Complementary Punctured Convolutional (CPC) Codes and Their Applications

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Abstract—**We present in this paper a new class of punctured convolutional codes that are complementary (CPC codes). A set of punctured convolutional codes derived from the same original low rate code are said to be complementary if they are equivalent (in terms of their distance properties) and if when combined yield at least the original low rate code. Based on these CPC codes we propose and analyze a variation of the type II hybrid ARQ scheme which we call type III hybrid ARQ scheme. With the type III hybrid ARQ scheme, the starting code rate can be chosen to match the channel noise requirements, and like with the type II scheme, packets that are detected in error are not discarded, but are combined with complementary transmissions provided by the transmitter to help recover the transmitted message. The main advantage is that any complementary sequence sent for a packet that is detected with errors is self decodable. That is the decoder does not have to rely on previously received sequences for the same data packet for decoding, as is generally the case with incremental redundancy ARQ schemes. This feature is desirable especially in situations where a transmitted packet can be lost or severely damaged as a result of interference. CPC codes can find applications in diversity transmissions systems. A novel complementary diversity scheme which makes use of CPC codes is briefly discussed.**

I. Introduction

In RECENT years there has been a great interest in convolutional codes and their use in modern communication systems [1], [2]. Convolutional codes can be used solely for forward error correction (FEC), and can be incorporated into systems using automatic repeat request (ARQ) schemes [1]. A popular scheme is the so-called type II hybrid ARQ, which uses a rate 1/2 convolutional code [3], [4]. The advent of high rate punctured convolutional codes has accentuated the interest in convolutional coding even further, as these codes are readily decodable and yet offer substantial coding gains without sacrificing much bandwidth [5]. Variable coding rate FEC schemes using a family of punctured convolutional codes derived from the same low rate code have been suggested [6].

Recently, Hagenauer extended the concept of punctured convolutional codes to the generation of a family of rate-compatible punctured convolutional codes (RCPC) [7]. The rate compatibility condition insures that all coded bits of any code of the family are used by all lower rate codes. Based on these RCPC codes, Hagenauer proposed an efficient hybrid ARQ technique [7]. In the ARQ scheme by Hagenauer, starting with the higher rate code of the family, incremental code bits are provided by the transmitter whenever it is necessary. Later, variations to Hagenauer's construction method of RCPC codes have been proposed [8] and various ARQ schemes have been suggested and analyzed [8]–[11]. We shall refer to these schemes as incremental redundancy ARQ schemes.

The main drawback of incremental redundancy ARQ schemes is that additional incremental code bits sent for a packet received with errors (or a packet that is lost) are not in general self decodable. That is the decoder must rely on both the initially transmitted packet as well as the additional incremental code bits for decoding. In situations where a packet can be lost or severely damaged as a result of interference, such as contention in multiple access protocols, it is desirable to have a scheme where any additional information sent is self decodable. Note that the type II hybrid ARQ scheme possesses this property.

In this paper, we present a new class of punctured convolutional codes that are complementary (CPC codes). A set of punctured convolutional codes derived from the same original low rate code are said to be complementary if they are equivalent (in terms of their distance properties) and if when combined yield at least the original low rate code. Note that the term complementary is also used to denote convolutional codes of rate $1/V$ in which the coded bits on two stemming branches are logical complements of each other. Based on these CPC codes we propose a variation of the type II hybrid ARQ scheme. With this scheme, the starting code rate can be chosen to match the channel noise requirements, and like with the type II scheme, packets that are detected in error are not discarded, but are combined with complementary transmissions provided by the transmitter to help recover the transmitted message. Since with the proposed scheme there is an additional level for error correction as compared to the conventional type II scheme, we choose to call it type III hybrid ARQ scheme. The main advantage is that any complementary sequence sent for a packet that is detected with errors is self decodable. That is the decoder does not have to rely on previously received sequences for the same data packet for decoding, as is generally the case with incremental redundancy ARQ schemes. The performance of the proposed type III hybrid ARQ scheme is analyzed and compared to that of a conventional type II hybrid ARQ scheme over an additive white Gaussian noise (AWGN) channel.

CPC codes can also find applications in frequency, code or time diversity transmissions schemes. A novel complementary diversity scheme which makes use of CPC codes is briefly discussed.

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II. Complementary Punctured Convolutional (CPC) Codes

A. Punctured and Repetition Convolutional Codes

A rate b/V punctured convolutional code can be obtained from a rate $1/V_0, (V_0 \leq V)$, code by deleting $(bV_0 - V)$ bits from every bV_0 coded bits corresponding to the encoding of b information bits by the original rate $1/V_0$ code, according to a perforation pattern [5]. The perforation pattern is usually represented by a "perforation matrix." The perforation matrix yielding a rate b/V code has V_0 rows and b columns. Each row corresponds to one of the V_0 encoded bits at the output of the rate $1/V_0$ encoder, and each column is associated to one encoding cycle. The elements of a perforation matrix are only zero's and one's, corresponding to deleting or keeping the coded bit at the output of the original rate $1/V_0$ encoder. For example, the perforation matrix P_0 of a rate 7/8 punctured convolutional code of memory $m = 6$, obtained from a rate 1/2 code is given [6] by

$$P_0 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}. \tag{1}$$

A low rate $b/(bV_0 + l), l \geq 1$, repetition code can be obtained form a rate $1/V_0$ code by repeating l bits among every bV_0 coded bits which result from the encoding of each group of b information bits [8]. The l bits that are repeated are determined by a well-selected repetition pattern. The repetition pattern is usually represented by a "repetition matrix" which has V_0 rows and b columns. In contrast to the perforation matrix, each element of a repetition matrix is greater than or equal to one and indicates the number of repeats of the corresponding coded bit. For example, the repetition matrix Q_1 of a rate 7/17 repetition convolutional code of memory $m = 6$, obtained from a rate 1/2 code is given [8] by

$$Q_1 = \begin{bmatrix} 2 & 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 \end{bmatrix}. \tag{2}$$

B. Equivalent Punctured and Repetition Convolutional Codes

Let C_1 and C_2 denote two punctured or repetition codes of the same rate obtained from the same original rate $1/V_0$ code. C_1 and C_2 are said to be equivalent if the b columns of the perforation or repetition matrix of one code are shifted versions of the b columns of the perforation or repetition matrix of the other code [12]. For example the three rate 5/6 codes of perforation matrices

$$P_1 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

are equivalent. Equivalent codes have the same distance properties and they hence yield the same error correction performance [12]. With a perforation or repetition matrix of b columns, one can construct at most b distinct codes that are equivalent.

C. Complementary Punctured Convolutional Codes

Let $C_i, i = 1, 2, \cdots, p$, be p equivalent codes of the same rate b/V, all obtained from the same original rate $1/V_0$ code, where $p = \lceil \frac{bV_0}{V} \rceil$. Let P_i denote the perforation matrix of code C_i. Define the matrix P as

$$P = \sum_{i=1}^{p} P_i. \tag{4}$$

The p codes $C_i, i = 1, 2, \cdots, p$, are said to be complementary if every element of matrix P is greater or equal than one. This means that when complementary codes are combined together according to (4), the code obtained contains the original rate $1/V_0$ code. For the case $V_0 = V$, we have $p = b$, and the combined code is of rate $b/bV_0 = 1/V_0$, which is the original rate $1/V_0$ code, viewed b information bits at a time. For the case $V_0 < V$, if the p matrices are chosen to satisfy (4), then some elements of P would be greater than one, and the combined code would correspond to a repetition code of rate b/pV.

As an example, consider the rate 5/6 punctured code obtained from a rate 1/2 code with perforation matrix P_1 given by (3). From this code, four equivalent codes, including the two with perforation matrices P_2 and P_3 given by (3) can be constructed, as described above. Here, we have $p = 2$; that is two codes among all five can be combined to get a code that contains the original rate 1/2 code, but not any two such codes can serve the purpose. Take for example the two codes with perforation matrices P_1 and P_2. With these two codes P is

$$P = P_1 + P_2 = \begin{bmatrix} 1 & 2 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & 2 \end{bmatrix}. \tag{5}$$

Since matrix P contains some zero's, then the two codes with perforation matrices P_1 and P_2 do not satisfy the complementarity criterion. However, the two codes with perforation matrices P_1 and P_3 are complementary as

$$P = P_1 + P_3 = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{6}$$

does not have any zero elements. The combined code corresponding to this matrix P is a repetition code of rate 5/12. Obviously this code is better than the former one, as it contains the original rate 1/2 code which was used to obtain the rate 5/6 punctured code.

D. Construction of a Family of Complementary Punctured Convolutional (CPC) Codes

The construction of a family of rate b/V CPC codes from a rate $1/V_0$ code is as follows:

1) Select a perforation pattern P_1 that yields the best non-catastrophic rate b/V code. The selection criterion is based on maximal free distance (d_{free}) and minimal number of incorrect paths at d_{free}.
2) Obtain from P_1 the b equivalent perforation patterns $P_i, i = 1, 2, 3, \cdots b$.
3) Select $p = \lceil \frac{bV_0}{V} \rceil$ among the b equivalent codes found above that satisfy the complementarity criterion. If no

TABLE I
RATE 2/3 PUNCTURED CONVOLUTIONAL CODES DERIVED FROM BEST RATE 1/3 CODE

Original Code (R = 1/3)				Punctured Code (R = 2/3)		
M	G_1	G_2	G_3	P	d_f	$(a_n, n = d_f, d_{f+1}, d_{f+2}, \ldots)$ $[c_n, n = d_f, d_{f+1}, d_{f+2}, \ldots]$
2	5	7	7	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	3	(1, 4, 14, 40, 116, 339, 991, 2897, 8468, 24752) [1, 10, 54, 226, 856, 3072, 10647, 35998, 119478, 390904]
4	25	33	37	$\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	4	(2, 4, 17, 75, 256, 957, 3560, 13149, 48720) [6, 12, 104, 555, 2500, 11349, 49186, 208733, 872172]
6	133	145	175	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	6	(6, 12, 54, 197, 763 2879 10752 40992 154909) [31, 47, 404, 1643, 7875, 33454 143148 609684 2546016]

such p codes can be constructed, discard perforation pattern P_1 and repeat from 1.

Property: To insure that p complementary matrices using cyclic shifts of the b columns of matrix P_1 can be constructed, a necessary condition is that the number of one's in each row of P_1 must at least be equal to $\lceil \frac{b}{p} \rceil$.

Proof: Take any row of matrix P_1, and denote the number of one's in this row by x. The number of elements greater than one in the same row of matrix P, which is obtained by combining p cyclic shifts of P_1 according to (4), is less or equal than xp. Thus, if $x < \lceil \frac{b}{p} \rceil$, then $xp < b$, which means that some elements of matrix P are zero's, and this of course violates the complementarity criterion.

The choice for b, V, V_0, and hence p depends on the application where these CPC codes are to be used. Moreover, in certain applications, codes obtained from combining only a subset $p_0, 1 < p_0 < p$, of the p complementary codes may be used. For these applications, it is important to insure that such codes are also good.

Lemma: It is not possible to construct CPC codes from rate b/V codes which are derived from rate $1/V_0$ if $\lceil \frac{b}{p} \rceil \cdot V_0 > V$.

Proof: The proof of this lemma falls from the property above. Since the number of one's per row must at least be equal to $\lceil \frac{b}{p} \rceil$, and there are V_0 rows, then $\lceil \frac{b}{p} \rceil \cdot V_0$ cannot exceed V. As an example, CPC codes of rate 4/5 cannot be obtained from a rate 1/3 code, as here $p = 3$, and $\lceil \frac{b}{p} \rceil \cdot V_0 = 6 > V = 5$.

It should be pointed out however that this restriction is not quite severe, and a variety of CPC codes can indeed be constructed. We have verified that from most best known punctured convolutional codes of rate $(2 + i)/(3 + i), i = 0, 1, 2$, which are derived from best-known rate 1/2 codes, complementary codes can be readily found. Using a computer search procedure, we have found best rate 2/3, 3/4 and 5/6 punctured codes of memory $m = 2, 4$, and 6, obtained from best known rate 1/3 codes, from which CPC codes can be readily obtained. Results are given in Tables I, II, and III, for rate 2/3, 3/4, and 5/6 codes, respectively. Best CPC codes derived from rate 1/4 codes, and from lower rate codes, can be readily found in the same way.

Note that intermediate rate compatible convolutional (RCC) codes can be added to the set of CPC codes in the same way as described in [8]. As an example, consider the two complementary matrices P_1 and P_3 given by (3). Starting with P_1, the zero's of matrix P_1 are substituted by the one's of matrix P_3, h at a time, yielding RCC codes of rates $5/(6 + ih), i = 1, 2, \cdots$. If $h = 3$, we get the family of RCC codes given by matrices

$$P_1^{(1)} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix},$$
$$P_1^{(2)} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$
$$P_1^{(3)} = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}. \quad (7)$$

Lower rate compatible codes can be obtained by adding to matrix $P_1^{(3)}$ the elements one of yet another equivalent matrix, h at a time.

TABLE II
RATE 3/4 PUNCTURED CONVOLUTIONAL CODES DERIVED FROM BEST RATE 1/3 CODE

Original Code (R = 1/3)				Punctured Code (R = 3/4)		
M	G_1	G_2	G_3	P	d_f	$(a_n, n = d_f, d_{f+1}, d_{f+2}, \ldots)$ $[c_n, n = d_f, d_{f+1}, d_{f+2}, \ldots]$
2	5	7	7	$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$	3	(6, 23, 80, 284, 1027, 3724, 13480, 48768, 176445, 638422) [15, 104, 540, 2536, 11302, 48638, 203998, 839392, 3403522, 13640568]
4	25	33	37	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	4	(5, 33, 137, 674, 3377, 16517, 81052, 398633, 1958670, 9622756) [19, 180, 1130, 7026, 42350, 244915, 1383836, 7693698, 42190352, 228835889]
6	133	145	175	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	5	(8, 34, 158, 838, 4384, 22630) [36, 214, 1385, 9287, 58644, 355514]

TABLE III
RATE 5/6 PUNCTURED CONVOLUTIONAL CODES DERIVED FROM BEST RATE 1/3 CODE

Original Code (R = 1/3)				Punctured Code (R = 5/6)		
M	G_1	G_2	G_3	P	d_f	$(a_n, n = d_f, d_{f+1}, d_{f+2}, \ldots)$ $[c_n, n = d_f, d_{f+1}, d_{f+2}, \ldots]$
2	5	7	7	$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	2	(2, 26, 131, 657, 3423, 17776, 92215, 478560, 2483454) [6, 121, 1032, 7360, 48709, 307321, 1877873, 11214275, 65819387]
4	25	33	37	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$	2	(1, 6, 58, 421, 2951, 20986, 149369, 1062318, 7554050, 53721670) [4, 52, 518, 5143, 46507, 398520, 3327020, 27160749, 217991733, 1727002951]
6	133	145	175	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$	3	(1, 12, 120, 905, 7099, 56251) [9, 105, 1341, 13684, 132529, 1248007]

III. TYPE III HYBRID ARQ SCHEME

Let $P_i, i = 1, 2, \cdots, p$, denote the perforation patterns of p CPC codes of rate b/V obtained from a rate $1/V_0$ code, as described above. We refer to the code with perforation pattern P_i as C_i. The code obtained by combining j CPC codes $C_i, i = 1, 2, \cdots, j$, has perforation (or repetition) matrix $P^{(j)} = P_1 + P_2 + \cdots + P_j$, and we shall refer to it by $C^{(j)}$. Operations of the type III hybrid ARQ scheme are now described. To each k-bit information packet $\boldsymbol{B}$ to be transmitted are appended n_p parity bits for error detection and m known *tail* bits corresponding to the memory of the encoder. The transmission state of each data packet $\boldsymbol{B}$ is described by a level. Each packet of $n = (k + n_p + m)$ bits is encoded with the rate $1/V_0$ code and transmitted according to the following procedure.

Level 1: Initially, only coded bits in C_1 are sent to the receiver. At the receiver end, Viterbi decoding using perforation pattern P_1 is applied on the received sequence. The decoded sequence, which corresponds to the information and parity check bits, is then examined by the error detection decoder. If this sequence is declared error-free, transmission of $\boldsymbol{B}$ is complete. Otherwise, that sequence is saved for subsequent decoding attempts and $\boldsymbol{B}$ moves to the next level.

Level $i, 2 \le i < p$: Coded bits in C_i are sent to the receiver. Viterbi decoding is first applied on the received sequence, using perforation pattern P_i. If the decoded sequence is declared error free, transmission of $\boldsymbol{B}$ is complete. Otherwise Viterbi decoding is applied once again, but on the combination of the i sequences received up to this level, using the combined code $C^{(i)}$ which has perforation pattern $P^{(i)} = P_1 + P_2 + \cdots + P_i$. If now decoding is successful, transmission of $\boldsymbol{B}$ is complete. Otherwise, the i sequences received so far are saved and $\boldsymbol{B}$ moves to the next level.

Level p: Coded bits in C_p are sent to the receiver. The same decoding process as above is performed. That is, first using only the sequence received at this level, then, if this decoding is not successful, using all received sequences available at the receiver. At this level, all codes have been used. Should decoding still fail, the received sequence in C_1 is discarded and $\boldsymbol{B}$ moves to the next level.

Level $(p + j), j = 1, 3, \cdots$: Coded bits in C_j are sent to the receiver. If decoding in C_j is successful transmission of $\boldsymbol{B}$ is complete. Otherwise, decoding resumes using all p sequences available at the receiver. In the event that decoding is still not successful, the sequence received at level $(j + 1)$ in C_{j+1} is discarded and $\boldsymbol{B}$ moves to the next level.

We note that the above type III scheme can be viewed as the generalized type II scheme in [8], where the number of incremental code bits sent at each level per every b information bits is equal to V here, with of course the additional requirement that these additional bits must emanate from an equivalent and complementary code to the one used for the initial transmission. This makes any additional sequence sent for the same data packet self decodable, in the same way as with the type II hybrid ARQ scheme.

A. Throughput Evaluation

The throughput η is defined as $R/\overline{N}$ where R is the code rate and $\overline{N}$ is the average number of packets transmitted per correctly decoded packet. Let $F^{(i)}$ denote the event {decoding failure at level i of the ARQ scheme, i.e., after receiving i sequences for a given data packet}. Let $D_d(j), j = 1, 2, 3, \cdots$, denote the event {decoded sequence in $C^{(j)}$, obtained by combining j equivalent codes, is detected with errors}. The average number of packets transmitted per correctly decoded packet is given by

$$\overline{N} = 1 + \Pr\{F^{(1)}\} + \Pr\{F^{(1)}, F^{(2)}\} + \Pr\{F^{(1)}, F^{(2)}, F^{(3)}\} + \cdots + \Pr\{F^{(1)}, F^{(2)}, F^{(3)}, \cdots, F^{(i)}\} + \cdots. \quad (8)$$

Event $F^{(i)}$ is equivalent to the joint event $\{D_d(1), D_d(i)\}$ for $i \le p$, and to the joint event $\{D_d(1), D_d(p)\}$ for $i \ge p$. Due to the statistical dependency among the joint events $\{D_d(1), D_d(i)\}$, the exact evaluation of (8) is difficult. However, we can bound each term in (8) as

$$\Pr\{F^{(1)}, F^{(2)}, \cdots, F^{(i)}\} \le \begin{cases} \Pr\{D_d(i)\}, & i \le p \\ \Pr\{D_d(p)\}^j, & i = jp, \quad j = 1, 2, \cdots \\ \Pr\{D_d(p)\}^j \Pr\{D_d(jp - i)\}, & jp < i < (j+1)p, \quad j = 1, 2, \cdots. \end{cases} \quad (9)$$

Substituting (9) into (8) and rearranging terms, we obtain

$$\overline{N} \le \left(1 + \sum_{i=1}^{p-1} \Pr\{D_d(i)\}\right)\left(\sum_{j=0}^{\infty} \Pr\{D_d(p)\}^j\right) = \left(1 + \sum_{i=1}^{p-1} \Pr\{D_d(i)\}\right)\frac{1}{1 - \Pr\{D_d(p)\}}. \quad (10)$$

The probability $\Pr\{D_d(i)\}$ in (10) can be upperbounded as in [8], and thus a lower bound on the throughput can be obtained. Assuming antipodal signaling over an AWGN channel, we have computed the throughput of the type III hybrid ARQ scheme using CPC codes of rates 2/3, 3/4, and 5/6, of memory $m = 6$, all derived from the best known rate 1/2 code. Results are shown on Fig. 1. The throughput curve of the conventional type II hybrid ARQ scheme is also shown on the figure. As expected, the throughput of the type III hybrid ARQ scheme is higher than that of a conventional type II hybrid ARQ scheme, except over a small range of SNR values.

Fig. 2 compares the throughputs of our scheme using CPC codes of rate 3/4 and memory $m = 6$, derived from best known rates 1/2, 1/3 and 1/4 codes. It can be seen that for low SNR values, the throughput with CPC codes obtained from a low rate code is higher than that with CPC codes obtained from a higher rate code. This is due to the fact that the combined code from CPC codes derived from a lower rate code is better than from those derived from a higher rate code.

The type III hybrid ARQ scheme is quite flexible, and offers many variations. The starting code rate can be chosen to overcome the nominal noise always present on the channel, and as the channel degrades, a useful throughput is maintained due to the transmission of complementary coded sequences. For time varying channels, the code rate used with the type III hybrid ARQ scheme can be made adaptive accordingly to the channel conditions. That is during good channel conditions, a high code rate can be used, and as the channel degrades, the code rate can be lowered accordingly.

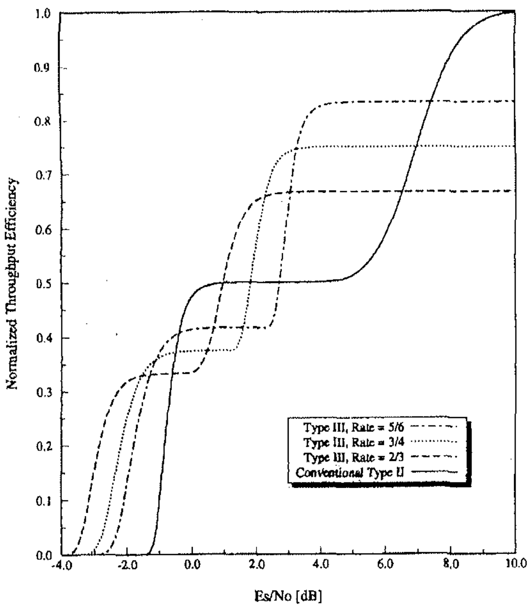

Fig. 1. Throughput of type III hybrid ARQ scheme with CPC codes derived from rate 1/2 codes.

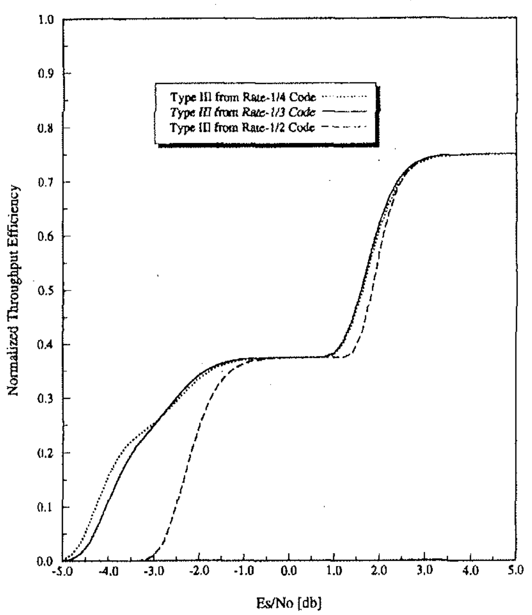

Fig. 2. Throughput of type III hybrid ARQ scheme with rate 3/4 CPC Codes derived from rate 1/4, 1/3, and 1/2 codes.

IV. Conclusions

We have proposed in this paper a new class of punctured convolutional codes called CPC codes. A set of punctured convolutional codes derived from the same original low rate code are said to be complementary if they are equivalent (in terms of their distance properties) and if when combined yield at least the original low rate code. Based on these CPC codes we have proposed and analyzed a so called type III hybrid ARQ scheme. With the type III hybrid ARQ scheme, the starting code rate can be chosen to match the channel noise requirements, and like with the type II scheme, packets that are detected in error are not discarded, but are combined with complementary transmissions provided by the transmitter to help recover the transmitted message. The main advantage is that any complementary sequence sent for a packet that is detected with errors is self-decodable. That is the decoder does not have to rely on previously received sequences for the same data packet for decoding, as is generally the case with incremental redundancy ARQ schemes. This feature is desirable especially in situations where a transmitted packet can be lost or severely damaged as a result of interference. The code rate used with the type III hybrid ARQ scheme can be made adaptive according to the channel conditions. That is during good channel conditions, a high code rate can be used, and as the channel degrades, the code rate can be lowered accordingly.

CPC codes can find applications in diversity transmissions systems. Assume that t channels are available for frequency, code or time diversity transmissions. In conventional diversity transmissions techniques the same packet (encoded or not) is sent over t channels, and a combining scheme is applied on the t received sequences for that packet. In what we call a complementary diversity (CD) scheme, t CPC codes of rate b/V are used, one code per channel. At the receiver end the t received sequences are combined and decoded using a Viterbi algorithm based on the combined code of rate b/tV.

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