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Pipelined H-Trees for High-Speed Clocking of Large Integrated Systems in Presence of Process Variations

Mohamed Nekili, Guy Bois, and Yvon Savaria, *Member, IEEE*



Abstract—This paper addresses the problem of clocking large high-speed digital systems, as well as deterministic skew modeling, a related problem. A conventional method for clocking a large digital system is to use a set of metallic lines organized as a tree. This method is limited by the bandwidth of the clock network. Another limitation of existing solutions is that available skew models do not directly take into account process variations. In order to provide a reliable skew model, and to avoid the frequency limitation, we propose a novel approach that distributes the clock with an H-tree, whose branches are composed of minimum-sized inverters rather than metal. With such a structure, we obtain the highest clocking rate achievable with a given technology. Indeed, clock rates around 1 GHz are possible with a 1.2 μm CMOS technology. From the skew modeling standpoint, we derive an analytic expression of the skew between two leaves of the H-tree, which we consider to be the difference in root-to-leaf delay pairs. The skew upper bound obtained has an order of complexity which, with respect to the H-tree size D , is the same as the one that may be derived from the Fisher and Kung model for both side-to-side and neighbor-to-neighbor communications, i.e., a $\Omega(D^2)$, whereas, the Steiglitz and Kugelmass probabilistic model predicts $\Theta(D \times \sqrt{\log D})$. In an H-tree implemented with metallic lines, the leaf-to-leaf skew is obviously bounded by the delay between the root and the leaves. However, with the logic based H-tree proposed in this paper, we arrive at a nonobvious result, which states that the leaf-to-leaf skew grows faster than the root-to-leaf delay in presence of a uniform transistor time constant gradient. This paper also proposes generalizations of the skew model to 1) the case of chips in a wafer subject to a smooth, but nonuniform gradient and 2) the case of H-tree configurations mixing logic and interconnections; in this respect, this paper covers the H-tree configurations based on the combination of logic and interconnections.

Index Terms—H-tree, high-speed clocking, pipelining, process variations, skew.

I. INTRODUCTION

THE evolution of VLSI chips toward larger die sizes and faster clock speeds makes clock design an increasingly important issue. A striking example of what can be accomplished with aggressive clock design is the DEC alpha chip [1],

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designed to operate at more than 200 MHz. At such speeds, clock skew becomes a very significant problem. Available literature dealing with skew [2]–[8], [10], [11] approaches the problem both from deterministic and probabilistic standpoints.

In the deterministic approaches, Friedmann and Powell [6] emphasize the use of a hierarchical clock distribution, while others [2], [3], [8], [11] suggest the length equalization of the different paths followed by the clock throughout the circuit. Shoji [5] suggests an approach that guarantees a symmetry between paths that contribute to propagate "0" and "1." This symmetry ensures proper operation despite some types of process variations [5]. Except for the work of Fisher and Kung [4], which provides bounds on skew, the other authors do not deal with the analytic modeling of system skew.

In the probabilistic approaches, Kugelmass and Steiglitz [7] consider the delay of a clock signal along a given path as a sum of delays along path segments, each of these segments behaving according to a probabilistic law. Then, by assuming independence between these delays, the total delay, as well as the skew, can then be described by a normal law. By assuming independence and the linearity of delay with line length, their approach becomes an oversimplification of the reality. Other authors [10] consider the skew as a dispersion in the physical parameters of a circuit (e.g., geometrical dimensions) and in the process (e.g., sensitivity to temperature).

The work that is most directly related to that presented in this paper is the work of Fisher and Kung [4]. These authors have developed two deterministic skew models (the difference model and the summation model), from which they determined bounds on skew. However, these models do not directly refer to a process variation model. The difference model tends to be unrealistically optimistic, whereas, under the summation model, Fisher and Kung reached a pessimistic result which states that, from a skew standpoint, synchronous systems are not feasible with large two-dimensional arrays.

In order to avoid the frequency limitation when using metallic lines, we propose a logic-based H-tree structure that provides the highest clocking rate achievable with a given technology in Section II. To provide a reliable skew model, Section III suggests a model based on delay differences combined with a model of electrical variations in the process parameters. Under this model, we derive an analytic expression of the skew between any leaf pair, which we consider to be the difference in root-to-leaf delay pairs. Even though the model of electrical variations described in this paper assumes

VLSI Array Algorithms and Architectures for RSA Modular Multiplication

Yong-Jin Jeong, *Member, IEEE*, and Wayne P. Burleson, *Member, IEEE*

Abstract—We present two novel iterative algorithms and their array structures for integer modular multiplication. The algorithms are designed for Rivest–Shamir–Adelman (RSA) cryptography and are based on the familiar iterative *Horner's rule*, but use precalculated complements of the modulus. The problem of deciding which multiples of the modulus to subtract in intermediate iteration stages has been simplified using simple look-up of precalculated complement numbers, thus allowing a finer-grain pipeline. Both algorithms use a carry save adder scheme with modulo reduction performed on each intermediate partial product which results in an output in carry-save format. Regularity and local connections make both algorithms suitable for high-performance array implementation in FPGA's or deep submicron VLSI. The processing nodes consist of just one or two full adders and a simple multiplexor. The stored complement numbers need to be precalculated only when the modulus is changed, thus not affecting the performance of the main computation. In both cases, there exists a bit-level systolic schedule, which means the array can be fully pipelined for high performance and can also easily be mapped to linear arrays for various space/time tradeoffs.

Index Terms—Cryptography, modular multiplication, RSA, systolic arrays, VLSI.

I. INTRODUCTION

CRYPTOGRAPHY systems have been growing in importance recently as a method for improving data security. *Public key cryptography* (PKC) systems are generally preferred to traditional *secret key cryptography* systems like the *data encryption standard* due to the safety of key distribution [3]. The Rivest–Shamir–Adelman (RSA) [10] system is one of the most widely used public key cryptography systems, and its core arithmetic is modular multiplication over a positive integer. Modular multiplication is also a major computation of *residue number systems* [13] as well as other cryptography systems (e.g., *international data encryption algorithm* [8], [16], *Diffie–Hellman key exchange* [3]). In this paper, we develop an array modular multiplier with applications to, but not restricted to, RSA systems.

In RSA, the modulus is a product of two large prime numbers, usually more than 500 bits, and should be changeable for security reasons. But, since the modulus (or key) is not changed very often, we can use precomputation and look-up in our array modular multipliers. We are not aware of anyone

who has utilized this special property of *multirate input data* in the RSA algorithm, that is, the *input message* changes rapidly while the *key* remains unchanged for a long period. In practice, the key is updated infrequently, for example, a few months, weeks, or days, depending on the security requirements. In order to satisfy the ever growing security requirements of high-speed communications, such as personal communication services and wireless local area networks, a *dedicated* VLSI hardware solution is needed because of 1) high throughput requirements, 2) low-power requirements, 3) a high-volume market, 4) the computation is poorly suited to microprocessors or DSP's, and 5) the problem size is expected to continue to grow rather than saturate.

Modular multiplication is generally considered a complicated arithmetic operation because of the inherent multiplication and division operations. There are two main approaches to computing modular multiplication: 1) perform the modulo operation *after* multiplication or 2) *during* multiplication. The modulo operation is accomplished by integer division in which only the remainder is needed for further computation. The first approach requires a $n \times n$ bit multiplier with a $2n$ -bit register followed by a $2n \times n$ bit divider. In the second approach, the modulo operation occurs in each iteration step of integer multiplication. Therefore the first approach requires more hardware while the second requires more addition/subtraction computations due to $O(n)$ modulo reduction steps. In both cases, most previous research has focused on the fast calculation of a long carry chain. Redundant number systems and a higher radix carry-save form are some of the different number representations that have been used for this purpose [12], [14]. A carry prediction technique has also been used for fast calculation of modular multiplication [1].

Since PKC was introduced, many algorithms and hardware structures have been proposed for modular multiplication, and [4] contains a good review on this topic. Several array structures suited for VLSI implementation have been discussed in [4], [5], [14], and [15]. In [14], Vandemeulebroecke *et al.*, use a *modulo after multiplication* approach using a *signed digit* number representation. It consists of two arrays: one for multiplication and the other for integer division. In [5], Koc and Hung apply Blakley's algorithm [2] and use a sign-estimation method by looking at the five most significant bits in each iteration stage. Although they derive a bit-level systolic array structure, the latency and clock cycle are relatively long due to the control node which estimates the sign of the intermediate result in each stage. In [4] and [15], Eldridge and

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