## **Distinctive Image Features from Scale-Invariant Keypoints**

### DAVID G. LOWE

Computer Science Department, University of British Columbia, Vancouver, B.C., Canada Lowe@cs.ubc.ca

Received January 10, 2003; Revised January 7, 2004; Accepted January 22, 2004

**Abstract.** This paper presents a method for extracting distinctive invariant features from images that can be used to perform reliable matching between different views of an object or scene. The features are invariant to image scale and rotation, and are shown to provide robust matching across a substantial range of affine distortion, change in 3D viewpoint, addition of noise, and change in illumination. The features are highly distinctive, in the sense that a single feature can be correctly matched with high probability against a large database of features from many images. This paper also describes an approach to using these features for object recognition. The recognition proceeds by matching individual features to a database of features from known objects using a fast nearest-neighbor algorithm, followed by a Hough transform to identify clusters belonging to a single object, and finally performing verification through least-squares solution for consistent pose parameters. This approach to recognition can robustly identify objects among clutter and occlusion while achieving near real-time performance.

**Keywords:** invariant features, object recognition, scale invariance, image matching

#### 1. Introduction

Image matching is a fundamental aspect of many problems in computer vision, including object or scene recognition, solving for 3D structure from multiple images, stereo correspondence, and motion tracking. This paper describes image features that have many properties that make them suitable for matching differing images of an object or scene. The features are invariant to image scaling and rotation, and partially invariant to change in illumination and 3D camera viewpoint. They are well localized in both the spatial and frequency domains, reducing the probability of disruption by occlusion, clutter, or noise. Large numbers of features can be extracted from typical images with efficient algorithms. In addition, the features are highly distinctive, which allows a single feature to be correctly matched with high probability against a large database of features, providing a basis for object and scene recognition.

The cost of extracting these features is minimized by taking a cascade filtering approach, in which the more

expensive operations are applied only at locations that pass an initial test. Following are the major stages of computation used to generate the set of image features:

- Scale-space extrema detection: The first stage of computation searches over all scales and image locations. It is implemented efficiently by using a difference-of-Gaussian function to identify potential interest points that are invariant to scale and orientation.
- Keypoint localization: At each candidate location, a
  detailed model is fit to determine location and scale.
  Keypoints are selected based on measures of their
  stability.
- 3. Orientation assignment: One or more orientations are assigned to each keypoint location based on local image gradient directions. All future operations are performed on image data that has been transformed relative to the assigned orientation, scale, and location for each feature, thereby providing invariance to these transformations.



4. Keypoint descriptor: The local image gradients are measured at the selected scale in the region around each keypoint. These are transformed into a representation that allows for significant levels of local shape distortion and change in illumination.

This approach has been named the Scale Invariant Feature Transform (SIFT), as it transforms image data into scale-invariant coordinates relative to local features.

An important aspect of this approach is that it generates large numbers of features that densely cover the image over the full range of scales and locations. A typical image of size 500 × 500 pixels will give rise to about 2000 stable features (although this number depends on both image content and choices for various parameters). The quantity of features is particularly important for object recognition, where the ability to detect small objects in cluttered backgrounds requires that at least 3 features be correctly matched from each object for reliable identification.

For image matching and recognition, SIFT features are first extracted from a set of reference images and stored in a database. A new image is matched by individually comparing each feature from the new image to this previous database and finding candidate matching features based on Euclidean distance of their feature vectors. This paper will discuss fast nearest-neighbor algorithms that can perform this computation rapidly against large databases.

The keypoint descriptors are highly distinctive, which allows a single feature to find its correct match with good probability in a large database of features. However, in a cluttered image, many features from the background will not have any correct match in the database, giving rise to many false matches in addition to the correct ones. The correct matches can be filtered from the full set of matches by identifying subsets of keypoints that agree on the object and its location, scale, and orientation in the new image. The probability that several features will agree on these parameters by chance is much lower than the probability that any individual feature match will be in error. The determination of these consistent clusters can be performed rapidly by using an efficient hash table implementation of the generalized Hough transform.

Each cluster of 3 or more features that agree on an object and its pose is then subject to further detailed verification. First, a least-squared estimate is made for an affine approximation to the object pose. Any other

image features consistent with this pose are identified, and outliers are discarded. Finally, a detailed computation is made of the probability that a particular set of features indicates the presence of an object, given the accuracy of fit and number of probable false matches. Object matches that pass all these tests can be identified as correct with high confidence.

#### 2. Related Research

The development of image matching by using a set of local interest points can be traced back to the work of Moravec (1981) on stereo matching using a corner detector. The Moravec detector was improved by Harris and Stephens (1988) to make it more repeatable under small image variations and near edges. Harris also showed its value for efficient motion tracking and 3D structure from motion recovery (Harris, 1992), and the Harris corner detector has since been widely used for many other image matching tasks. While these feature detectors are usually called corner detectors, they are not selecting just corners, but rather any image location that has large gradients in all directions at a predetermined scale.

The initial applications were to stereo and short-range motion tracking, but the approach was later extended to more difficult problems. Zhang et al. (1995) showed that it was possible to match Harris corners over a large image range by using a correlation window around each corner to select likely matches. Outliers were then removed by solving for a fundamental matrix describing the geometric constraints between the two views of rigid scene and removing matches that did not agree with the majority solution. At the same time, a similar approach was developed by Torr (1995) for long-range motion matching, in which geometric constraints were used to remove outliers for rigid objects moving within an image.

The ground-breaking work of Schmid and Mohr (1997) showed that invariant local feature matching could be extended to general image recognition problems in which a feature was matched against a large database of images. They also used Harris corners to select interest points, but rather than matching with a correlation window, they used a rotationally invariant descriptor of the local image region. This allowed features to be matched under arbitrary orientation change between the two images. Furthermore, they demonstrated that multiple feature matches could accomplish general recognition under occlusion and



clutter by identifying consistent clusters of matched features.

The Harris corner detector is very sensitive to changes in image scale, so it does not provide a good basis for matching images of different sizes. Earlier work by the author (Lowe, 1999) extended the local feature approach to achieve scale invariance. This work also described a new local descriptor that provided more distinctive features while being less sensitive to local image distortions such as 3D viewpoint change. This current paper provides a more in-depth development and analysis of this earlier work, while also presenting a number of improvements in stability and feature invariance.

There is a considerable body of previous research on identifying representations that are stable under scale change. Some of the first work in this area was by Crowley and Parker (1984), who developed a representation that identified peaks and ridges in scale space and linked these into a tree structure. The tree structure could then be matched between images with arbitrary scale change. More recent work on graph-based matching by Shokoufandeh et al. (1999) provides more distinctive feature descriptors using wavelet coefficients. The problem of identifying an appropriate and consistent scale for feature detection has been studied in depth by Lindeberg (1993, 1994). He describes this as a problem of scale selection, and we make use of his results below.

Recently, there has been an impressive body of work on extending local features to be invariant to full affine transformations (Baumberg, 2000; Tuytelaars and Van Gool, 2000; Mikolajczyk and Schmid, 2002; Schaffalitzky and Zisserman, 2002; Brown and Lowe, 2002). This allows for invariant matching to features on a planar surface under changes in orthographic 3D projection, in most cases by resampling the image in a local affine frame. However, none of these approaches are yet fully affine invariant, as they start with initial feature scales and locations selected in a non-affineinvariant manner due to the prohibitive cost of exploring the full affine space. The affine frames are are also more sensitive to noise than those of the scale-invariant features, so in practice the affine features have lower repeatability than the scale-invariant features unless the affine distortion is greater than about a 40 degree tilt of a planar surface (Mikolajczyk, 2002). Wider affine invariance may not be important for many applications, as training views are best taken at least every 30 degrees rotation in viewpoint (meaning that recognition

is within 15 degrees of the closest training view) in order to capture non-planar changes and occlusion effects for 3D objects.

While the method to be presented in this paper is not fully affine invariant, a different approach is used in which the local descriptor allows relative feature positions to shift significantly with only small changes in the descriptor. This approach not only allows the descriptors to be reliably matched across a considerable range of affine distortion, but it also makes the features more robust against changes in 3D viewpoint for nonplanar surfaces. Other advantages include much more efficient feature extraction and the ability to identify larger numbers of features. On the other hand, affine invariance is a valuable property for matching planar surfaces under very large view changes, and further research should be performed on the best ways to combine this with non-planar 3D viewpoint invariance in an efficient and stable manner.

Many other feature types have been proposed for use in recognition, some of which could be used in addition to the features described in this paper to provide further matches under differing circumstances. One class of features are those that make use of image contours or region boundaries, which should make them less likely to be disrupted by cluttered backgrounds near object boundaries. Matas et al. (2002) have shown that their maximally-stable extremal regions can produce large numbers of matching features with good stability. Mikolajczyk et al. (2003) have developed a new descriptor that uses local edges while ignoring unrelated nearby edges, providing the ability to find stable features even near the boundaries of narrow shapes superimposed on background clutter. Nelson and Selinger (1998) have shown good results with local features based on groupings of image contours. Similarly, Pope and Lowe (2000) used features based on the hierarchical grouping of image contours, which are particularly useful for objects lacking detailed texture.

The history of research on visual recognition contains work on a diverse set of other image properties that can be used as feature measurements. Carneiro and Jepson (2002) describe phase-based local features that represent the phase rather than the magnitude of local spatial frequencies, which is likely to provide improved invariance to illumination. Schiele and Crowley (2000) have proposed the use of multidimensional histograms summarizing the distribution of measurements within image regions. This type of feature may be particularly useful for recognition of textured objects with



deformable shapes. Basri and Jacobs (1997) have demonstrated the value of extracting local region boundaries for recognition. Other useful properties to incorporate include color, motion, figure-ground discrimination, region shape descriptors, and stereo depth cues. The local feature approach can easily incorporate novel feature types because extra features contribute to robustness when they provide correct matches, but otherwise do little harm other than their cost of computation. Therefore, future systems are likely to combine many feature types.

### 3. Detection of Scale-Space Extrema

As described in the introduction, we will detect keypoints using a cascade filtering approach that uses efficient algorithms to identify candidate locations that are then examined in further detail. The first stage of keypoint detection is to identify locations and scales that can be repeatably assigned under differing views of the same object. Detecting locations that are invariant to scale change of the image can be accomplished by searching for stable features across all possible scales, using a continuous function of scale known as scale space (Witkin, 1983).

It has been shown by Koenderink (1984) and Lindeberg (1994) that under a variety of reasonable assumptions the only possible scale-space kernel is the Gaussian function. Therefore, the scale space of an image is defined as a function,  $L(x, y, \sigma)$ , that is produced from the convolution of a variable-scale Gaussian,  $G(x, y, \sigma)$ , with an input image, I(x, y):

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y),$$

where \* is the convolution operation in x and y, and

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}.$$

To efficiently detect stable keypoint locations in scale space, we have proposed (Lowe, 1999) using scale-space extrema in the difference-of-Gaussian function convolved with the image,  $D(x, y, \sigma)$ , which can be computed from the difference of two nearby scales separated by a constant multiplicative factor k:

$$D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y)$$
$$= L(x, y, k\sigma) - L(x, y, \sigma). \tag{1}$$

There are a number of reasons for choosing this function. First, it is a particularly efficient function to compute, as the smoothed images, L, need to be computed in any case for scale space feature description, and D can therefore be computed by simple image subtraction.

In addition, the difference-of-Gaussian function provides a close approximation to the scale-normalized Laplacian of Gaussian,  $\sigma^2 \nabla^2 G$ , as studied by Lindeberg (1994). Lindeberg showed that the normalization of the Laplacian with the factor  $\sigma^2$  is required for true scale invariance. In detailed experimental comparisons, Mikolajczyk (2002) found that the maxima and minima of  $\sigma^2 \nabla^2 G$  produce the most stable image features compared to a range of other possible image functions, such as the gradient, Hessian, or Harris corner function.

The relationship between D and  $\sigma^2 \nabla^2 G$  can be understood from the heat diffusion equation (parameterized in terms of  $\sigma$  rather than the more usual  $t = \sigma^2$ ):

$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G.$$

From this, we see that  $\nabla^2 G$  can be computed from the finite difference approximation to  $\partial G/\partial \sigma$ , using the difference of nearby scales at  $k\sigma$  and  $\sigma$ :

$$\sigma \nabla^2 G = \frac{\partial G}{\partial \sigma} \approx \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

and therefore,

$$G(x, y, k\sigma) - G(x, y, \sigma) \approx (k-1)\sigma^2 \nabla^2 G$$
.

This shows that when the difference-of-Gaussian function has scales differing by a constant factor it already incorporates the  $\sigma^2$  scale normalization required for the scale-invariant Laplacian. The factor (k-1) in the equation is a constant over all scales and therefore does not influence extrema location. The approximation error will go to zero as k goes to 1, but in practice we have found that the approximation has almost no impact on the stability of extrema detection or localization for even significant differences in scale, such as  $k = \sqrt{2}$ .

An efficient approach to construction of  $D(x, y, \sigma)$  is shown in Fig. 1. The initial image is incrementally convolved with Gaussians to produce images separated by a constant factor k in scale space, shown stacked in the left column. We choose to divide each octave of scale space (i.e., doubling of  $\sigma$ ) into an integer number, s, of intervals, so  $k = 2^{1/s}$ . We must produce s + 3



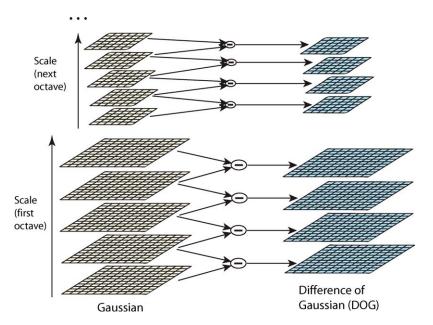


Figure 1. For each octave of scale space, the initial image is repeatedly convolved with Gaussians to produce the set of scale space images shown on the left. Adjacent Gaussian images are subtracted to produce the difference-of-Gaussian images on the right. After each octave, the Gaussian image is down-sampled by a factor of 2, and the process repeated.

images in the stack of blurred images for each octave, so that final extrema detection covers a complete octave. Adjacent image scales are subtracted to produce the difference-of-Gaussian images shown on the right. Once a complete octave has been processed, we resample the Gaussian image that has twice the initial value of  $\sigma$  (it will be 2 images from the top of the stack) by taking every second pixel in each row and column. The accuracy of sampling relative to  $\sigma$  is no different than for the start of the previous octave, while computation is greatly reduced.

#### 3.1. Local Extrema Detection

In order to detect the local maxima and minima of  $D(x, y, \sigma)$ , each sample point is compared to its eight neighbors in the current image and nine neighbors in the scale above and below (see Fig. 2). It is selected only if it is larger than all of these neighbors or smaller than all of them. The cost of this check is reasonably low due to the fact that most sample points will be eliminated following the first few checks.

An important issue is to determine the frequency of sampling in the image and scale domains that is needed to reliably detect the extrema. Unfortunately, it turns out that there is no minimum spacing of sam-

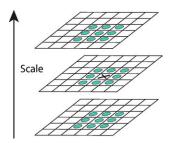


Figure 2. Maxima and minima of the difference-of-Gaussian images are detected by comparing a pixel (marked with X) to its 26 neighbors in  $3 \times 3$  regions at the current and adjacent scales (marked with circles).

ples that will detect all extrema, as the extrema can be arbitrarily close together. This can be seen by considering a white circle on a black background, which will have a single scale space maximum where the circular positive central region of the difference-of-Gaussian function matches the size and location of the circle. For a very elongated ellipse, there will be two maxima near each end of the ellipse. As the locations of maxima are a continuous function of the image, for some ellipse with intermediate elongation there will be a transition from a single maximum to two, with the maxima arbitrarily close to each other near the transition.



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