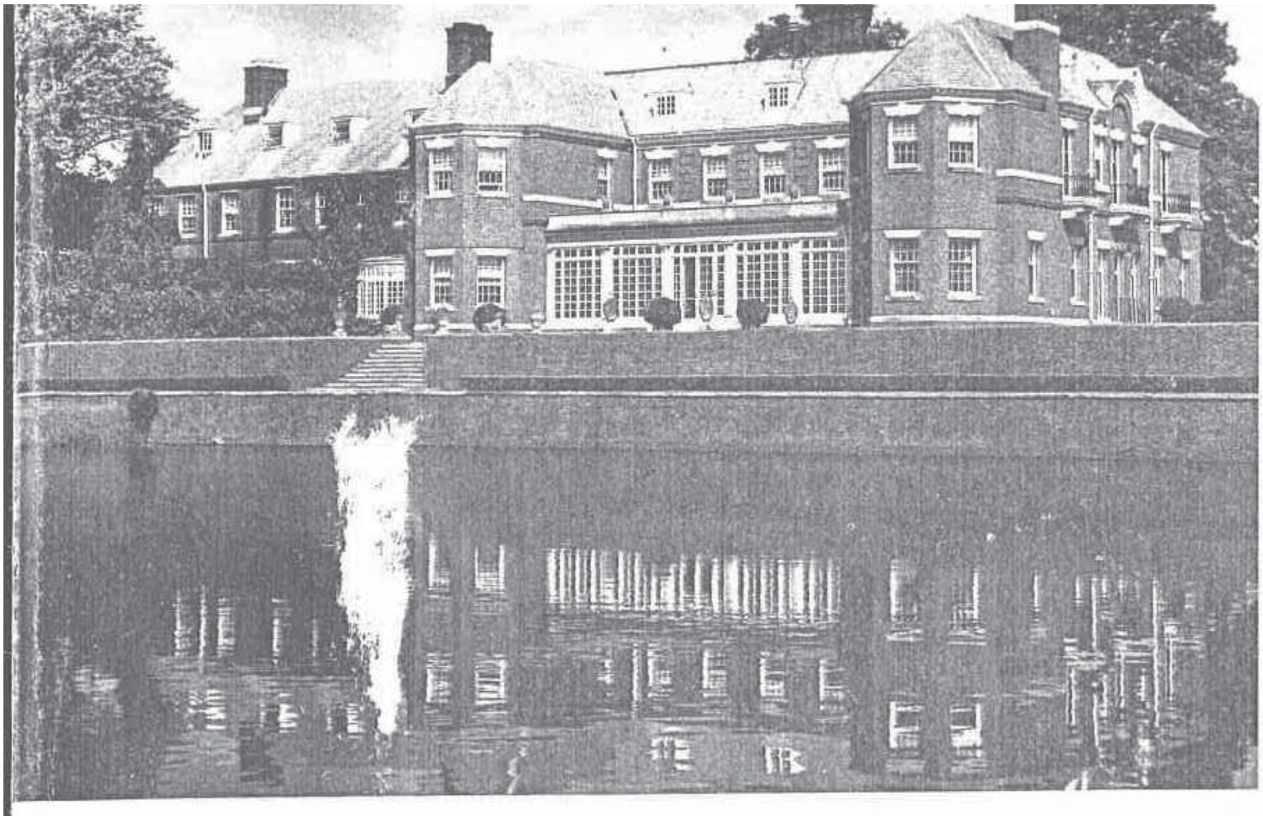

Proceedings

THIRTY-SIXTH ANNUAL ALLERTON CONFERENCE ON COMMUNICATION, CONTROL AND COMPUTING

September 23 - 25, 1998

Allerton House, Monticello, Illinois
Sponsored by the
Coordinated Science Laboratory and the
Department of Electrical and Computer Engineering of the
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PROCEEDINGS

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Coding Theorems for "Turbo-Like" Codes*

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Abstract.

In this paper we discuss AWGN coding theorems for ensembles of coding systems which are built from fixed convolutional codes interconnected with random interleavers. We call these systems "turbo-like" codes and they include as special cases both the classical turbo codes [1,2,3] and the serial concatentation of interleaved convolutional codes [4]. We offer a general conjecture about the behavior of the ensemble (maximum-likelihood decoder) word error probability as the word length approches infinity. We prove this conjecture for a simple class of rate $1/q$ serially concatenated codes where the outer code is a q-fold repetition code and the inner code is a rate 1 convolutional code with transfer function $1/(1+D)$. We believe this represents the first rigorous proof of a coding theorem for turbo-like codes.

1. Introduction.

The 1993 discovery of turbo codes by Berrou, Glavieux, and Thitimajshima [1] has revolutionized the field of error-correcting codes. In brief, turbo codes have enough randomness to achieve reliable communication at data rates near capacity, yet enough structure to allow practical encoding and decoding algorithms. This paper is an attempt to illuminate the first of these two attributes, i.e., the "near Shannon limit" capabilities of turbo-like codes on the AWGN channel.

Our specific goal is to prove AWGN coding theorems for a class of generalized concatenated convolutional coding systems with interleavers, which we call "turbo-like" codes. This class includes both parallel concatenated convolutional codes (classical turbo codes) [1, 2, 3] and serial concatenated convolutional codes [4] as special cases. Beginning with a code structure of this type, with fixed component codes and interconnection topology, we attempt to show that as the block length approaches infinity, the ensemble (over all possible interleavers) maximum likelihood error probability approaches zero if E_b/N_0 exceeds some threshold. Our proof technique is to derive an explicit expression for the ensemble input-output weight enumerator (IOWE) and then to use this expression, in combination with either the classical union bound, or the recent "improved" union bound of Viterbi and Viterbi [9], to show that the maximum likelihood word error probability approaches zero as $N \to \infty$. Unfortunately the difficulty of the first step, i.e., the computation of the ensemble IOWE, has kept us from full success, except for some very simple coding systems, which we call *repeat and accumulate* codes. Still, we are optimistic that this technique will yield coding theorems for a much wider class of interleaved concatenated codes. In any case, it is satisfying to have rigorously proved coding theorems for even a restricted class of turbo-like codes.

Here is an outline of the paper. In Section 2 we quickly review the classical union bound on maximum-likelihood word error probability for block codes on the AWGN

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channel, which is seen to depend on the code's weight enumerator. In Section 3 we define the class of "turbo-like" codes, and give a formula for the average input-output weight enumerator for such a code. In Section 4 we state a conjecture (the interleaver gain exponent conjecture) about the ML decoder performance of turbo-like codes. In Section 5, we define a special class of turbo-like codes, the repeat-and-accumulate codes, and prove the IGE conjecture for them. Finally, in Section 6 we present performance curves for some RA codes, using an iterative, turbo-like, decoding algorithm. This performance is seen to be remarkably good, despite the simplicity of the codes and the suboptimality of the decoding algorithm.

2. Union Bounds on the Performance of Block Codes.

In this section we will review the classical union bound on the maximum-likelihood word error probability for block codes.

Consider a binary linear (n, k) block code C with code rate $r = k/n$. The *(output) weight enumerator* (WE) for C is the sequence of numbers $A_0, \ldots, A_n$, where A_h denotes the number of codewords in C with (output) weight h. The *input-output weight enumerator* (IOWE) for C is the array of numbers $A_{w,h}$, $w = 0, 1, \ldots, k$, $h = 0, 1, \ldots, n$: $A_{w,h}$ denotes the number of codewords in C with input weight w and output weight h.

The union bound on the word error probability P_W of the code C over a memoryless binary-input channel, using maximum likelihood decoding, has the well-known form

$$P_W \leq \sum_{h=1}^{n} A_h z^h \tag{2.1}$$

$$= \sum_{h=1}^{n} \left(\sum_{w=1}^{k} A_{w,h} \right) z^h. \tag{2.2}$$

In (2.1) and (2.2), the function z^h represents an upper bound on the pairwise error probability for two codewords separated by Hamming (output) distance h. For AWGN channels, $z = e^{-rE_b/N_0}$ where E_b/N_0 is the signal-to-noise ratio per bit.

3. The Class of "Turbo-Like" Codes.

In this section, we consider a general class of concatenated coding systems of the type depicted in Figure 1, with q encoders (circles) and $q-1$ interleavers (boxes). The ith code C_i is an (n_i, N_i) linear block code, and the ith encoder is preceded by an interleaver (permuter) P_i of size N_i, except C_1 which is not preceded by an interleaver, but rather is connected to the input. The overall structure must have no loops, i.e., it must be a graph-theoretic tree. We call a code of this type a "turbo-like" code.

Define $s_q = \{1, 2, \ldots, q\}$ and subsets of s_q by $s_I = \{i \in s_q : C_i \text{ connected to input}\}$, $s_O = \{i \in s_q : C_i \text{ connected to output }\}$, and its complement $\overline{s}_O$. The overall system depicted in Figure 1 is then an encoder for an (n, N) block code with $n = \sum_{i \in s_O} n_i$.

If we know the IOWE $A^{(i)}_{w_i,h_i}$'s for the constituent codes C_i, we can calculate the *average* IOWE $A_{w,h}$ for the overall system (averaged over the set of all possible interleavers), using the *uniform interleaver* technique [2]. (A uniform interleaver is defined as a probabilistic device that maps a given input word of weight w into all distinct $\binom{N_i}{w}$ permutations of it with equal probability $p = 1/\binom{N_i}{w}$.) The result is

$$A_{w,h} = \sum_{\substack{h_i : i \in s_O \\ \sum h_i = h}} \sum_{h_i : i \in \overline{s}_O} A^{(1)}_{w_1,h_1} \prod_{i=2}^{q} \frac{A^{(i)}_{w_i,h_i}}{\binom{N_i}{w_i}} \tag{3.1}$$

In (3.1) we have $w_i = w$ if $i \in s_I$, and $w_i = h_j$ if C_i is preceeded by C_j (see Figure 2.). We do not give a proof of formula (3.1), but it is intuitively plausible if we note that the term $A^{(i)}_{w_i,h_i}/\binom{N_i}{w_i}$ is the probability that a random input word to C_i of weight w_i will produce an output word of weight h_i.

For example, for the $(n_2+n_3+n_4, N)$ encoder of Figure 1 the formula (3.1) becomes

$$A_{w,h} = \sum_{\substack{h_1,h_2,h_3,h_4 \\ (h_2+h_3+h_4=h)}} A^{(1)}_{w_1,h_1} \frac{A^{(2)}_{w_2,h_2}}{\binom{N_2}{w_2}} \frac{A^{(3)}_{w_3,h_3}}{\binom{N_3}{w_3}} \frac{A^{(4)}_{w_4,h_4}}{\binom{N_4}{w_4}}$$

$$= \sum_{\substack{h_1,h_2,h_3,h_4 \\ (h_2+h_3+h_4=h)}} A^{(1)}_{w,h_1} \frac{A^{(2)}_{w,h_2}}{\binom{N}{w}} \frac{A^{(3)}_{h_1,h_3}}{\binom{n_1}{h_1}} \frac{A^{(4)}_{h_1,h_4}}{\binom{n_1}{h_1}}.$$

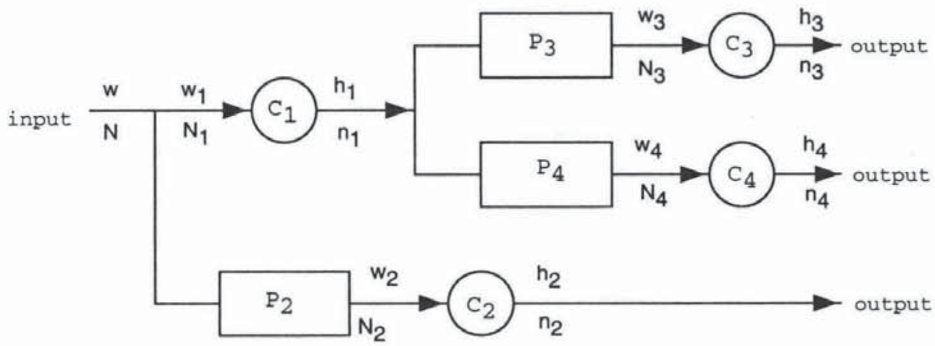

Figure 1. A "turbo-like" code with $s_I = \{1,2\}$, $s_O = \{2,3,4\}$, $\overline{s}_O = \{1\}$.

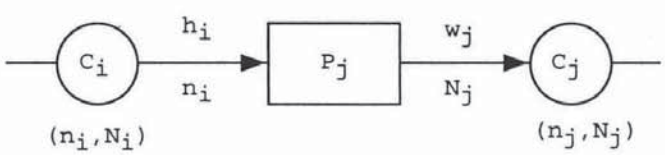

Figure 2. C_i (an (n_i, N_i) encoder) is connected to C_j (an (n_j, N_j) encoder) by an interleaver of size N_j. We have the "boundary conditions" $N_j = n_i$ and $w_j = h_i$.

4. The Interleaving Gain Exponent Conjecture.

In this section we will consider systems of the form depicted in Figure 1, in which the individual encoders are truncated convolutional encoders, and study the behavior of the average ML decoder error probability as the input block length N approaches

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