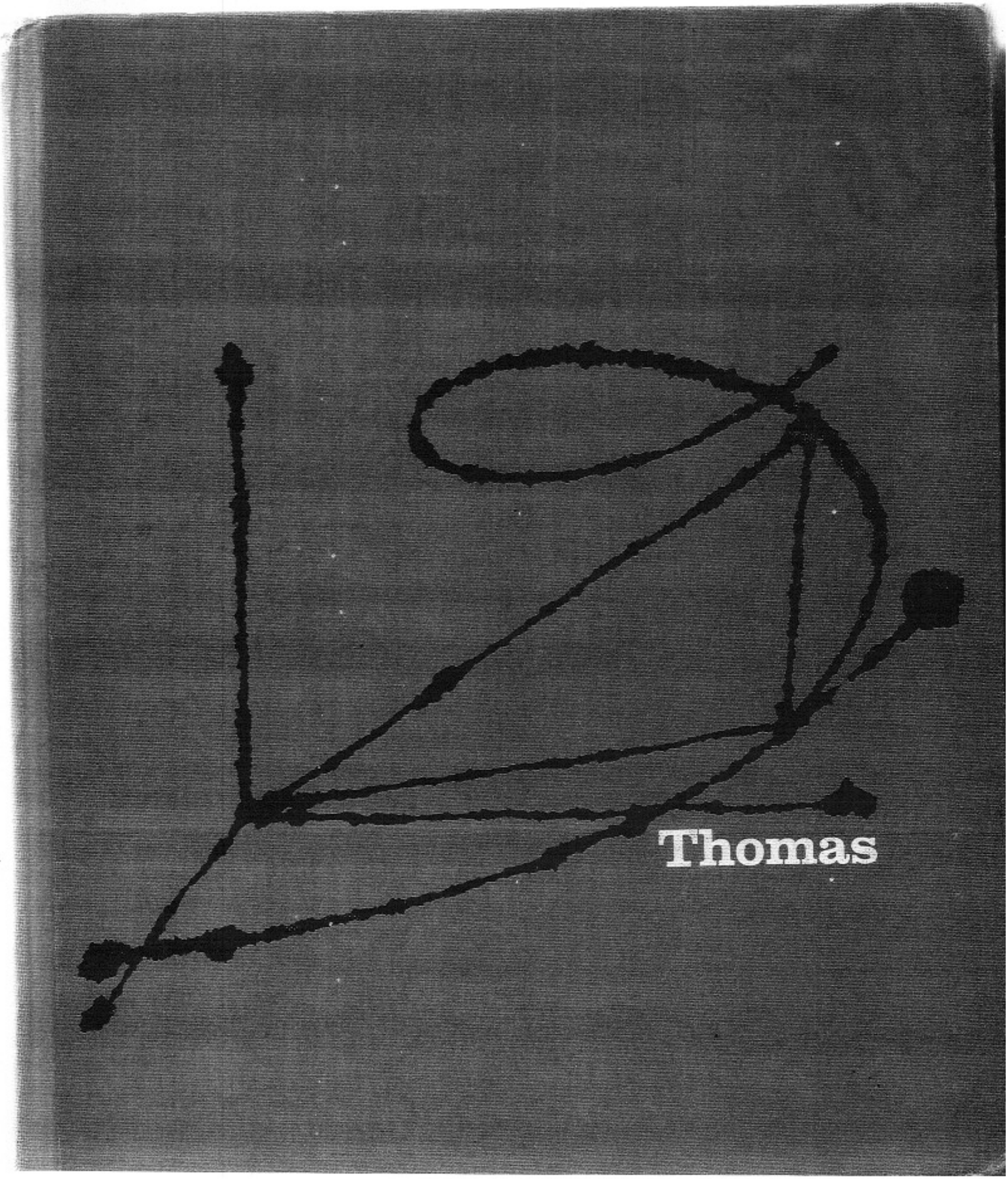
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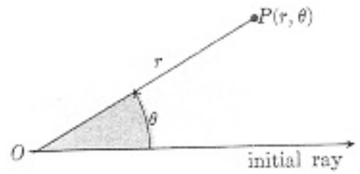
POLAR COORDINATES

CHAPTER 11

11.1 THE POLAR COORDINATE SYSTEM

We know that a point can be located in a plane by giving its abscissa and ordinate relative to a given coordinate system. Such x- and y-coordinates are called *Cartesian* coordinates, in honor of the French mathematician-philosopher René Descartes* (1596–1650), who is credited with discovering this method of fixing the position of a point in a plane.



11.1

Another useful way to locate a point in a plane is by *polar coordinates* (see Fig. 11.1). First, we fix an *origin* O and an *initial ray*† from O. The point P has polar coordinates r, θ, with

$$r = \text{directed distance from } O \text{ to } P, \tag{1a}$$

and

$$\theta = \text{directed angle from initial ray to } OP. \tag{1b}$$

As in trigonometry, the angle θ is *positive* when measured counterclockwise and negative when measured clockwise (Fig. 11.1). But the angle associated with a given point is not unique (Fig. 11.2). For instance, the point 2 units from the origin, along the ray $\theta = 30°$, has polar coordinates $r = 2$, $\theta = 30°$. It also has coordinates $r = 2$, $\theta = -330°$, or $r = 2$, $\theta = 390°$.

There are occasions when we wish to allow r to be negative. That's why we say "directed distance"

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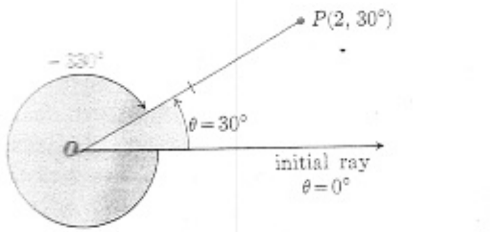
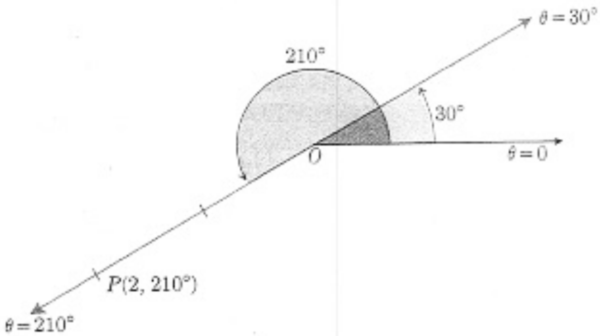
* For an interesting biographical account together with an excerpt from Descartes' own writings, see *World of Mathematics*, Vol. 1, pp. 235–253.

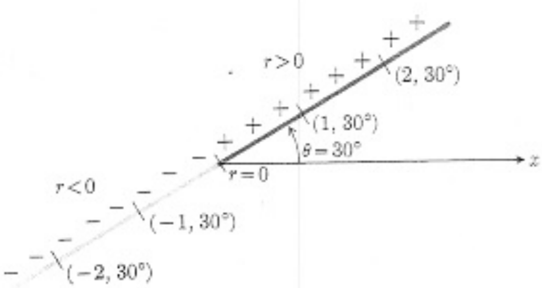
† A *ray* is a half-line consisting of a vertex and points of a line on one side of the vertex. For example, the origin and positive x-axis is a ray. The points on the line $y = 2x + 3$ with $x \geq 1$ is another ray; its vertex is $(1, 5)$.



11.2 The ray $\theta = 30°$ is the same as the ray $\theta = -330°$.



11.3 The rays $\theta = 30°$ and $\theta = 210°$ make a line.



11.4 The terminal ray $\theta = \pi/6$ and its negative.

in Eq. (1a). The ray $\theta = 30°$ and the ray $\theta = 210°$ together make up a complete line through O (see Fig. 11.3). The point $P(2, 210°)$ 2 units from O on the ray $\theta = 210°$ has polar coordinates $r = 2$, $\theta = 210°$. It can be reached by a person standing at O and facing out along the initial ray, if he first turns 210° counterclockwise, and then goes forward 2 units. He would reach the same point by turning only 30° counterclockwise from the initial ray and then going *backward* 2 units. So we say that the point also has polar coordinates $r = -2$, $\theta = 30°$.

Whenever the angle between two rays is 180°, the rays actually make a straight line. We then say that either ray is the negative of the other. Points on the ray $\theta = \alpha$ have polar coordinates (r, α) with $r \geq 0$. Points on the negative ray, $\theta = \alpha + 180°$, have coordinates (r, α) with $r \leq 0$. The origin is $r = 0$. (See Fig. 11.4 for the ray $\theta = 30°$ and its negative. A word of caution: The "negative" of the ray $\theta = 30°$ is the ray $\theta = 30° + 180° = 210°$ and *not* the ray $\theta = -30°$. "Negative" refers to the directed distance r.)

There is a great advantage in being able to use both polar and Cartesian coordinates at once. To do this, we use a common origin and take the initial ray as the positive x-axis, and take the ray $\theta = 90°$ as the positive y-axis. The coordinates, shown in Fig. 11.5, are then related by the equations

$$x = r \cos \theta, \quad y = r \sin \theta. \tag{2}$$

These are the equations that define $\sin \theta$ and $\cos \theta$ when r is positive. They are also valid if r is negative, because

$$\cos(\theta + 180°) = -\cos \theta,$$
$$\sin(\theta + 180°) = -\sin \theta,$$

so positive r's on the $(\theta + 180°)$-ray correspond to negative r's associated with the θ-ray. When $r = 0$, then $x = y = 0$, and P is the origin.

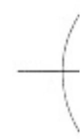
If we impose the condition

$$r = a \quad (a \text{ constant}), \tag{3}$$

then the locus of P is a circle with center O and radius a, and P describes the circle once as θ varies from 0 to 360° (see Fig. 11.6). On the other hand, if we let r vary and hold θ fixed, say

$$\theta = 30°, \tag{4}$$

the locus of P is the straight line shown in Fig. 11.4.



11.5 Polar and C…

11.6 The circle r =…

We adopt the c… number, $-\infty <$ … on $x = 0$, $y = 0$ in…

$r =$ …

the origin, $x =$ …
The same point … different ways in … the point $(2, 30°)$ … representations: (… $-2, -150°$). The … the two formula…

$(2, 30° + n$ …
$(-2, 210° + n$ …

… if we represent … formulas

$(2, \frac{1}{6}\pi + 2n$ …
$(-2, \frac{7}{6}\pi + 2n$ …

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