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THEORIES AND APPLICATIONS

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Edited by

Luis TORRES
Enrique MASGRAU
Miguel A. LAGUNAS
Department of Signal Theory and Communications
ETSIT-UPC
Barcelona, Spain

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DIFFERENTIALLY CODED MULTI-FREQUENCY MODULATION FOR DIGITAL COMMUNICATIONS

Paul H. Moose

Department of Electrical and Computer Engineering
Naval Postgraduate School
Monterey, CA 93943
and
Mercury Digital Communications, Inc.
243 Eldorado St., Suite 201,
Monterey, CA, 93940

Multi-frequency Modulation (MFM™), utilizing a multiplicity of orthogonal carrier tones simultaneously, produces a robust, bandwidth efficient signal for digital communications. Signals are generated at baseband or bandpass with minimal hardware requirements. Differential coding between adjacent carrier tones, providing the tones are closely spaced, eliminates the need for coherent carrier reference signals and for channel equalization. Encoding/decoding take place utilizing fast Fourier transforms.

1. INTRODUCTION

Multi-frequency Modulation (MFM™) is a new method for data communications that relys on digital signal processing capabilities resident in the host sending and receiving microcomputers to generate and demodulate the actual physical analog signals sent over the link. Interfacing to the link is via digital-to-analog (D/A), and analog-to-digital (A/D) converters. The frequency spectrum of the signal, either bandpass or baseband, is controlled by an externally supplied clock to the D/A and A/D. MFM is a packet oriented signalling format that sends K tones per baud for L bauds. These KL signals form an orthogonal signal set. Data are encoded in the amplitude and phase of each of the KL signals. In differentially encoded MFM, data are encoded as the change in amplitude and/or of phase between two adjacent tones within the same baud. Differential encoding of MFM signals is extremely effective when successive bauds or adjacent frequencies are subject to identical but unknown amplitude and/or phase changes between the transmit and receive computers. In this paper we describe the encoding, generation, demodulation, decoding and performance of Multi-Frequency Differential Quadrature Phase Shift Keyed (MFDQPSK), and of Multi-Frequency Differential 16-QAM (MFD16-QAM).

2. THEORY

MFM signals are generated inside the host transmit microcomputer using an Inverse Discrete Fourier Transform (IDFT). The DFT technique was first suggested by Weinstein and Ebert [1] and has subsequently been further described by others {see, for example, [2],[3], and [4]}. Each baud consists of a digital signal of k_x real values. When clocked out upon command through an I/O port to a D/A converter at f_x samples per second, a baud of length $\Delta T = k_x/f_x$ seconds is sent over the channel. The signal consists of tones spaced at intervals $\Delta f = 1/\Delta T$ Hz. A bandpass signal is generated in the band $f_1 = k_1\Delta f$ to $f_2 = k_2\Delta f$ by assigning non-zero amplitudes only to those digital frequencies between k_1 and $k_2 = k_1+K$. A baseband signal is generated by assigning all digital frequencies between one and $k_x/2-1$ non-zero amplitudes. In both cases, the actual frequency spectrum occupied by the signal is controlled by the clock frequency f_x. Concatenation of L signal bauds produced by an L-fold repetition of this process creates a signal packet of length $L\Delta T$.

From the discussion above, we see that in MFM the data to be transmitted with each baud are encoded directly in the frequency domain as complex numbers. In MFQPSK, two bits (a di-bit) are sent with each digital frequency using the state diagram of Figure 1. Since the signal occupies a bandwidth of $K/\Delta T$, the throughput rate is 2 bits per Hz of occupied channel bandwidth. In MF16-QAM, four bits are sent with each digital frequency using the state diagram shown in Figure 1. Three bits are encoded into the 8 phases and one bit is encoded as amplitude of the digital frequencies. Using this constellation, data is transmitted at a throughput rate of 4 bits per Hz of channel bandwidth. An MFM packet is illustrated in Figure 2.

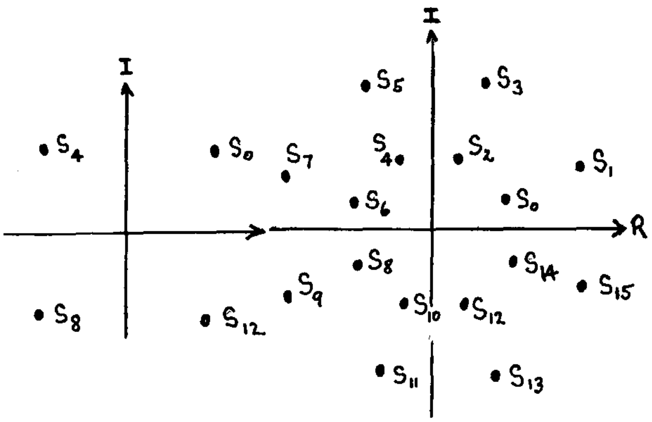

Figure 1
MFM Constellations

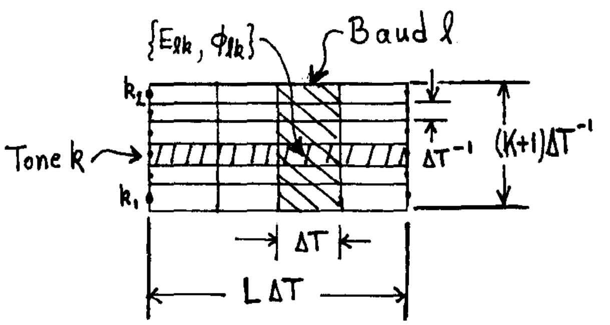

Figure 2
An MFM Signal Packet

The demodulation and decoding of MFM is accomplished as the inverse of the encoding and modulation process. That is, at the receiver L real valued sequences of k_x points are obtained from the packet of L bauds by sampling the received analog signal at f_x samples per second and storing the samples in the host receiver microcomputer's RAM. The k_x point DFTs are obtained of the L sequences, but only those complex coefficients that correspond to transmitted digital frequencies are retained for decoding. In an additive white gaussian noise memoryless channel, the 2KL values obtained in this manner are statistically independent gaussian random variables with identical standard deviations and with means that depend on the transmitted data values. Dividing each of these values by the standard deviation yields a set of 2KL statistically independent, unit variance, gaussian random variables that have mean values given by

$$E[R_l(k)] = \{2E_{kl}/N_o\}^{1/2} \cos\phi_{kl}$$
$$E[I_l(k)] = \{2E_{kl}/N_o\}^{1/2} \sin\phi_{kl} \qquad (1)$$

where E_{kl} is the energy and ϕ_{lk} is the phase of the kth tone during the lth signal baud and the white noise has power spectral density $N_o/2$ [See the Appendix].

The bit error rate for MFQPSK is identical to the bit error rate of ordinary QPSK and the symbol error rate of the 3 phase bits of MF16-QAM is the same as the symbol error rate of ordinary 8 PSK given the same E_{kl}/N_o.

3. DIFFERENTIAL CODING IN THE FREQUENCY DOMAIN

Demodulation of MFM is strictly coherent and requires that phase synchronization between the transmitter and receivers be maintained for each of the multiplicity of carrier frequencies in the MFM signal. For links involving radio frequency (or acoustic) propagation between the transmitting and receiving microcomputers these requirements may be difficult or impossible to meet. In such cases, differential encoding should be employed.

In Multi-frequency Differential Modulation (MFDM), symbols are differentially encoded within each baud between adjacent tones. The differential encoding algorithims that we employ are given in Table I. The first digital frequency, k_1, is always assigned state S_0. K+1 digital frequencies are sent with each signal baud. At the receiver, following the DFT, the complex product between the DFT coefficient of digital frequency k and the complex conjugate of the DFT coefficient of digital frequency k-1 is formed. In the case of MFDQPSK, the result is multiplied by $\exp(j\pi/4)$; in the case of MFD16-QAM, the result is multiplied by $\exp(j\pi/8)$. Consideration of Table I shows that this realigns the differentially encoded phase-bits to the constellations of Figure 1. It is shown in the Appendix that the 2KL values thus obtained are approximately gaussian random variables and that after normalization by dividing by their standard deviations they have mean values given by

$$E[R_l(k)] = A_k\cos\phi_{kl}$$
$$E[I_l(k)] = A_k\sin\phi_{kl} \qquad (2)$$

with,

$$A_k = \{2E_kE_{k-1}/N_o(E_k+E_{k-1}+N_o)\}^{1/2}. \quad (3)$$

Comparing (1) and (2), we see that when adjacent frequencies have equal E_k there will be a theoretical loss of 3 db in output signal-to-noise ratio compared to coherent demodulation of each tone separately. However, actual measurements on a prototype system show that MFDQPSK performs within one to two db of MFQPSK over an E_k/N_o range of 5 to 20 db [Ref 6]. The reasons for this are as of now not completely clear, but are believed to be due to positive correlation of the noise of adjacent tones.

As can be seen from (2), the signal-to-noise ratio for decoding the phase bits in MFD16-QAM depends on the amplitude bit. If the amplitude bit is a zero, then one amplitude is high and one is low; if the amplitude bit is one, then adjacent tones have equal amplitudes which are either both high or both low. Phase bit decoding errors are dominated by the case when both amplitudes are low. The probability of a differential phase symbol decoding error, given both amplitudes low, is [See, for example, Hakin [5], pg. 317]

$$P_{\phi e} = 2Q\{(E_{Lo}/N_o)^{1/2}\sin(\pi/8)\}. \quad (4)$$

For moderately high signal-to-noise ratios, the probability of an amplitude bit error is closely approximated by

$$P_{Ae} = 1.5Q\{(E_{Lo}/N_o)^{1/2}3/8\} \quad (5)$$

where we have used high energy symbols with 25/4 the energy of the low ones. Overall, the symbol error probability for MFD16-QAM is bounded by

$$P_{se} \le P_{Ae} + P_{\phi e}. \quad (6)$$

Figure 3 shows theoretical upper bounds for 4-bit symbol error probabilities for MFDQPSK and MFD16-QAM versus average E_b/N_o. MFQPSK is shown for comparison.

4. CONCLUSIONS

Multi-frequency modulation is an extremely robust, bandwidth efficient technique for digital data communications. It relys on DFT algorithims to modulate and demodulate the data. The practical application of MFM is greatly enhanced by differentially encoding the information to be transmitted between adjacent digital frequencies. Differential coding\decoding algorithims have been described and theoretical performance results have been given herein for MFDQPSK and MFD16-QAM. The theoretical performance is approximately 3db lower than for non-differential MFM. Actual data suggest these results may in fact be too pessimistic; however, even with the 3 db reduction, differential coding should be employed because it reduces or eliminates the need for channel equalization.

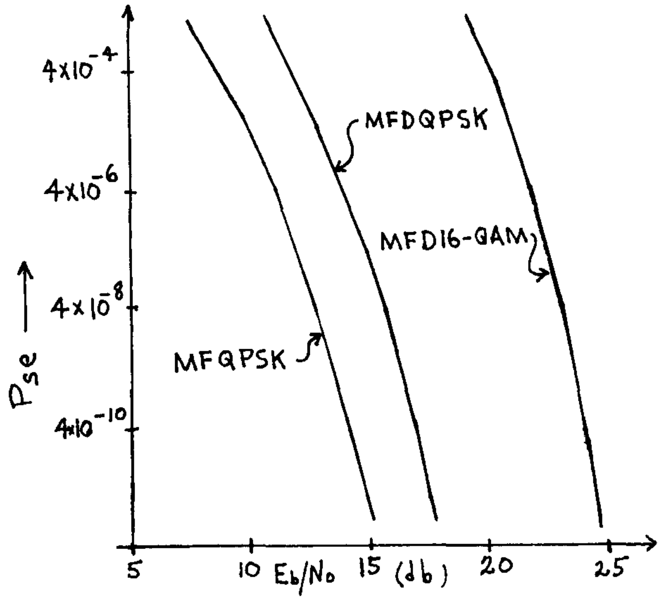

Figure 3
MFDM 4 Bit Symbol Error Probabilities

INPUT DATA FOR MFD16-QAM/MFDQPSK	STATE FOR TONE k GIVEN TONE k-1 IS IN STATE S_{2n}/S_{2n+1}
0000	S_{2n+1}/S_{2n}
0001/00	S_{2n}/S_{2n+1}
0010	S_{2n+3}/S_{2n+2}
0011	S_{2n+2}/S_{2n+3}
0110	S_{2n+5}/S_{2n+4}
0111/01	S_{2n+4}/S_{2n+5}
0100	S_{2n+7}/S_{2n+6}
0101	S_{2n+6}/S_{2n+7}
1100	S_{2n+9}/S_{2n+8}
1101/11	S_{2n+8}/S_{2n+9}
1110	S_{2n+11}/S_{2n+10}
1111	S_{2n+10}/S_{2n+11}
1010	S_{2n+13}/S_{2n+12}
1011/10	S_{2n+12}/S_{2n+13}
1000	S_{2n+15}/S_{2n+14}
1001	S_{2n+14}/S_{2n+15}

TABLE I
(Index addition modulo 16)
Differential Encoding

APPENDIX

Let the lth baud of an MFM signal at the receiver be given by

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