Formulas for Stress and Strain

FIFTH EDITION

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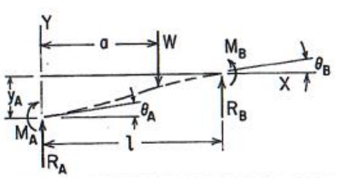
TABLE 3 ***Shear, moment, slope, and deflection formulas for beams***

NOTATION: W = load (pounds); w = unit load (pounds per linear inch); M_o = applied couple (inch-pounds); θ_o = externally created concentrated angular displacement (radians); Δ_o = externally created concentrated lateral displacement (inches); T_1 and T_2 = temperatures on the top and bottom surfaces, respectively (degrees). R_A and R_B are the vertical end reactions at the left and right, respectively, and are positive upward. M_A and M_B are the reaction end moments at the left and right, respectively. All moments are positive when producing compression on the upper portion of the beam cross section. The transverse shear force V is positive when acting upward on the left end of a portion of the beam. All applied loads, couples, and displacements are positive as shown. All forces are in pounds, all moments in inch-pounds, all deflections and beam dimensions in inches, all slopes in radians, and all temperatures in degrees. All deflections are positive upward, and all slopes are positive when up and to the right. E is the modulus of elasticity of the beam material, and I is the area moment of inertia about the centroidal axis of the beam cross section. γ is the temperature coefficient of expansion (inches per inch per degree)

1. Concentrated intermediate load



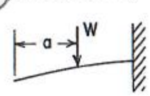
Transverse shear $= V = R_A - W\langle x-a\rangle^0$

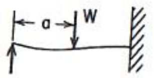
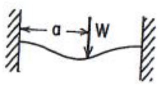
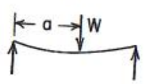
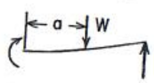
Bending moment $= M = M_A + R_A x - W\langle x-a\rangle$

Slope $= \theta = \theta_A + \frac{M_A x}{EI} + \frac{R_A x^2}{2EI} - \frac{W}{2EI}\langle x-a\rangle^2$

Deflection $= y = y_A + \theta_A x + \frac{M_A x^2}{2EI} + \frac{R_A x^3}{6EI} - \frac{W}{6EI}\langle x-a\rangle^3$

(*Note:* see page 94 for a definition of the term $\langle x-a\rangle^n$.)

End restraints, reference no.	Boundary values	Selected maximum values of moments and deformations
1a. Left end free, right end fixed (cantilever) 	$R_A = 0$ $M_A = 0$ $\theta_A = \frac{W(l-a)^2}{2EI}$ $y_A = \frac{-W}{6EI}(2l^3 - 3l^2a + a^3)$ $R_B = W$ $M_B = -W(l-a)$ $\theta_B = 0$ $y_B = 0$	Max $M = M_B$; max possible value $= -Wl$ when $a = 0$ Max $\theta = \theta_A$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$
	$W(l-a)^2$ ...	... max possible value $= \frac{Wl}{\ldots}$ when $a = 0$

1c. Left end simply supported, right end fixed  	$R_A = \frac{W}{2l^3}(l-a)^2(2l+a) \quad M_A = 0$ $\theta_A = \frac{-Wa}{4EIl}(l-a)^2 \quad y_A = 0$ $R_B = \frac{Wa}{2l^3}(3l^2 - a^2) \quad \theta_B = 0$ $M_B = \frac{-Wa}{2l^2}(l^2 - a^2) \quad y_B = 0$	Max $+M = \frac{Wa}{2l^3}(l-a)^2(2l+a)$ at $x = a$; max possible value $= 0.174Wl$ when $a = 0.366l$ Max $-M = M_B$; max possible value $= -0.1924Wl$ when $a = 0.5773l$ Max $y = \frac{-Wa}{6EI}(l-a)^2\left(\frac{a}{2l+a}\right)^{1/2}$ at $x = l\left(\frac{a}{2l+a}\right)^{1/2}$ when $a > 0.414l$ Max $y = \frac{-Wa(l^2-a^2)^3}{3EI(3l^2-a^2)^2}$ at $x = \frac{l(l^2+a^2)}{3l^2-a^2}$ when $a < 0.414l$; max possible $y = -0.0098\frac{Wl^3}{EI}$ when $x = a = 0.414l$
1d. Left end fixed, right end fixed  	$R_A = \frac{W}{l^3}(l-a)^2(l+2a)$ $M_A = \frac{-Wa}{l^2}(l-a)^2$ $\theta_A = 0 \quad y_A = 0$ $R_B = \frac{Wa^2}{l^3}(3l - 2a)$ $M_B = \frac{-Wa^2}{l^2}(l-a)$ $\theta_B = 0 \quad y_B = 0$	Max $+M = \frac{2Wa^2}{l^3}(l-a)^2$ at $x = a$; max possible value $= \frac{Wl}{8}$ when $a = \frac{l}{2}$ Max $-M = M_A$ if $a < \frac{l}{2}$; max possible value $= -0.1481Wl$ when $a = \frac{l}{3}$ Max $y = \frac{-2W(l-a)^2a^3}{3EI(l+2a)^2}$ at $x = \frac{2al}{l+2a}$ if $a > \frac{l}{2}$; max possible value $= \frac{-Wl^3}{192EI}$ when $x = a = \frac{l}{2}$
1e. Left end simply supported, right end simply supported  	$R_A = \frac{W}{l}(l-a) \quad M_A = 0$ $\theta_A = \frac{-Wa}{6EIl}(2l-a)(l-a) \quad y_A = 0$ $R_B = \frac{Wa}{l} \quad M_B = 0$ $\theta_B = \frac{Wa}{6EIl}(l^2 - a^2) \quad y_B = 0$	Max $M = R_A a$ at $x = a$; max possible value $= \frac{Wl}{4}$ when $a = \frac{l}{2}$ Max $y = \frac{-Wa}{3EIl}\left(\frac{l^2-a^2}{3}\right)^{3/2}$ at $x = l - \left(\frac{l^2-a^2}{3}\right)^{1/2}$ when $a < \frac{l}{2}$; max possible value $= \frac{-Wl^3}{48EI}$ at $x = \frac{l}{2}$ when $a = \frac{l}{2}$ Max $\theta = \theta_A$ when $a < \frac{l}{2}$; max possible value $= -0.0642\frac{Wl^2}{EI}$ when $a = 0.423l$
1f. Left end guided, right end simply supported  	$R_A = 0 \quad M_A = W(l-a) \quad \theta_A = 0$ $y_A = \frac{-W(l-a)}{6EI}(2l^2 + 2al - a^2)$ $R_B = W \quad M_B = 0$ $\theta_B = \frac{W}{2EI}(l^2 - a^2) \quad y_B = 0$	Max $M = M_A$ for $0 < x < a$; max possible value $= Wl$ when $a = 0$ Max $\theta = \theta_B$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$