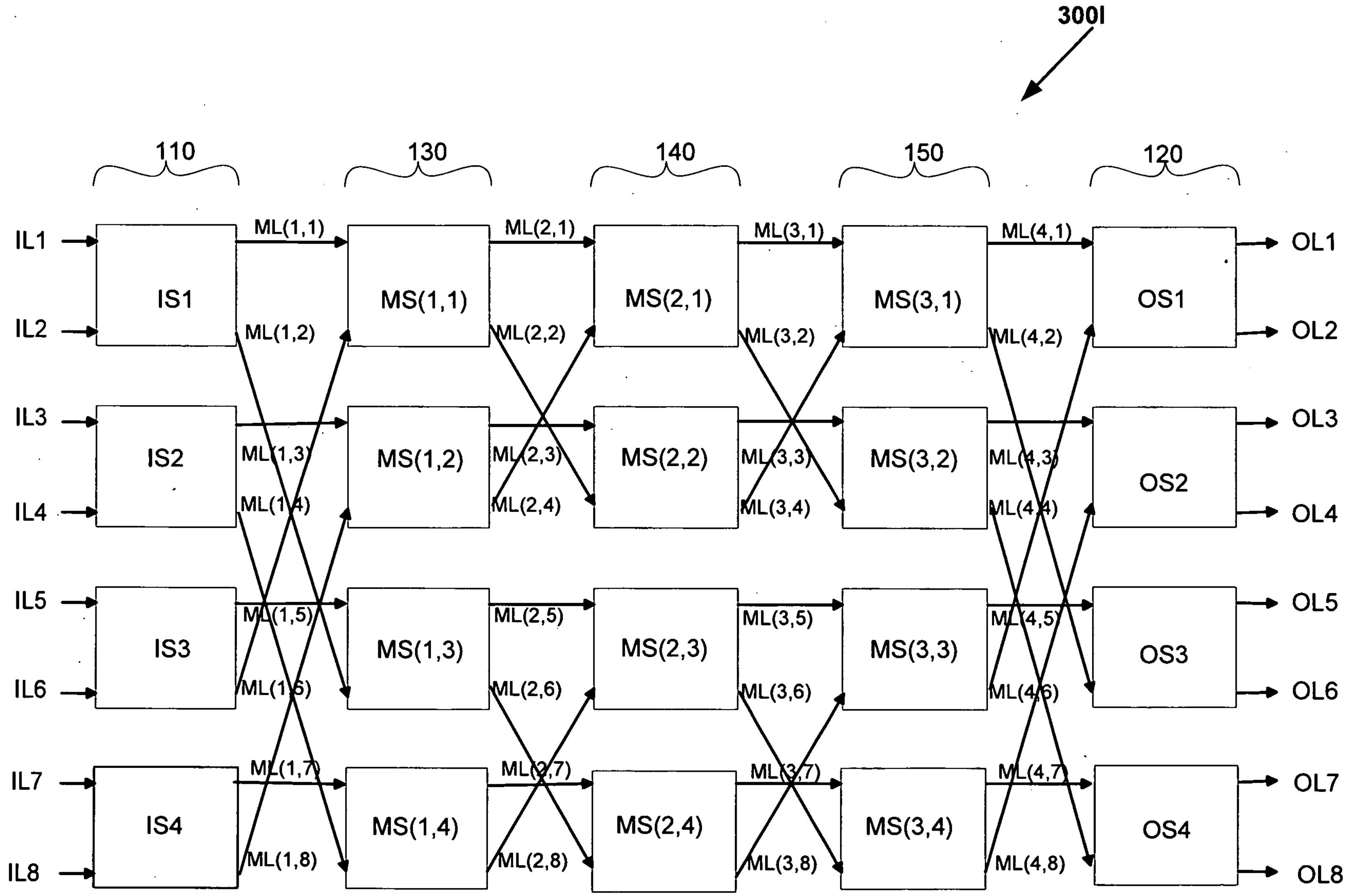
FIG. 3I


300I
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IL1
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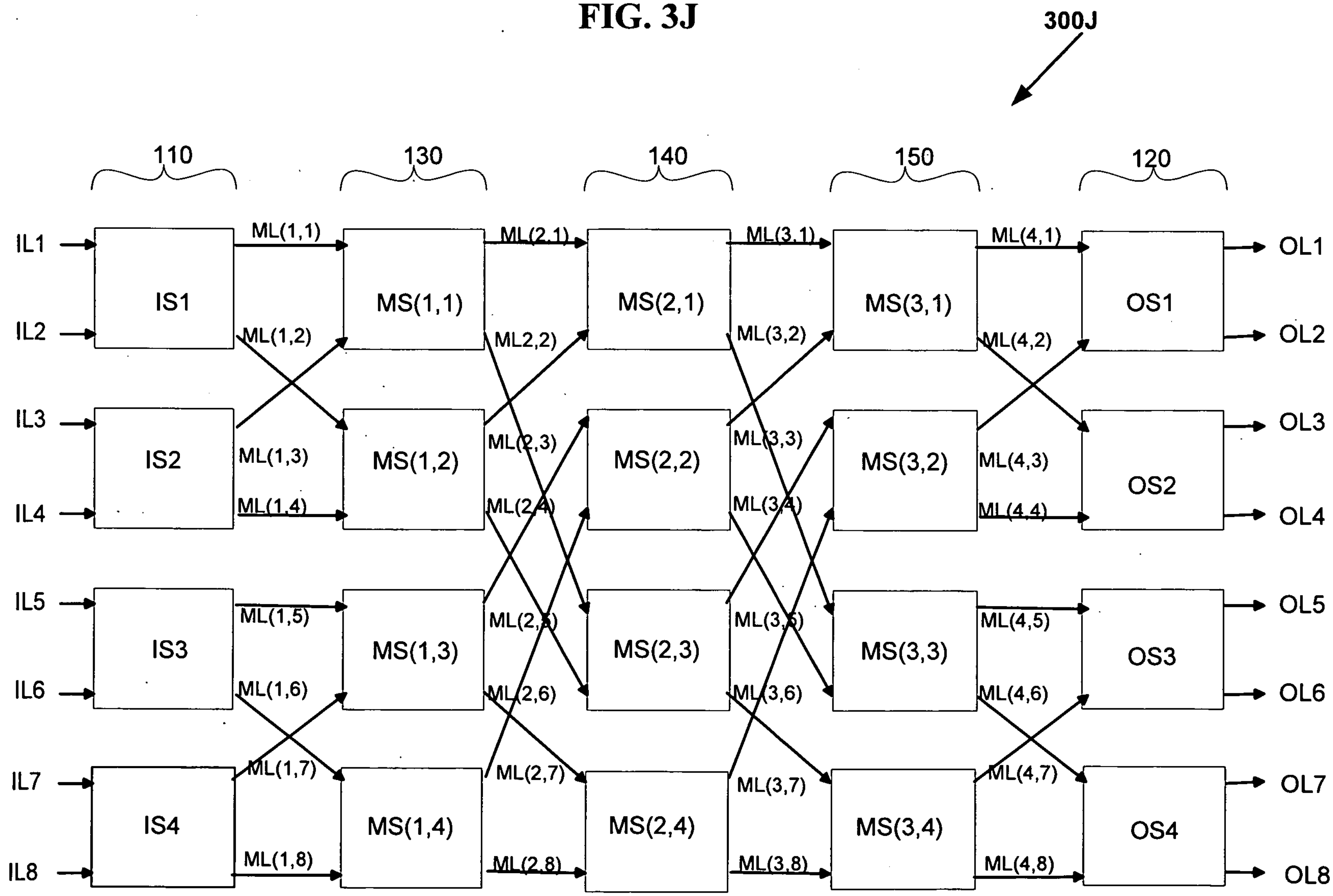
FIG. 3J


300J
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IL1
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MS(1,1)
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FIG. 3K

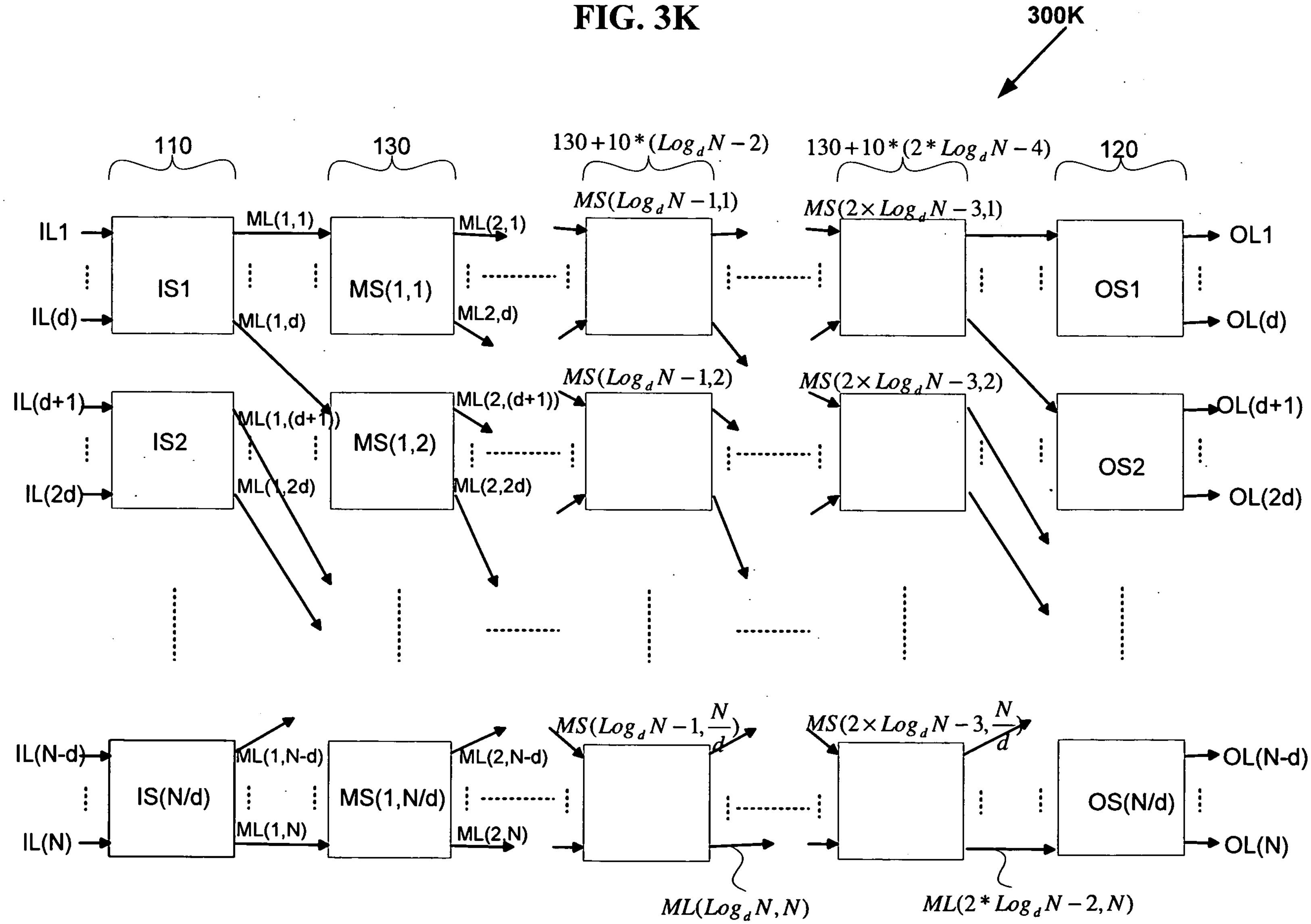
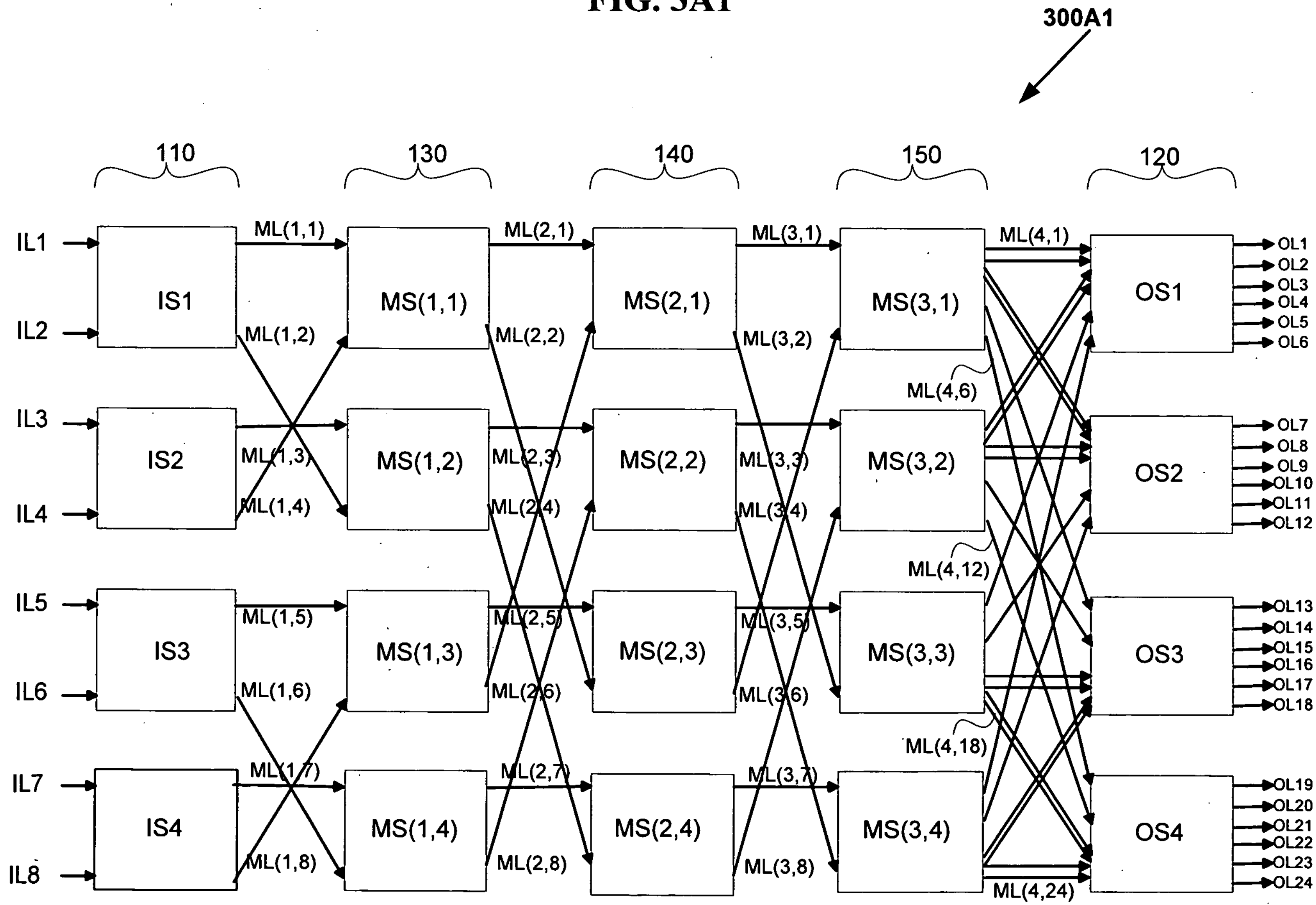

300K
110
130
130+10*(Log_d N-2)
130+10*(2*Log_d N-4)
120
IL1
IL(d)
IL(d+1)
IL(2d)
IL(N-d)
IL(N)
IS1
IS2
IS(N/d)
ML(1,1)
ML(1,d)
ML(1,(d+1))
ML(1,2d)
ML(1,N-d)
ML(1,N)
MS(1,1)
MS(1,2)
MS(1,N/d)
ML(2,1)
ML2,d)
ML(2,(d+1))
ML(2,2d)
ML(2,N-d)
ML(2,N)
MS(Log_d N-1,1)
MS(Log_d N-1,2)
MS(Log_d N-1,N/d)
ML(Log_d N,N)
MS(2×Log_d N-3,1)
MS(2×Log_d N-3,2)
MS(2×Log_d N-3,N/d)
ML(2*Log_d N-2,N)
OS1
OS2
OS(N/d)
OL1
OL(d)
OL(d+1)
OL(2d)
OL(N-d)
OL(N)

FIG. 3A1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IS2
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MS(1,2)
MS(1,3)
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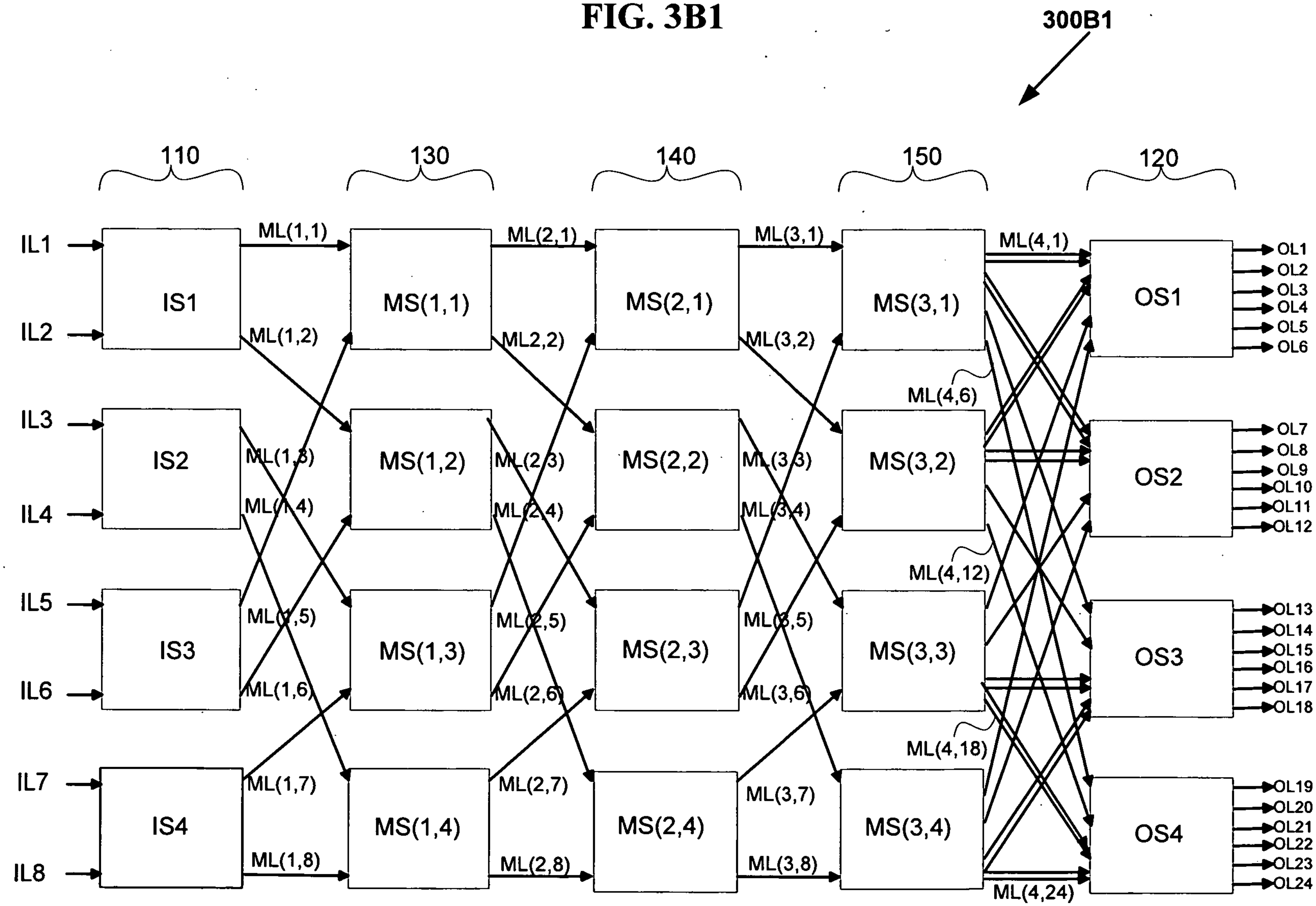
FIG. 3B1


300B1
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IL1
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FIG. 3C1

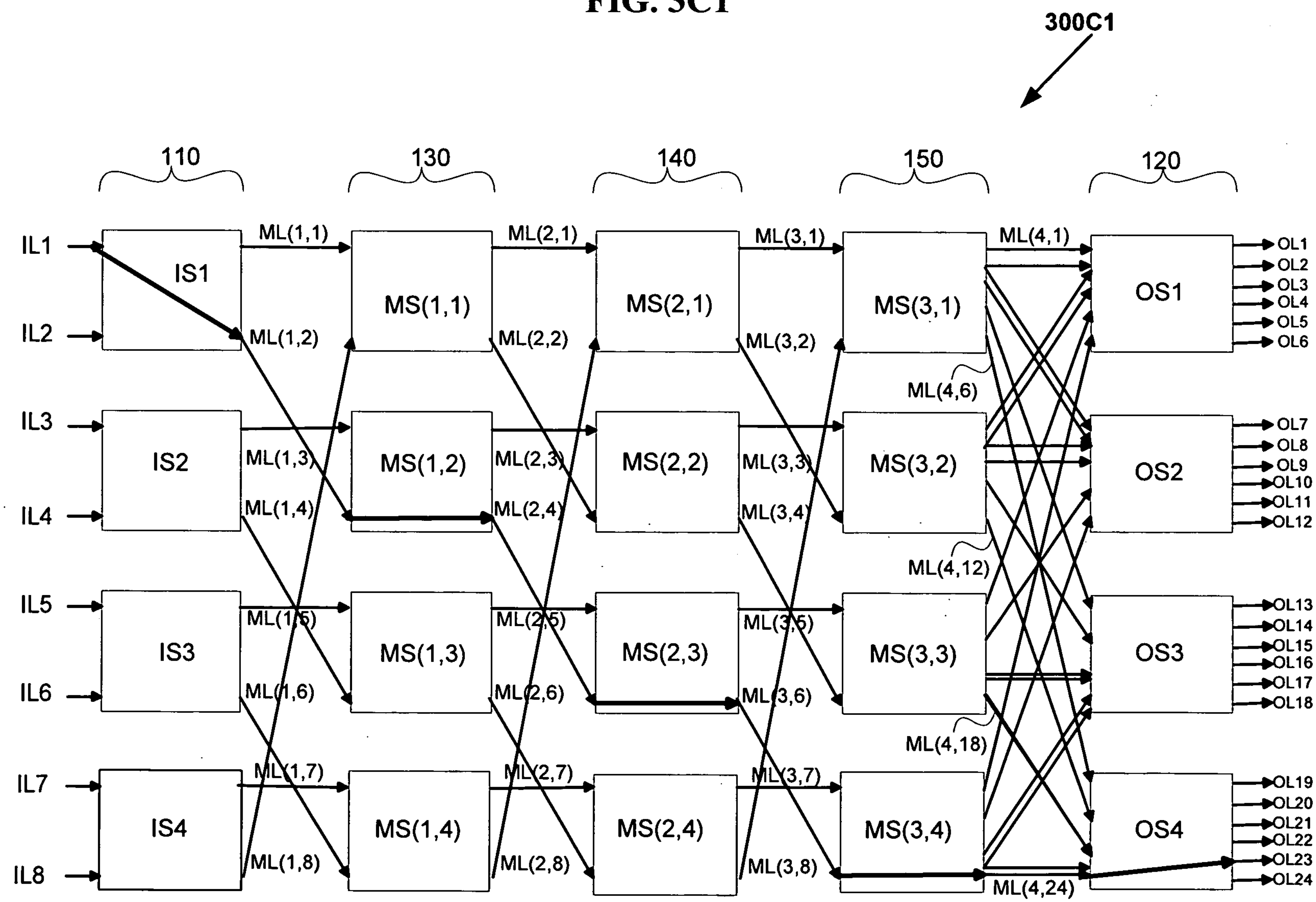
300C1
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FIG. 3D1

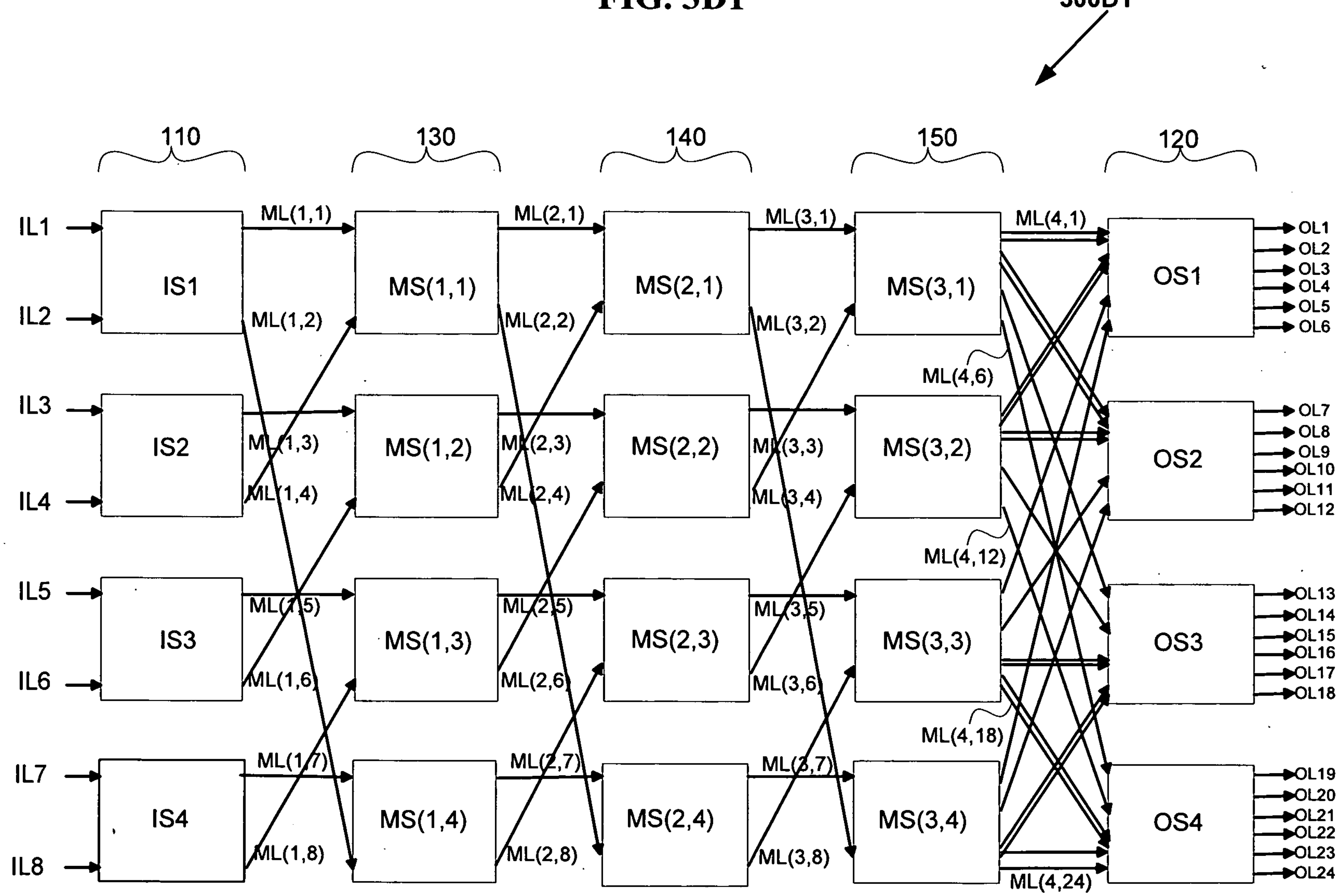
300D1
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FIG. 3E1

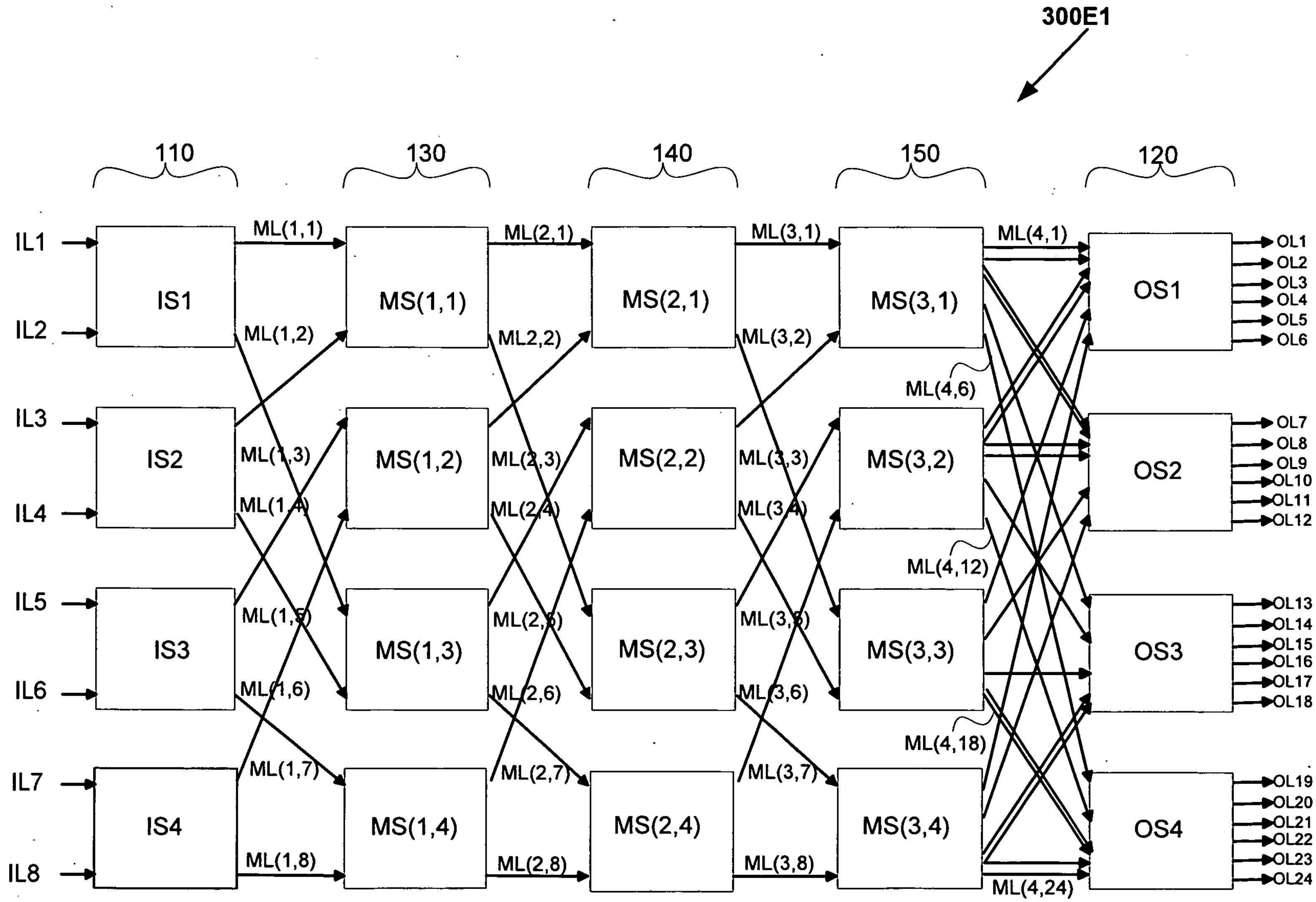
300E1
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FIG. 3F1

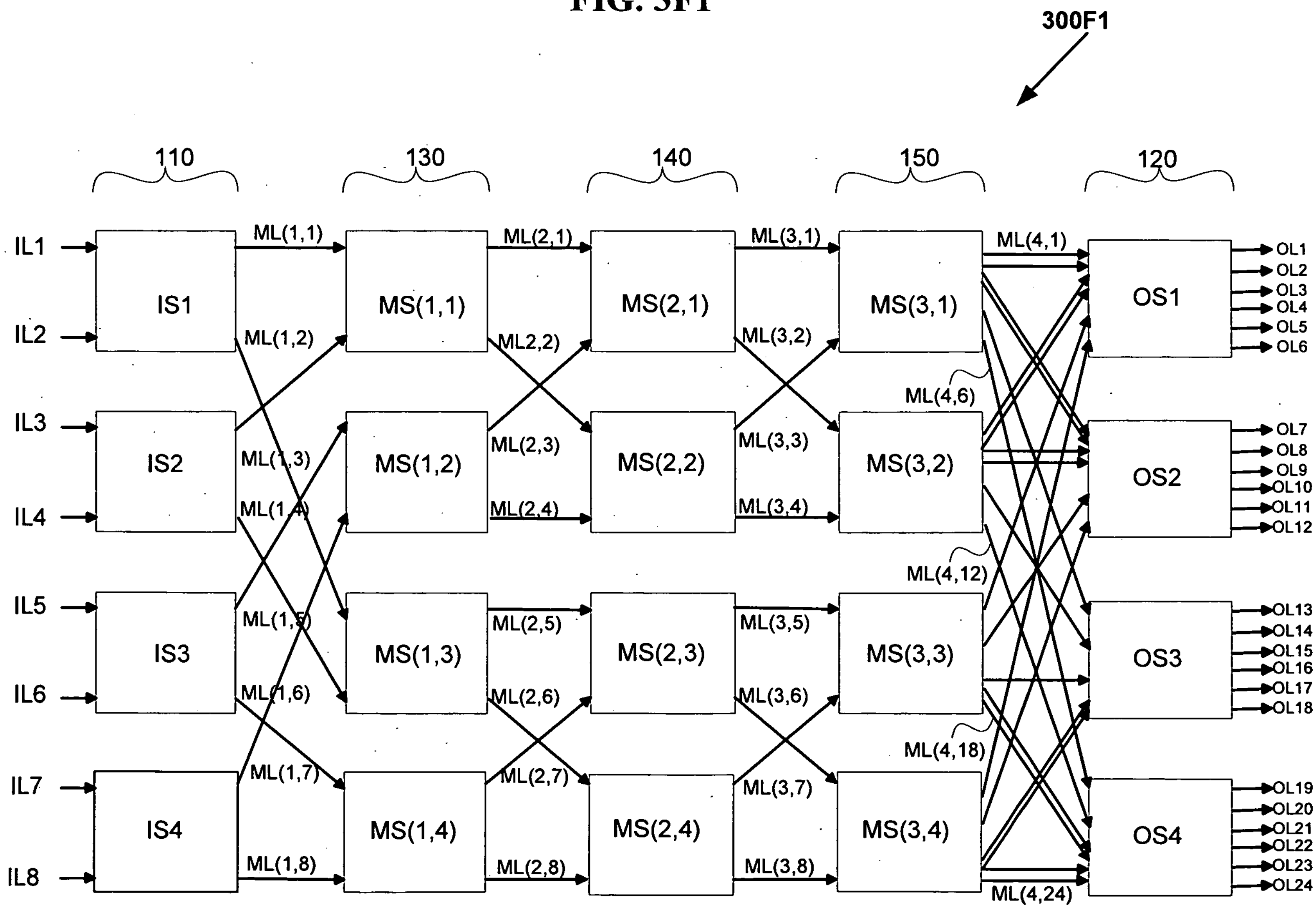
300F1
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FIG. 3G1

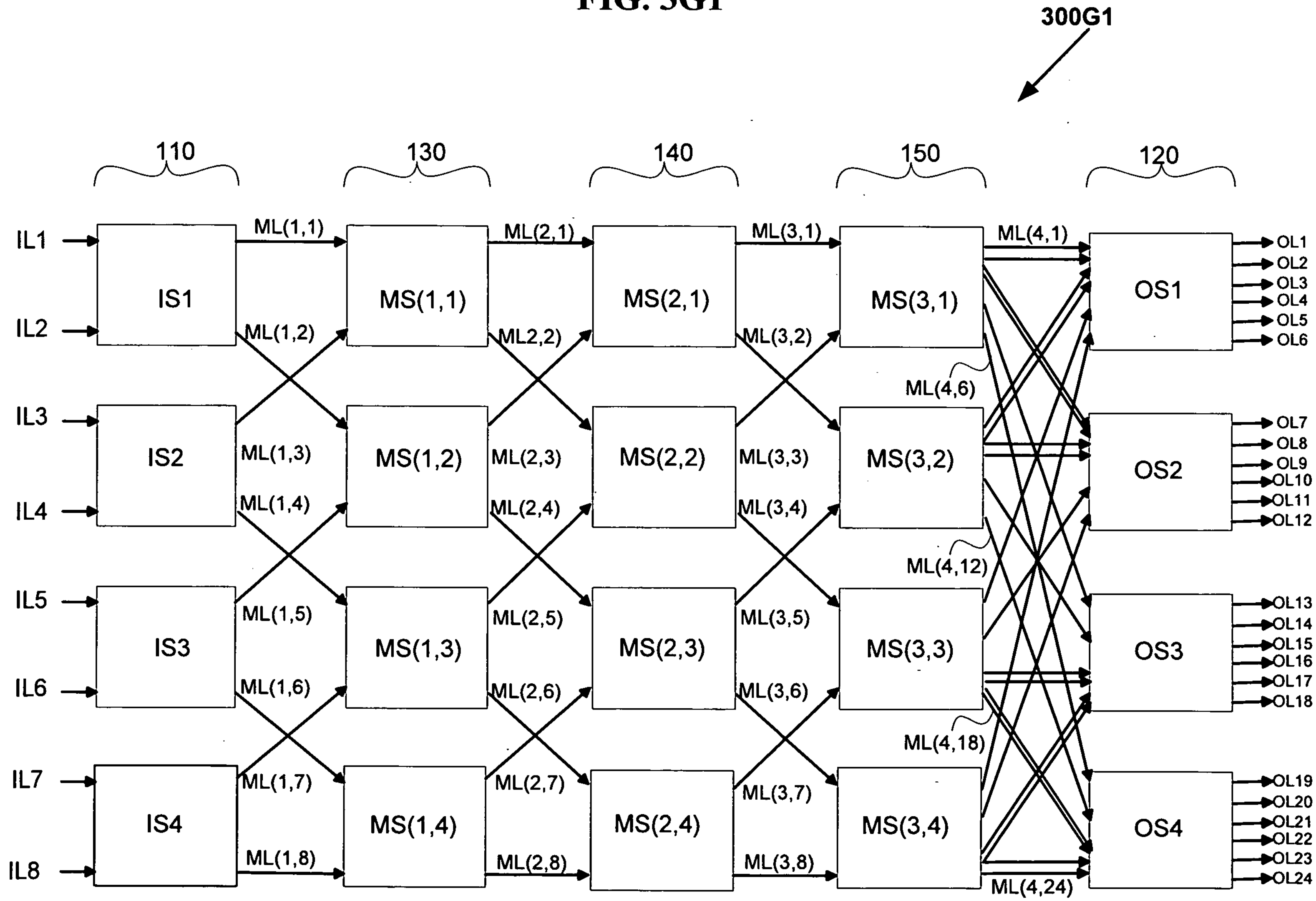
300G1
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FIG. 3H1

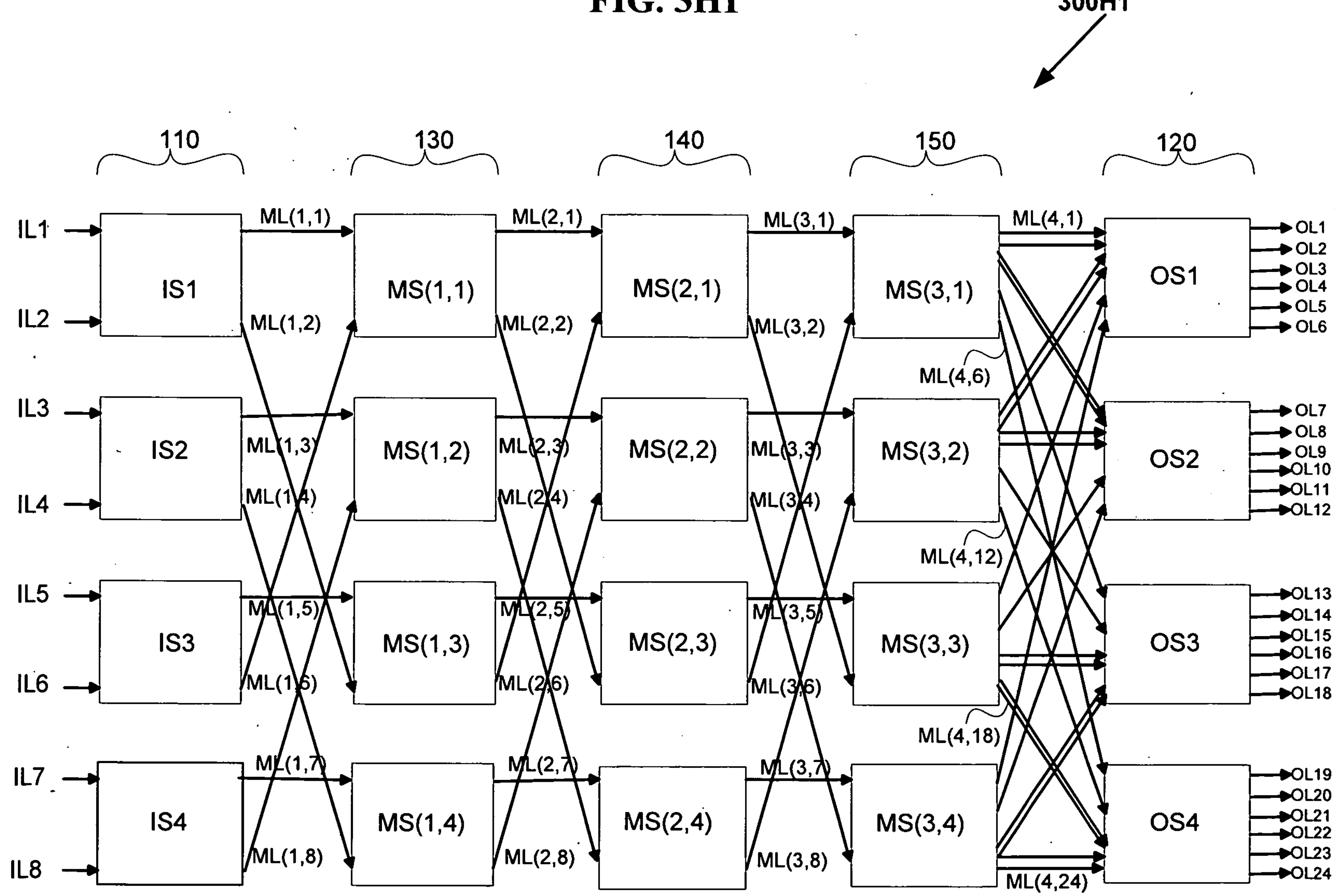
300H1
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FIG. 3I1

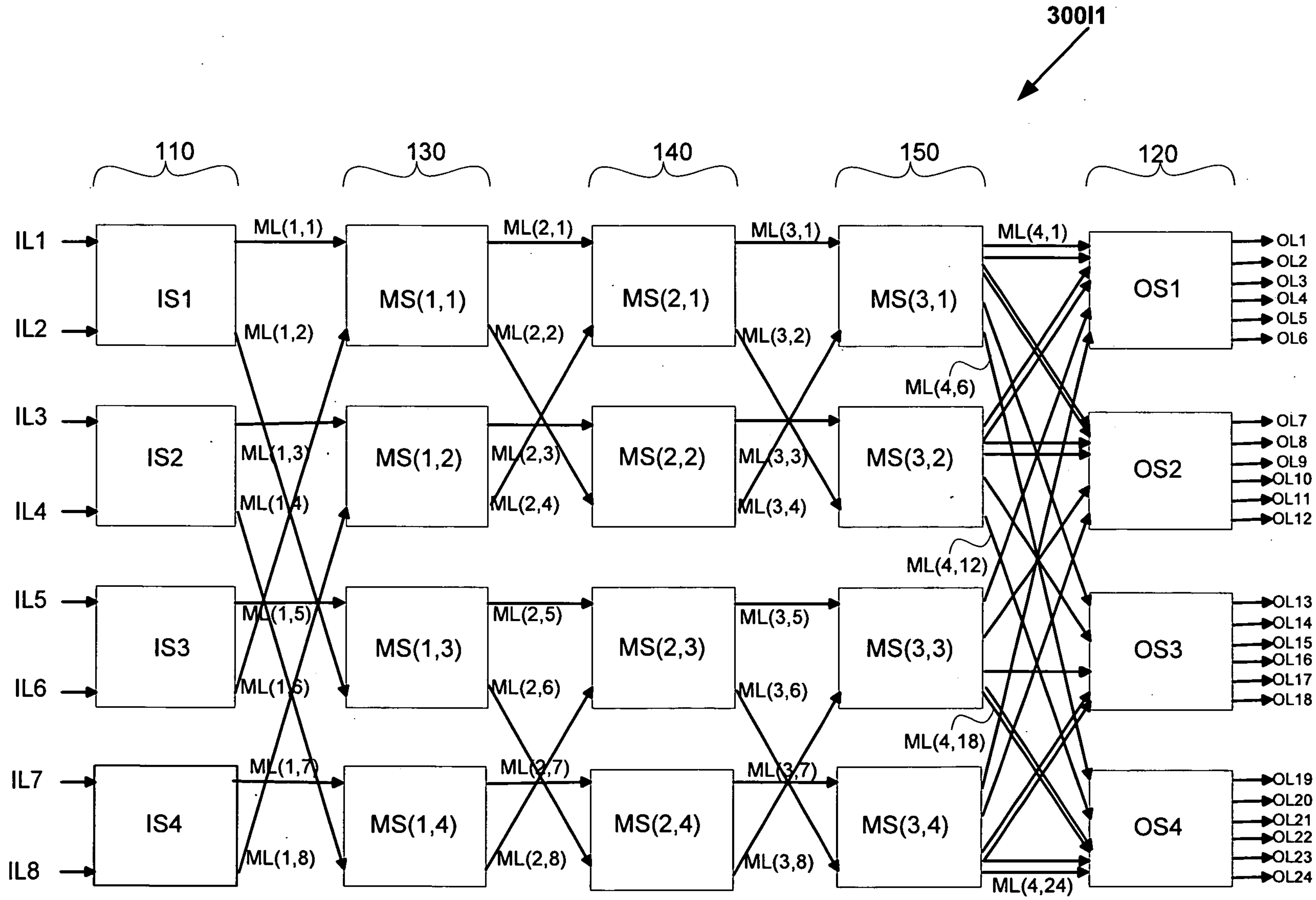
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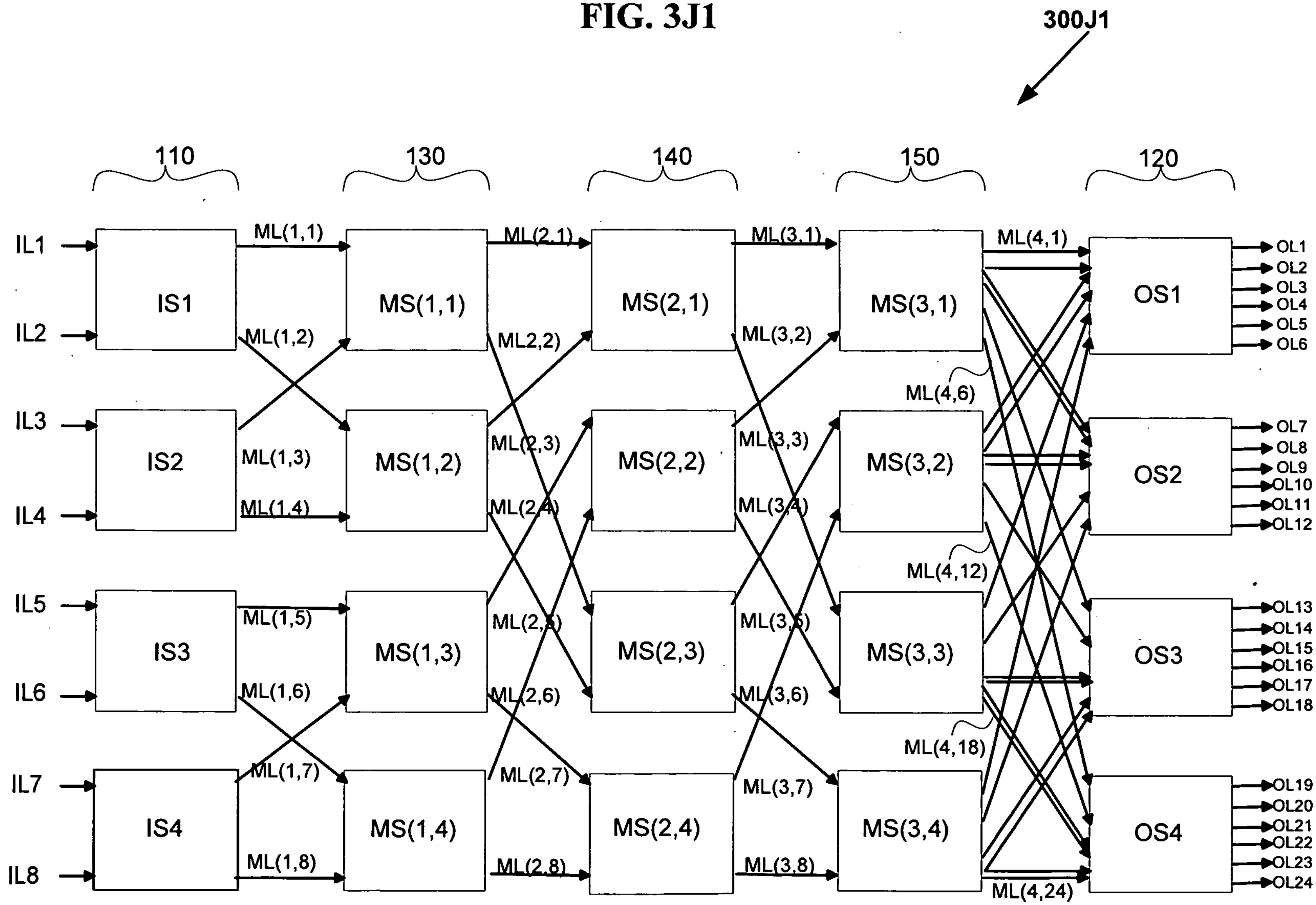
FIG. 3J1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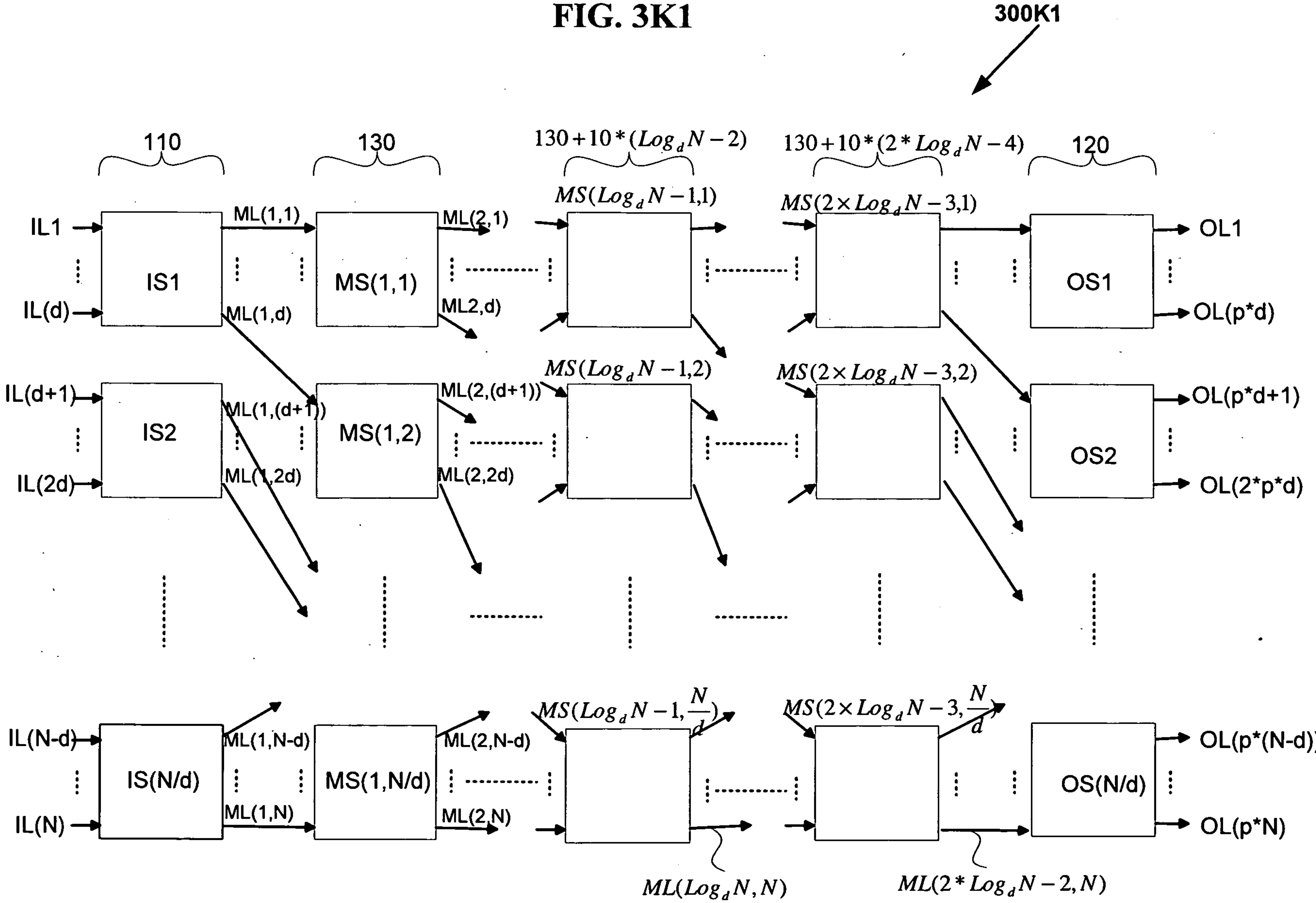
OL24

FIG. 3K1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130
130+10*(Log_d N−2)
130+10*(2*Log_d N−4)
120
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IL(d+1)
IL(2d)
IL(N-d)
IL(N)
IS1
IS2
IS(N/d)
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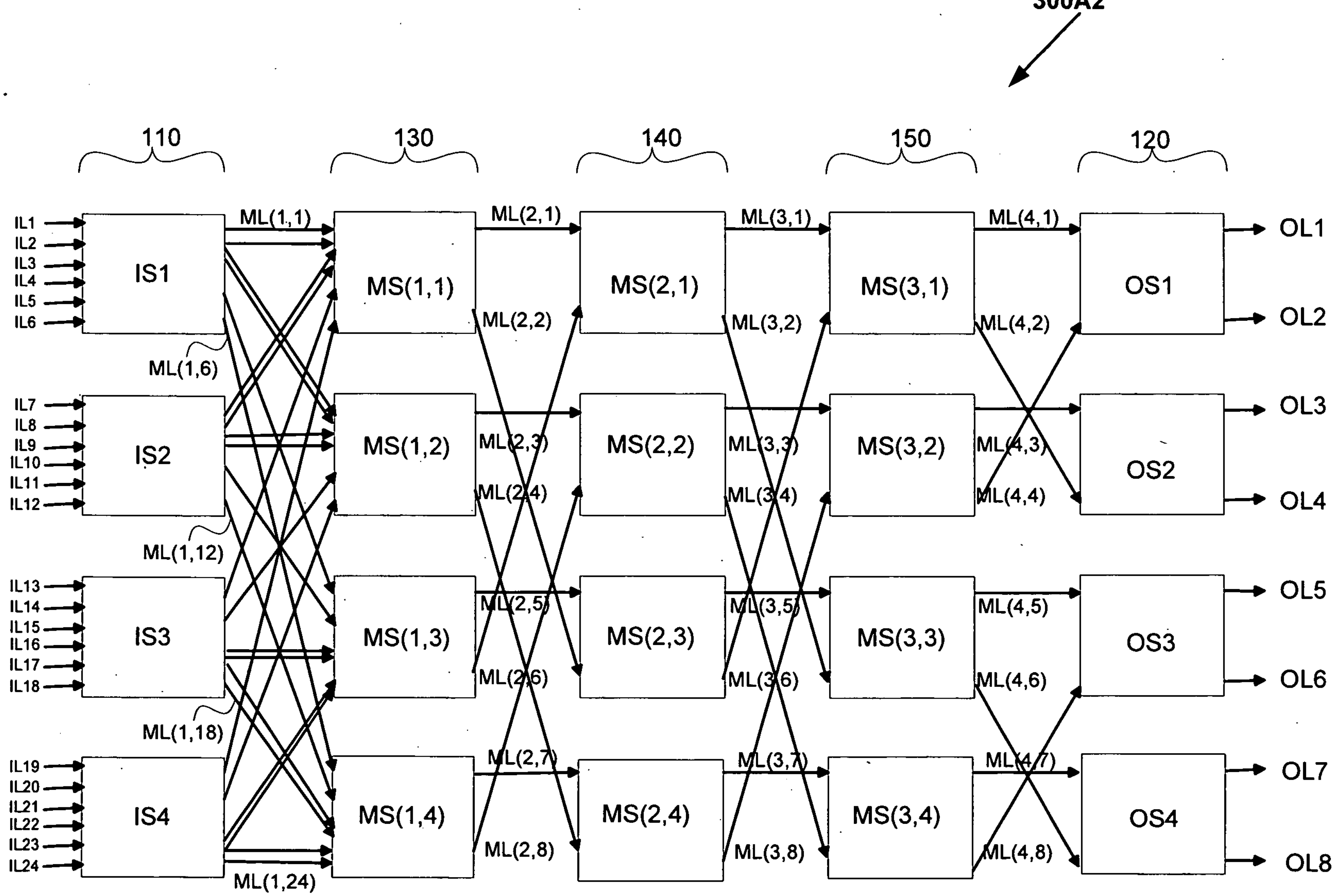
ML(1,d)
ML(1,(d+1))
ML(1,2d)
ML(1,N-d)
ML(1,N)
MS(1,1)
MS(1,2)
MS(1,N/d)
ML(2,1)
ML2,d)
ML(2,(d+1))
ML(2,2d)
ML(2,N-d)
ML(2,N)
MS(Log_d N−1,1)
MS(Log_d N−1,2)
MS(Log_d N−1, N/d)
MS(2×Log_d N−3,1)
MS(2×Log_d N−3,2)
MS(2×Log_d N−3, N/d)
ML(Log_d N, N)
ML(2*Log_d N−2, N)
OS1
OS2
OS(N/d)
OL1
OL(p*d)
OL(p*d+1)
OL(2*p*d)
OL(p*(N-d))
OL(p*N)

FIG. 3A2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IL7
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IL11
IL12
IL13
IL14
IL15
IL16
IL17
IL18
IL19
IL20
IL21
IL22
IL23
IL24
IS1
IS2
IS3
IS4
ML(1,1)
ML(1,6)
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ML(1,18)
ML(1,24)
MS(1,1)
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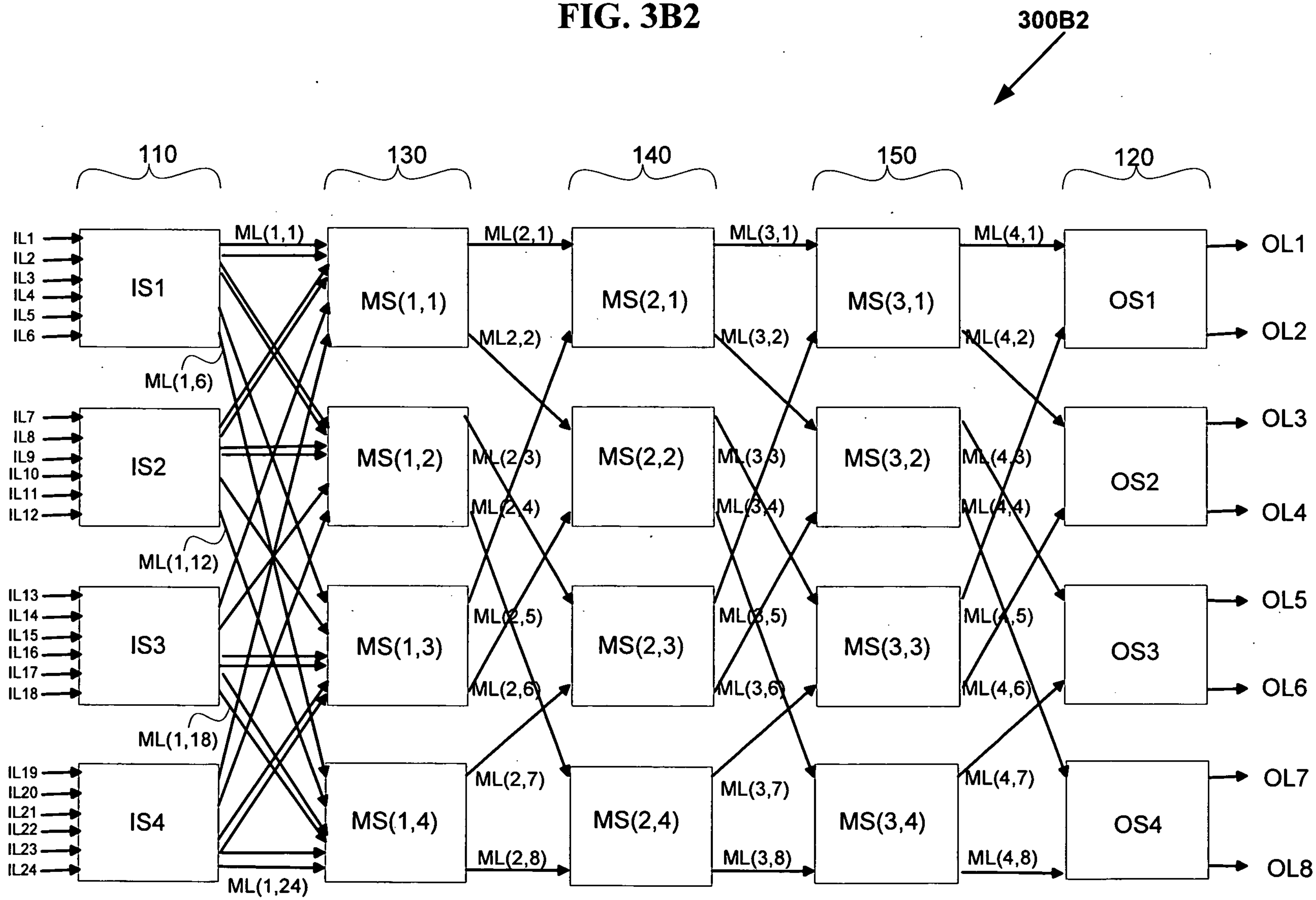
ML(4,5)
ML(4,6)
ML(4,7)
ML(4,8)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3B2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110
130
140
150
120
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MS(3,3)
MS(3,4)
OS1
OS2
OS3
OS4
IL1
IL2
IL3
IL4
IL5
IL6
IL7
IL8
IL9
IL10
IL11
IL12
IL13
IL14
IL15
IL16
IL17
IL18
IL19
IL20
IL21
IL22
IL23
IL24
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ML(1,24)
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ML2,2)
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ML(4,5)
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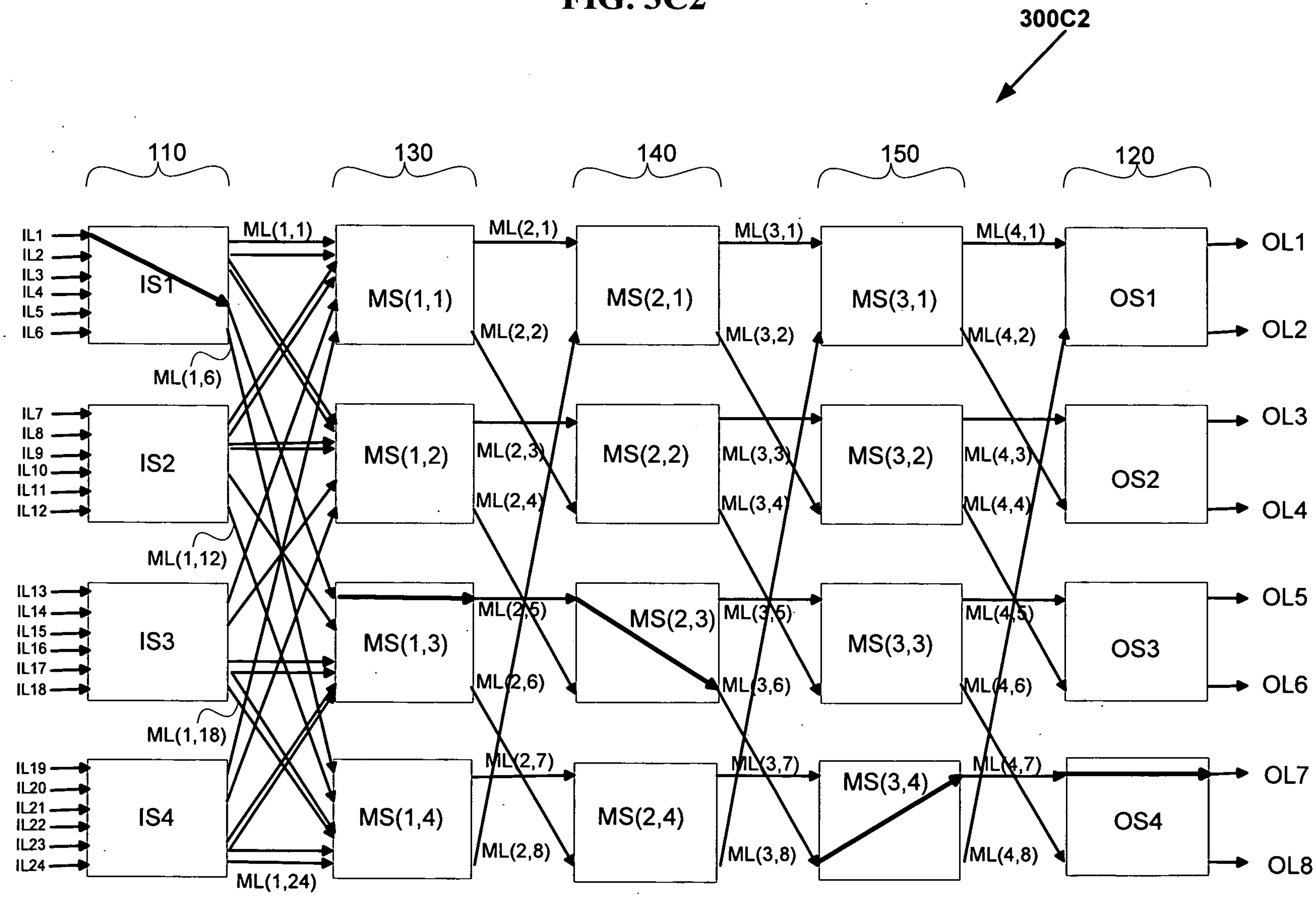
ML(4,8)
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3C2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140
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IL22
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IL24
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IS2
IS3
IS4
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ML(4,3)
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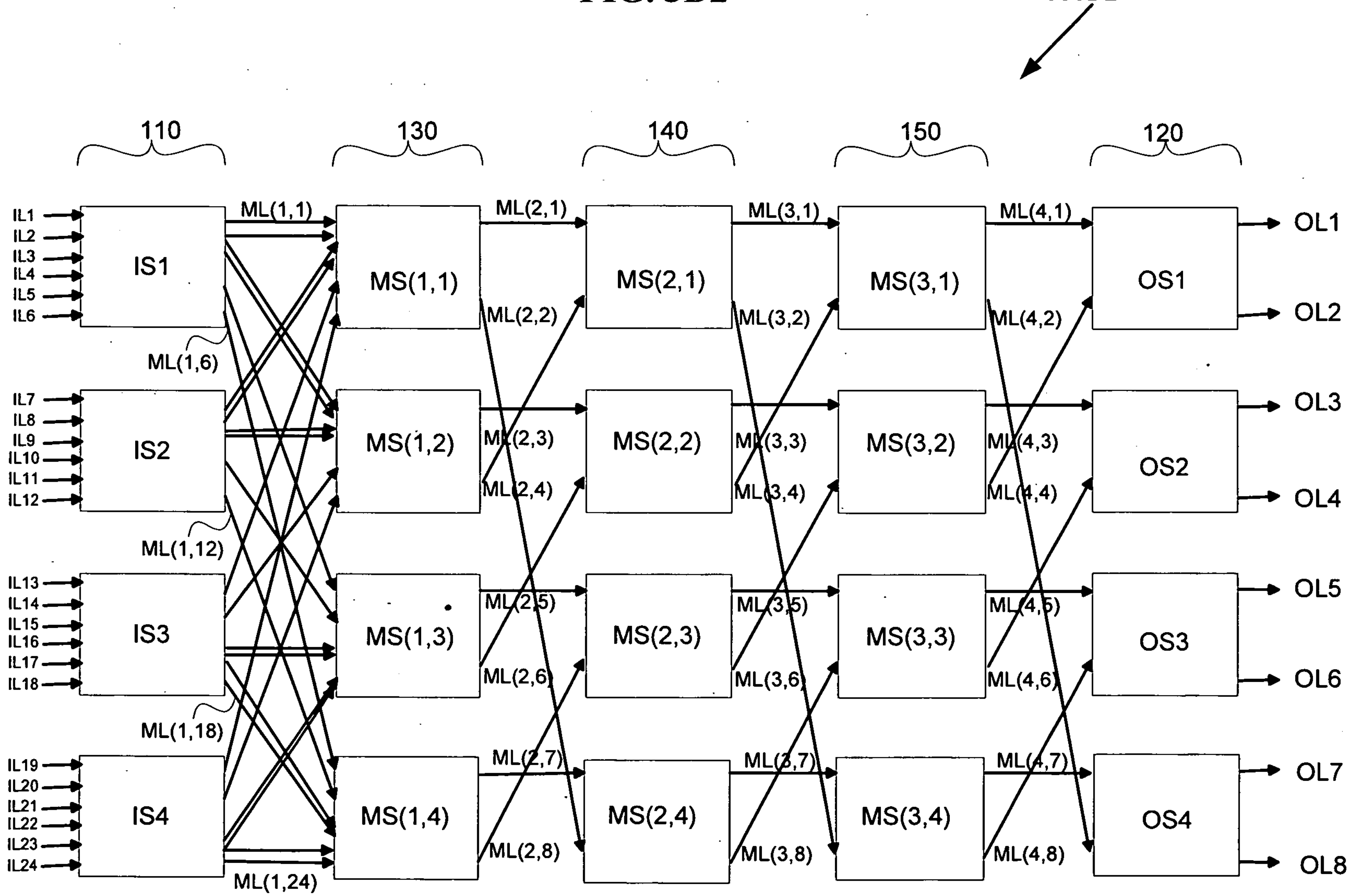
ML(4,7)
ML(4,8)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3D2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IL24
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OS2
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OS4
OL1
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FIG. 3E2

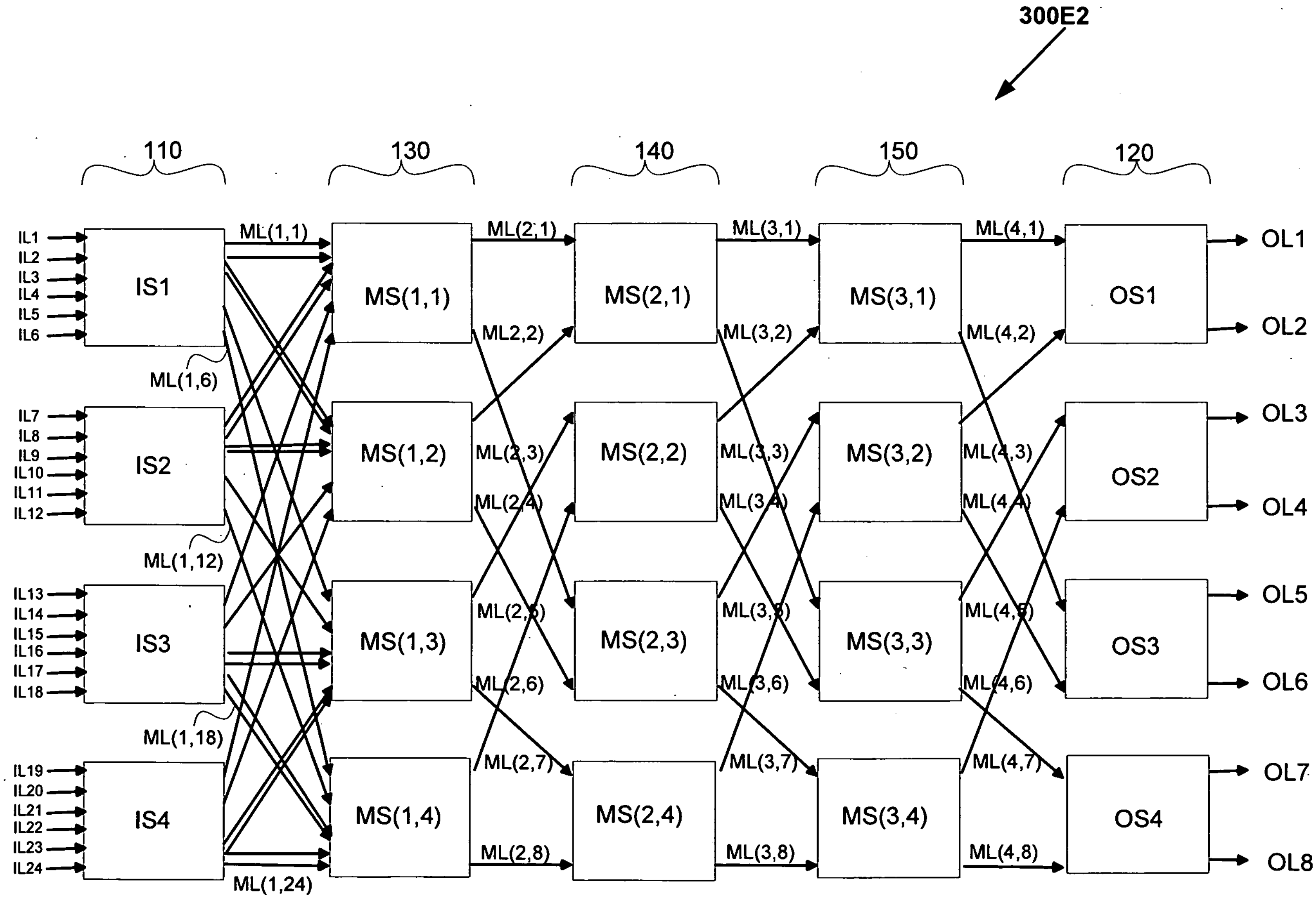
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FIG. 3F2

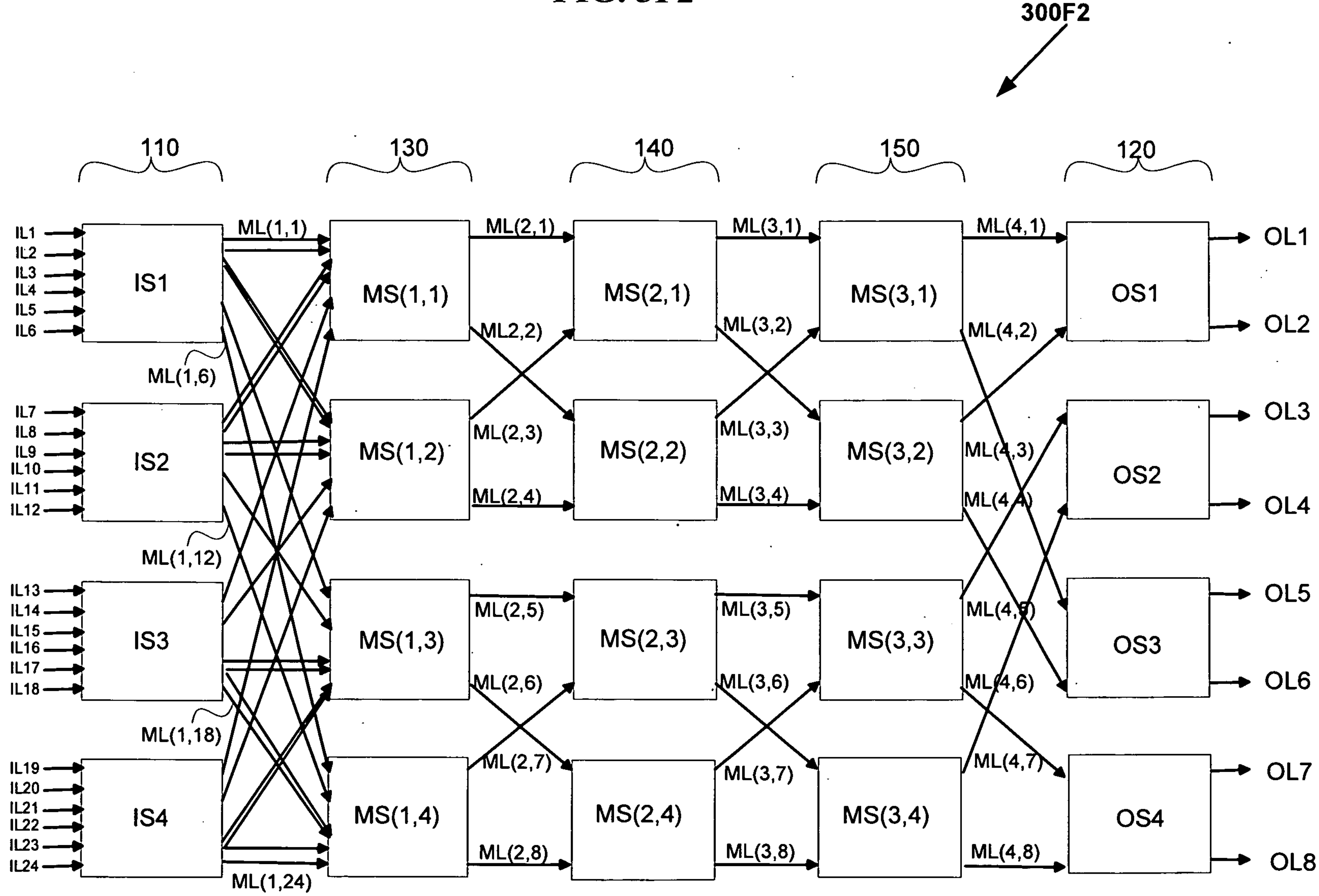
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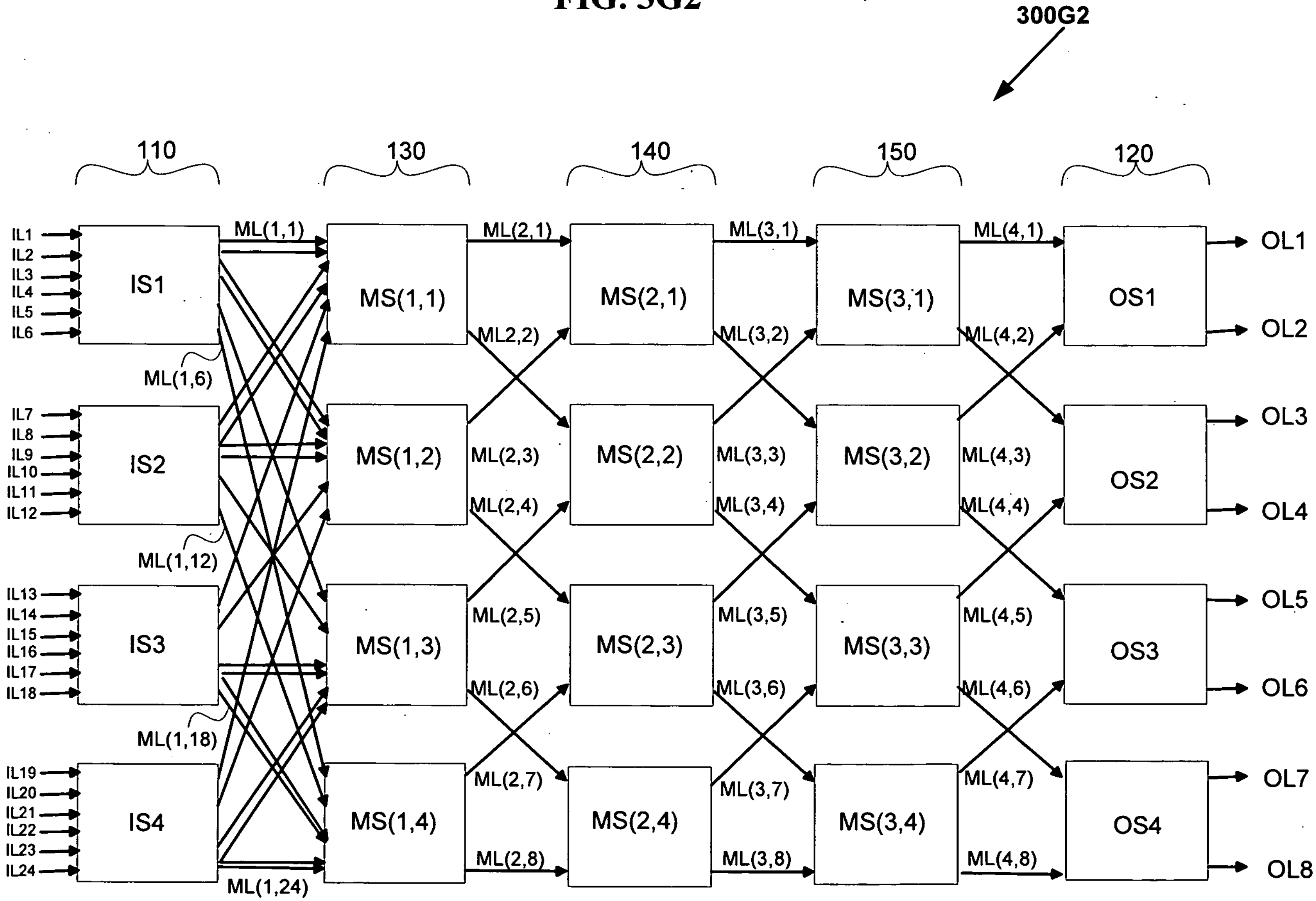
FIG. 3G2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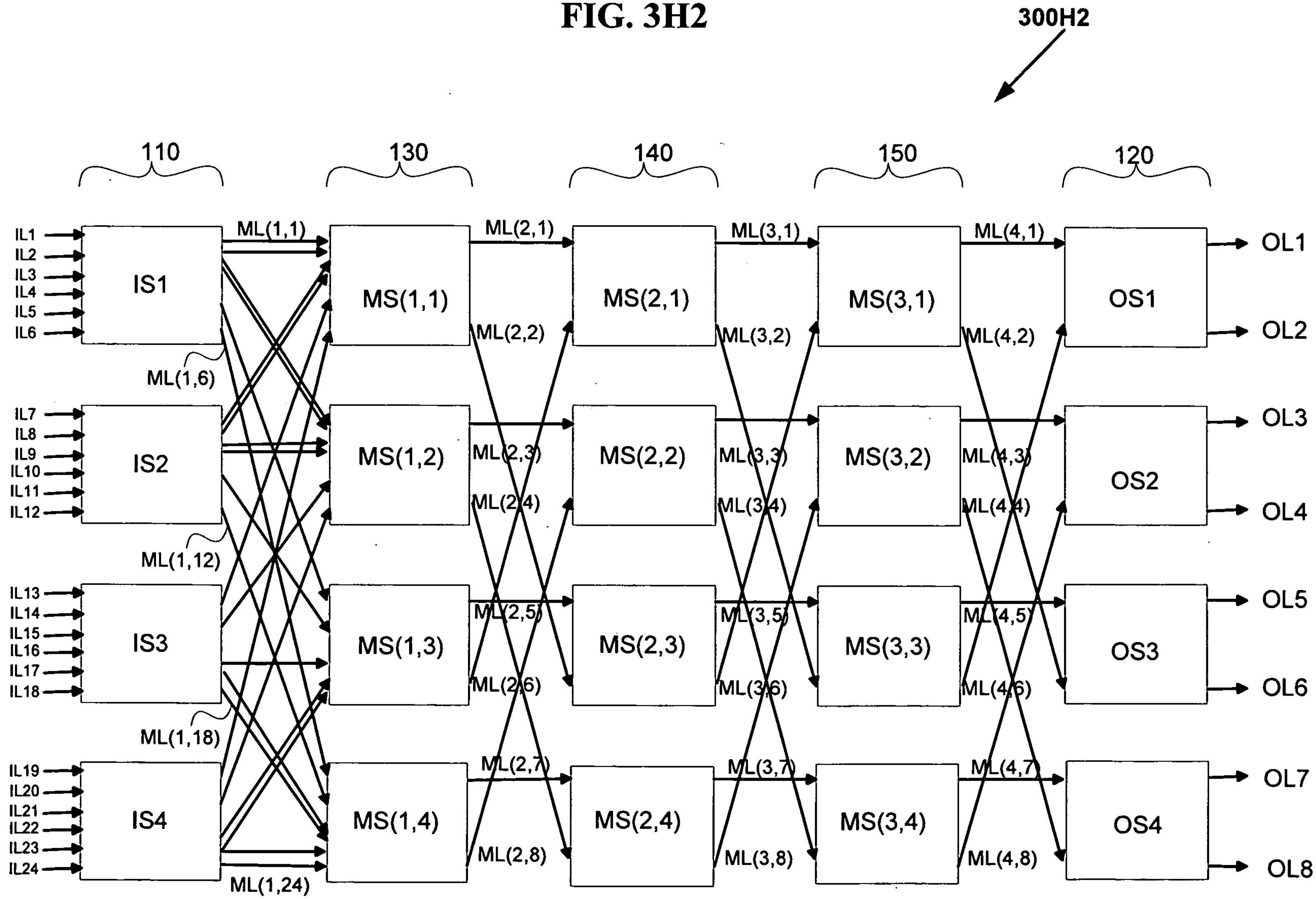
OL7
OL8

FIG. 3H2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OS1
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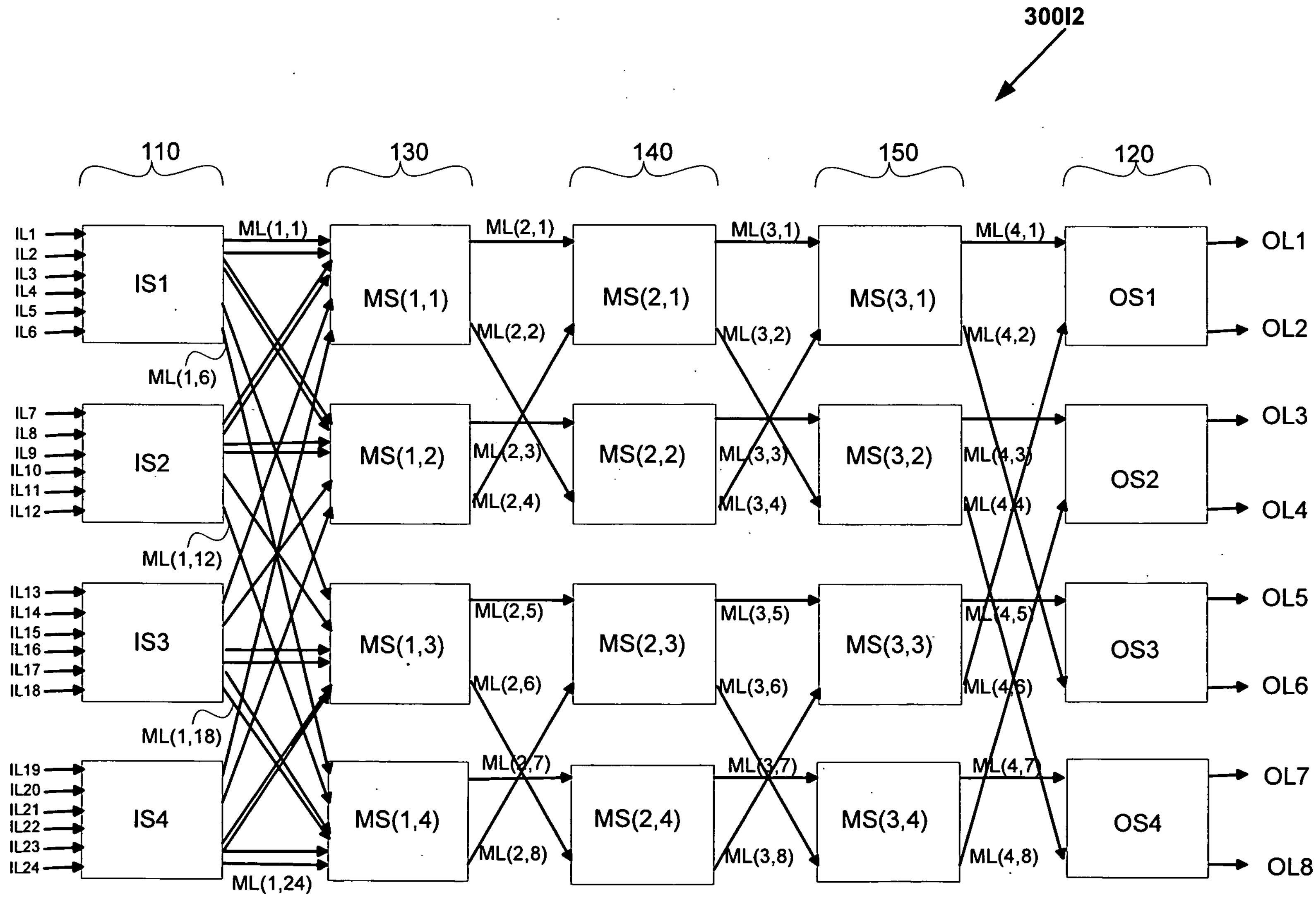
OL5
OL6
OL7
OL8

FIG. 3I2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IL11
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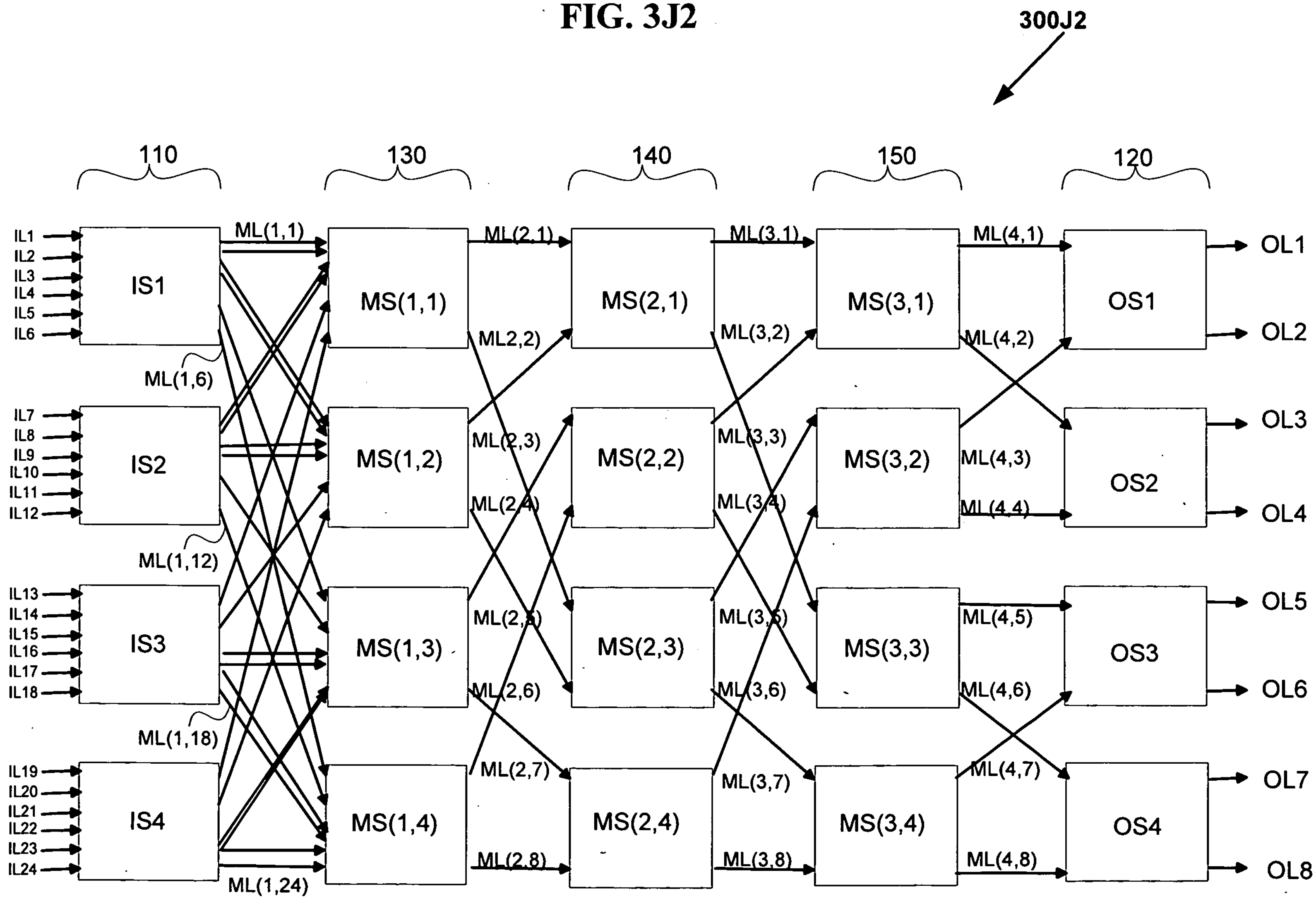
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3J2


300J2
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140
150
120
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IL4
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IL18
IS3
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IL23
IL24
IS4
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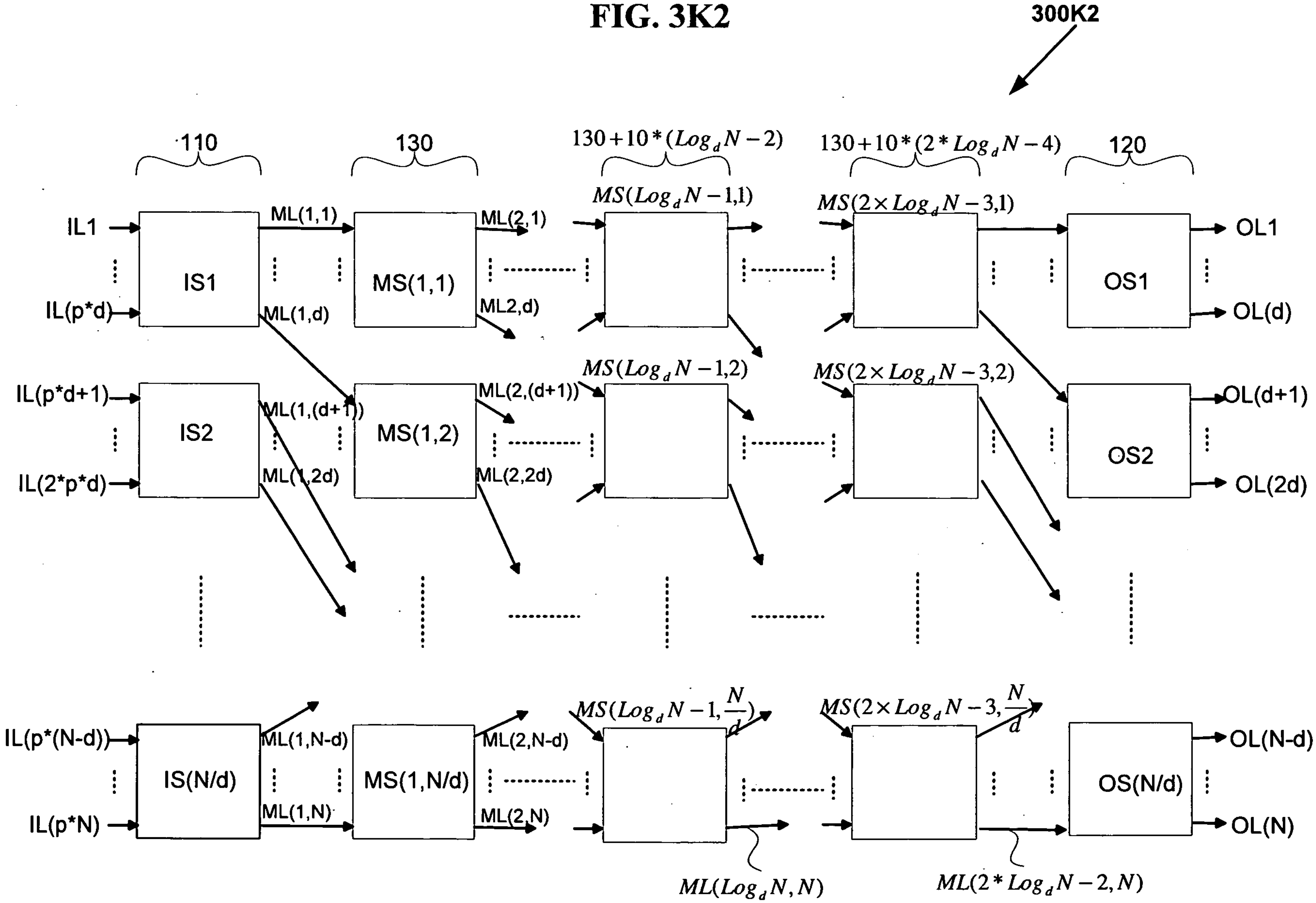
ML(4,7)
ML(4,8)
OS1
OS2
OS3
OS4
OL1
OL2
OL3
OL4
OL5
OL6
OL7
OL8

FIG. 3K2


300K2
110
130
130+10*(Log_d N-2)
130+10*(2*Log_d N-4)
120
IL1
IL(p*d)
IL(p*d+1)
IL(2*p*d)
IL(p*(N-d))
IL(p*N)
IS1
IS2
IS(N/d)
ML(1,1)
ML(1,d)
ML(1,(d+1))
ML(1,2d)
ML(1,N-d)
ML(1,N)
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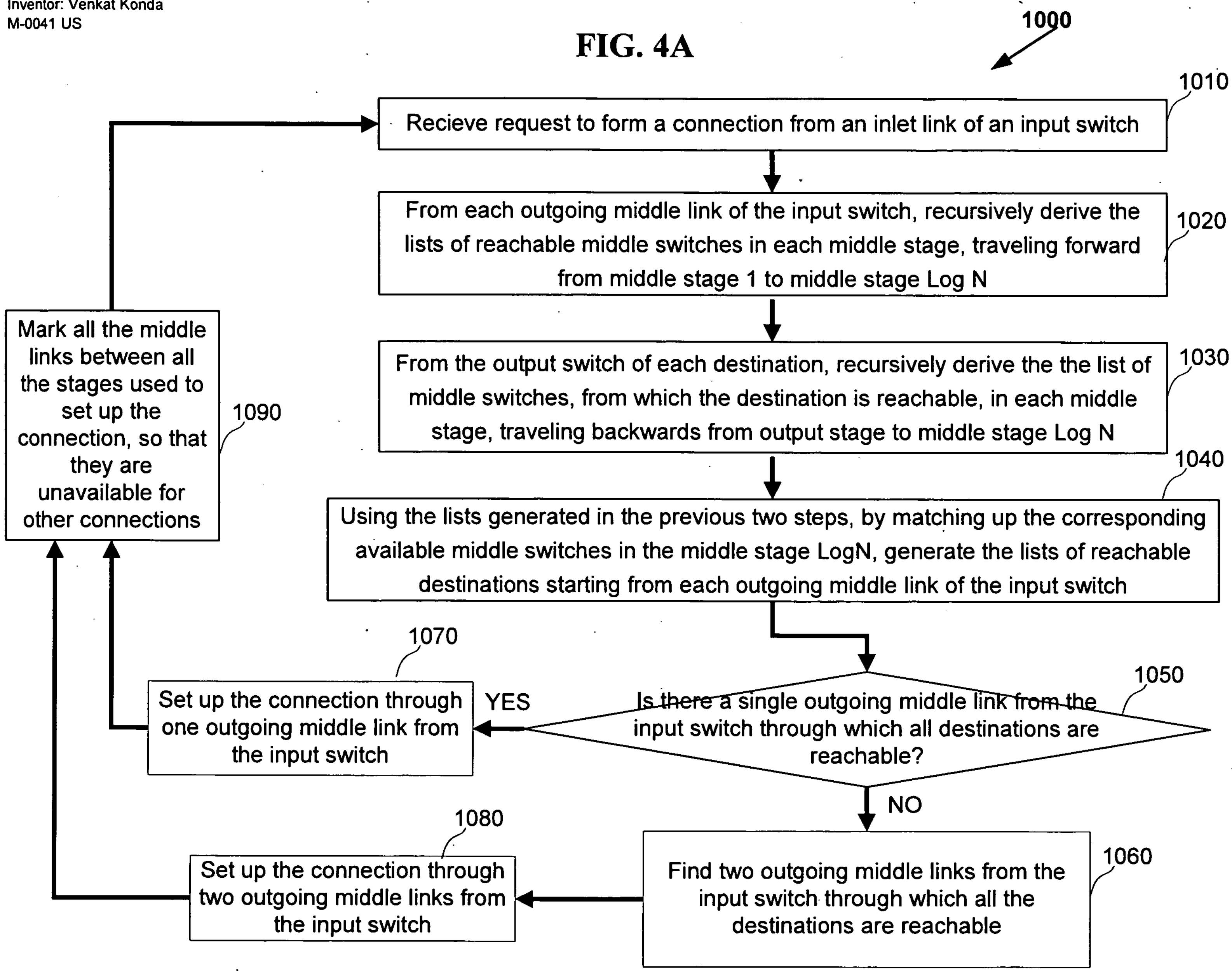
MS(1,N/d)
ML(2,1)
ML2,d)
ML(2,(d+1))
ML(2,2d)
ML(2,N-d)
ML(2,N)
MS(Log_d N-1,1)
MS(Log_d N-1,2)
MS(Log_d N-1,N/d)
MS(2×Log_d N-3,1)
MS(2×Log_d N-3,2)
MS(2×Log_d N-3,N/d)
ML(Log_d N,N)
ML(2*Log_d N-2,N)
OS1
OS2
OS(N/d)
OL1
OL(d)
OL(d+1)
OL(2d)
OL(N-d)
OL(N)

FIG. 4A


1000
1010
Recieve request to form a connection from an inlet link of an input switch
1020
From each outgoing middle link of the input switch, recursively derive the lists of reachable middle switches in each middle stage, traveling forward from middle stage 1 to middle stage Log N
1030
From the output switch of each destination, recursively derive the the list of middle switches, from which the destination is reachable, in each middle stage, traveling backwards from output stage to middle stage Log N
1040
Using the lists generated in the previous two steps, by matching up the corresponding available middle switches in the middle stage LogN, generate the lists of reachable destinations starting from each outgoing middle link of the input switch
1050
Is there a single outgoing middle link from the input switch through which all destinations are reachable?
YES
NO
1060
Find two outgoing middle links from the input switch through which all the destinations are reachable
1070
Set up the connection through one outgoing middle link from the input switch
1080
Set up the connection through two outgoing middle links from the input switch
1090
Mark all the middle links between all the stages used to set up the connection, so that they are unavailable for other connections

UNITED STATES DEPARTMENT OF COMMERCE
United States Patent and Trademark Office
Address: COMMISSIONER FOR PATENTS
P.O. Box 1450
Alexandria, Virginia 22313-1450
www.uspto.gov

APPLICATION NUMBER	FILING or 371(c) DATE	GRP ART UNIT	FIL FEE REC'D	ATTY.DOCKET.NO	TOT CLAIMS	IND CLAIMS
60/940,391	05/25/2007		100	M-0041US		

CONFIRMATION NO. 7097

38139
TEAK NETWORKS, INC.
6278 GRAND OAK WAY
SAN JOSE, CA95135

FILING RECEIPT

Date Mailed: 06/21/2007

Receipt is acknowledged of this provisional Patent Application. It will not be examined for patentability and will become abandoned not later than twelve months after its filing date. Be sure to provide the U.S. APPLICATION NUMBER, FILING DATE, NAME OF APPLICANT, and TITLE OF INVENTION when inquiring about this application. Fees transmitted by check or draft are subject to collection. Please verify the accuracy of the data presented on this receipt. **If an error is noted on this Filing Receipt, please mail to the Commissioner for Patents P.O. Box 1450 Alexandria Va 22313-1450. Please provide a copy of this Filing Receipt with the changes noted thereon. If you received a "Notice to File Missing Parts" for this application, please submit any corrections to this Filing Receipt with your reply to the Notice. When the USPTO processes the reply to the Notice, the USPTO will generate another Filing Receipt incorporating the requested corrections (if appropriate).**

Applicant(s)

Venkat Konda, Residence Not Provided;

Power of Attorney: None

If Required, Foreign Filing License Granted: 06/20/2007

The country code and number of your priority application, to be used for filing abroad under the Paris Convention, is **US60/940,391**

Projected Publication Date: None, application is not eligible for pre-grant publication

Non-Publication Request: No

Early Publication Request: No

** SMALL ENTITY **

Title

FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS

PROTECTING YOUR INVENTION OUTSIDE THE UNITED STATES

Since the rights granted by a U.S. patent extend only throughout the territory of the United States and have no effect in a foreign country, an inventor who wishes patent protection in another country must apply for a patent in a specific country or in regional patent offices. Applicants may wish to consider the filing of an international application under the Patent Cooperation Treaty (PCT). An international (PCT) application generally has the same effect as a regular national patent application in each PCT-member country. The PCT process **simplifies** the filing of patent applications on the same invention in member countries, but **does not result** in a grant of "an international patent" and does not eliminate the need of applicants to file

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For information on preventing theft of your intellectual property (patents, trademarks and copyrights), you may wish to consult the U.S. Government website, http://www.stopfakes.gov. Part of a Department of Commerce initiative, this website includes self-help "toolkits" giving innovators guidance on how to protect intellectual property in specific countries such as China, Korea and Mexico. For questions regarding patent enforcement issues, applicants may call the U.S. Government hotline at 1-866-999-HALT (1-866-999-4158).

LICENSE FOR FOREIGN FILING UNDER

Title 35, United States Code, Section 184

Title 37, Code of Federal Regulations, 5.11 & 5.15

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The applicant has been granted a license under 35 U.S.C. 184, if the phrase "IF REQUIRED, FOREIGN FILING LICENSE GRANTED" followed by a date appears on this form. Such licenses are issued in all applications where the conditions for issuance of a license have been met, regardless of whether or not a license may be required as set forth in 37 CFR 5.15. The scope and limitations of this license are set forth in 37 CFR 5.15(a) unless an earlier license has been issued under 37 CFR 5.15(b). The license is subject to revocation upon written notification. The date indicated is the effective date of the license, unless an earlier license of similar scope has been granted under 37 CFR 5.13 or 5.14.

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The grant of a license does not in any way lessen the responsibility of a licensee for the security of the subject matter as imposed by any Government contract or the provisions of existing laws relating to espionage and the national security or the export of technical data. Licensees should apprise themselves of current regulations especially with respect to certain countries, of other agencies, particularly the Office of Defense Trade Controls, Department of State (with respect to Arms, Munitions and Implements of War (22 CFR 121-128)); the Bureau of Industry and Security, Department of Commerce (15 CFR parts 730-774); the Office of Foreign AssetsControl, Department of Treasury (31 CFR Parts 500+) and the Department of Energy.

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APPLICATION NUMBER	FILING or 371(c) DATE	GRP ART UNIT	FIL FEE REC'D	ATTY.DOCKET.NO	TOT CLAIMS	IND CLAIMS
60/940,391	05/25/2007		100	M-0041US		

CONFIRMATION NO. 7097

FILING RECEIPT

38139
TEAK NETWORKS, INC.
6278 GRAND OAK WAY
SAN JOSE, CA95135

Date Mailed: 06/22/2007

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Applicant(s)

Venkat Konda, Residence Not Provided;

Power of Attorney: None

If Required, Foreign Filing License Granted: 06/20/2007

The country code and number of your priority application, to be used for filing abroad under the Paris Convention, is **US60/940,391**

Projected Publication Date: None, application is not eligible for pre-grant publication

Non-Publication Request: No

Early Publication Request: No

** SMALL ENTITY **

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APPLICATION NUMBER	FILING OR 371 (c) DATE	FIRST NAMED APPLICANT	ATTORNEY DOCKET NUMBER
60/940,391	05/25/2007	Venkat Konda	M-0041US

CONFIRMATION NO. 7097

FORMALITIES LETTER

38139
TEAK NETWORKS, INC.
6278 GRAND OAK WAY
SAN JOSE, CA 95135

Date Mailed: 06/22/2007

NOTICE TO FILE MISSING PARTS OF PROVISIONAL APPLICATION

FILED UNDER 37 CFR 1.53(c)

Filing Date Granted

An application number and filing date have been accorded to this provisional application. The items indicated below, however, are missing. Applicant is given **TWO MONTHS** from the date of this Notice within which to file all required items and pay any fees required below to avoid abandonment. Extensions of time may be obtained by filing a petition accompanied by the extension fee under the provisions of 37 CFR 1.136(a).

- The provisional application cover sheet under 37 CFR 1.51(c)(1), which may be an application data sheet (37 CFR 1.76), is required identifying either city and state or city and foreign country of the residence of each inventor and the name(s) of the inventor(s).

The applicant needs to satisfy supplemental fees problems indicated below.

The required item(s) identified below must be timely submitted to avoid abandonment:

- To avoid abandonment, a surcharge (for late submission of filing fee or cover sheet) as set forth in 37 CFR 1.16(g) of $25 for a small entity in compliance with 37 CFR 1.27, must be submitted with the missing items identified in this letter.

SUMMARY OF FEES DUE:

Total additional fee(s) required for this application is **$25** for a small entity

- **$25** Surcharge.

Replies should be mailed to: Mail Stop Missing Parts
Commissioner for Patents
P.O. Box 1450
Alexandria VA 22313-1450

Registered users of EFS-Web may alternatively submit their reply to this notice via EFS-Web.
https://sportal.uspto.gov/authenticate/AuthenticateUserLocalEPF.html

For more information about EFS-Web please call the USPTO Electronic Business Center at **1-866-217-9197** or visit our website at http://www.uspto.gov/ebc.

If you are not using EFS-Web to submit your reply, you must include a copy of this notice.

Office of Initial Patent Examination (571) 272-4000, or 1-800-PTO-9199
PART 3 - OFFICE COPY

PTO/SB/16 (02-07)
Approved for use through 02/28/2007. OMB 0651-0032
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PROVISIONAL APPLICATION FOR PATENT COVER SHEET – Page 1 of 2

This is a request for filing a PROVISIONAL APPLICATION FOR PATENT under 37 CFR 1.53(c).

Express Mail Label No. ______________________________

INVENTOR(S)

Given Name (first and middle [if any])	Family Name or Surname	Residence (City and either State or Foreign Country)
Venkat	Konda	San Jose, CA

Additional inventors are being named on the ____________ *separately numbered sheets attached hereto*

TITLE OF THE INVENTION (500 characters max):

FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS

Direct all correspondence to: **CORRESPONDENCE ADDRESS**

☑ The address corresponding to Customer Number: 38139

OR

☐ Firm or Individual Name Teak Networks, Inc.

Address 6278, Grand Oak Way

City San Jose	State CA	Zip 95135
Country USA	Telephone 408-472-3273	Email venkat@teaknetworks.com

ENCLOSED APPLICATION PARTS *(check all that apply)*

☐ Application Data Sheet. See 37 CFR 1.76

☐ CD(s), Number of CDs ____________

☐ Drawing(s) *Number of Sheets* ______________

☐ Other (specify) ________________________

☐ Specification (e.g. description of the invention) *Number of Pages* ______________

Fees Due: Filing Fee of $200 ($100 for small entity). If the specification and drawings exceed 100 sheets of paper, an application size fee is also due, which is $250 ($125 for small entity) for each additional 50 sheets or fraction thereof. See 35 U.S.C. 41(a)(1)(G) and 37 CFR 1.16(s).

METHOD OF PAYMENT OF THE FILING FEE AND APPLICATION SIZE FEE FOR THIS PROVISIONAL APPLICATION FOR PATENT

☐ Applicant claims small entity status. See 37 CFR 1.27.

☐ A check or money order is enclosed to cover the filing fee and application size fee (if applicable).

☐ Payment by credit card. Form PTO-2038 is attached

TOTAL FEE AMOUNT ($) ____________

☐ The Director is hereby authorized to charge the filing fee and application size fee (if applicable) or credit any overpayment to Deposit Account Number: ________________. A duplicative copy of this form is enclosed for fee processing.

USE ONLY FOR FILING A PROVISIONAL APPLICATION FOR PATENT

PROVISIONAL APPLICATION COVER SHEET
Page 2 of 2

PTO/SB/16 (02-07)
Approved for use through 02/28/2007. OMB 0651-0032
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The invention was made by an agency of the United States Government or under a contract with an agency of the United States Government.

- [✓] No.
- [ ] Yes, the name of the U.S. Government agency and the Government contract number are: ______

WARNING:

Petitioner/applicant is cautioned to avoid submitting personal information in documents filed in a patent application that may contribute to identity theft. Personal information such as social security numbers, bank account numbers, or credit card numbers (other than a check or credit card authorization form PTO-2038 submitted for payment purposes) is never required by the USPTO to support a petition or an application. If this type of personal information is included in documents submitted to the USPTO, petitioners/applicants should consider redacting such personal information from the documents before submitting them to the USPTO. Petitioner/applicant is advised that the record of a patent application is available to the public after publication of the application (unless a non-publication request in compliance with 37 CFR 1.213(a) is made in the application) or issuance of a patent. Furthermore, the record from an abandoned application may also be available to the public if the application is referenced in a published application or an issued patent (see 37 CFR 1.14). Checks and credit card authorization forms PTO-2038 submitted for payment purposes are not retained in the application file and therefore are not publicly available.

SIGNATURE Venkat Konda Date 8/1/2007

TYPED or PRINTED NAME Venkat Konda REGISTRATION NO. ______
(if appropriate)

TELEPHONE 408-472-3273 Docket Number: M-0041 US

APPLICATION # 60/940,391

Electronic Patent Application Fee Transmittal

Application Number:	60940391
Filing Date:	25-May-2007
Title of Invention:	FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS
First Named Inventor/Applicant Name:	Venkat Konda
Filer:	Venkar Konda
Attorney Docket Number:	M-0041US

Filed as Small Entity

Provisional Filing Fees

Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Basic Filing:				
Pages:				
Claims:				
Miscellaneous-Filing:				
Late provisional filing fee/cover sheet	2052	1	25	25
Petition:				
Patent-Appeals-and-Interference:				
Post-Allowance-and-Post-Issuance:				
Extension-of-Time:				

Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Miscellaneous:				
		Total in USD ($)		**25**

Electronic Acknowledgement Receipt

EFS ID:	2038490
Application Number:	60940391
International Application Number:	
Confirmation Number:	7097
Title of Invention:	FULLY CONNECTED GENERALIZED FOLDED MULTI-STAGE NETWORKS
First Named Inventor/Applicant Name:	Venkat Konda
Customer Number:	38139
Filer:	Venkar Konda
Filer Authorized By:	
Attorney Docket Number:	M-0041US
Receipt Date:	01-AUG-2007
Filing Date:	25-MAY-2007
Time Stamp:	21:42:04
Application Type:	Provisional

Payment information:

Submitted with Payment	yes
Payment was successfully received in RAM	$25
RAM confirmation Number	12363
Deposit Account	

File Listing:

Document Number	Document Description	File Name	File Size(Bytes) /Message Digest	Multi Part /.zip	Pages (if appl.)

1	Application Data Sheet	M-0041US-CoverSheet.PDF	146518 061af4022c637c818bb03be41b0bc34a924f1f4b	no	2
Warnings:					
Information:					
This is not an USPTO supplied ADS fillable form					
2	Fee Worksheet (PTO-06)	fee-info.pdf	8168 f42d1c3ed0474bad12402287ef60c2f9c82eb994	no	2
Warnings:					
Information:					
			Total Files Size (in bytes):	154686	

This Acknowledgement Receipt evidences receipt on the noted date by the USPTO of the indicated documents, characterized by the applicant, and including page counts, where applicable. It serves as evidence of receipt similar to a Post Card, as described in MPEP 503.

New Applications Under 35 U.S.C. 111
If a new application is being filed and the application includes the necessary components for a filing date (see 37 CFR 1.53(b)-(d) and MPEP 506), a Filing Receipt (37 CFR 1.54) will be issued in due course and the date shown on this Acknowledgement Receipt will establish the filing date of the application.

National Stage of an International Application under 35 U.S.C. 371
If a timely submission to enter the national stage of an international application is compliant with the conditions of 35 U.S.C. 371 and other applicable requirements a Form PCT/DO/EO/903 indicating acceptance of the application as a national stage submission under 35 U.S.C. 371 will be issued in addition to the Filing Receipt, in due course.

New International Application Filed with the USPTO as a Receiving Office
If a new international application is being filed and the international application includes the necessary components for an international filing date (see PCT Article 11 and MPEP 1810), a Notification of the International Application Number and of the International Filing Date (Form PCT/RO/105) will be issued in due course, subject to prescriptions concerning national security, and the date shown on this Acknowledgement Receipt will establish the international filing date of the application.

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APPLICATION NUMBER	FILING or 371(c) DATE	GRP ART UNIT	FIL FEE REC'D	ATTY.DOCKET.NO	TOT CLAIMS	IND CLAIMS
60/940,391	05/25/2007		125	M-0041US		

CONFIRMATION NO. 7097

UPDATED FILING RECEIPT

38139
TEAK NETWORKS, INC.
6278 GRAND OAK WAY
SAN JOSE, CA95135

Date Mailed: 09/24/2007

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Applicant(s)

Venkat Konda, San Jose, CA;

Power of Attorney: None

If Required, Foreign Filing License Granted: 06/20/2007

The country code and number of your priority application, to be used for filing abroad under the Paris Convention, is **US60/940,391**

Projected Publication Date: None, application is not eligible for pre-grant publication

Non-Publication Request: No

Early Publication Request: No

** SMALL ENTITY **

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Venkat Konda, San Jose, CA;

Power of Attorney: None

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Since the rights granted by a U.S. patent extend only throughout the territory of the United States and have no effect in a foreign country, an inventor who wishes patent protection in another country must apply for a patent in a specific country or in regional patent offices. Applicants may wish to consider the filing of an international application under the Patent Cooperation Treaty (PCT). An international (PCT) application generally has the same effect as a regular national patent application in each PCT-member country. The PCT process **simplifies** the filing of patent applications on the same invention in member countries, but **does not result** in a grant of "an international patent" and does not eliminate the need of applicants to file

additional documents and fees in countries where patent protection is desired.

Almost every country has its own patent law, and a person desiring a patent in a particular country must make an application for patent in that country in accordance with its particular laws. Since the laws of many countries differ in various respects from the patent law of the United States, applicants are advised to seek guidance from specific foreign countries to ensure that patent rights are not lost prematurely.

Applicants also are advised that in the case of inventions made in the United States, the Director of the USPTO must issue a license before applicants can apply for a patent in a foreign country. The filing of a U.S. patent application serves as a request for a foreign filing license. The application's filing receipt contains further information and guidance as to the status of applicant's license for foreign filing.

Applicants may wish to consult the USPTO booklet, "General Information Concerning Patents" (specifically, the section entitled "Treaties and Foreign Patents") for more information on timeframes and deadlines for filing foreign patent applications. The guide is available either by contacting the USPTO Contact Center at 800-786-9199, or it can be viewed on the USPTO website at http://www.uspto.gov/web/offices/pac/doc/general/index.html.

For information on preventing theft of your intellectual property (patents, trademarks and copyrights), you may wish to consult the U.S. Government website, http://www.stopfakes.gov. Part of a Department of Commerce initiative, this website includes self-help "toolkits" giving innovators guidance on how to protect intellectual property in specific countries such as China, Korea and Mexico. For questions regarding patent enforcement issues, applicants may call the U.S. Government hotline at 1-866-999-HALT (1-866-999-4158).

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Title 35, United States Code, Section 184

Title 37, Code of Federal Regulations, 5.11 & 5.15

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