DECLARATION OF ERIC PEPPER

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Exhibit	Reference	Date of Public Availability
A	Berthold K. P. Horn and Brian G. Schunk, "Determining optical flow," SPIE Vol. 281, Techniques and Applications of Image Understanding pp. 319–331 (1981)	November 12, 1981

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Executed on March 21, 2017 at Bellingham, Washington.

By: ______________

Eric A. Pepper

EXHIBIT A

Determining optical flow

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Abstract

Optical flow cannot be computed locally, since only one independent measurement is available from the image sequence at a point, while the flow velocity has two components. A second constraint is needed. A method for finding the optical flow pattern is presented which assumes that the apparent velocity of the brightness pattern varies smoothly almost everywhere in the image. An iterative implementation is shown which successfully computes the optical flow for a number of synthetic image sequences. The algorithm is robust in that it can handle image sequences that are quantized rather coarsely in space and time. It is also insensitive to quantization of brightness levels and additive noise. Examples are included where the assumption of smoothness is violated at singular points or along lines in the image.

1. Introduction

Optical flow is the distribution of apparent velocities of movement of brightness patterns in an image. Optical flow can arise from relative motion of objects and the viewer [8, 9]. Consequently, optical flow can give important information about the spatial arrangement of the objects viewed and the rate of change of this arrangement [10]. Discontinuities in the optical flow can help in segmenting images into regions that correspond to different objects [29]. Attempts have been made to perform such segmentation using differences between successive image frames [17, 18, 19, 22, 3, 27]. Several papers address the problem of recovering the motions of objects relative to the viewer from the optical flow [12, 20, 21, 23, 31]. Some recent papers provide a clear exposition of this enterprise [32, 33]. The mathematics can be made rather difficult, by the way, by chosing an inconvenient coordinate system. In some cases information about the shape of an object may also be recovered [4, 20, 21].

These papers begin by assuming that the optical flow has already been determined. Although some reference has been made to schemes for computing the flow from successive views of a scene [7, 12], the specifics of a scheme for determining the flow from the image have not been described. Related work has been done in an attempt to formulate a model for the short range motion detection processes in human vision [2, 24]. The pixel recursive equations of Netravali and Robbins [30], designed for coding motion in television signals, bear some similarity to the iterative equations developed in this paper. A recent review [28] of computational techniques for the analysis of image sequences contains over 150 references.

The optical flow cannot be computed at a point in the image independently of neighboring points without introducing additional constraints, because the velocity field at each image point has two components while the change in image brightness at a point in the image plane due to motion yields only one constraint. Consider, for example, a patch of a pattern where brightness[1] varies as a function of one image coordinate but not the other. Movement of the pattern in one direction alters the brightness at a particular point, but motion in the other direction yields no change. Thus components of movement in the latter direction cannot be determined locally.

2. Relationship to Object Motion

The relationship between the optical flow in the image plane and the velocities of objects in the three dimensional world is not necessarily obvious. We perceive motion when a changing picture is projected onto a stationary screen, for example. Conversely, a moving object may give rise to a constant brightness pattern. Consider, for example, a uniform sphere which exhibits shading because its surface elements are oriented in many different directions. Yet, when it is rotated, the optical flow is zero at all points in the image, since the shading does not move with the surface. Also, specular reflections move with a velocity characteristic of the virtual image, not the surface in which light is reflected.

For convenience, we tackle a particularly simple world where the apparent velocity of brightness patterns can be directly identified with the movement of surfaces in the scene.

3. The Restricted Problem Domain

To avoid variations in brightness due to shading effects we initially assume that the surface being imaged is flat. We further assume that the incident illumination is uniform across the surface. The brightness

[1] In this paper, the term brightness means image irradiance. The brightness pattern is the distribution of irradiance in the image.

at a point in the image is then proportional to the reflectance of the surface at the corresponding point on the object. Also, we assume at first that reflectance varies smoothly and has no spatial discontinuities. This latter condition assures us that the image brightness is differentiable. We exclude situations where objects occlude one another, in part, because discontinuities in reflectance are found at object boundaries. In two of the experiments discussed later, some of the problems occasioned by occluding edges are exposed.

In the simple situation described, the motion of the brightness patterns in the image is determined directly by the motions of corresponding points on the surface of the object. Computing the velocities of points on the object is a matter of simple geometry once the optical flow is known.

4. Constraints

We will derive an equation that relates the change in image brightness at a point to the motion of the brightness pattern. Let the image brightness at the point (x, y) in the image plane at time t be denoted by $E(x, y, t)$. Now consider what happens when the pattern moves. The brightness of a particular point in the pattern is constant, so that

$$\frac{dE}{dt} = 0.$$

Using the chain rule for differentiation we see that,

$$\frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

(See Appendix A for a more detailed derivation.) If we let

$$u = \frac{dx}{dt} \quad \text{and} \quad v = \frac{dy}{dt},$$

then it is easy to see that we have a single linear equation in the two unknowns u and v,

$$E_x u + E_y v + E_t = 0,$$

where we have also introduced the additional abbreviations E_x, E_y, and E_t for the partial derivatives of image brightness with respect to x, y and t, respectively. The constraint on the local flow velocity expressed by this equation is illustrated in Figure 1. Writing the equation in still another way,

$$(E_x, E_y) \cdot (u, v) = -E_t.$$

Thus the component of the movement in the direction of the brightness gradient (E_x, E_y) equals

$$-\frac{E_t}{\sqrt{E_x^2 + E_y^2}}.$$

We cannot, however determine the component of the movement in the direction of the iso-brightness contours, at right angles to the brightness gradient. As a consequence, the flow velocity (u, v) cannot be computed locally without introducing additional constraints.

5. The Smoothness Constraint

If every point of the brightness pattern can move independently, there is little hope of recovering the velocities. More commonly we view opaque objects of finite size undergoing rigid motion or deformation. In this case neighboring points on the objects have similar velocities and the velocity field of the brightness patterns in the image varies smoothly almost everywhere. Discontinuities in flow can be expected where one object occludes another. An algorithm based on a smoothness constraint is likely to have difficulties with occluding edges as a result.

One way to express the additional constraint is to minimize the square of the magnitude of the gradient of the optical flow velocity:

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \quad \text{and} \quad \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2.$$

Another measure of the smoothness of the optical flow field is the sum of the squares of the Laplacians of the two velocity components. The Laplacians of u and v are defined as

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \text{and} \quad \nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}.$$

In simple situations, both Laplacians are zero. If the viewer translates parallel to a flat object, rotates about a line perpendicular to the surface or travels orthogonally to the surface, then the second partial derivatives of both u and v vanish (assuming perspective projection in the image formation.)

In this paper, we will use the square of the magnitide of the gradient as our smoothness measure. Note that our approach is in contrast with that of [7], who propose an algorithm that incorporates additional assumptions such as constant flow velocities within discrete regions of the image. Their method, based on cluster analysis, cannot deal with rotating objects, since these give rise to a continuum of flow velocities.

6. Quantization and Noise

Images may be sampled at intervals on a fixed grid of points. While tesselations other than the obvious one have certain advantages [11, 25], for convenience we will assume that the image is sampled on a square grid at regular intervals. Let the measured brightness be $E_{i,j,k}$ at the intersection of the i-th row and j-th column in the k-th image frame. Ideally, each measurement should be an average over the area of a picture cell and over the length of the time interval. In the experiments cited here we have taken samples at discrete points in space and time instead.

In addition to being quantized in space and time, the measurements will in practice be quantized in brightness as well. Further, noise will be apparent in measurements obtained in any real system.

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