Declaration of Steve Wasserman

I, Steve Wasserman, declare that:

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2. I am the owner of Retriev-it, located at 324 South Beverly Drive Suite 200, Beverly Hills, CA 90212 ("Retriev-it"). Retriev-it is an information retrieval company. I have owned and worked for Retriev-it since approximately 2000.

3. As part of my duties, I retrieve information related to publications from various libraries in California. For example, if a customer needs to find out when a publication was added to a certain library's catalog, but there is no online record, I will go the library, search for the book through a library's catalog, retrieve the book from the shelf, and inspect the book to determine if there is a date stamp.

4. On August 10, 2018, I went to the library of the University of California, Los Angeles ("UCLA"). While at UCLA, I reviewed the article entitled "Modified Generalized Concatenated Codes and Their Application To The Construction And Decoding of LUEP Codes," included as part of the IEEE Transaction on Information Theory, Vol. 41, Issue 5, September 1995 (the "Information Theory"). I copied the cover page and introductory pages of the Information Theory. The cover page and page 1217 contain stamps showing that the Information Theory was entered into UCLA's collection on September 20, 1995. Attached hereto as Exhibit A are true and correct copies of the cover page and page 1217 of the Information Theory.

5. On August 13, 2018, I returned to UCLA, where I reviewed the article entitled "Some Constructions of Optimal Binary Linear Unequal Error Protection Codes," included as part of the Philips Journal of Research, Vol. 39, no. 6, 1984

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6. On August 15, 2018, I once again returned to UCLA, where I reviewed the article entitled "Future Trends in Optical Recording," included as part of the Philips Technical Review, Vol. 44, no. 2, April 1988 (the "Philips Review"). I copied the cover page that contains a stamp showing that the Philips Review was entered into UCLA's collection on June 8, 1988. Attached hereto as Exhibit C is a true and correct copy of the cover page of the Philips Review.

I declare under the penalty of perjury under the laws of the United States of America that the foregoing is true and correct.

Executed on August 16, 2018 in Los Angeles, California.

Steve Wasserman

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CORRESPONDENCE

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Modified Generalized Concatenated Codes and their Application to the Construction and Decoding of LUEP Codes

Uwe Dettmar, Yan Gao, and Ulrich K. Sorger

Abstract—**We propose a modification of generalized concatenated codes, which allows us to construct some of the best known binary codes in a simple way. Furthermore, a large class of optimal linear unequal error protection codes (LUEP codes) can easily be generated. All constructed codes can be efficiently decoded by the Blokh–Zyablov–Zinov'ev algorithm if an appropriate metric is used.**

Index Terms—**Linear unequal error protection codes, generalized concatenated codes, multistage decoding.**

I. Introduction

Many of the best known codes can be constructed as Generalized Concatenated (GC) codes [2], [3]. Generally, the constructions of GC codes use different outer codes A_i of constant length n_a but only one inner code $B^{(1)}$ together with its partition. However, this restriction is not necessary. In this correspondence we construct GC codes consisting of outer codes A_i with different lengths $n_{a,i}$, and inner codes $B_j^{(1)}$ in the columns of the code matrix with different lengths $n_{b,j}$ and distances $d_{b,j}^{(i)}$ together with their partitions.

In Section II, modified GC codes are defined and a lower bound on their minimum distance, a designed minimum distance, is derived. Two examples of the construction of good binary codes as modified GC codes are given. In Section III, the modification is used to construct binary optimal linear unequal error protection (LUEP) codes. In Section IV, a decoding algorithm for the modified GC codes is presented, which allows decoding up to half their designed minimum distance. Moreover, decoding of the constructed LUEP codes up to half the separation vector is discussed.

For the sake of simplicity only binary codes are considered in this correspondence. An extension to the nonbinary case is straightforward.

II. Definition of Modified GC Codes

GC codes are a generalization of concatenated codes defined by Forney [4]. The inner code is multiply partitioned, and this partitioning into subcodes is protected by different outer codes. Denote the m outer codes by $A_i = A_i(q_i;\, n_a,\, k_{a,i},\, d_{a,i})$, where $q_i = 2^{r_i}$ is the alphabet size, n_a is the code length, $k_{a,i}$ is the number of information symbols, and $d_{a,i}$ is the Hamming distance of the code. Further, denote by $B^{(i)} = B^{(i)}(2;\, n_b,\, k_{b,i},\, d_{b,i})$, for $i = 1, 2, \cdots, m$, the binary inner codes, where

$$B^{(i+1)} \subset B^{(i)} \quad \text{and} \quad k_b^{(i)} - k_b^{(i+1)} = \log_2(q_i).$$

By concatenating inner and outer codes we get a GC code with the

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TABLE I
EXAMPLE 1

Inner codes				Outer codes			GC code
	$j = 1, 2, \ldots, 8$	$j = 9$	$j = 10$				
$B_j^{(1)}$	$(8, 4, 4)$	$(4, 1, 4)$	$(7, 3, 4)$	A_1	$(2^3; 10, 7bit, 8)$	$\mathcal{J}_1 = \{1, ..., 10\}$	$(75, 11, 32)$
$B_j^{(2)}$	$(8, 1, 8)$	$(4, 0, \infty)$	$(7, 0, \infty)$	A_2	$(2; 8, 4, 4)$	$\mathcal{J}_2 = \{1, ..., 8\}$	.
$B_j^{(1)}$	$(8, 4, 4)$	$(4, 3, 2)$	$(7, 3, 4)$	A_1	$(2^3; 10, 3, 8)$	$\mathcal{J}_1 = \{1, ..., 10\}$	$(75, 13, 30)$
$B_j^{(2)}$	$(8, 1, 8)$	$(4, 0, \infty)$	$(7, 0, \infty)$	A_2	$(2; 8, 4, 4)$	$\mathcal{J}_2 = \{1, ..., 8\}$	

parameters [3]

$$n = n_a n_b,$$

$$k = \sum_{i=1}^{m} k_{a,\,i} \log_2(q_i)$$

$$d \geq \min\{d_{a,1} d_{b,1},\, d_{a,2} d_{b,2},\, \cdots,\, d_{a,m} d_{b,m}\}.$$

The lower bound on the minimum distance is derived as follows [3]: since the codes are linear, the minimum weight and distance are equal. If $a_1 \neq 0$, with $a_1 \in A_1$, then not less than $d_{a,\,1}$ codewords of the inner code $B^{(1)}$ are different from the zero word. However, this inner code has a minimum weight $d_{b,\,1}$ which leads to a minimum weight of $d_{a,\,1} d_{b,\,1}$ for the appropriate codeword of the GC code. Similar considerations hold for the other stages.

In the above definition the outer codes all have equal length n_a. However, this restriction is not necessary: denote the outer codes by

$$A_i = A_i(q_i;\, n_{a,\,i},\, k_{a,\,i},\, d_{a,\,i})$$

where the $n_{a,\,i}$ now can be different. Define a set of indices $\mathcal{J}_i$ with $|\mathcal{J}_i| = n_{a,\,i}$ and $j_{\max}$ so that

$$1 \leq j \leq j_{\max}, \quad \forall j \in \mathcal{J}_i$$

holds. The inner codes are given by

$$B_j^{(i)} = B_j^{(i)}(2;\, n_{b,\,j},\, k_{b,\,j}^{(i)},\, d_{b,\,j}^{(i)})$$

for $i = 1, 2, \cdots, m$ and $j = 1, \cdots, j_{\max}$, with

$$B_j^{(i+1)} \subseteq B_j^{(i)} \text{ and } k_{b,\,j}^{(i)} - k_{b,\,j}^{(i+1)} = \begin{cases} \log_2(q_i), & \text{if } j \in \mathcal{J}_i \\ 0, & \text{if } j \notin \mathcal{J}_i. \end{cases}$$

We assume that $\cup_i \mathcal{J}_i = \mathcal{J}$, with $j \in \mathcal{J}$ for $j = 1, 2, \cdots, j_{\max}$, because in this case all inner codes are concatenated with some outer codes.

The GC code has the parameters

$$n = \sum_{j=1}^{j_{\max}} n_{b,\,j} \qquad k = \sum_{i=1}^{m} k_{a,\,i} \log_2(q_i).$$

The minimum distance of the GC code is given by

$$d \geq \min_i \min_{\mathcal{S}_i} \sum_{j \in \mathcal{S}_i} d_{b,\,j}^{(i)} \tag{1}$$

where $\mathcal{S}_i \subseteq \mathcal{J}_i$ with $|\mathcal{S}_i| = d_{a,\,i}$.

The lower bound for the minimum distance is derived similarly to that for conventional GC codes: the minimum weight in stage i is attained if the positions where $a_i \neq 0$ coincide with codewords of the inner codes $B_j^{(i)}$ with the smallest minimum distances. This leads to (1).

To simplify this expression, we define by $\Pi(j)$ the permutation of the indices $j = 1, \cdots, n_{a,\,i}$ so that

$$d_{b,\,\Pi(1)} \leq d_{b,\,\Pi(2)} \leq \cdots \leq d_{b,\,\Pi(n_{a,\,i})}.$$

This results in

$$d \geq \min_{\forall i} \sum_{j=1}^{d_{a,\,i}} d_{b,\,\Pi(j)}^{(i)}.$$

TABLE II
CONSTRUCTION 1

Inner codes		Outer codes		
	$j = 1, ..., n_1$			
$B_j^{(1)}$	$(n_2, k_{21} + k_{22}, d_{21})$	A_1	$(2^{k_{21}}; n_1, k_{11}, \mathbf{s}_1)$	$\mathcal{J}_1 = \{1, ..., n_1\}$
$B_j^{(2)}$	(n_2, k_{22}, d_{22})	A_2	$(2^{k_{22}}; n_1, k_{12}, \mathbf{s}_2)$	$\mathcal{J}_2 = \{1, ..., n_1\}$

Example 1: The $(75, 11, 32)$ and the $(75, 13, 30)$ code from [5] can be constructed as given in Table I.

III. OPTIMAL LUEP CODES CONSTRUCTED AS GC CODES

Linear unequal error protection (LUEP) codes can be useful if different information symbols have different importance. In [1] and [6], van Gils proposed constructions for some special classes of LUEP codes (some of them based on product or concatenated codes). In [7], Zinov'ev investigated the application of GC codes on the construction of LUEP codes, but these constructions only work for composite code length, i.e., $n = n_a n_b$. The modified construction yields a large class of binary LUEP codes which contains most of van Gils constructions and which can be easily decoded.

The LUEP code is characterized by its separation vector [6].

Definition 1: For a linear (n, k) code C over the alphabet GF(q), the separation vector $\boldsymbol{s} = (s_1, s_2, \cdots, s_k)$ with respect to a generator matrix G of C, is defined as

$$s_i := \min\{\text{wt}\{\boldsymbol{m}G | \boldsymbol{m} \in \text{GF}(q)^k,\, m_i \neq 0\},\, i = 1, \cdots, k\}$$

where $\boldsymbol{m}$ is an information vector and wt$\{\cdot\}$ denotes the Hamming weight function.

We assume, without loss of generality, that $\boldsymbol{s}$ is nonincreasing, i.e., $s_i \geq s_j$ if $i < j$ $\forall i, j \in \{1, \cdots, k\}$. Note that this definition is different from that in [8] as it deals with information symbols instead of code symbols. The separation vector guarantees the correct interpretation of the ith information symbol whenever nearest neighbor decoding [9] is applied and not more than $\lfloor (s(G)_i - 1)/2 \rfloor$ errors have occurred in the transmitted codeword [10].

An $(n, k, \boldsymbol{s})$ code is called optimal if an $(n, k, \boldsymbol{t})$ code with $\boldsymbol{t} > \boldsymbol{s}$, i.e., $t_i \geq s_i$ $\forall i \in \{1, \cdots, k\}$ and $\exists j \in \{1, \cdots, k\}$: $t_j > s_j$, does not exist. Denote by $n(\boldsymbol{s})$ the length of the shortest linear binary code of dimension k with separation vector at least $\boldsymbol{s}$ and denote $n^{\text{ex}}(\boldsymbol{s})$ the length of the shortest linear binary code of dimension k with separation vector (exactly) $\boldsymbol{s}$. Van Gils [1], [11], has derived the following lower bounds on $n(\boldsymbol{s})$:

Theorem 1: For any $k \in \mathcal{N}$, and nonincreasing $\boldsymbol{s} \in \mathcal{N}^k$

$$n^{ex}(s_1, s_2, \cdots, s_k) \geq s_i + n(\hat{s}_1, \cdots, \hat{s}_{i-1}, \hat{s}_{i+1}, \cdots, \hat{s}_k) \tag{2}$$

holds for any $i \in \{1, \cdots, k\}$, where

$$\hat{s}_j := \begin{cases} s_j - \left\lfloor \frac{s_i}{2} \right\rfloor, & \text{for } j < i \\ \left\lceil \frac{s_j}{2} \right\rceil, & \text{for } j > i. \end{cases} \tag{3}$$

$\lceil x \rceil$ denotes the smallest integer larger than or equal to x.

TABLE III
CONSTRUCTION 2

	Inner codes		Outer codes		
	$j = 1, \ldots, n_1$	$j = n_1+1, \ldots, n_1+n'$			
$B_j^{(0)}$	$(n_2, n_2, 1)$	$(n_2-k_2, n_2-k_2, 1)$	A_1	$(2^{n_2-k_2}; n_1+n', k_{11}, \mathbf{s}_1)$	$\mathcal{J}_1 = \{1, \ldots, n_1+n'\}$
$B_j^{(1)}$	(n_2, k_2, d_{22})	$(n_2-k_2, 0, \infty)$	A_2	$(2^{k_2}; n_1, k_{12}, \mathbf{s}_2)$	$\mathcal{J}_2 = \{1, \ldots, n_1\}$

TABLE IV
CONSTRUCTION 3

	Inner codes		Outer codes		
	$j = 1, \ldots, n_1$	$j = n_1 + 1$			
$B_j^{(1)}$	$(n_2, k_2 + 1, d_2)$	$(1, 1, 1)$	A_1	$(2^{k_2}; n_1, k_{11}, \mathbf{s}_1)$	$\mathcal{J}_1 = \{1, \ldots, n_1\}$
$B_j^{(2)}$	$(n_2, 1, n_2)$	$(1, 1, 1)$	A_2	$(2; n_1 + 1, k_{12}, 2\lceil \mathbf{s}_2'/2 \rceil)$	$\mathcal{J}_2 = \{1, \ldots, n_1 + 1\}$

Theorem 2: For any $k \in \mathcal{N}$, and nonincreasing $\boldsymbol{s} \in \mathcal{N}^k$, $n(\boldsymbol{s})$ satisfies the inequalities

$$n(s_1, \cdots, s_k) \geq s_1 + n\left(\left\lceil \frac{s_2}{2} \right\rceil, \cdots, \left\lceil \frac{s_k}{2} \right\rceil\right) \tag{4}$$

$$n(s_1, \cdots, s_k) \geq \sum_{i=1}^{k} \left\lceil \frac{s_i}{2^{i-1}} \right\rceil. \tag{5}$$

Construction 1: First we construct a two-level GC code as shown in Table II. A_1 and A_2 are LUEP codes with nonincreasing separation vectors

$$\boldsymbol{s}_1 = (s_{11}, s_{12}, \cdots, s_{1k_{11}}) \quad \boldsymbol{s}_2 = (s_{21}, s_{22}, \cdots, s_{2k_{12}}).$$

As a special case, both A_1 and A_2, or one of them, may be chosen as equal error protection codes. If $d_{21}s_{1k_{11}} \geq d_{22}s_{21}$, then the GC code is a binary $(n_1n_2, k_{11}k_{21} + k_{12}k_{22}, \boldsymbol{s})$ LUEP code, where

$$\boldsymbol{s} = (d_{21}s_{11}\mathbf{1}_{k_{21}}, \cdots, d_{21}s_{1k_{11}}\mathbf{1}_{k_{21}}, d_{22}s_{21}\mathbf{1}_{k_{22}}, \cdots, d_{22}s_{2k_{12}}\mathbf{1}_{k_{22}})$$

where $s\mathbf{1}_{k_{2i}}$ denotes the k_{2i} vector with all components equal to s.

Obviously, [6, Construction 5] is a special case of the above construction.

If A_1 is an $(n, 1, n)$ repetition code, A_2 is an optimal $(n, k, \boldsymbol{s})$ LUEP code, $B_j^{(1)}$ and $B_j^{(2)}$ are Reed–Muller codes with parameters $(2^m, m + 1, 2^{m-1})$ and $(2^m, 1, 2^m)$ an optimal LUEP code equivalent to the code of [6, Construction 1] is obtained. Choosing $B_j^{(1)}$ as the $(2, 2, 1)$ code and $B_j^{(2)}$ as the $(2, 1, 2)$ code, the above construction is equivalent to [6, Construction 3A].

Construction 2: The GC Code given in Table III has the parameters

$$n = n_1n_2 + (n_2 - k_2)n'$$
$$k = (n_2 - k_2)k_{11} + k_2k_{12}$$

and

$$\boldsymbol{s} = (s_{11}\mathbf{1}_{(n_2-k_2)}, \cdots s_{1k_{11}}\mathbf{1}_{(n_2-k_2)}, d_2s_{21}\mathbf{1}_{k_2}, \cdots, d_2s_{2k_{21}}\mathbf{1}_{k_2})$$

where $s_{1k_{11}} \geq d_2s_{21}$. If $B_j^{(1)}$ is a $(2, 2, 1)$ code and $B_j^{(2)}$ is a $(2, 1, 2)$ code for $j = 1, \cdots, n_1$, we obtain all the LUEP codes of *Constructions A,C,E,F,I,J* and *K* from van Gils in [1] and a class of LUEP code which is better than the codes of [6, Construction 2] with the same code rate.

In fact, choosing for A_1 the $(n_1 + n', 1, n_1 + n')$ repetition code and for A_2 an optimal $(n_1, k, \boldsymbol{s})$ LUEP code, we get with the inner codes of length 2, as above, an optimal $(2n_1 + n', 1 + k, (n_1 + n', 2\boldsymbol{s}))$ GC code. The optimality can be shown as follows:

TABLE V
EXAMPLE 2

	Inner codes			Outer codes		
	$j = 1$	$j = 2, 3$	$j = 4$			
$B_j^{(1)}$	$(4, 3, 2)$	$(3, 2, 2)$	$(2, 2, 1)$	A_1	$(2^2; 4, 2, 3)$	$\mathcal{J}_1 = \{1, \ldots, 4\}$
$B_j^{(2)}$	$(4, 1, 4)$	$(3, 0, \infty)$	$(1, 0, \infty)$	A_2	$(2; 1, 1, 1)$	$\mathcal{J}_2 = \{1\}$

TABLE VI
EXAMPLE 3

	Inner codes			Outer codes		
	$j = 1, 2$	$j = 3$	$j = 4$			
$B_j^{(1)}$	$(4, 3, 2)$	$(3, 2, 2)$	$(2, 2, 1)$	A_1	$(2^2; 4, 2, 3)$	$\mathcal{J}_1 = \{1, \ldots, 4\}$
$B_j^{(2)}$	$(4, 1, 4)$	$(3, 0, \infty)$	$(1, 0, \infty)$	A_2	$(2; 2, 2, 1)$	$\mathcal{J}_2 = \{1, 2\}$

Proof: For the proof we use Theorem 1. First we show that the GC code is optimal in length

$$n^{ex}(n_1 + n', 2\boldsymbol{s}) \geq n_1 + n' + n(\boldsymbol{s}) \geq 2n_1 + n'.$$

We now have to show that all codes with a greater separation vector also have greater length

$$\begin{aligned} n^{ex}(n_1 + n' + 1, 2\boldsymbol{s}) &\geq n_1 + n' + 1 + n(\boldsymbol{s}) \\ &\geq 2n_1 + n' + 1 \\ &> 2n_1 + n' \end{aligned}$$

and

$$\begin{aligned} n^{ex}(n_1 + n', 2\boldsymbol{s} + \boldsymbol{u}) &\geq n_1 + n' + n \\ &\left(\left\lceil \frac{s_1 + u_1}{2} \right\rceil, \cdots, \lceil s_k + u_k \rceil\right) \\ &\geq 2n_1 + n' + 1. \end{aligned}$$

Construction 3: The construction is given in Table IV, where $\lceil \mathbf{s}_2/2 \rceil$ denotes $\lceil s_{2i}/2 \rceil$ for all $i = 1, 2, \cdots, k_{12}$.

We suppose here that the outer code A_2 is the code A_2' : $(2; n_1, k_{12}, \boldsymbol{s}_2')$ with an added overall parity bit. With $s_{1k_1}d_2' > n_2$ we obtain a new

$$\{n_1n_2 + 1, k_{11}k_2 + k_{12}, [s_1d_2\mathbf{1}_{k_2}, \cdots, s_{k_1}d_2\mathbf{1}_{k_2}, (n_2 - 1)\boldsymbol{s}_2' + 2\lceil \boldsymbol{s}_2'/2 \rceil]\}$$

LUEP code. Using the $(2, 2, 1)$ and the $(2, 1, 2)$ code as inner codes for $j = 1, \cdots, n_1$, we obtain the LUEP codes [6, Construction 3B] of van Gils. If at the same time A_1 is a repetition code and A_2' is uncoded, the LUEP codes are the same as in [1, Construction B], i.e., they are optimal.

Choosing A_1 as a repetition code and A_2 as uncoded and the $(4, 3, 2)$ and $(4, 1, 4)$ codes as inner codes for $j = 1, \cdots, n_1$, results in [1, Construction H] of van Gils.

In the following examples we show the construction of some codes from [1] as GC codes.

Example 2: The GC code given by Table V has the parameters $n = 12$, $k = 6$ $\boldsymbol{s} = (555544)$.

Example 3: The GC code given by Table VI has the parameters $n = 13$, $k = 6$ $\boldsymbol{s} = (555544)$.

Example 4: The GC code given by Table VII has the parameters $n = 14$, $k = 7$ $\boldsymbol{s} = (5555444)$.

For these three codes, see [1, Construction M].

TABLE VII
EXAMPLE 4

	Inner codes		Outer codes		
	$j = 1,2,3$	$j = 4$			
$B_j^{(1)}$	$(4,3,2)$	$(2,2,1)$	A_1	$(2^2;4,2,3)$	$\mathcal{J}_1 = \{1,\ldots,4\}$
$B_j^{(2)}$	$(4,1,4)$	$(1,0,\infty)$	A_2	$(2;3,3,1)$	$\mathcal{J}_2 = \{1,2,3\}$

TABLE VIII
EXAMPLE 5

	Inner codes		Outer codes		
	$j = 1,2,3$	$j = 4$			
$B_j^{(1)}$	$(4,3,2)$	$(3,3,1)$	A_1	$(2^2;4,2,3)$	$\mathcal{J}_1 = \{1,\ldots,4\}$
$B_j^{(2)}$	$(4,1,4)$	$(3,1,3)$	A_2	$(2;4,4,1)$	$\mathcal{J}_2 = \{1,\ldots,4\}$

Example 5: The GC code given by Table VIII has the parameters $n = 15, \quad k = 8 \quad \boldsymbol{s} = (55554443)$ (See [1, Construction R]).

IV. A Decoding Algorithm

GC codes can be decoded by the well-known Blokh–Zyablov–Zinov'ev (BZZ) algorithm up to half their designed minimum distance.

For decoding GC codes with different inner codes we use the BZZ algorithm together with an appropriate metric.

As shown in Section II, the designed minimum distance of the constructed codes is given by

$$d \geq \min_{\forall i} \sum_{j=1}^{d_{a,\,i}} d_{b,\,\Pi(j)}^{(i)}.$$

Define c to be the transmitted codeword, with $c_j \in B_j^{(1)}$ for $j = 1, \cdots, j_{\max}$. Denote by $\check{a}_i$ the transmitted codeword of the outer code A_i with respect to c, and by $\hat{a}_i$ the estimate for $\check{a}_i$ calculated by decoding the inner codes of stage i and mapping the result to symbols of the outer code alphabet. Let a_i be a codeword of the outer code A_i and $\mathcal{E}(a_i, \hat{a}_i)$ be the set of indices j such that $a_i \neq \hat{a}_i$. Define w to be the cardinality of $\mathcal{E}(a_i, \hat{a}_i)$. Denote by $\bar{\mathcal{E}}^*(a_i, \hat{a}_i)$ the $d_{a,i} - w$ components in

$$\bar{\mathcal{E}} = \{1, \cdots, n_{a,i}\}/\mathcal{E}(a_i, \hat{a}_i)$$

with the smallest $\alpha_j^{(i)}$.

An appropriate reliability function is given by the following definition:

Definition 2: Let $d_{b,j}^{(i)}$ be the minimum distance of the inner code $B_j^{(i)}$, and $d_H(r_j, b_j^{(i)})$ be the distance between the received word r_j and the estimated codeword $b_j^{(i)} \in B_j^{(i)}$. Then the reliability $\alpha_j^{(i)}$ for the jth position in $\hat{a}_i$ is given by

$$\alpha_j^{(i)} = \max\,[0,\, d_{b,j}^{(i)} - 2d_H(r_j, b_j^{(i)})].$$

This definition takes into account the different distances of the inner codes. It may be interpreted as the minimum number of additional errors that occurred in case of a wrong decision in the inner code. During the decoding process the following condition has to be checked:

$$\text{(Condition 1)} \qquad \sum_{j \in \bar{\mathcal{E}}^*(a_i, \hat{a}_i)} \alpha_j^{(i)} > \sum_{j \in \mathcal{E}(a_i, \hat{a}_i)} \alpha_j^{(i)}. \qquad (6)$$

Decoding Algorithm: For each stage i, with $i = 1, 2, \cdots, m$, do:

1) Decode the received word $\boldsymbol{r}$ in the inner codes to get an estimated codeword $b_j^{(i)}$ of code $B_j^{(i)}$ for $j \in \mathcal{J}_i$.
2) Map these estimates $b_j^{(i)}, \quad j = 1, \cdots, n_{a,i}$ to the outer code symbols to get $\hat{a}_i$ and use $\alpha_j^{(i)}$ as reliability for position j of $\hat{a}_i$.
3) For $l = 1, \cdots, d_{a,i}$ set the l positions with smallest reliabilities $\alpha_j^{(i)}$ to erasures and use an Error-and-Erasure Decoder (EED) to find $a_i\{l\}$.
4) Check if any $a_i\{l\}$ satisfies *Condition 1*. If yes, $a_i = a_i\{l\}$ is the final decision. If no, signal decoding failure.
5) Continue with the next stage.

In the Appendix, we prove the following theorem:

Theorem 3: The algorithm will find the transmitted codeword as long as less than $d/2$ errors have occurred during transmission.

For an LUEP code constructed as a (modified) GC code, it is desirable to be able to guarantee a correct decoding as long as at most $\lfloor(s(G)_i - 1)/2\rfloor$ errors have occurred in the transmitted codeword. If the outer codes in the above constructions have an equal protection of their information symbols, the described decoding algorithm can be applied directly and guarantees a decoding up to $\lfloor(s(G)_i - 1)/2\rfloor$ errors. This is due to the properties of the multistage decoding. Similar to the BZZ algorithm, many error patterns of higher weight are decoded too.

The authors are not aware of a general BMD decoder for LUEP codes that corrects errors and erasures, and issues complete codewords. Such a decoder would be necessary for the general case. However, if we again assume that the outer codes A_i in the constructions are also GC LUEP codes with inner codes that are not UEP codes, the decoding up to $\lfloor(s(G)_i - 1)/2\rfloor$ errors can be achieved by the algorithm for modified GC codes as described above. This is demonstrated for the first information symbol m_1 with separation $s(G)_1$: first the inner codes $B_j^{(0)'}$, for $j = 1, \cdots, n_{a,1}$, are BMD decoded and an estimation for the outer code symbols $a_{1,j}$, together with the estimation of the reliability for these symbols $\alpha_j^{(0)}$, are transmitted to the outer code A_1. Since this code is again a GC LUEP code with inner codes that are not UEP codes, the $\alpha_j^{(0)}$ represent the reliabilities for the code symbols of the inner codes $B_j^{(0)'}$ of the GC code A_1. Proceeding in this way until the outer code is no longer an UEP code (this occurs in the worst case after $k_{a,1}$ steps) finally allows the application of the above algorithm and a decoding of m_1.

V. Summary

In this correspondence we use modified GC codes for the construction of binary codes of noncomposite length and of LUEP codes. Using this construction, it is not only possible to construct good codes but also to decode them efficiently. Especially, this holds true for the constructed LUEP codes. A decoding algorithm which guarantees decoding up to half the designed minimum distance, similar to the BZZ algorithm, is derived.

Appendix
Proof of Theorem 3

We will prove Theorem 3 by using the following two theorems from [12]:

Theorem 4: There is only one codeword a_i in a code with minimum distance $d_{a,i}$ that satisfies *Condition 1*.

Theorem 5: If any codeword a_i satisfies *Condition 1*, it will be found by an Error-and-Erasure Decoder.

Notice that for the proof of these two theorems, the reliability $\alpha_j^{(i)}$ does not have to be specified. It only has to be greater or equal to zero. Using *Condition 1* we prove Theorem 3:

Proof: Denote by T the total number of errors and consider the following inequalities:

$$2T < \sum_{j=1}^{d_{a,i}} d_{b,\Pi(j)}^{(i)}$$

$$\Leftrightarrow \quad 2\sum_{j=1}^{n_{a,i}} d_H[r_j, c_j] < \sum_{j=1}^{d_{a,i}} d_{b,\Pi(j)}^{(i)}$$

$$\Rightarrow \quad 2\sum_{j=1}^{d_{a,i}} d_H[r_{\Pi(j)}, c_{\Pi(j)}] < \sum_{j=1}^{d_{a,i}} d_{b,\Pi(j)}^{(i)}$$

$$\Rightarrow \quad \sum_{j\in\mathcal{E}(\check{a}_i, \hat{a}_i)} \alpha_j^{(i)} < \sum_{j\in\bar{\mathcal{E}}^*(\check{a}_i, \hat{a}_i)} \alpha_j^{(i)}.$$

The last step follows from the following considerations:

1) If $b_j^{(i)} = c_j$ then $\alpha_j^{(i)} \ge d_{b,j} - 2d_H(c_j, r_j)$.
2) If $b_j^{(i)} \ne c_j$ then $\alpha_j^{(i)} \le 2d_H(c_j, r_j) - d_{b,j}$, because

$$d_{b,j} \le d_H(c_j, r_j) + d_H[b_j^{(i)}, r_j]$$

$$\Leftrightarrow \quad d_{b,j} - 2d_H[b_j^{(i)}, r_j] \le 2d_H(c_j, r_j) - d_{b,j}.$$

So

$$\sum_{j\in\bar{\mathcal{E}}^*(\check{a}_i, \hat{a}_i)} \alpha_j^{(i)} - \sum_{j\in\mathcal{E}(\check{a}_i, \hat{a}_i)} \alpha_j^{(i)}$$
$$\ge \sum_{j\in\bar{\mathcal{E}}^*(\check{a}_i, \hat{a}_i)} [d_{b,j} - 2d_H(b_j^{(i)}, r_j)]$$
$$- \sum_{j\in\mathcal{E}(\check{a}_i, \hat{a}_i)} [2d_H(c_j, r_j) - d_{b,j}]$$
$$= \sum_{j\in\bar{\mathcal{E}}^*(\check{a}_i, \hat{a}_i)\cup\mathcal{E}(\check{a}_i, \hat{a}_i)} d_{b,j} - 2d_H(c_j, r_j)$$
$$= \sum_{j=1}^{d_{a,i}} d_{b,\Pi(j)} - 2d_H[r_{\Pi(j)}, c_{\Pi(j)}]$$

i.e., as long as less than $d/2$ errors have occurred, only the transmitted codeword satisfies *Condition 1* and the EED will find it. □

For LUEP codes the following theorem holds:

Corollary 1: All codewords a_i in an LUEP code with separation vector $s = (s_1, \cdots, s_k)$ that satisfy *Condition 1* with $d = s_1$, have the same information symbol m_1.

Proof: Codewords generated from information vectors which differ in component m_1 have minimum distance s_1 by definition of s. The theorem follows from the same arguments as in [12, proof of Theorem 1]. □

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A New Approach to the Design of Codes for Synchronous-CDMA Systems

Gurgen H. Khachatrian and Samvel S. Martirossian

***Abstract*— In this correspondence a new approach to increase the sum rate for conventional synchronous code-division multiple-access (S-CDMA) systems is presented. It is shown that it can be done by joint processing of the outputs of matched filters, when one considers the system of codes for S-CDMA to be the codes for the usual adder channel. An example of construction and decoding of such a system is also given.**

***Index Terms*—Multiuser spread-spectrum system, code-division multiple access, adder channel, matched filter.**

I. Introduction

Recent developments in multiuser spread-spectrum communication systems show the need to increase their sum rate. In cellular systems this means increasing the number of users, that can be simultaneously active inside each cell. In code-division multiple-access (CDMA) systems each of the users is assigned a binary ± 1-valued spreading sequence of the same length.

In a synchronous CDMA (S-CDMA) system, all users are in exact synchronism in the sense that not only are their carrier frequencies and phases the same, but also their expanded data symbols are aligned in time. It is also assumed that all the sequences have equal energy.

In a conventional CDMA receiver, the demodulator output for each symbol interval is further processed separately by each user. This procedure is called matched filtering. Mathematically, this corresponds to computing the scalar product between the spreading sequence of the ith user and the vector which represents the demodulator output of the CDMA system. Interuser interference then is defined by the crosscorrelation function between the spreading sequences of

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Exhibit B

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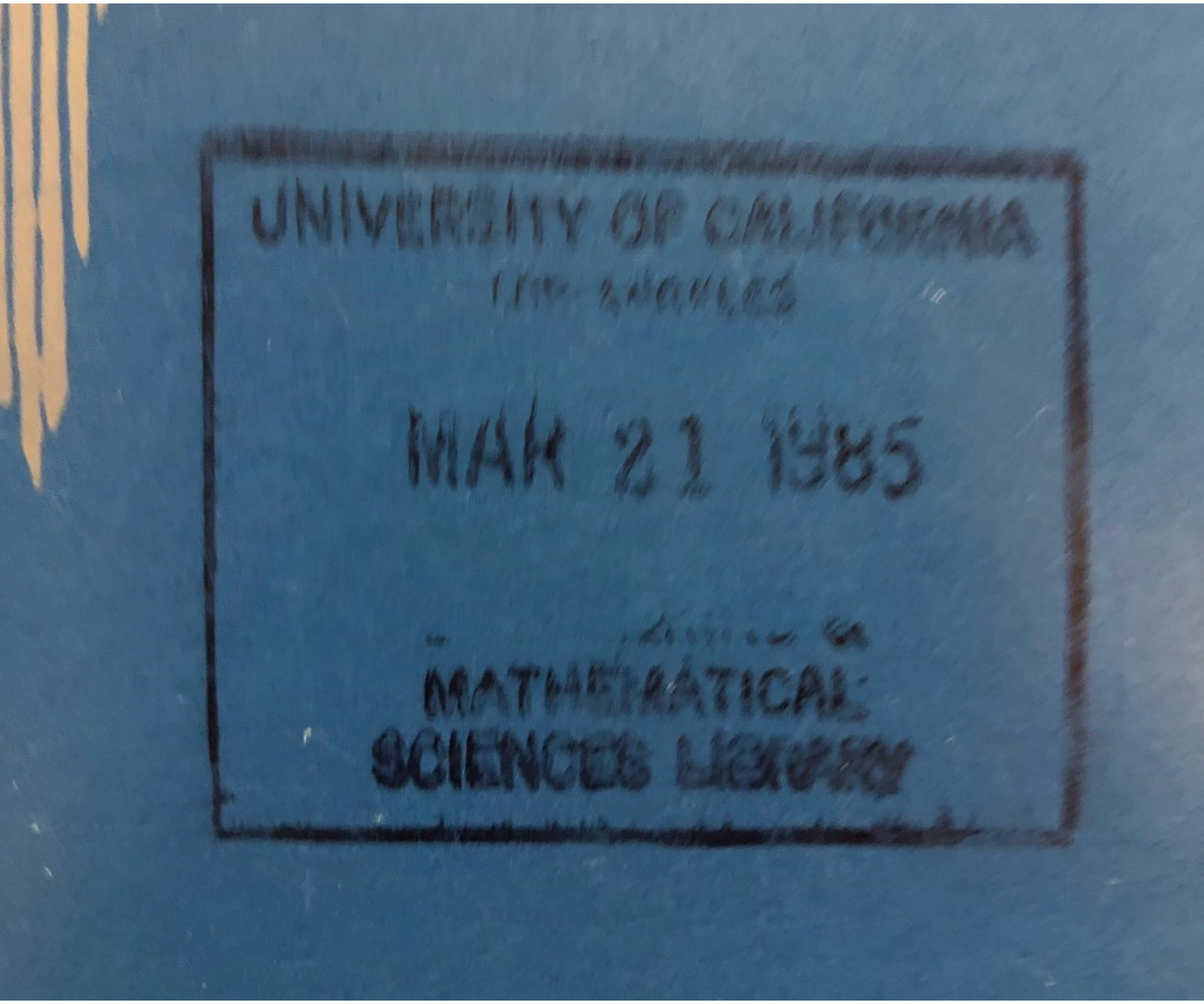
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SOME CONSTRUCTIONS OF OPTIMAL BINARY LINEAR UNEQUAL ERROR PROTECTION CODES

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Abstract

This paper describes a number of constructions of binary Linear Unequal Error Protection (LUEP) codes. The separation vectors of the constructed codes include those of all optimal binary LUEP codes of length less than or equal to 15.

AMS: 94B05, 94B60.

1. Introduction

Consider a binary linear code C of length n and dimension k with generator matrix G to be used on a binary symmetric channel. In many applications it is necessary to provide different protection levels for different components m_i of the input message word $\boldsymbol{m}$. For example in transmitting numerical (binary) data, errors in the more significant bits are more serious than are errors in the less significant bits, and therefore more significant bits should have more protection than less significant bits.

A suitable measure for these protection levels for separate positions in input message words is the separation vector [1]).

Definition

For a binary linear $[n,k]$ code C the separation vector $\boldsymbol{s}(G) = (s(G)_1, s(G)_2, \ldots, s(G)_k)$ with respect to a generator matrix G of C is defined by

$$s(G)_i := \min \{\mathrm{wt}(\boldsymbol{m}\, G) \mid \boldsymbol{m} \in \{0, 1\}^k, m_i = 1\},$$

where wt(.) denotes the Hamming weight function.

This separation vector $\boldsymbol{s}(G)$ guarantees the correct interpretation of the i^{th} message bit whenever Nearest Neighbour Decoding (ref. 2 p. 11) is applied and no more than $(s(G)_i - 1)/2$ errors have occurred in the transmitted codeword [1]).

A linear code that has a generator matrix G such that the components of the corresponding separation vector $\boldsymbol{s}(G)$ are not mutually equal is called a Linear

Unequal Error Protection (LUEP) code[1]). By permuting the rows of a generator matrix G we may obtain a generator matrix G' for the code such that $s(G')$ is nonincreasing, i.e. $s(G')_i \geqslant s(G')_{i+1}$ for $i = 1, 2, \ldots, k-1$. In this paper we always assume that the rows in generator matrices are so ordered that the corresponding separation vectors are nonincreasing.

Any LUEP code C has a so-called optimal generator matrix G^*. This means that the separation vector $\boldsymbol{s}(G^*)$ is componentwise larger than or equal to the separation vector $\boldsymbol{s}(G)$ of any generator matrix G of C[1]), denoted by $\boldsymbol{s}(G^*) \geqslant \boldsymbol{s}(G)$ ($\boldsymbol{x} \geqslant \boldsymbol{y}$ means $x_i \geqslant y_i$ for all i). The vector $\boldsymbol{s} = \boldsymbol{s}(G^*)$ is called the separation vector of the linear code C. We use the notation $[n,k,\boldsymbol{s}]$ for C.

For any $k \in \mathbb{N}$ and $\boldsymbol{s} \in \mathbb{N}^k$ we define $n(\boldsymbol{s})$ to be the length of the shortest binary linear code of dimension k with a separation vector of at least $\boldsymbol{s}$, and $n^{ex}(\boldsymbol{s})$ to be the length of the shortest binary linear code of dimension k with separation vector (exactly) $\boldsymbol{s}$[3,4]). An $[n(\boldsymbol{s}),k,\boldsymbol{s}]$ code is called length-optimal[3]). It is called optimal if an $[n(\boldsymbol{s}),k,\boldsymbol{t}]$ code with $\boldsymbol{t} \geqslant \boldsymbol{s}$, $\boldsymbol{t} \neq \boldsymbol{s}$ does not exist[3,4]). In refs 3 and 4 a number of bounds for the functions $n(\boldsymbol{s})$ and $n^{ex}(\boldsymbol{s})$ are derived. In ref. 5 methods for constructing LUEP codes from shorther codes are described.

In refs 3 and 4 an incomplete list of the separation vectors of the optimal binary LUEP codes of length less than or equal to 15 is given. In this paper we provide the complete list of the separation vectors of all optimal binary LUEP codes of length less than or equal to 15, together with examples of generator matrices having these separation vectors. Furthermore, we give a number of constructions of infinite series optimal binary LUEP codes.

2. Constructions

Table I provides the separation vectors of all optimal binary LUEP codes of length less than or equal to 15. In this table, n denotes the length of the code, k denotes the dimension, and $d(n,k)$ denotes the maximal minimum distance of a binary code of length n and dimension k. The brackets and commas commonly appearing in separation vectors have been deleted. Only in the cases where a component of a separation vector is larger than 9, it is followed by a point (.). Examples of codes having the parameters given in table I are constructed below. The bounds in ref. 4 can be used to show that certain LUEP codes are optimal. They are also useful in showing that table I is complete. In cases where these bounds did not work, methods of exhaustive search were used to show that codes with certain parameters do not exist. Table I is the same table as table I in ref. 4, extended by the parameters [14,10,(4333322222)], [15,3,(994)], [15,8,(73333333)], [15,8,(55554443)], [15,8,(55544444)] and [15,11,(43333222222)]. In (ref. 4 table I) no references to constructions were given, which has been done in this paper.

TABLE I

The separation vector of all binary optimal LUEP codes of length less than or equal to 15.

n	k	$d(n,k)$	separation vector
4	2	2	*A* 32
5	2	3	*A* 42
5	3	2	*A* 322
6	2	4	*A* 52
6	3	3	*A* 422
6	4	2	*A* 3222
7	2	4	*A* 62, *I* 54
7	3	4	*A* 522
7	4	3	*A* 4222
7	5	2	*A* 32222
8	2	5	*A* 72, *I* 64
8	3	4	*A* 622, *C* 544
8	4	4	*A* 5222
8	5	2	*A* 42222, *J* 33332
8	6	2	*A* 322222
9	2	6	*A* 82, *I* 74
9	3	4	*A* 722, *C* 644, *G* 554
9	4	4	*A* 6222, *C* 5444
9	5	3	*A* 52222, *J* 44442, *B* 43333
9	6	2	*A* 422222, *J* 333322
9	7	2	*A* 3222222
10	2	6	*A* 92, *I* 84, *I* 76
10	3	5	*A* 822, *C* 744, *L* 664
10	4	4	*A* 7222, *C* 6444, *G* 5544
10	5	4	*A* 62222, *C* 54444
10	6	3	*A* 522222, *J* 444422, *J* 433332
10	7	2	*A* 4222222, *J* 3333222
10	8	2	*A* 32222222
11	2	7	*A* 10.2, *I* 94, *I* 86
11	3	6	*A* 922, *C* 844, K_1 764
11	4	5	*A* 8222, *C* 7444, *E* 6644
11	5	4	*A* 72222, *C* 64444, *G* 55444
11	6	4	*A* 622222, *J* 544442, *B* 533333
11	7	3	*A* 5222222, *J* 4444222, *J* 4333322
11	8	2	*A* 42222222, J 33332222
11	9	2	*A* 322222222
12	2	8	*A* 11.2, *I* 10.4, *I* 96
12	3	6	*A* 10.22, *C* 944, *E* 864, K_2 774, K_1 766
12	4	6	*A* 9222, *C* 8444, K_1 7644
12	5	4	*A* 82222, *C* 74444, *E* 66444, *M* 55554
12	6	4	*A* 722222, *C* 644444, *G* 554444
12	7	4	*A* 6222222, *J* 5444422, *J* 5333332

TABLE I (cont.)

n	k	$d(n,k)$	separation vector
12	8	3	A 52222222, J 44442222, J 43333222
12	9	2	A 422222222, J 333322222
12	10	2	A 3222222222
13	2	8	A 12.2, I 11.4, I 10.6, I 98
13	3	7	A 11.22, C 10.44, K_1 964, E 884, L 866
13	4	6	A 10.222, C 9444, L 8644, F 7744, K_1 7666
13	5	5	A 92222, C 84444, K_1 76444, L 66664, H 66555
13	6	4	A 822222, C 744444, D 664444, M 555544
13	7	4	A 7222222, J 6444442, B 6333333, J 5544442, K_1 5444444
13	8	4	A 62222222, J 54444222, J 53333322
13	9	3	A 522222222, J 444422222, J 433332222
13	10	2	A 4222222222, J 3333222222,
13	11	2	A 32222222222
14	2	9	A 13.2, I 12.4, I 11.6, I 10.8
14	3	8	A 12.22, C 11.44, L 10.64, K_1 984, K_1 966
14	4	7	A 11.222, C 10.444, K_1 9644, L 8844, L 8666
14	5	6	A 10.2222, C 94444, L 86444, F 77444, N 76666
14	6	5	A 922222, C 844444, E 764444, L 666644, J 665552
14	7	4	A 8222222, C 7444444, J 6644442, Q 6544444, M 5555444
14	8	4	A 72222222, J 64444422, J 63333332, J 55444422, K_1 54444444
14	9	4	A 622222222, J 544442222, J 533333222
14	10	3	A 5222222222, J 4444222222, J 4333322222
14	11	2	A 42222222222, J 33332222222
14	12	2	A 322222222222
15	2	10	A 14.2, I 13.4, I 12.6, I 11.8
15	3	8	A 13.22, C 12.44, K_1 11.64, K_1 10.84, L 10.66, K_2 994, K_1 988
15	4	8	A 12.222, C 11.444, L 10.644, K_1 9844, K_1 9666
15	5	7	A 11.2222, C 10.4444, K_1 96444, L 88444, L 86666
15	6	6	A 10.22222, C 944444, L 864444, K_2 774444, J 766662, K_1 766644, O 765554
15	7	5	A 9222222, C 8444444, P 7644444, L 6666444, J 6655522
15	8	4	A 82222222, J 74444442, B 73333333, J 66444422, J 65444442, L 64444444, R 55554443, S 55544444
15	9	4	A 722222222, J 644444222, J 633333322, J 554444222, K_1 544444444
15	10	4	A 6222222222, J 5444422222, J 5333332222
15	11	3	A 52222222222, J 44442222222, J 43333222222
15	12	2	A 422222222222, J 333322222222
15	13	2	A 3222222222222

In this paper we frequently use two results of ref. 4. Hence we repeat these results in the following two theorems.

Theorem 1 (ref. 4, theorem 12)

For any $k \in \mathbb{N}$ and nonincreasing $\boldsymbol{s} \in \mathbb{N}^k$ we have that

$$n^{\mathrm{ex}}(s_1, \ldots, s_k) \geqslant s_i + n(\hat{s}_1, \ldots, \hat{s}_{i-1}, \hat{s}_{i+1}, \ldots, \hat{s}_k)$$

holds for any $i \in \{1, \ldots, k\}$, where

$$\hat{s}_j := \begin{cases} s_j - \lfloor s_j/2 \rfloor & \text{for } j < i \\ \lceil s_j/2 \rceil & \text{for } j > i, \end{cases}$$

(where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x, and $\lceil x \rceil$ denotes the smallest integer larger than or equal to x).

Theorem 2 (ref. 4, corollary 14)

For any $k \in \mathbb{N}$ and any nonincreasing $\boldsymbol{s} \in \mathbb{N}^k$, the function $n(\boldsymbol{s})$ satisfies the following inequalities,

a. $n(s_1, s_2, \ldots, s_k) \geqslant s_1 + n(\lceil s_2/2 \rceil, \ldots, \lceil s_k/2 \rceil)$,

b. $n(s_1, s_2, \ldots, s_k) \geqslant \sum_{i=1}^{k} \lceil s_i/2^{i-1} \rceil$.

Construction A

For $n, k \in \mathbb{N}$, $n \geqslant k+1$, the k by n matrix

$$\left[\begin{array}{c|cc} & \multicolumn{2}{c}{11111\ldots\ldots111} \\ & 1 & \\ I_k & \vdots & 0_{k-1,n-k-1} \\ & \vdots & \\ & 1 & \end{array}\right] \qquad (1)$$

is a generator matrix of an optimal binary $[n, k, (n-k+1, 2, 2, \ldots, 2)]$ code (I_k denotes the identity matrix of order k, $0_{k-1,n-k-1}$ denotes the all-zero $k-1$ by $n-k-1$ matrix).

Proof

It is easy to check that the parameters of the code are correct. Furthermore by theorem 2*b* the length of a k-dimensional binary code with separation vector $(n-k+1, 2, 2, \ldots, 2)$ is at least n, and with separation vector larger than $(n-k+1, 2, 2, \ldots, 2)$ is at least $n+1$ (by $\boldsymbol{s} > \boldsymbol{t}$ ($\boldsymbol{s}$ larger than $\boldsymbol{t}$) we mean $\boldsymbol{s} \geqslant \boldsymbol{t}, \boldsymbol{s} \neq \boldsymbol{t}$).

Construction B

For $k \in \mathbb{N}$, $k \geq 4$, the k by $2k-1$ matrix

$$\left[\begin{array}{c|c|c} 00\ldots\ldots0 & 11\ldots\ldots1 & 0 \\ \hline & & 1 \\ I_{k-1} & I_{k-1} & 1 \\ & & \vdots \\ & & 1 \end{array}\right] \tag{2}$$

is a generator matrix of an optimal binary $[2k-1, k, (k-1, 3, 3, \ldots, 3)]$ code.

Proof

It is easy to verify that the parameters of the code are correct. By theorem 2b, we have that the length of a k–dimensional binary code with separation vector $(k-1, 3, 3, \ldots, 3)$ is at least $2k-1$. Application of theorem 2b to a k–vector $\boldsymbol{s}$ with $s_1 \geq k$ and $s_i \geq 3$ for $i = 2, \ldots, k$ shows that $n(\boldsymbol{s}) \geq 2k$. Application of the theorems 1 and 2 to a k–vector $\boldsymbol{s}$ such that $s_1 = k-1$, $s_2 \geq 4$, $s_i \geq 3$ for $i = 3, \ldots, k-1$, and $s_k = 3$ shows that

$$\begin{aligned} n^{\text{ex}}(\boldsymbol{s}) &\geq 3 + n(s_1 - 1, \ldots, s_{k-1} - 1) \\ &\geq 3 + s_1 - 1 + n(\lceil (s_2 - 1)/2 \rceil, \ldots, \lceil (s_{k-1} - 1)/2 \rceil) \\ &\geq 3 + k - 2 + n(\underbrace{2, 1, 1, \ldots, 1}_{k-2}) \\ &\geq 3 + k - 2 + k - 1 = 2k. \end{aligned}$$

Furthermore it is not difficult to check that a binary $[2k-1, k, (k-1, 4, 4, \ldots, 4)]$ code does not exist. Finally, by theorem 2b the length of a k–dimensional binary code with a separation vector of at least $(k-1, 5, 4, 4, \ldots, 4)$ is at least $2k$. These observations show that the code in construction B is optimal.

Construction C

For $n, k \in \mathbb{N}$, $n \geq \max\{2k, k+4\}$, the k by n matrix

$$\left[\begin{array}{c|c|c|c} 00\ldots\ldots0 & 11\ldots\ldots1 & 11\ldots1 & 10 \\ \hline & & & 11 \\ I_{k-1} & I_{k-1} & 0 & 11 \\ & & & \vdots\vdots \\ & & & 11 \end{array}\right] \tag{3}$$

is a generator matrix of an optimal binary $[n, k, (n-k, 4, 4, \ldots, 4)]$ code.

Proof

Similar to the proof of construction A.

Construction D

For $p,q \in \mathbb{N}$, $p \geqslant q \geqslant 2$, the $p+q+2$ by $2p+3q+3$ matrix

$$\left[\begin{array}{c|c|c|c|c}
 & & & & \overset{q-1}{\longleftrightarrow} \\
00\ldots\ldots\ldots0 & 1\,1\,1\,0 & 11\ldots1 & 00\ldots0 & 11..1 \\
00\ldots\ldots\ldots0 & 1\,1\,0\,1 & 00\ldots0 & 11\ldots1 & 11..1 \\
\hline
 & 0\,1\,0\,1 & & & \\
 & 0\,1\,0\,1 & I_p & 0 & 0 \\
 & \vdots\;\vdots\;\vdots\;\vdots & & & \\
I_{p+q} & \vdots\;\vdots\;\vdots\;\vdots & & & \\
 & 0\,1\,0\,1 & & & \\
\cline{2-5}
 & 1\,0\,1\,0 & & & \\
 & 1\,0\,1\,0 & 0 & I_q & 0 \\
 & \vdots\;\vdots\;\vdots\;\vdots & & & \\
 & 1\,0\,1\,0 & & &
\end{array}\right] \qquad (4)$$

is a generator matrix of an optimal binary

$$[2p+3q+3,\, p+q+2,\, (p+q+2,\, 2q+2,\, 4,\, 4,\, \ldots,\, 4)]$$

code.

Proof

Similar to the proof of construction A.

Construction E

For $p,q,r \in \mathbb{N}$, $p \geqslant 3$, $r \geqslant 2$, $q \geqslant r-p+2$, the p by $(2p+q+2r-4)$ matrix

$$\left[\begin{array}{c|c|c|c|c}
11\ldots\ldots1 & 00\ldots\ldots0 & 11\ldots1 & 00\ldots0 & 11..1 \\
00\ldots\ldots0 & 00\ldots\ldots0 & 11\ldots1 & 11\ldots1 & 00..0 \\
\hline
 & & 1 & 1 & \\
I_{p-2} & I_{p-2} & 1 & 1 & \\
 & & \vdots\;\; 0 & \vdots\;\; 0 & 0 \\
 & & \vdots & \vdots & \\
 & & 1 & 1 & \\
 & & \underset{r}{\longleftrightarrow} & \underset{r}{\longleftrightarrow} & \underset{q}{\longleftrightarrow}
\end{array}\right] \qquad (5)$$

is a generator matrix of an optimal binary

$$[2p+q+2r-4,\, p,\, (p+q+r-2,\, 2r,\, 4,\, 4,\, \ldots,\, 4)]$$

code.

Proof

Similar to the proof of construction A.

Construction F

For $p,q \in \mathbb{N}$, $p \geqslant 3$, $q \geqslant 2$, $q \geqslant p-2$, the p by $p+3q$ matrix

$$\left[\begin{array}{c|c|c|c|c} 00.....0 & 11.....1 & 11...1 & 00...0 & 11....1 \\ 00.....0 & 11.....1 & 00...0 & 11...1 & 11....1 \\ \hline & & 1 & 1 & \\ I_{p-2} & I_{p-2} & 1 \quad 0 & 1 \quad 0 & 0 \\ & & \vdots & \vdots & \\ & & 1 & 1 & \\ & & \underbrace{\qquad}_{q+1} & \underbrace{\qquad}_{q+1} & \underbrace{\qquad}_{q-(p-2)} \end{array}\right] \tag{6}$$

is a generator matrix of an optimal binary

$$[p+3q, p, (2q+1, 2q+1, 4, 4, \ldots, 4)]$$

code.

Proof

Similar to the proof of construction A.

Construction G

For $p \in \mathbb{N}$, the $2p$ by $4p$ matrix

$$\left[\begin{array}{c|c|c|c} 00.......0 & 1\,1\,1\,0 & 11..1 & 00..0 \\ 00.......0 & 1\,1\,0\,1 & 00..0 & 11..1 \\ \hline & 1\,0\,1\,0 & & \\ & \vdots\,\vdots\,\vdots\,\vdots & 0 & I_{p-1} \\ I_{2p-2} & 1\,0\,1\,0 & & \\ \cline{2-4} & 0\,1\,0\,1 & & \\ & \vdots\,\vdots\,\vdots\,\vdots & I_{p-1} & 0 \\ & 0\,1\,0\,1 & & \end{array}\right] \tag{7}$$

is a generator matrix of a binary $[4p, 2p, (p+2, p+2, 4, 4, \ldots, 4)]$ code. For $p = 2, 3$ the codes are optimal, but in general they are not.

In ref. 6 the codes from construction G are treated extensively, the weight enumerators and automorphism groups are determined completely and a majority logic decoding method for these codes is given. For $p = 3$ we obtain a [12,6,(5,5,4,4,4,4)] optimal LUEP code. By deleting the row and column pairs (6,4), (5,3) and (4,2) successively we obtain [11,5,(5,5,4,4,4)], [10,4,(5,5,4,4)] and [9,3,(5,5,4)] optimal LUEP codes respectively.

Construction H

For $p \in \mathbb{N}$, $p \geqslant 3$, the $(p+2)$ by $(4p+1)$ matrix

$$\left[\begin{array}{c|c|c|c|c} 00\ldots0 & 11\ldots1 & 11\ldots1 & 00\ldots0 & 0 \\ 00\ldots0 & 11\ldots1 & 00\ldots0 & 11\ldots1 & 0 \\ \hline & & & & 1 \\ I_p & I_p & I_p & I_p & 1 \\ & & & & \vdots \\ & & & & 1 \end{array}\right] \tag{8}$$

is a generator matrix of a length-optimal binary

$$[4p+1, p+2, (2p, 2p, 5, 5, \ldots, 5)]$$

LUEP code.

Proof

It is easy to check that the code has the given parameters. By theorem 2b the length of a $(p+2)$-dimensional binary code with separation vector $(2p, 2p, 5, 5, \ldots, 5)$ is at least $4p+1$.

For $p = 3$ this construction gives a $[13, 5, (6, 6, 5, 5, 5)]$ optimal LUEP code. Furthermore table I refers to the following trivial constructions.

Construction I

For $p, q \in \mathbb{N}$, $p > q$, the 2 by $(p+2q)$ matrix

$$\left[\begin{array}{c|c|c} 11\ldots\ldots.1 & 00\ldots.0 & 11\ldots.1 \\ 00\ldots\ldots.0 & 11\ldots.1 & 11\ldots.1 \\ \longleftrightarrow & \longleftrightarrow & \longleftrightarrow \\ p & q & q \end{array}\right] \tag{9}$$

is a generator matrix of an optimal binary $[p+2q, 2, (p+q, 2q)]$ LUEP code.

Construction J

If the matrix G_1 has separation vector $\boldsymbol{s}(G_1)$ such that $s(G_1)_k \geqslant 2$, then the matrix

$$G_2 = \left[\begin{array}{c|c} 0 & \\ 0 & \\ \vdots & G_1 \\ \vdots & \\ 0 & \\ \hline 1 & 100000\ldots\ldots\ldots.0 \end{array}\right] \tag{10}$$

has separation vector $\boldsymbol{s}(G_2) = (\boldsymbol{s}(G_1), 2)$.

Construction K_i

If the matrix G_1 has separation vector $s(G_1)$ then the matrix

$$G_2 := [\, G_1 \,|\, e_i \,], \qquad (11)$$

where e_i is the vector with a 1 on the i^{th} position and zeros elsewhere, has separation vector $s(G_2) = s(G_1) + e_i$.

The following theorem can be used to determine whether construction K_i gives an optimal code.

Theorem 3

If s is such that for all $t \geqslant s$, $t \neq s$, it holds that $n(t) > n(s)$ and if G is a generator matrix of a binary optimal $[r + n(s), k, (r, 2s)]$ code, then the code generated by $[G|\underbrace{e_1|e_1|\dots|e_1}_{t}]$ is a binary optimal $[r + t + n(s), k, (r + t, 2s)]$ code for t in $\mathbb{N}$ arbitrary.

Proof

Let s and G fulfill the conditions mentioned above. By theorem 2a we have that

a) $n(r + t, 2s) \geqslant r + t + n(s)$.
b) $n(r + t + 1, 2s) \geqslant r + t + 1 + n(s) > r + t + n(s)$.
c) $n(r + t, 2s + u) \geqslant r + t + n(\lceil s_1 + u_1/2\rceil, \dots, \lceil s_{k-1} + u_{k-1}/2\rceil) \geqslant r + t + 1 + n(s)$ for $u \geqslant 0$, $u \neq 0$.

Combination of a), b) and c) shows that the code generated by $[G|\underbrace{e_1|e_1|\dots|e_1}_{t}]$ is optimal.

Construction L

Adding an overall parity-check bit to a binary $[n, k, s = (s_1, \dots, s_k)]$ code gives a binary $[n + 1, k, s' = (2\lfloor(s_1 + 1)/2\rfloor, \dots, 2\lfloor(s_k + 1)/2\rfloor)]$ code.

Sporadic constructions referred to in table I are the following.

Construction M

The 7 by 14 matrix

$$\begin{bmatrix} 00011111000000 \\ 00011000111000 \\ 00010100100101 \\ 00001010010011 \\ 00110001100000 \\ 01001010001000 \\ 10000000000111 \end{bmatrix} \qquad (12)$$

is a generator matrix of an optimal binary [14,7,(5,5,5,5,4,4,4)] LUEP code. Deleting the first column and the last row from the matrix in (12) gives an optimal binary [13,6,(5,5,5,5,4,4)] code. Deleting the first two columns and the last two rows from the matrix in (12) gives an optimal binary [12,5,(5,5,5,5,4)] LUEP code.

Construction N

Application of [5,construction 1] with $m = 1$, $q = 2$ and G_1 a generator matrix of the [7,4,(3,3,3,3)] Hamming code gives an optimal binary [14,5,(7,6,6,6,6)] LUEP code.

Construction O

The 6 by 15 matrix

$$\begin{bmatrix} 000011111000111 \\ 000000000111111 \\ 100011000100100 \\ 010010100010010 \\ 001010010001001 \\ 000100001001001 \end{bmatrix} \qquad (13)$$

is a generator matrix of an optimal binary [15,6,(7,6,5,5,5,4)] LUEP code.

Construction P

The 7 by 15 matrix

$$\begin{bmatrix} 000001111111000 \\ 000001110000111 \\ 100001001000100 \\ 010001001000010 \\ 001000100100001 \\ 000100100010001 \\ 000010100001001 \end{bmatrix} \qquad (14)$$

is a generator matrix of an optimal binary [15,7,(7,6,4,4,4,4,4)] LUEP code.

Construction Q

By deleting the 8th column from the matrix in (14) we obtain a generator matrix of an optimal binary [14,7,(6,5,4,4,4,4,4)] code.

Construction R

The 8 by 15 matrix

$$\left[\begin{array}{c|c} 0 & \\ 0 & \\ 0 & \\ 0 & G \\ 0 & \\ 0 & \\ 0 & \\ \hline 1 & 00000100010000 \end{array}\right] \qquad (15)$$

where G is the matrix in (12), is a generator matrix of an optimal binary [15,8,(5,5,5,5,4,4,4,3)] LUEP code.

Construction S

The 8 by 15 matrix

$$\begin{bmatrix} 100000001100110 \\ 010000001010101 \\ 001000001001111 \\ 000100000110100 \\ 000010001110000 \\ 000001001101000 \\ 000000101011000 \\ 000000010111000 \end{bmatrix} \qquad (16)$$

is a generator matrix of an optimal binary [15,8,(5,5,5,4,4,4,4,4)] LUEP code.

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Exhibit C

Volume 44 Number 2 April 1988

PHILIPS TECHNICAL REVIEW

256-kbit SRAM

Expert system

Optical recording

REVIEW

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PH

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PHILIPS TECHNICAL REVIEW

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Philips Tech. Rev. 44, No. 2, 51-57, April 1988

Future trends in optical recording

G. E. Thomas

The text of the article below is largely taken from the invited paper that the author presented at the International Symposium on Optical Memory 1987, held in Tokyo from the 16th to the 18th of September 1987. In this article he describes the most important trends that can be recognized today in the field of optical recording. Besides the well-known audio Compact Disc ('CD'), the LaserVision disc and the Digital Optical Recording ('DOR') disc for data storage, there will be new applications in the future, especially if erasable optical recording becomes a practical reality.

Introduction

It is undoubtedly only a coincidence that the symposium for which this material was originally put together coincided with the fifteenth anniversary of the first public demonstration of optical recording, in September 1972 at Philips Research Laboratories in Eindhoven. To resurrect a cliché, it will be clear to all who have been involved in optical recording [1] that the field has been characterized during its brief history by turbulent and, at times, explosive growth. Industrially, the field of optical recording is a curious mixture of extremely successful consumer applications, such as the video disc and — in particular — the Compact Disc, and professional data-storage applications in which the major breakthrough in the market is avidly predicted but still awaited. The word 'curious' has been deliberately chosen here, since the two outstanding advantages of optical disc recording — the high storage capacity per unit area and the random-access capabilities — which could be expected to ensure its breakthrough in the professional area, have not yet led to that goal. On the other hand, these advantages have been quite successfully exploited in the products for consumer entertainment. Against this background, it is clear that it is very difficult to pre-

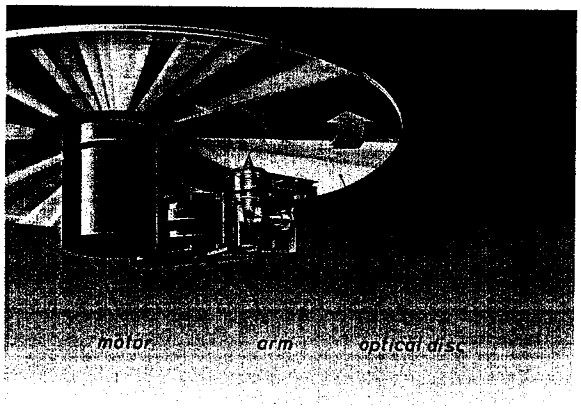

Fig. 1. An optical recording system consists of three main components: the optical disc, the arm and the motor. The information on the optical disc is contained in microscopically small optical details arranged on the disc in a spiral path, rather as in a conventional audio disc. These details are 'scanned' by a beam of light, from a semiconductor laser, which is reflected from the surface of the disc on to a detector. The arm contains all the facilities necessary for the light beam to follow the information track in the radial direction and for focusing the beam on to exactly the right position. The motor must always rotate the disc at the correct speed; in most optical recording systems this speed depends on the radial position of the arm. The unit containing the optical components is usually referred to as the 'light pen' of the system.

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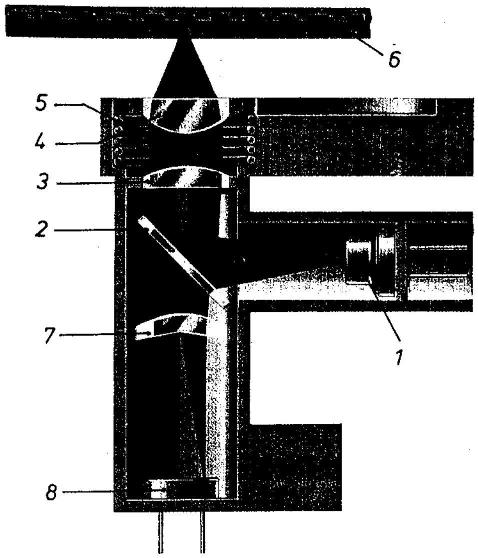


Fig. 2. Schematic drawing showing the most important components of the 'light pen' in an optical recording system. *1* semiconductor laser. *2* half-silvered mirror. *3* collimator lens, which makes the beam parallel. *4* objective lens for focusing the beam. *5* actuator, which determines the position of the objective lens. *6* optical disc. *7* beam-splitter. *8* detector for the beam reflected from the disc. The detector is in fact a multiple sensing device, and information about focusing and positioning of the beam with respect to the optical track on the disc is also obtained during the read-out. The variations in the light beam reflected from the disc can depend on various physical principles: differences in level ('pits') in a reflecting layer can give differences in intensity on reflection, differences in direction of magnetization of a recording film can affect the polarization of the reflected light ('magneto-optic recording'), or local differences in crystal structure can give differences in intensity on reflection ('phase-change recording').

dict future trends in optical recording. It is appropriate at this symposium to let the perceived requirements of sophisticated professional systems guide the predictions. However, it may well turn out that future adaptations of optical recording in the consumer area will provide much of the stimulus for new developments. In particular, the demands placed on storage systems for high-definition video signals are fairly stringent. There will certainly be a need for increased storage capacities and higher data-transfer rates.

As we look back on the brief history of optical recording systems, we can distinguish a number of research and development phases. The first phase was obviously concerned with investigating and defining the basic strategies to be followed in designing the optics, servomechanisms, coding and modulation methods, electronics and media for both analog (video) and digital (CD) read-only systems [1]-[3]. The following major surge in activity dealt with the development of media that were suitable for recording [4]-[6] and the associated investigation of higher-power laser technology and modulation methods adapted to these media. The current widespread efforts in the technology of erasable recording (based on both magneto-optics and amorphous/crystalline phase-change strategies) represent the third major development phase in optical recording [7] [8].

It is safe to predict that future developments will mainly centre on improvement of the existing systems, both in terms of a significantly higher storage density and of higher data rates. An improvement in the total performance assumes an improvement in the individual optical recording subsystems, from the laser to the disc itself; see *figs 1* and *2*.

Coding

Fig. 3 summarizes the current state of affairs with respect to storage density on optical media. In this figure, the bit density (in Mbit/cm^2) for a number of current systems is plotted as a function of the spatial optical cut-off frequency ($f_c = 2NA/\lambda$) for the optics and wavelengths used. As can be seen from the cluster of points in the lower left-hand corner, only one area of this diagram has been exploited to any degree. It is clear that the current situation is determined by the availability of semiconductor lasers in the wavelength range from 720 to 820 nm and by the relative ease of mass-producing diffraction-limited optics with a numerical aperture of 0.4 to 0.5. The spread in bit density is a measure of the effectiveness of various modulation methods in exploiting the characteristics of the optical recording channel. It is also interesting to note that the current CD system [2], in which pulse-length modulation is used, has already attained a respectable data density. If, in a hypothetical experiment, we go from the analog frequency-modulation methods like those used in video-disc systems (e.g. for PAL signals) to the digital domain via a conversion with 8 bits per pixel, we obtain a similar relatively high storage density. Both systems exploit the high accuracy with which the leading or trailing edges (or both) of optical effects on a disc can be located to encode the data stream.

Detailed investigations of the signal characteristics of the optical recording channel have only been made comparatively recently [9]. Such investigations require sophisticated optical recorders and signal-processing facilities. The channel capacity is ultimately limited by noise. For the present, however, the capacity is mainly limited by intersymbol interference and crosstalk. As better quantitative measurement techniques become available, so that the properties of various media (replicated, write-once and erasable) can be characterized in more detail, it is becoming increasingly probable that advanced coding and modulation strategies will lead to an increase in the bit density. This could amount to a factor of 4 to 5 with respect to the present situation.

An indication of some future trends is given in fig. 3. Starting with the characteristics of the CD system, for example, and assuming that noise is not the chief capacity-limiting factor of the channel, then the achievable bit density should increase with the square of the cut-off frequency. A number of reference lines (dashed) are shown for optics with a larger numerical aperture and for shorter recording wavelengths (possibly obtained by frequency doubling). The feasibility of the various combinations will be discussed briefly below.

Finally, fig. 3 shows the bit density resulting from a model calculation of the performance of the optical recording channel, based on the assumption that Gaussian white noise forms the ultimate limitation. This ultimate performance (it is proportional to $f_c^{3/2}$) is the theoretical limit for diffraction-limited optics. As can be seen, a considerable gap exists with respect to the current systems. It is unrealistic to presume that this gap can ever be completely eliminated, since practical considerations such as manufacturing tolerances and the like remain. Nevertheless, a combination of tactics (i.e. improving the coding techniques in combination with a higher f_c) will undoubtedly offer prospects of achieving recording densities that are higher by at least an order of magnitude.

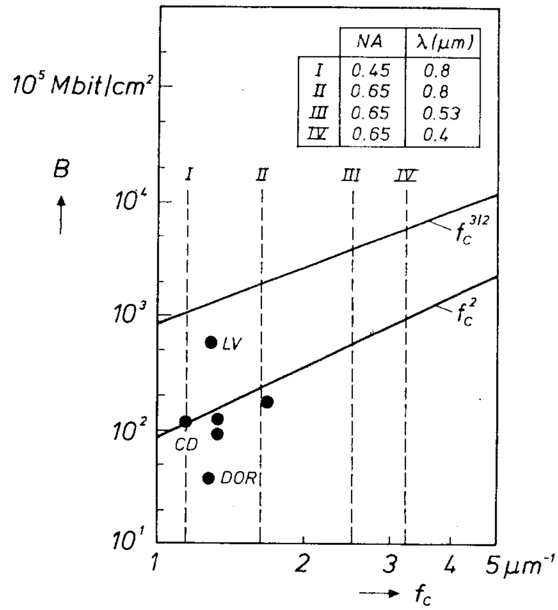


Fig. 3. Storage density B as a function of the spatial optical cut-off frequency $f_c = 2NA/\lambda$, where NA is the numerical aperture and λ is the wavelength of the light (in μm). The points represent the current state of the technology for a number of existing systems, such as Compact Disc (CD), LaserVision (LV) and Digital Optical Recording (DOR). The vertical dashed lines indicate f_c for particular combinations of NA and λ. The two sloping lines represent extrapolations, one based on the CD system, assuming a quadratic relation between f_c and B, the other on a model calculation of the noise-limited situation in which B is proportional to $f_c^{3/2}$.

Lasers

The future trends in the development of semiconductor lasers for optical recording are fairly easy to predict. A subject of great importance at the moment is the development of lasers in the wavelength range around 800 nm with a high useful power output. The term 'useful' here relates to the power in a beam of sufficiently high quality for application in the light pen of an optical recording system. The lifetime and the feedback characteristics of these lasers [10] are also critical parameters. The trend is clearly towards the use of higher powers for advanced write-once and erasable materials. This is a natural consequence of the general desire to attain higher data rates in optical recording. Here, of course, developments in the field of laser arrays (see *fig. 4*) in which each laser can be independently modulated are eagerly awaited, since the possibilities of achieving higher data rates through the use of parallel recording are obvious.

A second hoped-for trend in lasers is the reduction of the wavelength. This can be achieved by making

[2] Special issue on the Compact Disc, Philips Tech. Rev. **40**, 149-180, 1982.

[3] H. C. Haverkorn van Rijsewijk, P. E. J. Legierse and G. E. Thomas, Manufacture of LaserVision video discs by a photopolymerization process, Philips Tech. Rev. **40**, 287-297, 1982; J. G. Kloosterboer, G. J. M. Lippits and H. C. Meinders, Photopolymerizable lacquers for LaserVision video discs, Philips Tech. Rev. **40**, 298-309, 1982.

[4] G. C. Kenney *et al.*, An optical disk replaces 25 mag tapes, IEEE Spectrum **16**, No. 2 (February), 33-38, 1979; K. Bulthuis *et al.*, Ten billion bits on a disk, IEEE Spectrum **16**, No. 8 (August), 26-33, 1979; L. Vriens and B. A. J. Jacobs, Digital optical recording with tellurium alloys, Philips Tech. Rev. **41**, 313-324, 1983/84.

[5] D. J. Gravesteijn and J. van der Veen, Organic-dye films for optical recording, Philips Tech. Rev. **41**, 325-333, 1983/84.

[6] H. van Tongeren and M. Sens, Write-once phase-change recording in GaSb, Topical Meeting on Optical Data Storage, Technical Digest Series 1987, Vol. 10, (Optical Society of America, Washington D.C., 1987), pp. 50-53; D. J. Gravesteijn, H. M. van Tongeren, M. Sens, T. Bertens and C. J. van der Poel, Phase-change optical data storage in GaSb, Appl. Opt. **26**, 4772-4776, 1987.

[7] M. Hartmann, B. A. J. Jacobs and J. J. M. Braat, Erasable magneto-optical recording, Philips Tech. Rev. **42**, 37-47, 1985.

[8] D. J. Gravesteijn, New material developments for write-once and erasable phase-change optical recording, Topical Meeting on Optical Data Storage, Technical Digest Series 1987, Vol. 10, (Optical Society of America, Washington D.C., 1987), pp. 40-43; also published in Appl. Opt. **27**, 736-738, 1988.

[9] C. M. J. van Uijen, Measured performance of modulation strategies for digital optical recording media, Topical Meeting on Optical Data Storage, Technical Digest Series 1987, Vol. 10, (Optical Society of America, Washington D.C., 1987), pp. 102-105.

[10] B. H. Verbeek, D. Lenstra and A. J. den Boef, Noise due to optical feedback in semiconductor lasers, Philips Tech. Rev. **43**, 292-302, 1987.

use of new materials for semiconductor lasers [11]. Another way of doing this is to use the method of frequency doubling briefly mentioned earlier. A large world-wide effort is being devoted to frequency-doubled laser systems in which second harmonics are generated in nonlinear materials. The relative ease of producing III-V laser-pumped Nd:YAG lasers, in which frequency-doubling is applied to give a wavelength of 530 nm, has stimulated the search for similar techniques for versions of the 800-nm semiconductor laser with direct-doubling and high power (at 400 nm). In view of the power densities that this would require from the III-V laser, it would be necessary to consider the application of phase-coupled laser arrays (fig. 4). Success in this area would mean a breakthrough in achievable storage densities, although such a breakthrough would have to be accompanied by a great deal of development activity in recorder optics and on media that can be used in the 400-nm range.

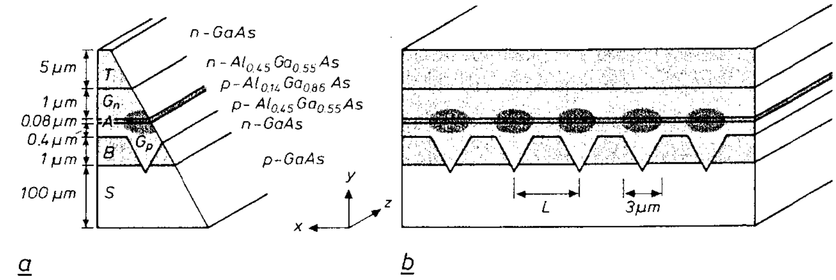


Fig. 4. *a*) Cross-section through the layer structure in a double-heterojunction semiconductor laser. On a substrate S of p-GaAs there are, in order, a current-isolating layer B, a confinement layer G_p, an active layer A, in which the laser operation takes place, a confinement layer G_n and a top layer T, to which contacts can be applied. The V groove etched into the current-isolating layer ensures that the current injected into the structure in the y-direction is confined in the x-direction. Dimensions and compositions of the various layers are indicated in the figure. The laser light is generated in the grey region and propagates in the z-direction. The length of the layer structure in the z-direction is much greater than the transverse dimension of the structure. *b*) A number of such lasers placed next to one another form a laser array. If the spacing L is large, the lasers can operate independently of one another, and at a spacing of say 150µm they can in principle be modulated independently, which is useful for parallel recording. If L is small (e.g. 5 µm), a composite laser with an increased power is formed because of phase coupling.

Optics and actuators

The growth of optical recording into the mass market in the consumer area has given an enormous stimulus to the optical industry. It has led to the development of various techniques for the mass production of inexpensive, high-quality optics. These techniques include the hybrid polymer-on-glass replication process and the precision moulding of glass and plastics for the production of aspheric collimator and objective lenses [12]; see *fig. 5*. In addition, the use of optical elements with a varying refractive index ('gradient index') has emerged as a viable technology. These new technologies have permitted the design and construction of light paths that are not only less expensive to manufacture, but are also relatively simple and light in weight. More recent developments in the area of light paths incorporating beam splitters based on grating operation will strengthen this trend. Finally, there are also the fascinating developments in integrated planar light paths and the use of holographic focusing techniques and optical-fibre technology. These are the forerunners of optical components that can easily be incorporated in high-performance actuators for use in systems in which the speed of rotation of the disc is increased for high-data-rate applications and in which the random-access facility is greatly improved.

Another challenge is that of the investigation of the strengths and weaknesses of the various modern optical technologies when they are combined with the predicted trends noted earlier towards higher numerical apertures and shorter laser wavelengths.

In the past the servo technology required for dynamic positioning to submicron accuracy has often been called the Achilles heel of optical recording. But now many authorities in this field agree that new servo techniques being developed, new actuator concepts [13] (see *fig. 6*) and new electronic possibilities will allow this area to keep pace with developments in the other subsystems. In other words, the control systems for optical recorders should not form a bottleneck for foreseeable developments in high-performance systems.

Recording media

Of the developments in the various subsystems associated with optical-recording technology, those in the recording media are the most diverse. Using the obvious subdivision of replicated, write-once and erasable discs, some general trends can be predicted.

Replicated media [3] will undoubtedly maintain their current status as carriers of information intended for large-scale distribution. In view of this, the necessity to define widely accepted system standards for this area will preclude unlimited diversification in the types of replicated discs. The pattern of development here will tend towards reduction of costs and improvement in quality — where quality means fewer inherent errors on the disc itself. The development of simpler mastering techniques and cost-effective replication technologies for the production of limited series of discs would improve market penetration in the areas of CD-ROM and CD-Audio. The development of new disc formats, which now range from 9 to 30 cm in diameter, will undoubtedly stabilize, since most application areas would now seem to be covered.

The prediction of the future trends in write-once media and systems is uncertain. Although there seems to be a niche in the marketplace for truly permanent

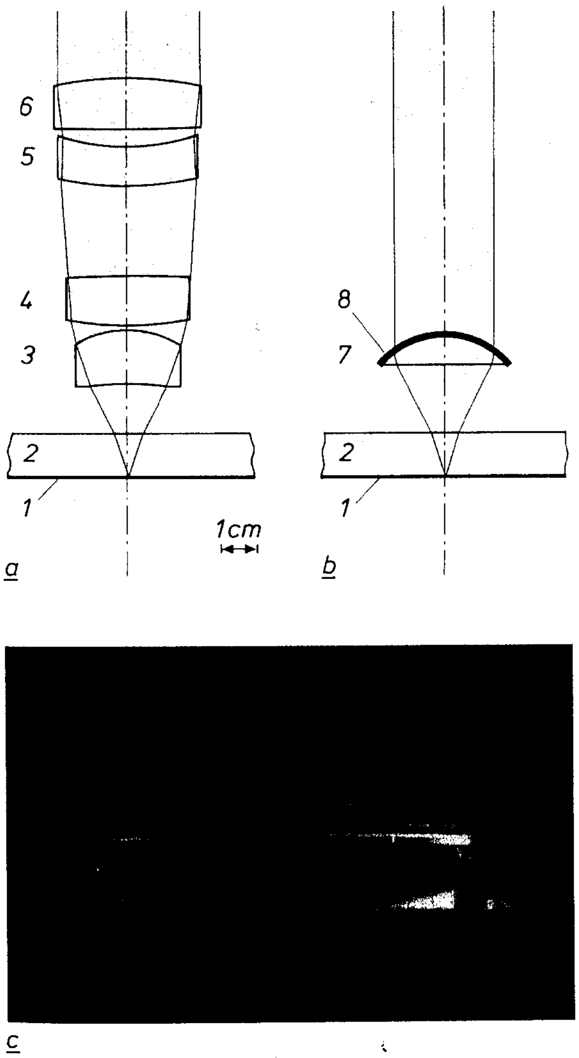


Fig. 5. In optical recording an objective lens focuses the laser beam on to the recording layer (*1*) on the 'reverse' side of the transparent optical disc (*2*). If the consequences of the lens aberrations are to be kept within acceptable limits, something other than a single *spherical* lens must be used. *a*) One known solution is to use a number of lenses (*3* to *6*), as in the classical microscope objective. The objective shown here has a numerical aperture of 0.45, and with its mount it weighs about 1.5 grams. *b*) If an *aspheric* lens is used it is possible to obtain an optically equivalent objective made in one piece and weighing only 20 mg. A lens of this type can be made by the 'polymer-on-glass' replication technology. A thin layer of polymer is applied to a spherical glass preform (*7*)[11]. The thickness of the polymer coating (*8*) is not constant, but is varied in an accurately predetermined way from 0 to 14 μm to produce a high-quality aspheric lens. *c*) Photograph of an aspheric lens made in this way.

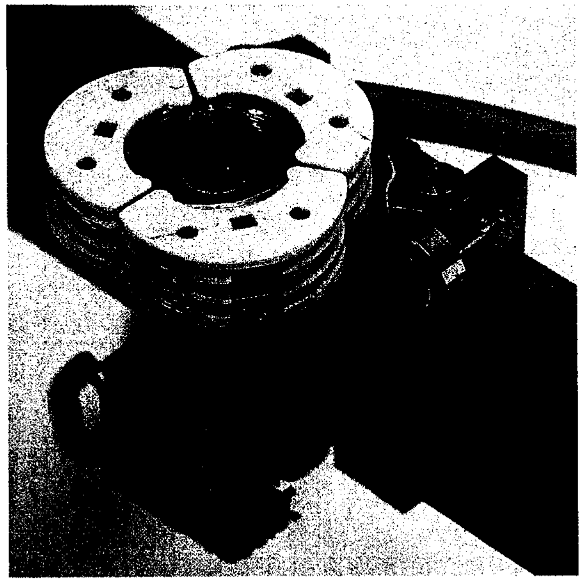


Fig. 6. Like any other rigid body, an objective lens has six degrees of freedom: three orthogonal translations and three rotations. In the actuators now in use only two or three degrees of freedom are normally employed (they are called 2D or 3D actuators). A recent development has led to the 5D actuator shown here, in which the objective lens 'floats' freely surrounded by six separate banana-shaped coils that can control all five of the relevant degrees of freedom (rotation about the axis of symmetry of the lens is irrelevant). The actual lens has a diameter of 3 mm, and is fixed in a permanent-magnet ring with a diameter of 5.5 mm. It can be moved 2 mm in the vertical direction and 1 mm in two perpendicular horizontal directions. The complete actuator is mounted on an arm that can be used to give larger displacements.

[11] See for example: Semiconductor laser for visible light, Philips Tech. Rev. 44, 23, 1988.

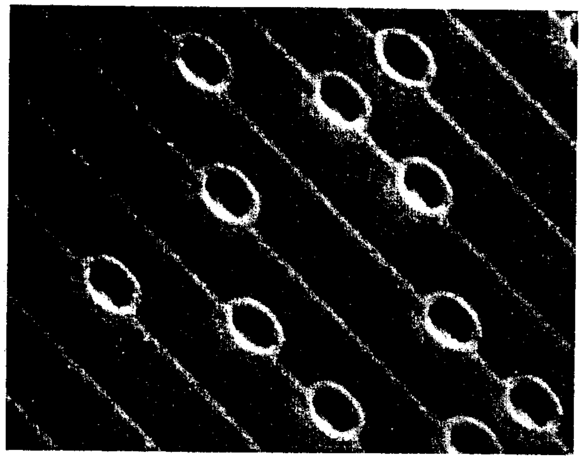
[12] J. Haisma, T. G. Gijsbers, J. J. M. Braat, W. Mesman, J. M. Oomen and J. C. Wijn, Aspherics, Philips Tech. Rev. 41, 285-303, 1983/84.

[13] G. E. van Rosmalen, A floating-lens actuator, paper TA7, Int. Symp. on Optical Memory (ISOM '87), Tokyo, Japan, Sept. 1987; also published in Jap. J. Appl. Phys. 26 (suppl. 26-4), 195-197, 1987.

archival information storage — a type of storage for which optical recording technology [4] offers unique possibilities — the further development of this market will become uncertain once erasable systems are widely available. Nevertheless, it should be mentioned that the signal characteristics of a number of proposed write-once systems — e.g. those using organic-dye materials [5] and an amorphous-to-crystalline phase-change [6] — are so good that they rival replicated discs in quality. Although in the past the properties of some of the 'ablative' (hole-forming) write-once systems could deteriorate because of the inherent chemical instability of the materials, a number of the more recently developed systems no longer suffer from this problem, so that truly archival recordings can be made (*fig. 7*).

The coming years will see the introduction of erasable optical-recording products (*fig. 8*). It is generally conceded that magneto-optics [7] is in a very advanced state of development and that this technology will lead the way into the marketplace. This introduction has become a possibility because of the development in recent years of procedures for direct overwriting, of simpler systems for polarization read-out of the signals and of optical-disc technologies that circumvent the problems of the limited chemical stability of the magneto-optical thin films. A great deal of work world-wide is now being concentrated on assessing the limits of magneto-optics in terms of signal characteristics, compatibility with various special modulation methods and useful lifetime.

In addition to magneto-optics, considerable progress has been made in the past year in the development of recording systems in which writing and erasure depend on a reversible amorphous/crystalline phase change [8]; fig. 8*b*. It is now clear that phase-change recording is feasible — this technology is attractive because of its relative simplicity and reasonable compatibility with the recording and read-out methods for replicated and write-once discs — even though it is not as mature a technology as magneto-optics. A great deal of comparative assessment of the various proposed phase-change systems still has to be done. In some quarters, however, predictions are being made that erasable phase-change systems will one day match the signal quality of known advanced write-once systems and thus become the choice for future generations of high-performance optical recording.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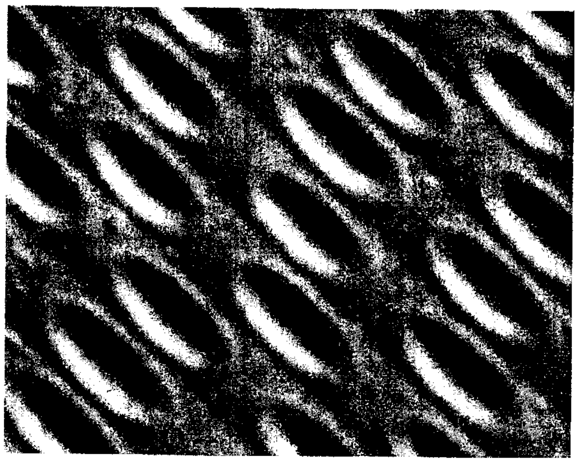
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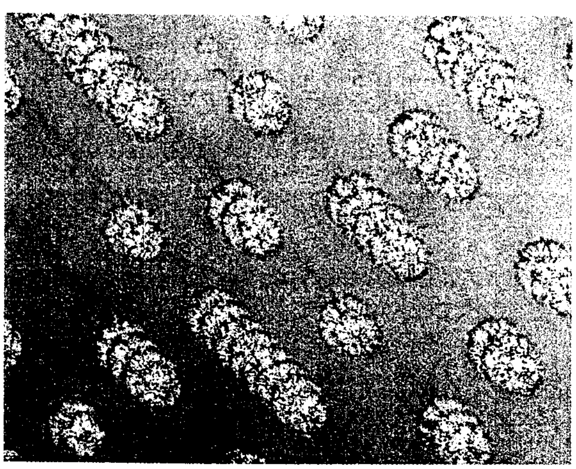
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Fig. 7. *a*) In 'ablative' write-once optical systems a 'hole' or 'pit' is formed in a recording layer by local heating with a laser beam. This photograph shows an example of digital optical recording in which a tellurium alloy is used as the recording layer. *b*) Example of ablative optical recording in which the recording layer is an organic dye. The photograph relates to analog optical recording, e.g. of video information. *c*) A modern alternative to ablative methods is based on the difference in optical properties between the amorphous and crystalline states of a material such as gallium antimonide. In this method a short pulse from a 'write' laser forms small domains with a *crystalline* structure in an *amorphous* layer. The result is extremely stable, both chemically and physically. It will therefore have a long life and be very suitable for archival recording. In the example of digital recording shown here the individual crystalline domains are so close together that they overlap. (In this figure and the next one the spacing between adjacent tracks is always about 1.6 μm.)


a

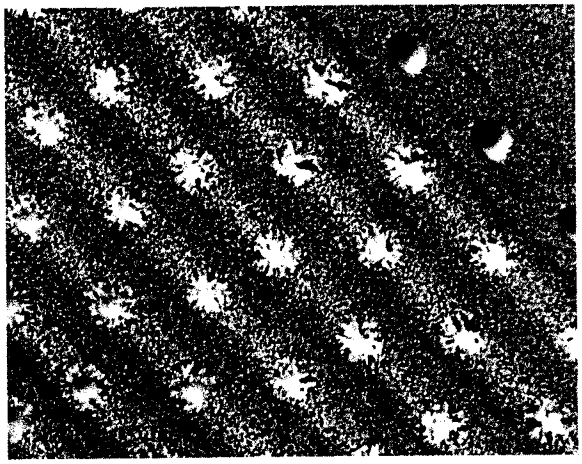
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Fig. 8. *a*) Magneto-optic effects can be used for erasable optical recording. A magnetic recording layer is initially magnetized perpendicular to the surface; a laser can then be used in conjunction with a constant external magnetic field to reverse the direction of magnetization locally. Read-out depends on the rotation of the plane of polarization of incident laser light. The information can be erased by irradiating with a laser beam in the presence of a constant external magnetic field, but now with the same direction as the original magnetization. *b*) A more recent development is erasable optical recording based on a reversible amorphous/crystalline phase change. Here the recording layer is of gallium antimonide or indium antimonide, doped with other elements. Initially the recording layer is in the *crystalline* state, and small *amorphous* domains can be created in it by heating rapidly with a laser beam to just above the melting point (the three 'black' areas in the upper right-hand corner of the photograph are amorphous domains). These have different reflection properties from the surrounding crystalline material. The crystalline structure can be restored by heating again with a laser beam to just below the melting point: this erases the stored information. Many erased domains can be identified in the photograph; their reflection properties are again the same as for the surrounding material.

As the demands for extremely high storage densities increase, the search for new optical recording systems that meet the requirements will continue. In addition to the developments mentioned above — which we can assess reasonably well in the light of the knowledge we now have — entirely new methods will undoubtedly emerge. It is almost certain that older ideas such as multilevel recording will be reinvestigated. New techniques such as 'spectral hole burning', in which the optical properties of a recording layer are modified wavelength-selectively and the bit density can increase by several orders of magnitude, offer exciting prospects. In this short summary the emphasis has been on the evolution of current systems to higher-performance systems. But in a field as young as optical recording we should not discount the probability of a revolution leading to entirely new possibilities.

Summary. The relatively short history of optical recording can be divided into a number of periods. First there is the development of the basic read-only systems that are intended for playing replicated discs, then come the systems and media that can also be used for recording, and finally there is the appearance of erasable optical recording. The future challenge will be to develop systems with higher densities and higher data rates. This article looks at the resultant consequences for the subsystems of optical recording — from laser to disc.