

# Comparison of Diversity Combining Techniques for GSM

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**ABSTRACT** This paper studies diversity combining techniques in the GSM based systems. A pre-detection maximal-ratio diversity receiver for GSM<sup>1</sup> is presented. It is an optimum diversity demodulator for band-limited channels with Gaussian noise and intersymbol interference. The performance of diversity combining techniques is studied in the GSM data channel. The effects of correlation between diversity channels is also studied. The best diversity combining technique is found to be the presented pre-detection maximal-ratio technique.

## 1. Introduction

The number of users in mobile radio is growing rapidly setting tighter requirements for frequency band usage. In the development of GSM to the phase 2+ and the development of UMTS, the ultimate performance of the GSM based systems must be researched. One limiting factor in the performance of the GSM radio interface is multipath fading. To reduce the effects of fading several well known diversity techniques can be applied either in the transmitter or the receiver. If the diversity branches are with low correlation significant performance gains can be achieved. The information in diversity signals must be combined as efficiently as possible, otherwise some decibels of potential diversity gain can be lost.

In this paper we focus on the diversity combining in receiver and assume that there are two diversity paths. In the uplink these paths can be created by using space diversity, which is already used in the GSM base stations. In the downlink pure space diversity is not so useful as in the uplink, because mobile station antennas can not be physically far enough separated. One promising diversity technique in mobiles is polarization diversity.

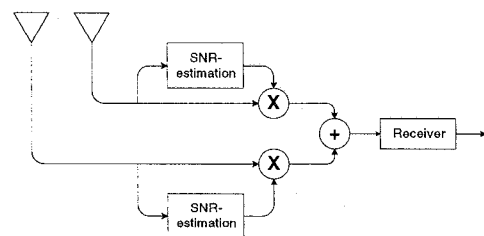
This paper is organized as follows. First we introduce the readers to the diversity combining and the performance of theoretical diversity. Next we present an optimal pre-detection diversity combining. Then the simulation models in the link level is presented. Next we show the simulation results and achieved diversity gains. Finally the conclusions are drawn.

## 2. Diversity Combining Techniques

Diversity combining can be done before or after signal detection. If diversity combining is done after signal detection, it is said to be post-detection combining, otherwise it is pre-detection combining. In the case of pre-detection combining diversity signals must be co-phased before they can be combined.

There are four major techniques to combine the signals from diversity paths:

1. Selection combining (SC) means that the best of two received signals is selected according some quality measurement, which can be e.g. signal level, power or signal-to-noise ratio. In the GSM based systems the burst-type transmission is used and it is best to do diversity selection burst by burst.
2. Switched combining means that one of the diversity signals is selected, based on a given threshold level in one receiver. If one signal above some threshold is selected, it is received until the signal falls below the threshold level.
3. Maximal-ratio combining (MRC), first proposed by Kahn [1], is a weighted sum of the input signals. Weighting factor can be signal level, power or signal-to-noise ratio. Following figure describes the maximal-ratio principle.



**Figure 1.** The principle of a two antenna maximal-ratio receiver

4. Equal-gain combining (EGC) means that baseband signals are summed. In the equal-gain combining there is no signal scaling like in the maximal-ratio combining.

If we have  $L$  diversity channels which are carrying the same information and which are independently

bit-error-probability for theoretical diversity. In the case of the binary PSK this can be expressed as follows [2]

$$P_2 = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k, \quad (1)$$

$$\mu = \sqrt{\frac{\gamma_c}{1+\gamma_c}}, \quad (2)$$

where  $\gamma_c$  represents the average signal-to-noise ratio per channel.

### 3. Optimal Pre-detection Diversity

In this section we show the optimum diversity demodulator for band-limited channel with Gaussian noise and intersymbol interference. This derivation is a generalization of the derivation given by Proakis [2]. The received signal can be written as

$$r(t) = \sum_n I_n h(t-nT) + z(t), \quad (3)$$

where  $h(t)$  represents the response of the channel,  $z(t)$  represents the additive Gaussian noise and  $I_n$  is the information sequence.

In the following we consider two branch diversity receiver, although the result can be generalized to any number of branches. Maximum-likelihood demodulator maximizes the joint probability function  $p(\mathbf{r}_N | \mathbf{I}_p)$  of the random variables  $\mathbf{r}_N$  conditioned by the transmitted sequence  $\mathbf{I}_p$ . In the case of two branch diversity receiver, which receives the uncorrelated sequences  $\mathbf{r}_k^{(1)}$  and  $\mathbf{r}_k^{(2)}$ , the joint probability can be expressed as

$$\begin{aligned} p(\mathbf{r}_N^{(1)}, \mathbf{r}_N^{(2)} | \mathbf{I}_p) &= p(\mathbf{r}_N^{(1)} | \mathbf{I}_p) p(\mathbf{r}_N^{(2)} | \mathbf{I}_p) \\ &= \left( \prod_{k=1}^N 2\pi\sigma^2 \right)^{-1} \exp \left( -\frac{1}{2\sigma^2} \sum_{k=1}^N \left( \left| r_k^{(1)} - \sum_n I_n h_{kn}^{(1)} \right|^2 + \left| r_k^{(2)} - \sum_n I_n h_{kn}^{(2)} \right|^2 \right) \right). \end{aligned} \quad (4)$$

The logarithm of the joint probability is proportional to the quantity  $J_0$ , defined as

$$J_0(\mathbf{I}_p) = -\int_{-\infty}^{\infty} \left( \left| r(t)^{(1)} - \sum_n I_n h(t-nT)^{(1)} \right|^2 + \left| r(t)^{(2)} - \sum_n I_n h(t-nT)^{(2)} \right|^2 \right) dt \quad (5)$$

The most probable transmitted sequence is the one which maximizes this quantity. A computationally

efficient method to implement the maximum likelihood estimation is the Viterbi algorithm. The usage of the Viterbi algorithm requires recursive formulation of (5). For ISI channels Forney [5] has first derived the recursive form of (5). In the following we use the alternative metrics computation defined by Ungerboeck [3]

$$\begin{aligned} J_n(\mathbf{I}_n) &= J_{n-1}(\mathbf{I}_{n-1}) + \text{Re} \left[ I_n^* \left( 2y_n^{(1)} + a_0^{(1)} I_{n-2} - \sum_{m \leq n-1} I_m a_{n-m}^{(1)} \right) \right] \\ &+ \text{Re} \left[ I_n^* \left( 2y_n^{(2)} + a_0^{(2)} I_{n-2} - \sum_{m \leq n-1} I_m a_{n-m}^{(2)} \right) \right] \\ &= J_{n-1}(\mathbf{I}_{n-1}) + \text{Re} \left[ I_n^* \left( 2(y_n^{(1)} + y_n^{(2)}) - I_n (a_0^{(1)} + a_0^{(2)}) - 2 \sum_{m \leq n-1} I_m (a_{n-m}^{(1)} + a_{n-m}^{(2)}) \right) \right] \end{aligned} \quad (6)$$

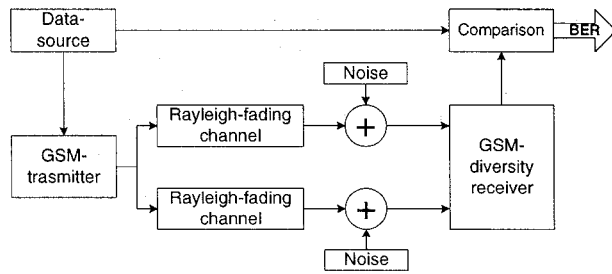
where  $\{a_k\}$ 's represent the autocorrelation values of the channel impulse response and  $\{y_k\}$ 's are the outputs of the channel matched filters. From (6) it is evident that an optimum maximum-likelihood diversity receiver can be implemented by summing the output signals of the two channel matched filters and summing the autocorrelation values (see fig. 3). This implementation has an advantage that the Viterbi metrics computation of no-diversity receiver can be used.

### 4. Simulation Model

The performance of diversity combining techniques is evaluated by means of bit level simulations of the GSM full rate data channel (TCH/F9.6). The simulation model includes blocks for transmitter, channel, noise and receiver. The simulation principle is the Monte Carlo method and COSSAP simulation tool is used. The following basic simplifications are done:

1. Complex envelope presentation for the radio channel and RF parts of the receiver and the transmitter.
2. Analog radio signals are presented in digital domain.
3. The time and frequency synchronization is assumed to be ideal.

The used channel propagation model is Typical Urban with mobile speed 50 km/h, specified in [4]. There is also a possibility to specify correlation between diversity channels. Random Gaussian noise is added in the both signals after Rayleigh fading channel. The carrier frequency 1750 MHz and ideal frequency hopping are used.



**Figure 2.** *The simulation environment*

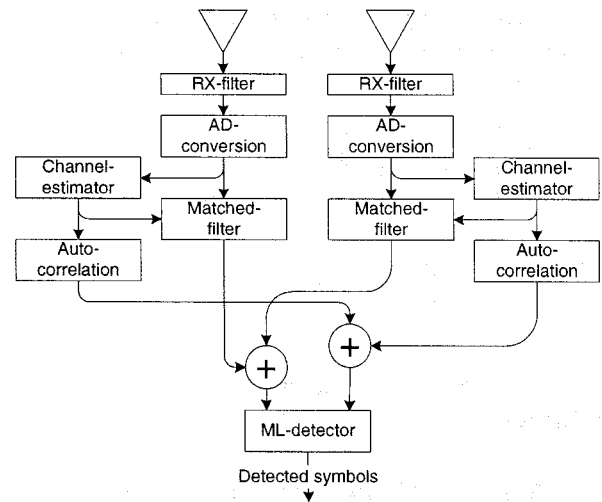
Four different diversity receivers are studied:

1. pre-detection maximal-ratio, defined by (6).
2. post-detection maximal-ratio
3. pre-detection equal-gain
4. selection

In the receivers signals are at the first RX-filtered and A/D-converted. After this channel impulse responses are estimated and signals are filtered by the channel matched filter and autocorrelation values of the channel impulse response estimates are calculated. In the pre-detection maximal-ratio receiver signals are summed at this point and after this combined signal is detected in the ML-detector. Pre-detection equal-gain receiver is similar, except the impulse response estimates of both channels are forced to have equal energy. In the post-detection maximal-ratio receiver both signals are detected separately and after this they are scaled by the signal-to-noise ratio and then they are summed. In the selection receiver both signals are detected separately, which is followed by the selection of the best signal according the signal-to-noise ratio. Fig. 3 illustrates a block diagram of the pre-detection maximal-ratio receiver.

## 5. Simulation Results

Diversity combining is simulated at the first in a single path Rayleigh fading channel without channel coding and the results are shown in Fig. 4. Also theoretical values according (1) are shown. The performance of the pre-detection diversity and no-diversity receivers are about 2 dB weaker than theoretical results. The reason for this is the imperfect channel estimation caused by noise and residual intersymbol interference.



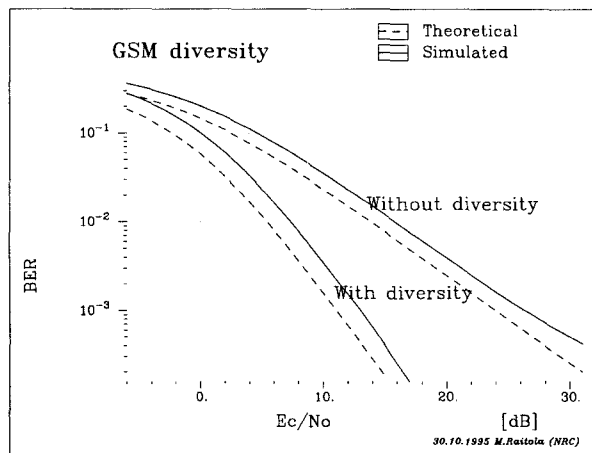
**Figure 3.** *Pre-detection maximal-ratio diversity receiver*

**Table 1.** Diversity gains in the noise limited environment

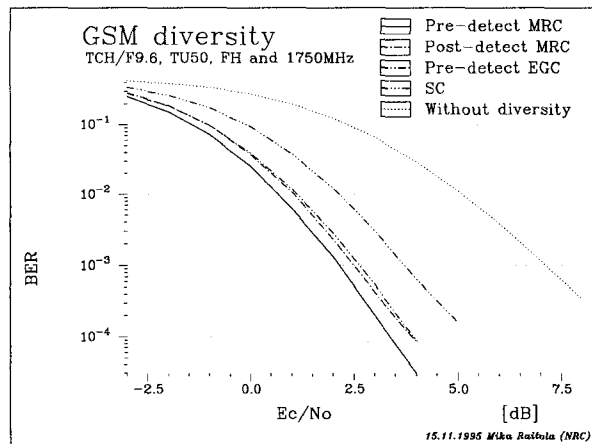
Receiver	Diversity Gain
Pre-detection maximal-ratio	4.8 dB
Post-detection maximal-ratio	4.4 dB
Pre-detection equal-gain	4.2 dB
Selection	3.2 dB

As a next step the performance of diversity combining is studied in the GSM 9.6 kbit/s data channel in the noise limited environment. The best combining technique is found to be the pre-detection maximal-ratio combining. The results are shown in the Fig. 5 and the diversity gains of various receivers in the GSM specified performance point (BER 0.3%) [4] are shown in the Table 1.

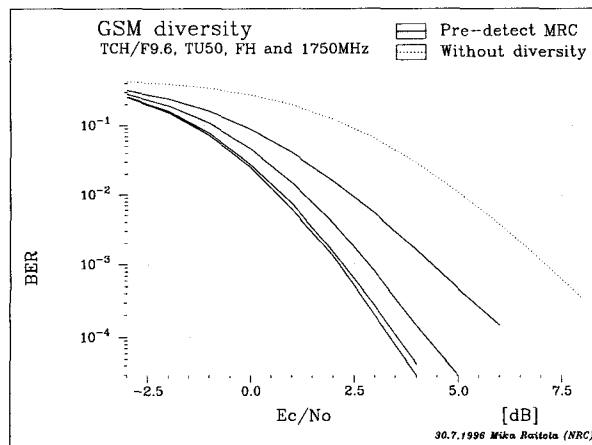
Next we study how presented pre-detection maximal-ratio receiver perform if there is correlation between the diversity channels. The receiver is simulated with 0.0, 0.3, 0.7 and 1.0 correlation between the diversity channels and the results are shown in the Fig. 6. When compared to the 0.0 correlation case the correlation of 0.3 decreases the performance of pre-detection receiver 0.1 dB and the correlation of 0.7 decreases the performance 0.7 dB. When correlation is 1.0 there still is 3.0 dB diversity gain left. The reason is that the signals are summed coherently and noise is summed non-coherently.



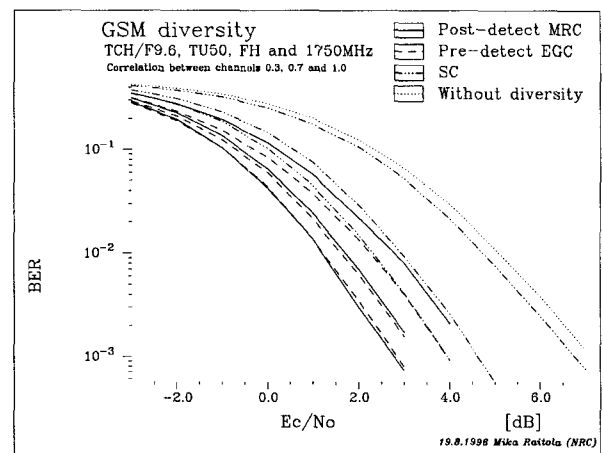
**Figure 4.** Theoretical diversity vs. simulated diversity in the single path Rayleigh fading channel.



**Figure 5.** The BER performance of diversity receivers in the noise limited GSM 9.6 kbit/s data channel (TCH/F9.6).



**Figure 6.** The BER performance of the pre-detection maximal-ratio receiver when correlation between diversity branches is 0.0, 0.3, 0.7 and 1.0.



**Figure 7.** The BER performance of the post-detection maximal-ratio, pre-detection equal-gain and selection receiver when correlation between diversity branches is 0.3, 0.7 and 1.0.

Finally we study how other three diversity receivers perform if there is correlation of 0.3, 0.7 and 1.0 between the diversity channels. The results are shown in Fig. 7. Post-detection maximal-ratio and pre-detection equal-gain receivers have almost the same performance when correlation is 0.3 or 0.7, but when correlation is 1.0 post-detection combining gives 2.3 dB diversity gain and pre-detection 3.0 dB gain.

## 6. Summary

In this study we have done a comparison between four diversity combining techniques. A pre-detection maximal-ratio diversity receiver for GSM has been presented.

The performance of different diversity combining techniques is studied by Monte Carlo simulations in the GSM data channel. The best diversity combining technique is found to be the pre-detection maximal-ratio combining which gives 4.8 dB diversity gain. The post-detection maximal-ratio and the pre-detection equal-gain receivers have diversity gains of 4.4 dB and 4.2 dB. The selection receiver is the weakest.

A small amount of correlation (0.3) between diversity branches has only a slight effect to the performance of diversity. When diversity signals are totally correlated post-detection combining gives 2.3 dB gain. In this case pre-detection combining sums signals coherently and noise non-coherently and thus gives theoretical 3.0 dB diversity gain.

## 7. References

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