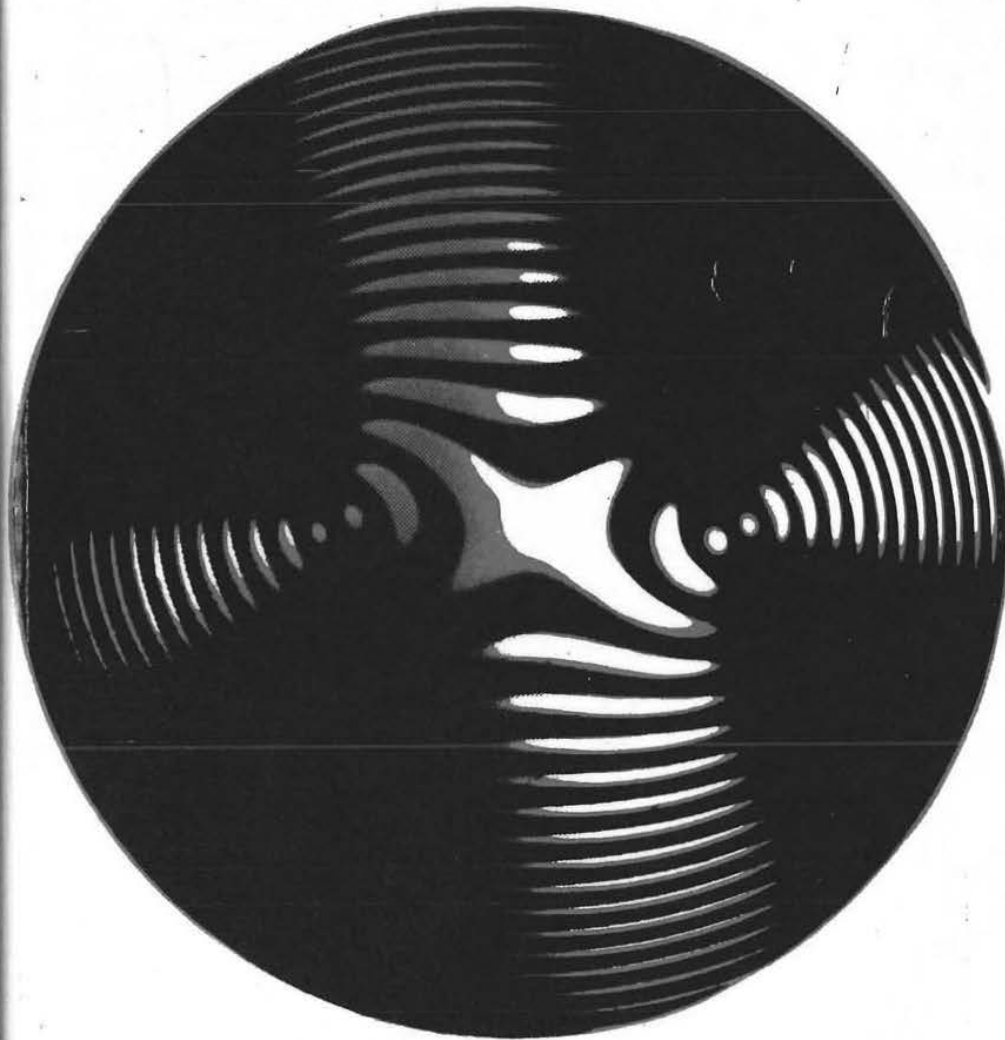


# Principles of Optics

ELECTROMAGNETIC THEORY OF PROPAGATION  
INTERFERENCE AND DIFFRACTION OF LIGHT

Sixth Edition

MAX BORN & EMIL WOLF



Pergamon Press

# Principles of Optics

*Electromagnetic Theory of Propagation,  
Interference and Diffraction of Light*

by

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Let  $c$  be the distance between the focal points  $F'_0$  and  $F_1$ . Since the image space of the first transformation coincides with the object space of the second,

$$Z_1 = Z'_0 - c, \quad Y_1 = Y'_0. \quad (23)$$

Elimination of the coordinates of the intermediate space from (22) by means of (23) gives

$$\left. \begin{aligned} Y'_1 &= \frac{Z'_1 Y_1}{f'_1} = \frac{Z'_1 Y'_0}{f'_1} = \frac{Z'_1 f_0 Y_0}{f'_1 Z_0} = \frac{f_0 f_1 Y_0}{f_0 f'_0 - c Z_0}, \\ Z'_1 &= \frac{f_1 f'_1}{Z_1} = \frac{f_1 f'_1}{Z'_0 - c} = \frac{f_1 f'_1}{\frac{f_0 f'_0}{Z_0} - c} = \frac{f_1 f'_1 Z_0}{f_0 f'_0 - c Z_0} \end{aligned} \right\} \quad (24)$$

Let

$$\left. \begin{aligned} Y &= Y_0, & Z &= Z_0 - \frac{f_0 f'_0}{c}, \\ Y' &= Y'_1, & Z' &= Z'_1 + \frac{f_1 f'_1}{c} \end{aligned} \right\} \quad (25)$$

Equations (25) express a change of coordinates, the origins of the two systems being shifted by distances  $f_0 f'_0 / c$  and  $-f_1 f'_1 / c$  respectively in the  $Z$ -direction. In terms of these variables, the equations of the combined transformation become

$$\frac{Y'}{Y} = \frac{f}{Z} = \frac{Z'}{f'}, \quad (26)$$

where

$$f = -\frac{f_0 f_1}{c}, \quad f' = \frac{f'_0 f'_1}{c}. \quad (27)$$

The distance between the origins of the new and the old systems of coordinates, i.e. the distances  $\delta = F_0 F'$  and  $\delta' = F'_1 F'$  of the foci of the equivalent transformation from the foci of the individual transformations are seen from (25) to be

$$\delta = \frac{f_0 f'_0}{c}, \quad \delta' = -\frac{f_1 f'_1}{c}. \quad (28)$$

If  $c = 0$ , then  $f = f' = \infty$  so that the equivalent collineation is telescopic. The equations (24) then reduce to

$$\left. \begin{aligned} Y'_1 &= \frac{f_1}{f'_0} Y_0, \\ Z'_1 &= \frac{f_1 f'_1}{f_0 f'_0} Z_0; \end{aligned} \right\} \quad (29)$$

the constants  $\alpha$  and  $\beta$  in (18) of the equivalent transformation, are therefore

$$\alpha = \frac{f_1}{f'_0}, \quad \beta = \frac{f_1 f'_1}{f_0 f'_0}. \quad (30)$$

The angular magnification is now

$$\frac{\tan \gamma'}{\tan \gamma} = \frac{\alpha}{\beta} = \frac{f_0}{f'_1}. \quad (31)$$

If one or both of the transformations are telescopic, the above considerations must be somewhat modified.

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**4.4.1 Refra**  
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## 4.4 GAUSSIAN OPTICS

We shall now study the elementary properties of lenses, mirrors, and their combinations. In this elementary theory only those points and rays will be considered which lie in the immediate neighbourhood of the axis; terms involving squares and higher powers of off-axis distances, or of the angles which the rays make with the axis, will be neglected. The resulting theory is known as Gaussian optics.\*

## 4.4.1 Refracting surface of revolution

Consider a pencil of rays incident on a refracting surface of revolution which separates two homogeneous media of refractive indices  $n_0$  and  $n_1$ . To begin with, points and rays in both media will be referred to the same Cartesian reference system, whose origin will be taken at the pole  $O$  of the surface, with the  $z$ -direction along the axis of symmetry.

Let  $P_0(x_0, y_0, z_0)$  and  $P_1(x_1, y_1, z_1)$  be points on the incident and on the refracted ray respectively. Neglecting terms of degree higher than first, it follows from § 4.1 (29), § 4.1 (40), and § 4.1 (44) that the coordinates of these points and the components of the two rays are connected by the relations

$$\left. \begin{aligned} x_0 - \frac{p_0}{n_0} z_0 &= \frac{\partial T^{(2)}}{\partial p_0} = 2\mathcal{A}p_0 + \mathcal{C}p_1, \\ x_1 - \frac{p_1}{n_1} z_1 &= -\frac{\partial T^{(2)}}{\partial p_1} = -2\mathcal{B}p_1 - \mathcal{C}p_0, \end{aligned} \right\} \quad (1a)$$

$$\left. \begin{aligned} y_0 - \frac{q_0}{n_0} z_0 &= \frac{\partial T^{(2)}}{\partial q_0} = 2\mathcal{A}q_0 + \mathcal{C}q_1, \\ y_1 - \frac{q_1}{n_1} z_1 &= -\frac{\partial T^{(2)}}{\partial q_1} = -2\mathcal{B}q_1 - \mathcal{C}q_0, \end{aligned} \right\} \quad (1b)$$

where, according to § 4.1 (45),

$$\mathcal{A} = \mathcal{B} = \frac{1}{2} \frac{r}{n_1 - n_0}, \quad \mathcal{C} = -\frac{r}{n_1 - n_0}, \quad (2)$$

$r$  being the paraxial radius of curvature of the surface.

Let us examine under what conditions all the rays from  $P_0$  (which may be assumed to lie in the plane  $x = 0$ ) will, after refraction, pass through  $P_1$ . The coordinates of  $P_1$  will then depend only on the coordinates of  $P_0$  and not on the components of the rays, so that when  $q_1$  is eliminated from (1b),  $q_0$  must also disappear.

Now from the first equation (1b)

$$q_1 = \frac{1}{\mathcal{C}} \left\{ y_0 - q_0 \left( 2\mathcal{A} + \frac{1}{n_0} z_0 \right) \right\}, \quad (3)$$

and substituting this into the second equation, we obtain

$$y_1 = - \left( 2\mathcal{B} - \frac{1}{n_1} z_1 \right) \frac{1}{\mathcal{C}} y_0 + \left[ \frac{1}{\mathcal{C}} \left( 2\mathcal{B} - \frac{1}{n_1} z_1 \right) \left( 2\mathcal{A} + \frac{1}{n_0} z_0 \right) - \mathcal{C} \right] q_0. \quad (4)$$

\* As before, the usual sign convention of analytical geometry (Cartesian sign convention) is used. The various sign conventions employed in practice are very fully discussed in a Report on the Teaching of Geometrical Optics published by the Physical Society (London) in 1934.

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