

ORTHOGONAL FREQUENCY MULTIPLEX DATA TRANSMISSION SYSTEM

Filed Nov. 14, 1966

3 Sheets-Sheet 1

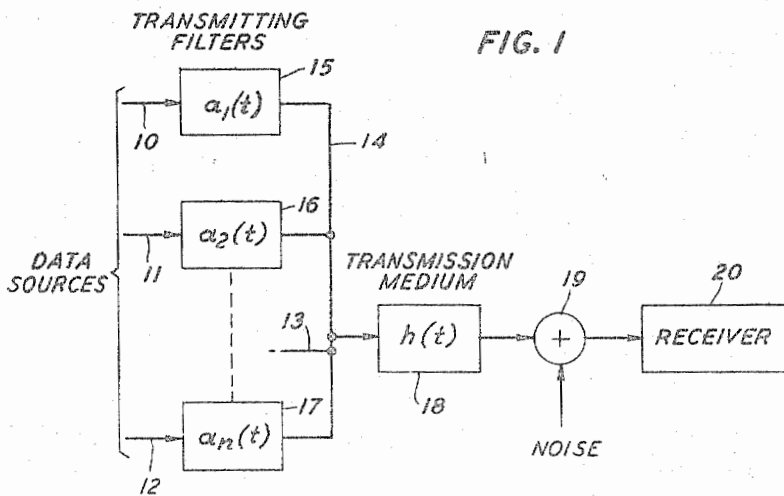


FIG. 1

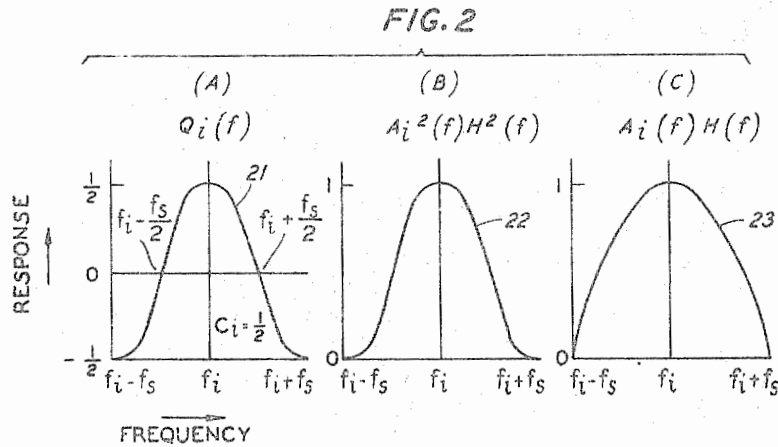


FIG. 2

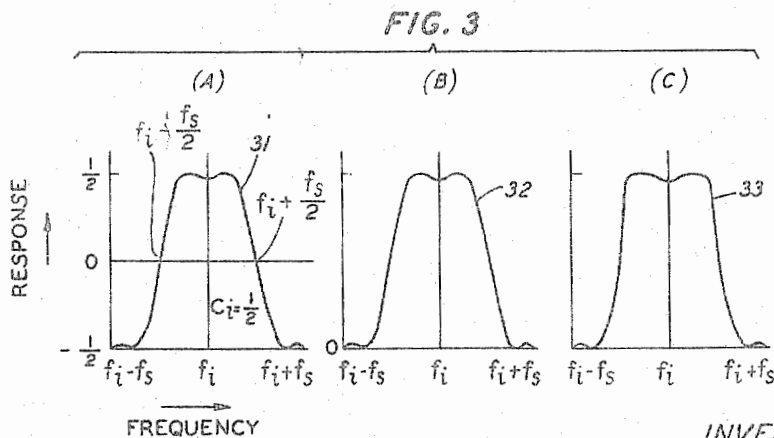
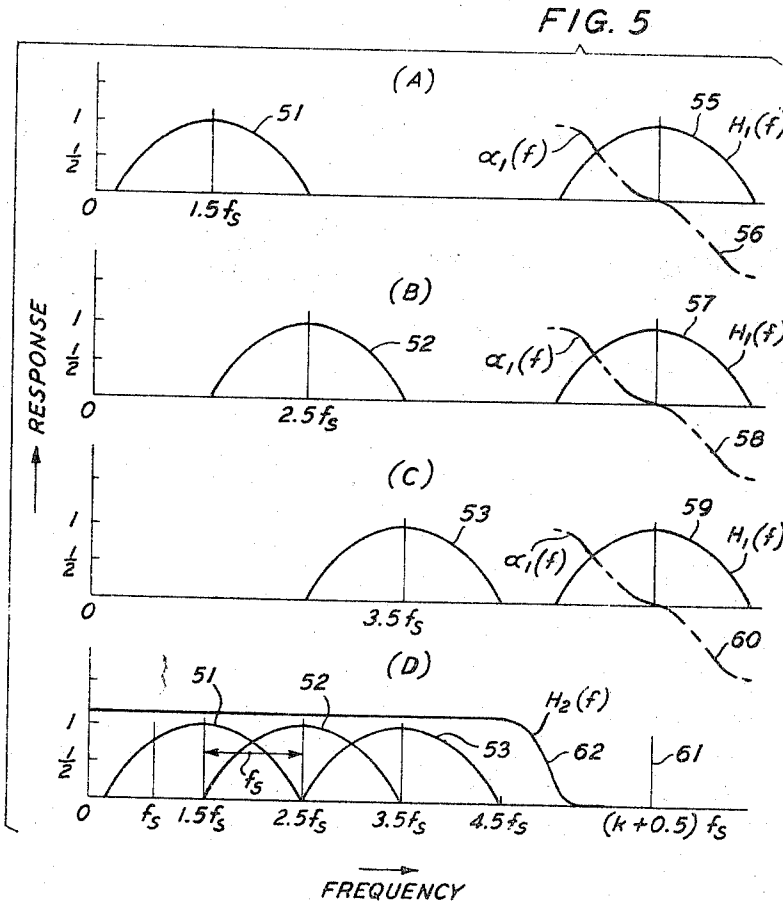
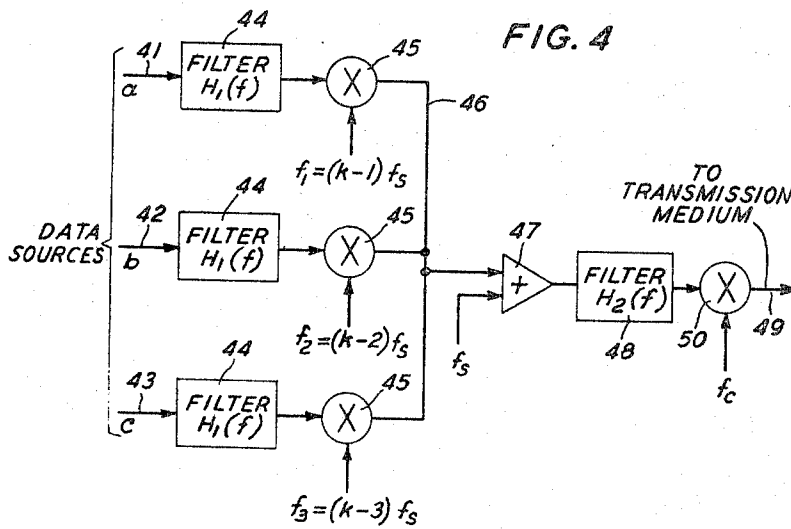


FIG. 3

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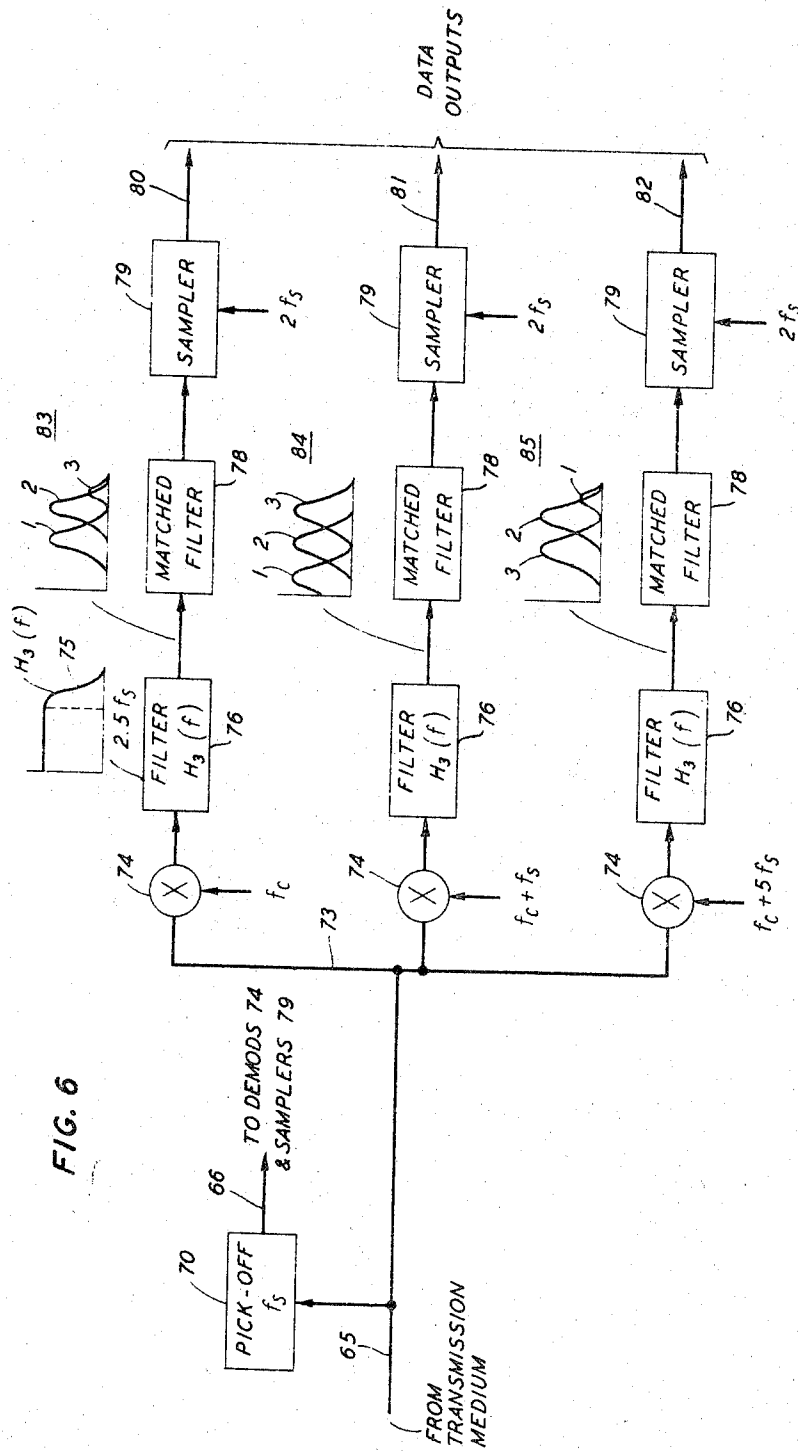


FIG. 6

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2

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**ORTHOGONAL FREQUENCY MULTIPLEX  
DATA TRANSMISSION SYSTEM**

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10 Claims

**ABSTRACT OF THE DISCLOSURE**

Apparatus and method for frequency multiplexing of a plurality of data signals simultaneously on a plurality of mutually orthogonal carrier waves such that overlapping, but band-limited, frequency spectra are produced without causing interchannel and intersymbol interference. Amplitude and phase characteristics of narrow-band filters are specified for each channel in terms of their symmetries alone. The same signal protection against channel noise is provided as though the signals in each channel were transmitted through an independent medium and intersymbol interference were eliminated by reducing the data rate. As the number of channels is increased, the overall data rate approaches the theoretical maximum.

This invention relates to systems for transmitting multiple channels of information signals over band-limited transmission media.

Multiplex transmission systems employing sinusoidal carriers separated in frequency, or rectangular pulse carriers separated in time, or combinations thereof, are well known. These known systems have the common characteristic that in order to avoid mutual interference among the channels guard bands of frequency or time are provided between channels. These guard bands represent a waste of valuable and limited bandwidth.

In digital data transmission, for example, it is common practice to transmit a plurality of data channels through a single band-limited transmission medium. In view of the limitation of frequency bandwidth in practical transmission media, the problem of maximization of the overall data rate and the concomitant minimization of interchannel and intersymbol interference arises. The general solution has been to center the individual channels on equally spaced carrier frequencies and to provide a finite guard band between channels. This has meant limiting the usable bandwidth of each channel to somewhat less than the carrier wave spacing in order to avoid interchannel interference in the frequency domain. The overall data rate is therefore much less than that attainable if the guard space could be eliminated without causing interference.

In the time domain, on the other hand, because the impulse response of band-limited transmission media is spread out in time, the signalling rate is generally held below the theoretical maximum in order to avoid intersymbol interference.

It is one object of this invention to define a new class of band-limited signals capable of being transmitted in parallel channels at substantially the maximum possible data rate without incurring either interchannel or intersymbol interference.

It is another object of this invention to so shape the spectra of individual signaling channels that the spectra of adjacent channels by virtue of their orthogonality can overlap without producing interchannel interference.

It is still another object of this invention to render the

independent of the phase characteristic of the transmission medium.

It is yet another object of this invention to achieve an overall data rate in a band-limited transmission medium approaching the theoretical maximum rate with physically realizable filters having smooth amplitude rolloffs and arbitrary phase characteristics.

It is a further object of this invention to so shape the response functions of adjacent channels in a frequency multiplex transmission system that the distance between any two sets of received signals in the signal space available defined by vectors representing all possible signals present at one time and which must be individually distinguishable is the same as if the signals in each channel were transmitted through independent media and intersymbol interference were eliminated by reducing the signaling rate. The concept of signal space is discussed more fully by J. R. Davey in his paper "Digital Data Signal Space Diagrams" published in the Bell System Technical Journal (vol. XLIII, No. 6, November 1964) at p. 2973.

According to this invention, a plurality of data signal samples are orthogonally multiplexed on equally spaced carrier frequencies for transmission over a band-limited transmission medium in channels having overlapping frequency spectra. Because of the orthogonal relationships achieved within and between channels intersymbol and interchannel interferences are avoided and a theoretically maximum data transmission rate is attained in each channel.

Orthogonality is a mathematical concept derived from the vector representation of time-dependent waveforms. Any two vectors are orthogonal if the cosine of the angle between them is zero, i.e., they are perpendicular to each other. The test for orthogonality between vectors is that the product of their amplitudes (lengths) and the cosine of the angle formed between them when their points of beginning are brought to a common origin without changing their relative directions is zero. Periodic waveforms, such as sine and cosine waves, are commonly represented by vectors. More complex waveforms can by the well-known methods of Fourier analysis be represented by summations of sine and cosine terms. Both simple and complex waveforms can be tested for orthogonality by analogy with the vector multiplication mentioned above. If the periodic waveforms to be compared are laid out on the time axis and the average of the integral of the products of pairs of values for all instants of time extending over their common period is taken, and this average is found to be zero, then the waveforms are said to be orthogonal. Thus, orthogonality becomes a broader concept than perpendicularity. In general, two time-dependent waveforms  $S_m(t)$  and  $S_n(t)$  and deemed to be orthogonal if

$$\frac{1}{2T} \int_{-T}^T S_m(t) S_n(t) dt = 0$$

for  $m \neq n$  over the interval  $2T$ , the common repetition period.

Closely allied with the orthogonality concept is that of symmetry. A function  $f(t)$ , which can be represented graphically by a waveform and is defined on an interval centered at the origin ( $t=0$ ), is said to be even if

$$f(-t) = f(t)$$

for all values of  $t$  in the assigned interval, and odd if

$$f(-t) = -f(t)$$

In graphic terms even functions are symmetric about vertical axis erected at the origin, i.e., the negative half is the mirror image of the positive half. Odd functions

functions is even whenever both functions are even or both are odd, and is odd whenever one of the functions is even and the other is odd. Summarizing,

$$\begin{aligned} (\text{Even})(\text{Even}) &= (\text{Odd})(\text{Odd}) = \text{Even} \\ (\text{Even})(\text{Odd}) &= (\text{Odd})(\text{Even}) = \text{Odd} \end{aligned}$$

It can therefore be further stated from the orthogonality integral above that whenever the function  $S_m(t)$  is of opposite parity to the function  $S_n(t)$  and both are centered in a common interval, they are mutually orthogonal. Since the interval is common to both functions and both functions are periodic with respect to this interval, the implication is that the two functions are synchronized.

The orthogonality concept is not limited to two functions. Any number of functions can be mutually orthogonal and mutually synchronized in a common interval.

Orthogonality with respect to time within each channel and with respect to frequency between channels is preserved by shaping the signals applied to each channel such that the integral of the mathematically transformed product of the squares of the shaping function applied to the individual channel and the channel transfer function and the integral of the transformed products of the shaping functions applied to adjacent channels and the square of the channel transfer function are each zero. These conditions are met in practical cases by shaping functions whose squares have even symmetry about the channel center frequencies and odd symmetry about frequencies located halfway between the channel center frequency and the channel band-edge frequencies. At the same time the phase characteristics of adjacent channels may be arbitrary, provided only that their phase characteristics differ by ninety electrical degrees plus an arbitrary phase function with odd symmetry about the frequency midway between the channel center frequencies.

The required symmetries are achievable in a half-cycle of the cosine wave whose square is the raised cosine shaping function as one readily definable illustrative example.

Preservation of orthogonality within each channel permits establishing individual channel data transmission rates equal to the channel bandwidth. This is half the ideal Nyquist rate. However, due to the fact that adjacent channels are synchronized, they can be overlapped by 50 percent. The overall data transmission rate for the full channel bandwidth then becomes the ideal Nyquist rate times the ratio of the number of channels to the number of channels plus one.

Inasmuch as the amplitudes of the shaping functions are proportional to the amplitudes of the samples by which they are multiplied, transmission is in no way restricted to binary digits. Multilevel symbols and symbols of arbitrary height derived from analog samples are equally transmissible.

Orthogonal signals are readily detectable by correlation procedures using matched filter techniques.

A feature of this invention is that the band-limited shaping filters for each channel can be identical.

Another feature of this invention is that the amplitude and phase characteristics of the transmitting filters can be synthesized independently.

A particular advantage of the orthogonal multiplex transmission system of this invention is that received signals can be recovered by using adaptive correlators regardless of the phase distortion arising in the transmission medium. In addition, synchronization problems are minimized because stationary phase differences between modulating and demodulating carrier waves are taken into account by the adaptive correlators.

Other objects, features and advantages of this invention will be readily appreciated from a consideration of

FIG. 1 is a block diagram of the basic orthogonal frequency-multiplex transmission system of this invention;

FIG. 2 is a waveform diagram showing the development of a shaping filter characteristic satisfying the condition of orthogonality according to this invention;

FIG. 3 is another waveform diagram showing the development of a shaping filter characteristic satisfying the condition of orthogonality according to this invention;

FIG. 4 is a block diagram of a representative three-channel orthogonal frequency multiplex transmitter according to this invention using identical shaping filters for all channels;

FIG. 5 is a series of waveform diagrams useful in explaining the operation of the system of FIG. 4; and

FIG. 6 is a block diagram of a representative correlation detection system capable of recovering the data signals generated in the transmitting system of FIG. 4.

FIG. 1 is a generalized block diagram of an orthogonal multiplex data transmission system according to this invention. From data sources on the left (not shown) impulse samples are applied in synchronism on a plurality of lines such as those designated 10, 11 and 12. Each impulse is shaped in associated transmitting filters 15, 16, and 17 and others not shown for additional sources. Line 13 symbolically indicates such other signaling channels. The passbands of the several transmitting filters are centered at equally spaced frequencies with the spacing equal to half the data rate per channel. Their outputs are combined on line 14 and applied to common transmission medium 18, having an impulse response  $h(t)$  and a transfer function

$$H(f)e^{j\eta(f)}$$

where  $H(f)$  and  $\eta(f)$  are respectively the amplitude and phase characteristics of medium 18,  $e$  is the base of natural logarithms and  $J$  is the imaginary number  $\sqrt{-1}$ . Noise is also added at various points in the system as indicated symbolically by adder 19. The several signaling channels are separately detected in receiver 20. It is assumed for the present that the channel with the lowest frequency is operating at baseband. Carrier modulation and demodulation at passband can be accomplished by standard techniques.

Channel shaping is the critical element here. Let  $b_0, b_1, b_2 \dots$  be a sequence of  $m$ -ary signal digits ( $m \geq 2$ ) or a sequence of analog samples to be transmitted over an arbitrary  $i$ th channel. Each of  $b_0, b_1, b_2 \dots$  can be represented by an impulse with height proportional to that of the corresponding sample. These impulses are applied to the  $i$ th transmitting filter at the rate of one impulse every  $T$  seconds (data rate per channel equals  $1/T$  bauds). Let  $a_i(t)$  be the impulse response of the associated  $i$ th transmitting filter. Then this filter transmits a sequence of signals as

$$b_0 a_i(t), b_1 a_i(t-T), b_2 a_i(t-2T) \dots$$

The received signals at the output of transmission medium 18 are

$$b_0 u_i(t), b_1 u_i(t-T), b_2 u_i(t-2T) \dots$$

where

$$u_i(t) = \int_{-\infty}^{\infty} h(t-\tau) a_i(\tau) d\tau$$

( $\tau$  is a dummy variable of integration.)

These received signals overlap in time, but they are orthogonal (noninterfering) if

$$\int_{-\infty}^{\infty} u_i(t) u_i(t-kT) dt = 0, k = \pm 1, \pm 2 \dots \quad (1)$$

Intersymbol interference in the  $i$ th channel is eliminated if Equation 1 is satisfied.

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