## The Serial Concatenation of Rate-1 Codes Through Uniform Random Interleavers

Henry Pfister and Paul H. Siegel Signal Transmission and Recording (STAR) Lab University of California, San Diego

{hpfister, psiegel}@ucsd.edu

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## **Outline**

- Union Bounds and Code Performance
- Serial Concatenation and Repeat-Accumulate (RA) Codes
- Serial Concatenation of Rate-1 Codes
- Repeat-Accumulate-Accumulate (RAA) Codes
- Summary



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#### **Union Bounds on Performance**

- Rate r = k/n, linear block code C
- Input Output Weight Enumerator Function (IOWEF):

 $A_{w,h} \stackrel{\mathrm{def}}{=} \#$  codewords, input weight w, output weight h

 $\bullet$  Union bound on word error probability  $P_{\it W}$  (binary-input, memoryless channel, maximum-likelihood decoding):

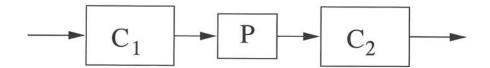
$$P_W \le \sum_{h=1}^n \sum_{w=1}^k A_{w,h} z^h$$

- z is channel dependent; e.g., for Gaussian channel,  $z=e^{-r(E_b/N_0)}$ .
- ullet For ensembles, replace  $A_{w,h}$  by average IOWEF  $\overline{A_{w,h}}$



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## Serial Concatenation through a Uniform Interleaver



- Let  $C_1$ ,  $C_2$  be  $(n_1,k_1)$ ,  $(n_2,k_2)$  linear block codes with  $n_1=k_2$ , and IOWEFs  $A_{w,h}^{(1)}$ ,  $A_{w,h}^{(2)}$ .
- Let C be the  $(n_2,k_1)$  code obtained by serial concatenation of  $C_1$  and  $C_2$  through a uniform interleaver of size  $n_1$ , with average IOWEF  $A_{w,h}$ :

$$A_{w,h} = \sum_{h_1=0}^{n_1} A_{w,h_1}^{(1)} \cdot \frac{A_{h_1,h}^{(2)}}{\begin{pmatrix} n_1 \\ h_1 \end{pmatrix}}$$



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## Repeat-Accumulate (RA) Codes (Divsalar, et al., Allerton'98)



- Repeat input block  $x_1x_2\cdots x_N$  a total of q times.
- Permute with random interleaver P of size n=qN.
- Accumulate over block:

$$u_1u_2\cdots u_n \rightarrow v_1v_2\cdots v_n$$

$$v_1 = u_1$$

$$v_2 = u_1 + u_2$$

$$\vdots$$

$$v_n = u_1 + \cdots + u_n$$



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