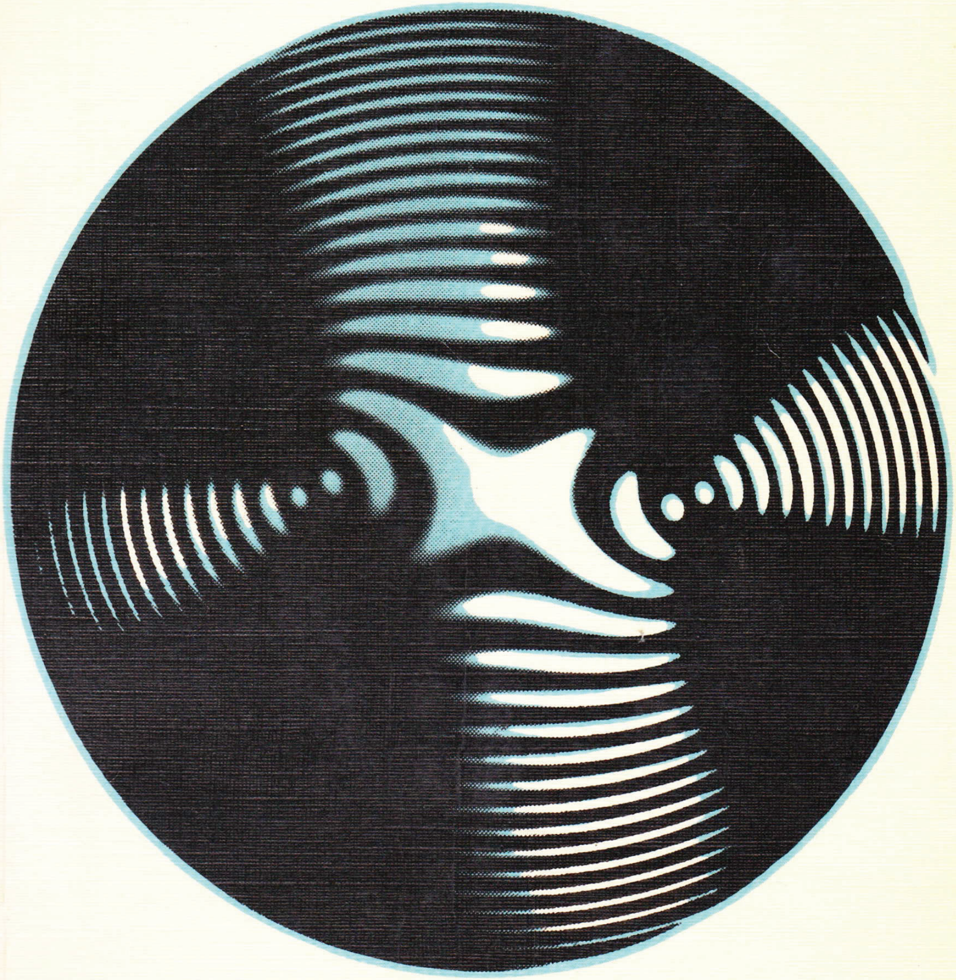
Principles of Optics

ELECTROMAGNETIC THEORY OF PROPAGATION INTERFERENCE AND DIFFRACTION OF LIGHT

Sixth Edition

MAX BORN & EMIL WOLF



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Principles of Optics

Electromagnetic Theory of Propagation, Interference and Diffraction of Light

by

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PREFACE

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* MAX BORN, *Optik* (Berlin,

This relation gives the position of the focus F_1 of the refracted rays. From (14a) it is seen that the focal line through F_1 is perpendicular to the yz-plane so that F_1 is a *primary focus*.

To find the position of the other focus, consider the rays which proceed from F_0'. Then $z_0 = \zeta_0'$, $\delta x_0 = \delta y_0 = \delta z_0 = 0$. Since all these rays intersect the focal line f_0, $\delta q_0 = \delta m_0 = 0$. Equations (14) and (16) now give

$$\frac{\zeta_0'}{n_0} \sec \theta_0 \delta p_0 - \frac{1}{\mu} r_z(\delta p_1 - \delta p_0) = 0, \tag{14b}$$

and

$$\delta q_1 = 0. \tag{16b}$$

(16b) shows that the refracted rays now lie in the xz-plane. All these rays will pass through the other focus $F_1'(z_1 = \zeta_1')$, so that (15) must be satisfied with $z_1 = \zeta_1'$, $\delta x_1 = \delta y_1 = \delta z_1 = 0$, whatever the value of δp_0. Hence,

$$\frac{\zeta_1'}{n_1} \sec \theta_1 \delta p_1 - \frac{1}{\mu} r_x(\delta p_1 - \delta p_0) = 0. \tag{15b}$$

Since (15b) and (14b) hold simultaneously for any arbitrary value of δp_0, it follows that

$$\frac{n_0 \cos \theta_0}{\zeta_0'} - \frac{n_1 \cos \theta_1}{\zeta_1'} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{r_x}. \tag{19}$$

This relation gives the position of the *secondary focus* F_1'.

It is often convenient to specify the position of the foci by means of their distances from O rather than by means of their z coordinates. If $OF_0 = d_0^{(t)}$, $OF_0' = d_0^{(s)}$, $OF_1 = d_1^{(t)}$, $OF_1' = d_1^{(s)}$ (in Fig. 4.22 $d_0^{(t)} < 0$, $d_0^{(s)} < 0$, $d_1^{(t)} > 0$, $d_1^{(s)} > 0$), then

$$\left.\begin{aligned} \zeta_0 &= d_0^{(t)} \cos \theta_0, & \zeta_1 &= d_1^{(t)} \cos \theta_1, \\ \zeta_0' &= d_0^{(s)} \cos \theta_0, & \zeta_1' &= d_1^{(s)} \cos \theta_1, \end{aligned}\right\} \tag{20}$$

and the two relations (18) and (19) become

$$\frac{n_0 \cos^2 \theta_0}{d_0^{(t)}} - \frac{n_1 \cos^2 \theta_1}{d_1^{(t)}} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{r_y}, \tag{21}$$

and

$$\frac{n_0}{d_0^{(s)}} - \frac{n_1}{d_1^{(s)}} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{r_x}. \tag{22}$$

The corresponding relations for reflection may be obtained by setting $n_1 = -n_0$.

4.7 CHROMATIC ABERRATION. DISPERSION BY A PRISM

In Chapter II it was shown that the refractive index is not a material constant but depends on colour, i.e. on the wavelength of light. We shall now discuss some elementary consequences of this result in relation to the performance of lenses and prisms.

4.7.1 Chromatic aberration

If a ray of polychromatic light is incident upon a refracting surface, it is split into a set of rays, each of which is associated with a different wavelength. In traversing an optical system, light of different wavelengths will therefore, after the first refraction, follow slightly different paths. In consequence, the image will not be sharp and the system is said to suffer from *chromatic aberration*.

We shall again confine our atten
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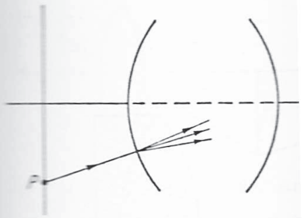

Fig. 4.23. The longit

where n_F, n_D and n_C are the refra
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Let us now examine under what
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We shall again confine our attention to points and rays in the immediate neighbourhood of the axis, i.e. it will be assumed that the imaging in each wavelength obeys the laws of Gaussian optics. The chromatic aberration is then said to be of the first order, or primary. If Q_α and Q_β are the images of a point P in two different wavelengths (Fig. 4.23), the projections of $Q_\alpha Q_\beta$ in the directions parallel and perpendicular to the axis are known as *longitudinal* and *lateral* chromatic aberration respectively.

Consider the change δf in the focal length of a thin lens, due to a change δn in the refractive index. According to § 4.4 (36) the quantity $(n-1)f$ will, for a given lens, be independent of the wavelength. Hence

$$\frac{\delta f}{f} + \frac{\delta n}{n-1} = 0. \tag{1}$$

The quantity

$$\Delta = \frac{n_F - n_C}{n_D - 1}, \tag{2}$$

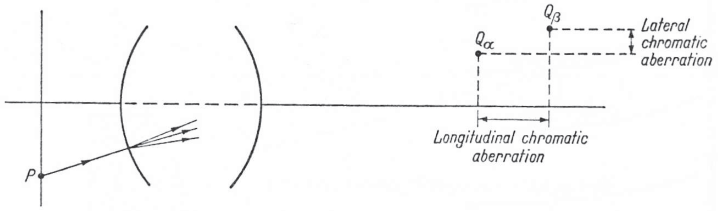

Fig. 4.23. The longitudinal and lateral chromatic aberration.

where n_F, n_D and n_C are the refractive indices for the Fraunhofer F, D and C lines ($\lambda = 4861$ Å, 5893 Å, 6563 Å respectively) is a rough measure of the dispersive properties of the glass, and is called the *dispersive power*. From (1) it is seen that it is approximately equal to the distance between the red and blue image divided by the focal length of the lens, when the object is at infinity. The variation with wavelength, of the refractive index of the usual types of glass employed in optical systems is shown in Fig. 4.24. The corresponding values of Δ lie between about 1/60 and 1/30.

To obtain an image of good quality, the monochromatic as well as the chromatic aberrations must be small. Usually a compromise has to be made, since in general it is impossible to eliminate all the aberrations simultaneously. Often it is sufficient to eliminate the chromatic aberration for two selected wavelengths only. The choice of these wavelengths will naturally depend on the purpose for which the system is designed; for example, since the ordinary photographic plate is more sensitive to the blue region than is the human eye, photographic objectives are usually "achromatized" for colours nearer to the blue end of the spectrum than is the case in visual instruments. Achromatization with respect to two wavelengths does, of course, not secure a complete removal of the colour error. The remaining chromatic aberration is known as *the secondary spectrum.*

Let us now examine under what conditions two thin lenses will form an achromatic combination with respect to their focal lengths. According to § 4.4 (39) the reciprocal of the focal length of a combination of two thin lenses separated by a distance l is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{l}{f_1 f_2}. \tag{3}$$