TIE-29: Refractive Index and Dispersion

0. Introduction

The most important property of optical glass is the refractive index and its dispersion behavior.
This technical information gives an overview of the following topics:

- Dispersion
  - Principal dispersion (page 2)
  - Secondary spectrum (page 3)
  - Sellmeier dispersion equation (page 4)
- Temperature dependence of refractive index (page 6)
- Influence of the fine annealing process on the refractive index and Abbe number (page 9)
- Tolerances (page 12)
- Refractive index measurement (page 13)

1. Refractive Index

If light enters a non-absorbing homogeneous materials reflection and refraction occurs at the boundary surface. The refractive index n is given by the ratio of the velocity of light in vacuum c to that of the medium v

$$n = \frac{c}{v} \qquad (1\text{-}1)$$

The refractive index data given in the glass catalogue are measured relative to the refractive index measured in air. The refractive index of air is very close to 1.

Practically speaking the refractive index is a measure for the strength of deflection occurring at the boundary surface due to the refraction of the light beam. The equation describing the amount of deflection is called Snell's law:

$$n_1 \cdot \sin(\alpha_1) = n_2 \cdot \sin(\alpha_2) \qquad (1\text{-}2)$$

The refractive index is a function of the wavelength. The most common characteristic quantity for characterization of an optical glass is the refractive index n in the middle range of the visible spectrum. This principal refractive index is usually denoted as n_d – the refractive index at the wavelength 587.56 nm or in many cases as n_e at the wavelength 546.07 nm.

2. Wavelength Dependence of Refractive Index: Dispersion

The dispersion is a measure of the change of the refractive index with wavelength. Dispersion can be explained by applying the electromagnetic theory to the molecular structure of matter. If an electromagnetic wave impinges on an atom or a molecule the bound charges vibrate at the frequency of the incident wave.

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The bound charges have resonance frequency at a certain wavelength. A plot of the refractive index as a function of the wavelength for fused silica can be seen in Figure 2-1. It can be seen that in the main spectral transmission region the refractive index increases towards shorter wavelength. Additionally the dotted line shows the absorption coefficient as a function of the wavelength.
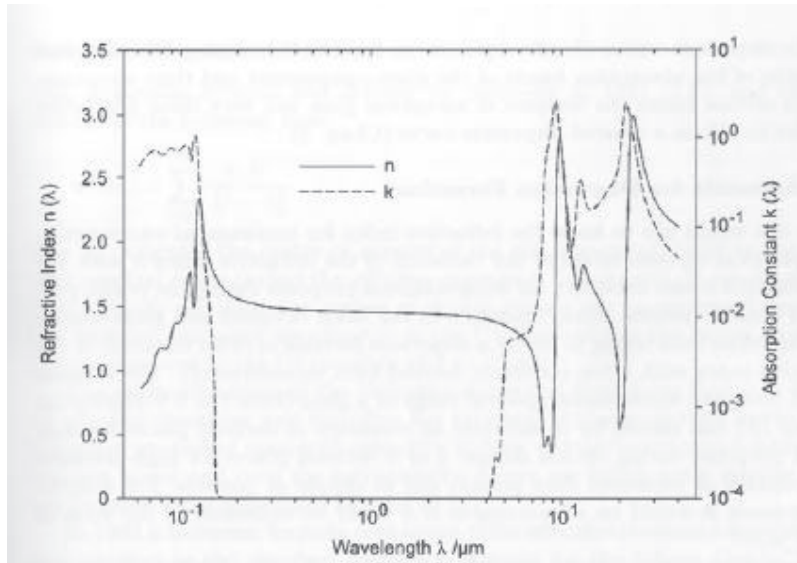

Figure 2-1: Measured optical constants of fused silica (SiO$_2$ glass) [1]

2.1 Principal Dispersion

The difference $(n_F - n_C)$ is called the principal dispersion. n_F and n_C are the refractive indices at the 486.13 nm and 656.27 nm wavelengths.
The most common characterization of the dispersion of optical glasses is the Abbe number. The Abbe number is defined as

$$v_d = (n_d - 1)/(n_F - n_C) \qquad\qquad (2.1\text{-}1)$$

Sometimes the Abbe number is defined according to the e line as

$$v_e = (n_e - 1)/(n_{F'} - n_{C'}) \qquad\qquad (2.1\text{-}2).$$

Traditionally optical glasses in the range of $v_d > 50$ are called crown glasses, the other ones as flint glasses.

Glasses having a low refractive index in general also have a low dispersion behaviour e.g. a high Abbe number. Glasses having a high refractive index have a high dispersion behaviour and a low Abbe number.

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2.2. Secondary Spectrum

The characterization of optical glass through refractive index and Abbe number alone is insufficient for high quality optical systems. A more accurate description of the glass properties is achievable with the aid of the relative partial dispersions.

The relative partial dispersion $P_{x,y}$ for the wavelengths x and y is defined by the equation:

$$P_{x,y} = (n_x - n_y)/(n_F - n_C) \tag{2.2-1}$$

As Abbe demonstrated, the following linear relationship will approximately apply to the majority of glasses, the so-called "normal glasses"

$$P_{x,y} \approx a_{xy} + b_{xy} \cdot v_d \tag{2.2-2}$$

a_{xy} and b_{xy} are specific constants for the given relative partial dispersion.
In order to correct the secondary spectrum (i.e. color correction for more than two wavelengths) glasses are required which do not conform to this rule. Therefore glass types having deviating partial dispersion from Abbe's empirical rule are especially interesting.

As a measure of the deviation of the partial dispersion from Abbe's rule the ordinate difference ΔP is introduced. Instead of relation (2.2-2) the following generally valid equation is used:

$$P_{x,y} = a_{xy} + b_{xy} \cdot v_d + \Delta P_{x,y} \tag{2.2-3}$$

The term $\Delta P_{x,y}$ therefore quantitatively describes a dispersion behavior that deviates from that of the "normal glasses."

The deviations $\Delta P_{x,y}$ from the "normal lines" are listed for the following five relative partial dispersions for each glass type in the data sheets.

$$P_{C,t} = (n_C - n_t)/(n_F - n_C)$$
$$P_{C,s} = (n_C - n_s)/(n_F - n_C)$$
$$P_{F,e} = (n_F - n_e)/(n_F - n_C) \tag{2.2-4}$$
$$P_{g,F} = (n_g - n_F)/(n_F - n_C)$$
$$P_{i,g} = (n_i - n_g)/(n_F - n_C)$$

The position of the normal lines is determined based on value pairs of the glass types K7 and F2. The explicit formulas for the deviations $\Delta P_{x,y}$ of the above-mentioned five relative partial dispersions are:

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$$\Delta P_{C,t} = (n_C - n_t)/(n_F - n_C) - (0.5450 + 0.004743 \cdot v_d)$$
$$\Delta P_{C,s} = (n_C - n_s)/(n_F - n_C) - (0.4029 + 0.002331 \cdot v_d)$$
$$\Delta P_{F,e} = (n_F - n_e)/(n_F - n_C) - (0.4884 - 0,000526 \cdot v_d) \qquad (2.2\text{-}5)$$
$$\Delta P_{g,F} = (n_g - n_F)/(n_F - n_C) - (0.6438 - 0.001682 \cdot v_d)$$
$$\Delta P_{i,g} = (n_i - n_g)/(n_F - n_C) - (1.7241 - 0.008382 \cdot v_d)$$

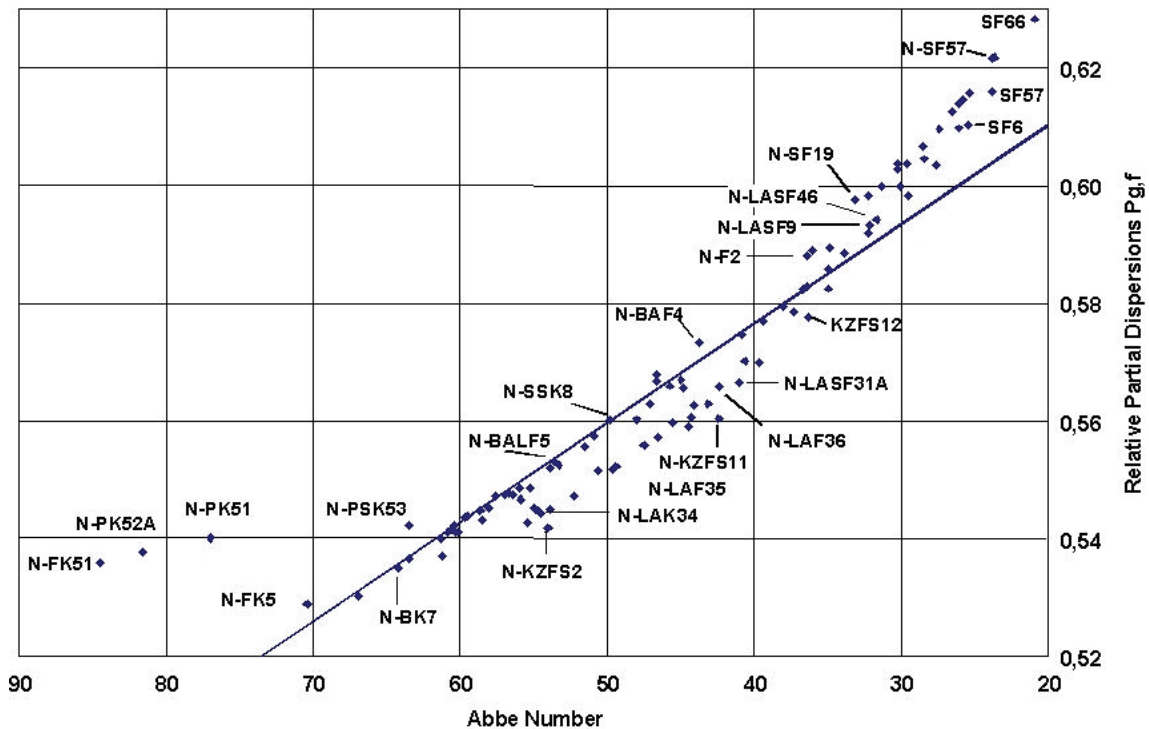Figure 2.2-1 shows the $P_{g,F}$ versus the Abbe number v_d diagram.

Figure 2.2-1: $P_{g,F}$ as a function of the Abbe number for Schott's optical glass sortiment. Additionally the normal line is given.

The relative partial dispersions listed in the catalog were calculated from refractive indices to 6 decimal places. The dispersion formula (2.3-1) can be used to interpolate additional unlisted refractive indices and relative partial dispersions (see chapter 2.3).

2.3. Sellmeier Dispersion Equation

The Sellmeier Equation is especially suitable for the progression of refractive index in the wavelength range from the UV through the visible to the IR area (to 2.3 μm). It is derived from the classical dispersion theory and allows the description of the progression of refractive index over the total transmission region with one set of data and to calculate accurate intermediate values.

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$$n^2(\lambda) - 1 = \frac{B_1 \cdot \lambda^2}{(\lambda^2 - C_1)} + \frac{B_2 \cdot \lambda^2}{(\lambda^2 - C_2)} + \frac{B_3 \cdot \lambda^2}{(\lambda^2 - C_3)} \qquad\qquad (2.3\text{-}1)$$

The determination of the coefficients was performed for all glass types on the basis of precision measurements by fitting the dispersion equation to the measurement values. The coefficients are listed in the data sheets.

The dispersion equation is only valid within the spectral region in which refractive indices are listed in the data sheets of each glass. Interpolation is possible within these limits. The wavelengths used in the equation have to be inserted in µm with the same number of digits as listed in Table 2.3-1. For practical purposes Equation 2.3-1 applies to refractive indices in air at room temperature. The achievable precision of this calculation is generally better than $1 \cdot 10^{-5}$ in the visible spectral range. The coefficients of the dispersion equation can be reported for individual glass parts upon request. This requires a precision measurement for the entire spectral region, provided the glass has sufficient transmission.

Wavelength [nm]	Designation	Spectral Line Used	Element
2325.42		infrared mercury line	Hg
1970.09		infrared mercury line	Hg
1529.582		infrared mercury line	Hg
1060.0		neodymium glass laser	Nd
1013.98	t	infrared mercury line	Hg
852.11	s	infrared cesium line	Cs
706.5188	r	red helium line	He
656.2725	C	red hydrogen line	H
643.8469	C'	red cadmium line	Cd
632.8		helium-neon-gas-laser	He-Ne
589.2938	D	yellow sodium line	Na
		(center of the double line)	
587.5618	d	yellow helium line	He
546.074	e	green mercury line	Hg
486.1327	F	blue hydrogen line	H
479.9914	F'	blue cadmium line	Cd
435.8343	g	blue mercury line	Hg
404.6561	h	violet mercury line	Hg
365.0146	i	ultraviolet mercury line	Hg
334.1478		ultraviolet mercury line	Hg
312.5663		ultraviolet mercury line	Hg
296.7278		ultraviolet mercury line	Hg
280.4		ultraviolet mercury line	Hg
248.3		ultraviolet mercury line	Hg

Table 2.3-1:   Wavelengths for a selection of frequently used spectral lines

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