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DIGITAL COMMUNICATIONS

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CTERIZATION OF LS AND MS

ategorized in a number of different ways such as random versus iscrete time versus continuous time, discrete amplitude versus plitude, lowpass versus bandpass, finite energy versus infinite erage power versus infinite average power, etc. In this chapter we terization of signals and systems that are usually encountered in of digital information over a communication channel. In partic- uce the representation of various forms of digitally modulated ribe their spectral characteristics.

with the characterization of bandpass signals and systems, nathematical representation of bandpass stationary stochastic , we present a vector space representation of signals. We conclude ntation of digitally modulated signals and their spectral charac-

ENTATION OF BANDPASS D SYSTEMS

formation-bearing signals are transmitted by some type of carrier e channel over which the signal is transmitted is limited in interval of frequencies centered about the carrier, as in double- ation, or adjacent to the carrier, as in single-sideband modula- nd channels (systems) which satisfy the condition that their uch smaller than the carrier frequency are termed *narrowband*

bandpass signals and channels (*systems*). The modulation performed at the transmitting end of the communication system to generate the bandpass signal and the demodulation performed at the receiving end to recover the digital information involve frequency translations. With no loss in generality and for mathematical convenience, it is desirable to reduce all bandpass signals and channels to equivalent lowpass signals and channels. As a consequence, the results of the performance of the various modulation and demodulation techniques presented in the subsequent chapters are independent of carrier frequencies and channel frequency bands. The representation of bandpass signals and systems in terms of equivalent lowpass waveforms and the characterization of bandpass stationary stochastic processes are the main topics of this section.

3.1.1 Representation of Bandpass Signals

A real-valued signal $s(t)$ with a frequency content concentrated in a narrow band of frequencies in the vicinity of a frequency f_c can be expressed in the form

$$s(t) = a(t)\cos\left[2\pi f_c t + \theta(t)\right] \tag{3.1.1}$$

where $a(t)$ denotes the amplitude (envelope) of $s(t)$, and $\theta(t)$ denotes the phase of $s(t)$. The frequency f_c is usually called the *carrier* of $s(t)$ and may be any convenient frequency within or near the frequency band occupied by the signal. When the band of frequencies occupied by $s(t)$ is small relative to f_c, the signal is called a *narrowband bandpass signal* or, simply, a *bandpass signal*.

By expanding the cosine function in (3.1.1) a second representation for $s(t)$ is obtained, namely,

$$\begin{aligned} s(t) &= a(t)\cos\theta(t)\cos 2\pi f_c t - a(t)\sin\theta(t)\sin 2\pi f_c t \\ &= x(t)\cos 2\pi f_c t - y(t)\sin 2\pi f_c t \end{aligned} \tag{3.1.2}$$

where the signals $x(t)$ and $y(t)$, termed the *quadrature components* of $s(t)$, are defined as

$$\begin{aligned} x(t) &= a(t)\cos\theta(t) \\ y(t) &= a(t)\sin\theta(t) \end{aligned} \tag{3.1.3}$$

The frequency content of the quadrature components $x(t)$ and $y(t)$ is concentrated at low frequencies (around $f = 0$, as shown below) and, hence, these components are appropriately called *lowpass signals*. Finally, a third representation for $s(t)$ is obtained from (3.1.1) by defining the complex envelope $u(t)$ as

$$\begin{aligned} u(t) &= a(t)e^{j\theta(t)} \\ &= x(t) + jy(t) \end{aligned} \tag{3.1.4}$$

so that

$$s(t) = \text{Re}\left[u(t)e^{j2\pi f_c t}\right] \tag{3.1.5}$$

where Re[ ] denotes the real part of the complex-valued quantity in the brackets. Thus a real bandpass signal is completely described by any one of the three equivalent forms given in (3.1.1), (3.1.2), or (3.1.5).

The Fourier transform of $s(t)$ is

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft}\, dt$$
$$= \int_{-\infty}^{\infty} \{\text{Re}\,[u(t) e^{j2\pi f_c t}]\} e^{-j2\pi ft}\, dt \tag{3.1.6}$$

Use of the identity

$$\text{Re}\,(\xi) = \tfrac{1}{2}(\xi + \xi^*) \tag{3.1.7}$$

in (3.1.6) yields the result

$$S(f) = \frac{1}{2}\int_{-\infty}^{\infty} [u(t) e^{j2\pi f_c t} + u^*(t) e^{-j2\pi f_c t}] e^{-j2\pi ft}\, dt$$
$$= \tfrac{1}{2}[U(f - f_c) + U^*(-f - f_c)] \tag{3.1.8}$$

where $U(f)$ is the Fourier transform of $u(t)$. Since the frequency content of the bandpass signal $s(t)$ is concentrated in the vicinity of the carrier f_c, the result in (3.1.8) indicates that the frequency content of $u(t)$ is concentrated in the vicinity of $f = 0$. Consequently, the complex-valued waveform $u(t)$ is basically a low-pass signal waveform and, hence, is called the *equivalent lowpass signal*.

The energy in the signal $s(t)$ is defined as

$$\mathscr{E} = \int_{-\infty}^{\infty} s^2(t)\, dt$$
$$= \int_{-\infty}^{\infty} \{\text{Re}\,[u(t) e^{j2\pi f_c t}]\}^2\, dt \tag{3.1.9}$$

When the identity in (3.1.7) is used in (3.1.9), we obtain the following result:

$$\mathscr{E} = \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2\, dt$$
$$+ \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 \cos\,[4\pi f_c t + 2\theta(t)]\, dt \tag{3.1.10}$$

Consider the second integral in (3.1.10). Since the signal $s(t)$ is narrowband, the real envelope $a(t) \equiv |u(t)|$ or, equivalently, $a^2(t)$ varies slowly relative to the rapid variations exhibited by the cosine function. A graphical illustration of the integrand in the second integral of (3.1.10) is shown in Fig. 3.1.1. The value of the integral is just the net area under the cosine function modulated by $a^2(t)$. Since the modulating waveform $a^2(t)$ varies slowly relative to the cosine function, the net area contributed by the second integral is very small relative to the value of the first integral in (3.1.10) and, hence, it can be neglected. Thus, for all practical purposes, the energy in the bandpass signal $s(t)$, expressed in terms of the equivalent lowpass signal $u(t)$, is

$$\mathscr{E} = \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2\, dt \tag{3.1.11}$$

where $|u(t)|$ is just the envelope $a(t)$ of $s(t)$.

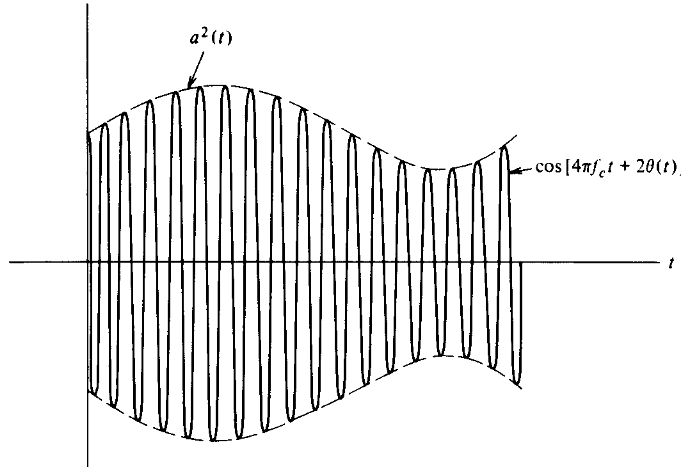

FIGURE 3.1.1
The signal $a^2(t)\cos\,[4\pi f_c t + 2\theta(t)]$.

3.1.2 Representation of Linear Bandpass Systems

A linear filter or system may be described either by its impulse by its frequency response $H(f)$, which is the Fourier transfor $h(t)$ is real,

$$H^*(-f) = H(f)$$

Let us define $C(f - f_c)$ as

$$C(f - f_c) = \begin{cases} H(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

Then

$$C^*(-f - f_c) = \begin{cases} 0 & f > 0 \\ H^*(-f) & f < 0 \end{cases}$$

Using (3.1.12), we have

$$H(f) = C(f - f_c) + C^*(-f - f_c)$$

The inverse transform of $H(f)$ in (3.1.15) yields $h(t)$ in the for

$$h(t) = c(t) e^{j2\pi f_c t} + c^*(t) e^{-j2\pi f_c t}$$
$$= 2\,\text{Re}\,[c(t) e^{j2\pi f_c t}]$$

where $c(t)$ is the inverse Fourier transform of $C(f)$. In gene response $c(t)$ of the equivalent lowpass system is complex-value form of $s(t)$ is

$$f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \{\mathrm{Re}[u(t)e^{j2\pi f_c t}]\} e^{-j2\pi ft}\, dt \qquad (3.1.6)$$

$$\mathrm{Re}(\xi) = \tfrac{1}{2}(\xi + \xi^*) \qquad (3.1.7)$$

ult

$$\tfrac{1}{2}\int_{-\infty}^{\infty} [u(t)e^{j2\pi f_c t} + u^*(t)e^{-j2\pi f_c t}] e^{-j2\pi ft}\, dt$$

$$[U(f - f_c) + U^*(-f - f_c)] \qquad (3.1.8)$$

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$$\mathscr{E} = \int_{-\infty}^{\infty} s^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} \{\mathrm{Re}[u(t)e^{j2\pi f_c t}]\}^2\, dt \qquad (3.1.9)$$

.1.7) is used in (3.1.9), we obtain the following result:

$$\frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2\, dt$$

$$+ \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2 \cos[4\pi f_c t + 2\theta(t)]\, dt \qquad (3.1.10)$$

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$$\mathscr{E} = \frac{1}{2}\int_{-\infty}^{\infty} |u(t)|^2\, dt \qquad (3.1.11)$$

nvelope $a(t)$ of $s(t)$.

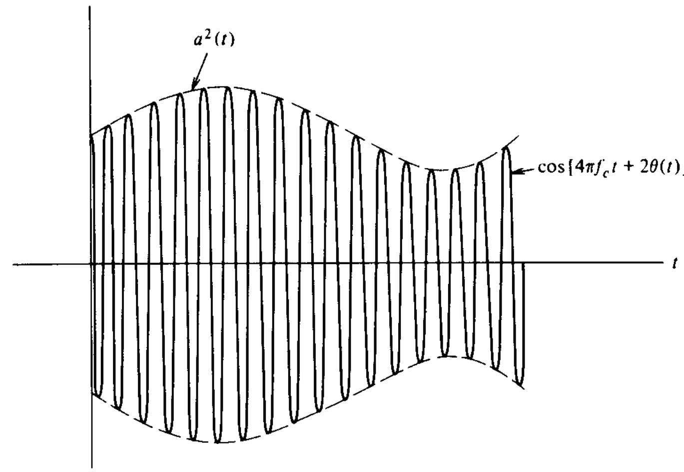

FIGURE 3.1.1
The signal $a^2(t)\cos[4\pi f_c t + 2\theta(t)]$.

3.1.2 Representation of Linear Bandpass Systems

A linear filter or system may be described either by its impulse response $h(t)$ or by its frequency response $H(f)$, which is the Fourier transform of $h(t)$. Since $h(t)$ is real,

$$H^*(-f) = H(f) \qquad (3.1.12)$$

Let us define $C(f - f_c)$ as

$$C(f - f_c) = \begin{cases} H(f) & f > 0 \\ 0 & f < 0 \end{cases} \qquad (3.1.13)$$

Then

$$C^*(-f - f_c) = \begin{cases} 0 & f > 0 \\ H^*(-f) & f < 0 \end{cases} \qquad (3.1.14)$$

Using (3.1.12), we have

$$H(f) = C(f - f_c) + C^*(-f - f_c) \qquad (3.1.15)$$

The inverse transform of $H(f)$ in (3.1.15) yields $h(t)$ in the form

$$h(t) = c(t)e^{j2\pi f_c t} + c^*(t)e^{-j2\pi f_c t}$$

$$= 2\,\mathrm{Re}[c(t)e^{j2\pi f_c t}] \qquad (3.1.16)$$

where $c(t)$ is the inverse Fourier transform of $C(f)$. In general, the impulse response $c(t)$ of the equivalent lowpass system is complex-valued.

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