DIFFERENTIAL MODULATION AND DEMODULATION OF MULTI-FREQUENCY DIGITAL COMMUNICATIONS SIGNALS

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Abstract

Multi-Frequency Modulation, is a bandwidth efficient digital communication signalling technique that may be employed effectively in mobile satellite communications links. A feature of MFM is that it requires good synchronization because it is coherent and signal symbols have high time-bandwidth products. In this paper we describe algorithims for generating and demodulating differentially encoded Multi-Frequency Quadrature Phased Shift Keyed signals using Discrete Fourier Transform techniques. Experimental results are presented showing good agreement with the theory.

INTRODUCTION

In order to increase the data rate of digital communications through bandwidth restricted channels, some MODEMS use multiple carrier frequencies spaced throughout the available bandwidth, each frequency independently modulated with digital information. The frequencies are transmitted simultaneously during each baud interval. This type of modulation is called Multiple Frequency Modulation (MFM). One advantage of MFM is that inter-baud interference can be reduced by introducing a small guard time between successive baud. In order to prevent inter-frequency interference, the frequencies must not be spaced too closely. Ideally, the frequencies are made orthogonal over one baud interval. The frequencies in the set will be mutually orthogonal so long as they are spaced at exact multiples of the reciprocal of the baud length, that is at the baud rate.

One problem with the MFM systems described above is that although they can effectively eliminate inter-baud interference and inter-frequency interference within the bauds, they must be demodulated using fully coherent receivers for each frequency. Since the frequencies are subject to different and unknown amplitude and phase changes introduced by the transmission channel, coherent reception by inclusion of a pilot tone will not be effective as was discovered by Alard et al [1] in a prototype model of a UHF satellite sound broadcasting utilizing MFM signalling techniques. Cimini has suggested a system that would correct for the phase of the different carriers by sending a training or pilot signal set through the mobile channel [2].

Differential encoding of the carriers provides a more practical solution to this problem with of course an attendent loss of performance against additive noise of approximately 3 db. The conventional way to differentially encode data is from baud to baud as is found in ordinary DPSK and in the MFM system of [1]. However, this is not desireable when the bauds are long, due to channel instability such as that introduced by fading or if asynchronous or packet transmissions are envisoned because of the potential loss of data rate when only two or three bauds are sent per packet. (Differential encoding in time requires using one baud as a reference). The solution to this problem is to differentially encode the symbols in frequency. Differential encoding between frequencies only requires that the channel be constant in phase over very narrow bands corresponding to the baud rate. In MFM the baud rate can be chosen independently of the data rate and so, in principle it can be chosen to make the differential coding effective for any given channel.

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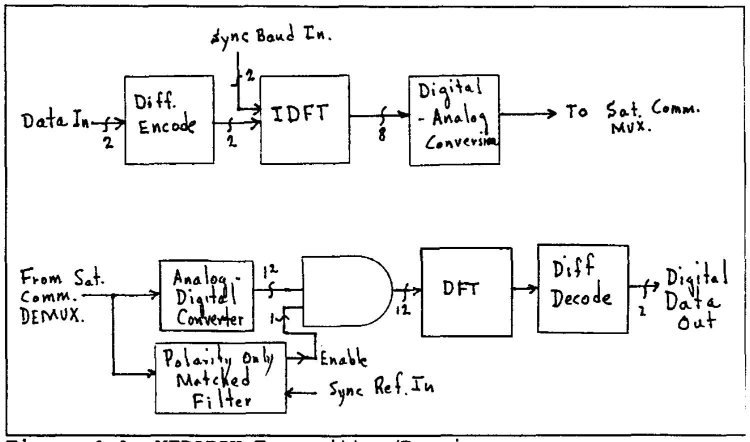

Figure 1 An MFDQPSK Transmitter/Receiver

Even though phase coherent reception of a differentially encoded MFM tone set is not required, baud (symbol) sync is nessecary to support asynchronous modes. We provide this synchronization by sending a known sync baud at the beginning of each packet of bauds. The sync baud is an ordinary MFM baud with randomly selected phases for the tones. The sync baud is detected using a polarity only matched filter. The matched filter peak output occurs exactly at the end of the baud. This peak is detected and used to initiate signal acquisition of the data bauds. A block diagram showing the principal components of a MFDQPSK MODEM is shown in Figure 1.

It is useful to note how generation and demodulation of an MFM baud using Inverse Discrete Fourier Transform (IDFT) and Discrete Fourier Transform techniques as suggested by Weinstein and Ebert [3] and later by Hirosaki [4] permits encoding and decoding directly in the frequency domain.

THEORY OF MFDQPSK

The following definitions are used in MFM ;

T: Packet length in seconds
T: Baud length in seconds
k_x: Baud length in number of samples
L: Number of bauds per packet
t: Time between samples in seconds
f_x=1/ t: Sampling or clock frequency for D/A and A/D conversion in Hz.
f=1/ T: Frequency spacing (minimum) between MFM tones.
K: Number of MFM tones.

Let the 1^{th} baud of the transmit signal be described by;

$$x(u) = x_k(u) \quad (1)$$

where,

$$x_k(u) = A_k\cos(2\pi k\ fu+\phi_k)\ ;0\le u\le\ T. \quad (2)$$

Here, u is time referenced to the beginning of the baud. Now the discrete time signal corresponding to the 1^{th} baud is found by sampling (1) and (2) at the sampling intervals t=1/f_x and is given by;

$$x(n) = \Sigma\ x_k(n) \quad (3)$$

where,

$$x_k(n) = A_k\cos(2\pi kn/k_x+\phi_k)\ ;0\le n\le k_x-1. \quad (4)$$

Here, n is discrete time referenced to the beginning of the baud. Note that, in general, k may take on all integer values between 0 and $k_x/2-1$. We will refer to k as the "harmonic number" of the MFM tone of frequency k f.

Consider the k_x point Discrete Fourier Transform (DFT) of (3). It is given by;

$$X(k_x/2) = 0 \quad (5)$$

and,

$$X(k) = \tfrac{1}{2}k_xA_k\{\exp(j\phi_k)\} \quad (6)$$
$$;1\le k\le k_x/2-1$$
$$X(k_x-k) = \tfrac{1}{2}k_xA_k\{\exp(+j\phi_k)\}. \quad (7)$$

Thus, it is apparent that the discrete time signal (3) is given by the k_x point Inverse Discrete Fourier Transform (IDFT) of (5) and (6), namely;

$$x(n) = IDFT[X(k)];\ 0\le k,n\le k_x-1. \quad (8)$$

Differential Encoding QPSK in the Frequency Domain

In QPSK, the real and imaginary parts of X(k) each carry one bit of information for each tone in the tone set within the transmission bandwidth. In satellite transmissions the original information band will be bandshifted to the appropriate channel location for combination and transmission with the other channels. Using MFM, the data band can be selected in such a way to simplify the band shifting operation. For example, a 45KHz wide MFM signal may be generated directly between 60KHz and 105KHz, leaving a 120KHz guard band between the alias spectrum to be filtered out in the bandshifting operation. In this example, we could select a sampling frequency f_x of 256KHz, a tone spacing and baud rate of 1KHz and select harmonic numbers 60 thru 105 to carry information. The amplitudes of all other harmonics between 0 and 127 would be set to zero.

Differential encoding in the frequency domain is accomplished by setting

$$X(k)=X(k-1)D(k) \qquad (9)$$

where D(k) are modulation values generated by input message di-bits in accordance with Table 1 for $k_1+1\le k\le k_1+K$.

Input Di-bit	D(k)
00	1+j0
01	0+j
10	0-j
11	-1+j0

TABLE 1
Differential Encoding Algorithim

The phase of the initial tone, k_1 is arbitrary.

Demodulation and Differential Decoding

Demodulation of MFM is accomplished by a process inverse to its generation. Given the time domain signal at the receiver y(u) on the interval $0\le u\le T$: The signal is sampled at f_x samples per second and converted to digital format with an A/D converter. The k_x real values thus obtained are loaded into a k_x point complex valued array (the imaginary parts are set to zero). This becomes the array for the 1^{th} baud. Its k_x point DFT yields the complex valued array Y(k) containing, in its first half, the amplitude and phase modulation information, of the K harmonics employed in the generation of the transmit signal.

To differentially decode MFDQPSK, we proceed as follows:

a. Compute

$$X_a(k)=Y(k)Y^*(k-1)(1+j) \quad ;k_1+1\le k\le k_1+K. \qquad (10)$$

b. If $Re[X_a(k)]\le 0$, then the least significant bit of the kth di-bit of the baud is 1; otherwise it is 0.

c. If $Im[X_a(k)]\le 0$, the the most significant bit of the kth di-bit of the baud is 1; otherwise it is 0.

Signal-to-Noise Ratios (SNR's) and the DFT

Let the receiver input signal-to-noise ratio, SNR_I, be defined as the signal power in bandwidth W divided by the noise power in bandwidth W. Also define the average tone signal power as;

$$P_o = P_k/K \qquad (11)$$

then,

$$[SNR]_I = KP_o/WN_o = P_o/ fN_o \qquad (12)$$

Note that the narrowband input SNR;

$$[SNR]_{NB} = P_k/ fN_o = mE_{bk}/N_o \qquad (13)$$

is the same as the wideband SNR of (11) if the power in tone k is the same as the average tone power. E_{bk} is the average received energy per bit carried by the tone k and m is the number of bits per tone.

Now consider the k_x-point DFT coefficients, Y(k,)of the input sequence y(n);

$$Y(k) = S(k) + W(k) \qquad (14)$$

where,

$$S(k) = \tfrac{1}{2}(2P_k)^{1/2}k_x\exp(j\phi_k) \quad ;k_1\le k\le k_2 \qquad (15)$$

and the W(k) are the DFT coefficients of the white noise sequence. It is easily shown that

$$E[W(k)] = 0 \qquad (16)$$

and,

$$E[Re\{W(k)\}^2]=E[Im\{W(k)\}^2]= fN_ok_x^2/4 \qquad (17)$$

$$E[Re\{W(k)\}Im\{W(k)\}] = 0. \qquad (18)$$

Furthermore, the Real and Imaginary parts of W(k) and W(i) are uncorrelated with one another for $k \ne i$.

Let us define the output signal-to-noise ratio for the kth tone, SNR_k, of the MFDQPSK receiver as the ratio of the square of the mean of the Real (or Imaginary) part of $X_a(k)$ to its variance. It can be shown [5], that these are given by

$$[SNR]_k = \frac{[SNR]_{NB}/2}{1+.5/[SNR]_{NB}} = \frac{E_{bk}/N_o}{1+.25/(E_{bk}/N_o)} \qquad (19)$$

where we have used the fact that for QPSK, m=2 ;that is two bits are sent with each tone. For relatively high values of E_b/N_o, the demodulated outputs, $Re[X_a(k)]$ and $Im[X_a(k)]$ are very nearly gaussian random variables with output signal-to-noise ratios just 1/2 those of ordinary QPSK (or MFQPSK). Under the gaussian approximation, the probability of a bit error using the decision rules of (12) and (13) is given by

$$P_{bk} = Q([SNR]_k^{1/2}) \qquad (20)$$

and Q(x) is the error probability Q-function. P_b is shown versus E_b/N_o in

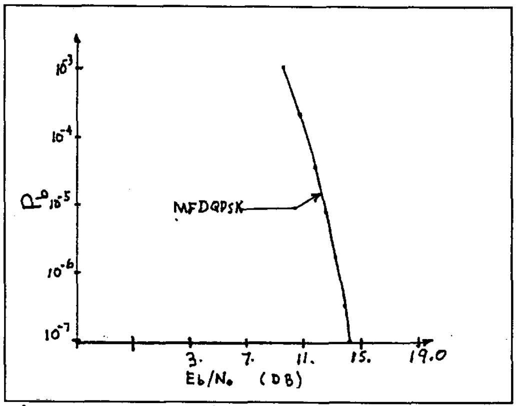

Figure 2 Probability of Bit Error for MFDQPSK

Figure 2.

Experimental Results

An MFM system has been configured to transmit MFDQPSK over a 4 KHz bandpass channel. Tone spacings, that is baud rates, of 15, 30, 60, 120, and 240 Hz were tested. Output signal-to-noise ratios were estimated by computing sample means and variances of the real and imaginary parts of X_a. Data were averaged across all tones in the MFM baud and over several bauds in order to reduce the variance of the estimates.

System noise is shown in Figure 3. Sources of system noise are numerical noise in the FFT's, assummed minimal, quantization noise induced by the 8 bit representation of the discrete time signal values x(n) at the transmitter and 12 bit representation of the y(n) at the receiver, and phase error in the differential decoding due to slightly different phase shifts of adjacent tones [6]. As expected, since the differential phase is less for tones closer together than for tones far apart, the maximum signal-to-noise as set by the system noise is highest, some 23.5 db, for the smallest tone spacing of 15 Hz. Since data rate in MFM is constant at m bits per Hz of channel bandwidth (2 bits per Hz for MFDQPSK), system performance in badly unequalized channels is enhanced significantly by using low baud rates, that is small tone spacings, so that adjacent tones incur nearly identical phase and amplitude changes through the channel.

White noise was added to the analog signal x(t) and output signal-to-noise ratios were measured versus E_b/N_o. The results are shown in Figure 4 for the same baud rates used in Figure 3. Note that as E_b/N_o increases, the output SNRs tend toward the system noise as expected. We also note that performance seems two to three db better than theory for most cases. The source of this unexpected benefit has not yet been determined but it may be due to the noise not being exactly white,so that there is some correlation in the phase noise from harmonic to harmonic in the receive DFT.

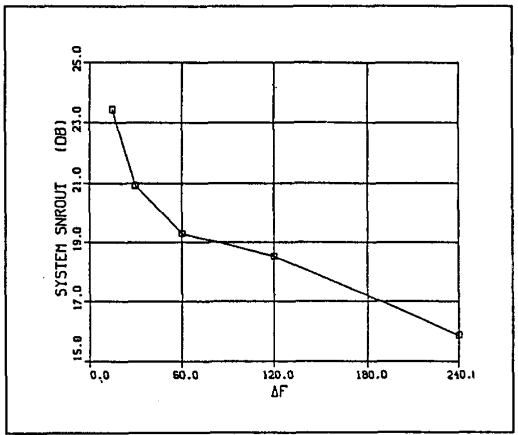

Figure 3 System Noise Signal-to-Noise Ratio vs. Baud Rate

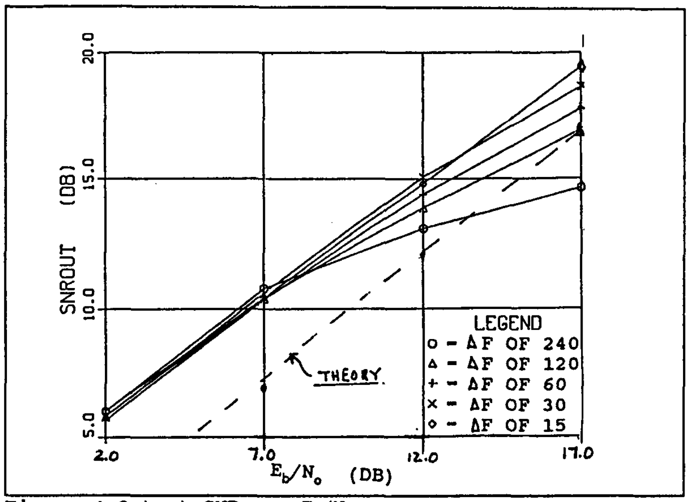

Figure 4 Output SNR vs. E_b/N_o

Conclusions

MFM has been shown to be an attractive approach for digital satellite communications to mobile receivers [1]. In this work, we have developed the theory and a prototype system for differentially encoding and decoding MFQPSK in the frequency domain, a practical way to solve problems of coherent reception with minimal performance loss. Furthermore, by using long baud intervals and corresponding small spacing of the carrier tones, problems associated with channel fading should be greatly relieved with respect to the previous method of differentially encoding the multiple carrier tones from baud to baud. A frequency differential encoded 16-QAM system capable of 4 bits/Hz transmission rate has been developed and is presently being evaluated.

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