A Practical Discrete Multitone Transceiver Loading Algorithm for Data Transmission over Spectrally Shaped Channels

Peter S. Chow, John M. Cioffi, and John A.C. Bingham

Abstract— **In this paper, we present a finite-granularity, loading algorithm for a Discrete Multitone (DMT) modulation system. The proposed algorithm offers significant implementational advantages over the well-known water-pouring method and the earlier Hughes-Hartogs algorithm, while typically suffering only negligible performance degradation relative to the optimal solution. We also present simulation results of this loading algorithm applied to the newly proposed Asymmetric Digital Subscriber Lines (ADSL) service.**

I. Introduction

In recent years, various digital subscriber line (DSL) [1] and high-speed voice-band modem applications [2] have generated a tremendous amount of research interest in designing high performance, yet cost effective, digital data transmission systems. One extremely efficient modulation and equalization technique that is particularly well suited for these types of applications is Discrete Multitone modulation (see [3], [4], and [5]). A crucial aspect in the design of a DMT system for data transmission over spectrally shaped channels is the need to optimize the system transmission bandwidth through an optimal loading algorithm. In this paper, we present a practical and efficient DMT loading algorithm.

II. A DMT Loading Algorithm

It has long been known that the capacity-achieving energy distribution for a spectrally shaped channel corresponds to a "water-pouring" distribution [6]. However, while the water-pouring energy allocation will indeed yield the optimal solution, it is often difficult to compute, and it tacitly assumes infinite granularity in constellation size, which is not realizable. One known finite-granularity multicarrier loading algorithm is the Hughes-Hartogs algorithm(see [7]). Unfortunately, the Hughes-Hartogs algorithm is very slow for applications like ADSL, where a large number of bits (in the range of 400 to 2000+) will be contained in each DMT symbol and transmitted over a large number of subchannels (typically 256). As an alternative to Hughes-Hartogs, we propose the following iterative algorithm. This algorithm consists of three main sections. It first finds the (approximately) optimal system performance margin, γ_{margin}[1], then it guarantees convergence with a suboptimal loop, and lastly it adjusts the energy distribution accordingly on a subchannel-by-subchannel basis[2]. Algorithmically, this procedure can be summarized as follows:

1. Compute subchannel signal-to-noise ratio, $SNR(i)$, $\forall i$, assuming that all subchannels are used with a normalized energy level of $\mathcal{E}(i) = 1 \ \forall i$.
2. Let $\gamma_{margin} = 0\ (dB)$, $IterateCount = 0$, and $UsedCarriers = N$, where γ_{margin} is the current system performance margin, and N is the maximum number of usable carriers.
3. For $i = 1$ to N, calculate $b(i)$, $\hat{b}(i)$, $diff(i)$, and $UsedCarriers$ according to:

$$b(i) = log_2(1 + \frac{SNR(i)}{\Gamma + \gamma_{margin}\ (dB)}) \quad (1)$$

$$\hat{b}(i) = round[b(i)] \quad (2)$$

$$diff(i) = b(i) - \hat{b}(i) \quad (3)$$

$$If\ \hat{b}(i) = 0,\ UsedCarriers = UsedCarriers - 1 \quad (4)$$

   where Γ in Equation (1) is the "SNR gap" in the well-known "gap approximation"[8].
4. Let $B_{total} = \sum_{i=1}^{N} \hat{b}(i)$. Stop and declare bad channel if $B_{total} = 0$.
5. Compute new γ_{margin} according to:

$$\gamma_{margin} = \gamma_{margin} + 10\ log_{10}(2^{\frac{B_{total} - B_{target}}{UsedCarriers}}) , \quad (5)$$

   where B_{target} is the desired number of bits per DMT symbol.
6. Let $IterateCount = IterateCount + 1$.
7. If $B_{total} \neq B_{target}$ and $IterateCount < MaxCount$, let $UsedCarriers = N$ and go to step 3, else go to step 8.
8. If $B_{total} > B_{target}$, then subtract one bit at a time from $\hat{b}(i)$ on the carrier that has the smallest $diff(i)$, adjust $diff(i)$ for that particular carrier, and repeat until $B_{total} = B_{target}$.
9. If $B_{total} < B_{target}$, then add one bit at a time to $\hat{b}(i)$ on the carrier that has the largest $diff(i)$, adjust $diff(i)$ for that particular carrier, and repeat until $B_{total} = B_{target}$.

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[1] System performance, or noise, margin is defined as the additional amount of noise (in dB) that the system can tolerate, while still achieving the minimum desired bit error rate requirement.

[2] The final energy adjustment section is only used when the transmission system is constrained by the total transmit power.

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10. Adjust input energy distribution accordingly so that $P_e(i) = P_{e,target}\ \forall i$ given the bit allocation $\hat{b}(i)$. At the end of this operation, the resulting transmit power mask will no longer be perfectly flat, but rather the transmit power mask will have a saw-tooth shape with approximately a 3 dB peak-to-peak deviation due to the integer bit constellation constraint.
11. Scale final energy distribution for all used carriers with a common scaling factor so that the total energy level, $\mathcal{E}_{total}$, equals to the target energy level, $\mathcal{E}_{target}$. This common scaling factor is equivalent to the final system performance margin at the specified bit error rate.

In essence, steps 1 to 7 will iteratively find the appropriate system performance margin, γ_{margin}. If the algorithm does not converge after $MaxCount$ iterations, convergence is forced with steps 8 and 9. Lastly, steps 10 and 11 fine tune the energy distribution on a subchannel-by-subchannel basis to assure equal and optimal system performance margin over all used subchannels at the specified bit error rate (BER). Furthermore, for an uncoded, zero-margin system with a BER of 10^{-7}, the SNR gap, Γ, in the calculation of $b(i)$ in step 3 above is approximately 9.8 dB[3].

While this algorithm may be slightly suboptimal relative to the Hughes-Hartogs algorithm at times[4], it will typically converge much faster than Hughes-Hartogs for applications like ADSL. Unlike Hughes-Hartogs, whose average running time is proportional to $\mathcal{O}\ (B_{total} \times N)$, the worst case running time for our proposed algorithm is proportional to $\mathcal{O}\ (MaxCount \times N + 2N)$. Based on actual system implementation with commercial DSP chips, it has been found that the number of iterations, $IterateCount$, necessary to bring this algorithm to convergence over real ADSL loops is no more than 10; i.e, $MaxCount = 10$ will suffice. Furthermore, this algorithm offers the added advantage of no additional penalty in terms of execution time when we reduce the granularity of constellation size from integer number of bits to fractional number of bits, say half bit increments. The running time for Hughes-Hartogs algorithm, on the other hand, will be doubled with half bit constellation sizes. More details on the general conditions for convergence on our proposed algorithm can be found in [9].

To illustrate the operation of this algorithm, in Figure 1 we plot the received SNR curve for ADSL canonical test loop 9[5] [10] with 100 mW of input power in the presence of 49 ADSL far-end crosstalkers and AWGN. We have assumed a sampling rate of 2.048 MHz, a lower bandedge of 40 kHz, and a FFT size of 512. The frequency domain "ripples" are due to unterminated bridged taps. In Figures 2 and 3, we show the resulting bit and input power distributions, respectively. The data rate for this particular simulation is 1.728 Mbps, which corresponds to a (216,200) Reed-Solomon coded transmission at a raw data rate of 1.6 Mbps.

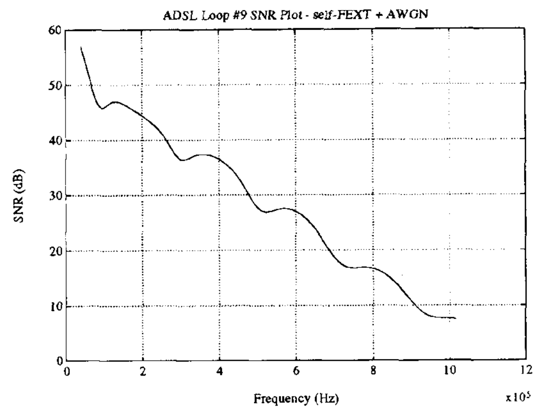

Fig. 1. ADSL Loop 9 Received SNR Curve with 49 ADSL FEXT Disturbers + AWGN

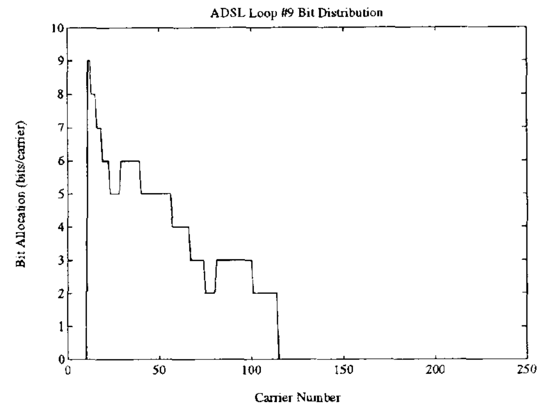

Fig. 2. ADSL Loop 9 Bit Distribution with 49 ADSL FEXT Disturbers + AWGN

III. Applications and Results

We now apply the loading algorithm presented earlier to the provisioning of ADSL service over copper twisted pairs. We will focus our simulation efforts on the ADSL downstream, or the high speed, direction of transmission, where the data rate is in the range of 1.6 Mbps to 6.4+ Mbps, and the required BER is 10^{-7} (see [11]). The ADSL environment introduces a variety of impairments, including ISI, AWGN from a number of possible sources, far-end crosstalk (FEXT) from adjacent twisted pairs within the same binder group, and impulse noise [12]. For the simulations, we will assume an AWGN floor of -143 dBm/Hz

[3]The SNR gap is a convenient single-parameter characterization of a transmission system, and it is a function of the chosen coding scheme, the target BER, and the desired minimum system performance margin. In particular, the SNR gap effectly estimates the difference between channel capacity and the actual achievable rate of a transmission system.

[4]In fact, often the two algorithms will yield the same solution, depending on the actual line and noise scenario.

[5]ADSL canonical test loop 9 consists of a segment of (3000 ft/26 AWG) wire, followed by a (1500 ft/26 AWG) bridged tap, followed by a segment of (6000 ft/26 AWG) wire, followed by another (1500 ft/26 AWG) bridged tap, followed by a segment of (1500 ft/26 AWG) wire, and followed by a last (1500 ft/26 AWG) bridged tap.

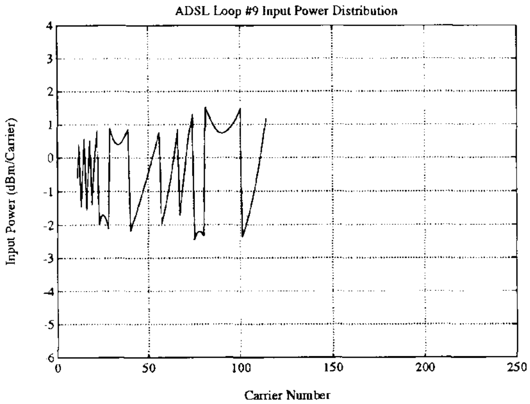

Fig. 3. ADSL Loop 9 Input Power Distribution with 49 ADSL FEXT Disturbers + AWGN

(two-sided) and 49 ADSL FEXT disturbers only. Impulse noise is not included in our simulation. The basic DMT system that is used for simulation has a sampling rate of 2.048 MHz and a FFT size of 512, resulting in a multicarrier symbol rate of 4 kHz. We will compare the performance of a DMT system implemented with the algorithm presented in this paper with that of an optimized, water-pouring, DMT system (see [3] and [4] for example). In fact, we will further restrict our integer bit constellation algorithm to have at least 2 bits and at most 10 bits per used carrier. We also assume a 40 kHz lower bandedge and use two representative ADSL loops; namely, the 9 kft, 26 gauge loop and the 18 kft, 24 gauge loop, with two possible data rates; i.e., 4.0 Mbps and 1.6 Mbps. Table I summarizes our computer simulation results of system performance margin. Clearly,

Rate	Length	Gauge	Water-Pour	Integer Bit	Difference
4.0 Mbps	9 kft	26 AWG	15.9 (dB)	15.7 (dB)	0.2 (dB)
1.6 Mbps	9 kft	26 AWG	27.5 (dB)	27.3 (dB)	0.2 (dB)
4.0 Mbps	18 kft	24 AWG	3.0 (dB)	1.7 (dB)	1.3 (dB)
1.6 Mbps	18 kft	24 AWG	20.9 (dB)	20.7 (dB)	0.2 (dB)

TABLE I

System Performance Margin of Water-Pouring, ∞ Granularity DMT vs. Integer Bit Granularity DMT

the performances of the two systems are virtually identical, except for the worst case scenario, where we are trying to transmit 4.0 Mbps over the 18 kft, 24 gauge loop, and in that case, we lose 1.3 dB. This degradation in performance turns out to be caused mainly by the 2 bit minimum/10 bit maximum constraint that we imposed on our system.

IV. Conclusion

In this paper, we have presented a practical DMT loading algorithm for high speed data transmission over channels with severe ISI, as in the ADSL transmission environment. We showed, through computer simulation, that the performance of our algorithm using only constellations with integer number of bits per 2D symbol is nearly as good as the optimal water-pouring solution with infinite granularity in constellation size, as long as we adjust the input energy appropriately on a subchannel-by-subchannel basis.

References

[1] J.W. Lechleider. High Bit Rate Digital Subscriber Lines: A Review of HDSL Progress. *IEEE Journal on Selected Areas in Communications*, 9(6):769–784, August 1991.

[2] CCITT Study Group XVII. Technical Contributions and Recommendations for the Standardization of V.fast Family of Modems. Period: 1989-1992.

[3] J.A.C. Bingham. Multicarrier Modulation for Data Transmission : An Idea Whose Time Has Come. *IEEE Communications Magazine*, 28(5):5–14, May 1990.

[4] I. Kalet. The Multitone Channel. *IEEE Transactions on Communications*, 37(2):119–124, February 1989.

[5] A. Ruiz, J.M. Cioffi, and S. Kasturia. Discrete Multiple Tone Modulation with Coset Coding for the Spectrally Shaped Channel. *IEEE Transactions on Communications*, 40(6):1012–1029, June 1992.

[6] R.G. Gallager. *Information Theory and Reliable Communication*. Wiley, New York, NY, 1968.

[7] D. Hughes-Hartogs. Emsemble Modem Structure for Imperfect Transmission Media. U.S. Patents Nos. 4,679,227 (July 1987), 4,731,816 (March 1988) and 4,833,706 (May 1989).

[8] G.D. Forney Jr. and M.V. Eyuboglu. Combined Equalization and Coding Using Precoding. *IEEE Communications Magazine*, 29(12):25–34, December 1991.

[9] Peter S. Chow. *Bandwidth Optimized Digital Transmission Techniques for Spectrally Shaped Channels with Impulse Noise*. PhD thesis, Stanford University, May 1993.

[10] K. Sistanizadeh. Proposed Canonical Loops for ADSL and Their Loss Characteristics. *ANSI T1E1.4 Technical Subcommittiee Working Group Contribution*, (91-116), August 1991. Montreal.

[11] P.S. Chow, J.C. Tu, and J.M. Cioffi. Performance Evaluation of a Multichannel Transceiver System for ADSL and VHDSL. *IEEE Journal on Selected Areas in Communications*, 9(6):909–919, August 1991.

[12] J.-J. Werner. The HDSL Environment. *IEEE Journal on Selected Areas in Communications*, 9(6):785–800, August 1991.