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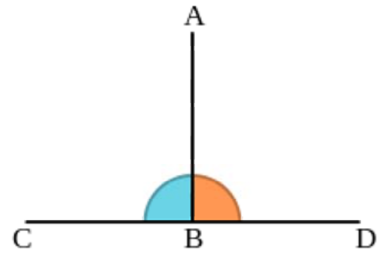
Orthogonality

In mathematics, **orthogonality** is the generalization of the notion of perpendicularity to the linear algebra of bilinear forms. Two elements u and v of a vector space with bilinear form B are **orthogonal** when $B(u, v) = 0$. Depending on the bilinear form, the vector space may contain nonzero self-orthogonal vectors. In the case of function spaces, families of orthogonal functions are used to form a basis.



The line segments AB and CD are orthogonal to each other.

By extension, orthogonality is also used to refer to the separation of specific features of a system. The term also has specialized meanings in other fields including art and chemistry.

Contents

Etymology

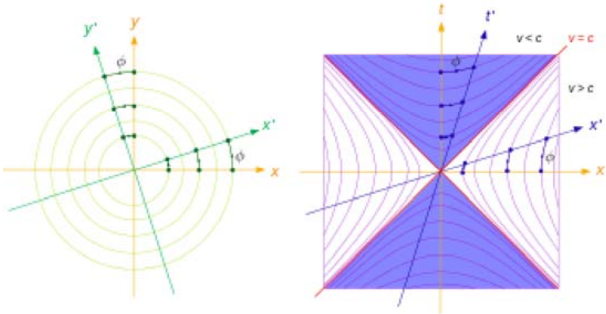
The word comes from the Greek *ὀρθός* (*orthos*), meaning "upright", and *γωνία* (*gonia*), meaning "angle". The ancient Greek ὀρθογώνιον *orthogōnion* (< ὀρθός *orthos* 'upright'[1] + γωνία *gōnia* 'angle'[2]) and classical Latin *orthogonium* originally denoted a rectangle.[3] Later, they came to mean a right triangle. In the 12th century, the post-classical Latin word *orthogonalis* came to mean a right angle or something related to a right angle.[4]

Mathematics and physics

Definitions

- In geometry, two Euclidean vectors are **orthogonal** if they are perpendicular, *i.e.*, they form a right angle.

- Two vectors, *x* and *y*, in an inner product space, *V*, are *orthogonal* if their inner product $\langle x, y \rangle$ is zero.[6] This relationship is denoted $x \perp y$.
- Two vector subspaces, *A* and *B*, of an inner product space *V*, are called **orthogonal subspaces** if each vector in *A* is orthogonal to each vector in *B*. The largest subspace of *V* that is orthogonal to a given subspace is its orthogonal complement.
- Given a module *M* and its dual *M**, an element *m*′ of *M** and an element *m* of *M* are *orthogonal* if their natural pairing is zero, i.e. ⟨*m*′, *m*⟩ = 0. Two sets *S*′ ⊆ *M** and *S* ⊆ *M* are orthogonal if each element of *S*′ is orthogonal to each element of *S*.[7]
- A term rewriting system is said to be orthogonal if it is left-linear and is non-ambiguous. Orthogonal term rewriting systems are confluent.



Orthogonality and rotation of coordinate systems compared between **left:** Euclidean space through circular angle *ϕ*, **right:** in Minkowski spacetime through hyperbolic angle *ϕ* (red lines labelled *c* denote the worldlines of a light signal, a vector is orthogonal to itself if it lies on this line).[5]

A set of vectors in an inner product space is called **pairwise orthogonal** if each pairing of them is orthogonal. Such a set is called an **orthogonal set**.

In certain cases, the word *normal* is used to mean *orthogonal*, particularly in the geometric sense as in the normal to a surface. For example, the *y*-axis is normal to the curve $y = x^2$ at the origin. However, *normal* may also refer to the magnitude of a vector. In particular, a set is called orthonormal (orthogonal plus normal) if it is an orthogonal set of unit vectors. As a result, use of the term *normal* to mean "orthogonal" is often avoided. The word "normal" also has a different meaning in probability and statistics.

A vector space with a bilinear form generalizes the case of an inner product. When the bilinear form applied to two vectors results in zero, then they are **orthogonal**. The case of a pseudo-Euclidean plane uses the term hyperbolic orthogonality. In the diagram, axes x′ and t′ are hyperbolic-orthogonal for any given *ϕ*.

Euclidean vector spaces

In Euclidean space, two vectors are orthogonal if and only if their dot product is zero, i.e. they make an angle of 90° (π/2 radians), or one of the vectors is zero.[8] Hence orthogonality of vectors is an extension of the concept of perpendicular vectors to spaces of any dimension.

The orthogonal complement of a subspace is the space of all vectors that are orthogonal to every vector in the subspace. In a three-dimensional Euclidean vector space, the orthogonal complement of a line through the origin is the plane through the origin perpendicular to it, and vice versa.[9]

Note that the geometric concept two planes being perpendicular does not correspond to the orthogonal complement, since in three dimensions a pair of vectors, one from each of a pair of perpendicular planes, might meet at any angle.

In four-dimensional Euclidean space, the orthogonal complement of a line is a hyperplane and vice versa, and that of a plane is a plane.[9]

Orthogonal functions

By using integral calculus, it is common to use the following to define the inner product of two functions *f* and *g* with respect to a nonnegative weight function *w* over an interval [*a*, *b*]:

$$\langle f, g\rangle_w = \int_a^b f(x)g(x)w(x)\,dx.$$

In simple cases, $w(x) = 1$.

We say that functions *f* and *g* are **orthogonal** if their inner product (equivalently, the value of this integral) is zero:

$$\langle f, g\rangle_w = 0.$$

Orthogonality of two functions with respect to one inner product does not imply orthogonality with respect to another inner product.

We write the norm with respect to this inner product as

$$\|f\|_w = \sqrt{\langle f, f\rangle_w}$$

The members of a set of functions $\{f_i : i = 1, 2, 3, \ldots\}$ are *orthogonal* with respect to *w* on the interval [*a*, *b*] if

$$\langle f_i, f_j\rangle_w = 0 \quad i \neq j.$$

The members of such a set of functions are *orthonormal* with respect to *w* on the interval [*a*, *b*] if

$$\langle f_i, f_j\rangle_w = \delta_{i,j},$$

where

$$\delta_{i,j} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

is the Kronecker delta. In other words, every pair of them (excluding pairing of a function with itself) is orthogonal, and the norm of each is 1. See in particular the orthogonal polynomials.

Examples

- The vectors $(1, 3, 2)^T$, $(3, -1, 0)^T$, $(1, 3, -5)^T$ are orthogonal to each other, since (1)(3) + (3)(−1) + (2)(0) = 0, (3)(1) + (−1)(3) + (0)(−5) = 0, and (1)(1) + (3)(3) + (2)(−5) = 0.

- The vectors $(1, 0, 1, 0, \ldots)^T$ and $(0, 1, 0, 1, \ldots)^T$ are orthogonal to each other. The dot product of these vectors is 0. We can then make the generalization to consider the vectors in $\mathbf{Z}_2{}^n$:

$$\mathbf{v}_k = \sum_{\substack{i=0 \\ ai+k<n}}^{n/a} \mathbf{e}_i$$

for some positive integer *a*, and for $1 \leq k \leq a - 1$, these vectors are orthogonal, for example $(1, 0, 0, 1, 0, 0, 1, 0)^T$, $(0, 1, 0, 0, 1, 0, 0, 1)^T$, $(0, 0, 1, 0, 0, 1, 0, 0)^T$ are orthogonal.

- The functions $2t + 3$ and $45t^2 + 9t - 17$ are orthogonal with respect to a unit weight function on the interval from −1 to 1:

$$\int_{-1}^{1} (2t+3)\left(45t^2+9t-17\right)\,dt = 0$$

- The functions 1, sin(*nx*), cos(*nx*) : *n* = 1, 2, 3, ... are orthogonal with respect to Riemann integration on the intervals [0, 2π], [−π, π], or any other closed interval of length 2π. This fact is a central one in Fourier series.

Orthogonal polynomials

- Various polynomial sequences named for mathematicians of the past are sequences of orthogonal polynomials. In particular:
  - The Hermite polynomials are orthogonal with respect to the Gaussian distribution with zero mean value.
  - The Legendre polynomials are orthogonal with respect to the uniform distribution on the interval [−1, 1].
  - The Laguerre polynomials are orthogonal with respect to the exponential distribution. Somewhat more general Laguerre polynomial sequences are orthogonal with respect to gamma distributions.
  - The Chebyshev polynomials of the first kind are orthogonal with respect to the measure $1/\sqrt{1-x^2}$.
  - The Chebyshev polynomials of the second kind are orthogonal with respect to the Wigner semicircle distribution.

Orthogonal states in quantum mechanics

- In quantum mechanics, a sufficient (but not necessary) condition that two eigenstates of a Hermitian operator, ψ_m and ψ_n, are orthogonal is that they correspond to different eigenvalues. This means, in Dirac notation, that $\langle \psi_m | \psi_n \rangle = 0$ if ψ_m and ψ_n correspond to different eigenvalues. This follows from the fact that Schrödinger's equation is a Sturm–Liouville equation (in Schrödinger's formulation) or that observables are given by hermitian operators (in Heisenberg's formulation).

Art

In art, the perspective (imaginary) lines pointing to the vanishing point are referred to as "orthogonal lines".

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