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Preface

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differ in two positions, etc. As in the simple example given previously, there will almost always be some patterns that are left over after assigning all those that differ in t or fewer places (thus accounting for the inequality).

At this point we are in a position to relate the amount of redundancy in a code to the number of errors that are correctable. First observe that there are 2^n possible sequences. Each column of the decoding table contains N_e of these sequences so that the number of code words, N_c, must obey the inequality

$$N_c \leq 2^n \bigg/ \left[1 + n + \binom{n}{2} + \cdots + \binom{n}{t}\right] \tag{1-3}$$

This is called a *Hamming bound* or "sphere-packing" bound. The equality in this bound can be achieved only for so-called *perfect codes*. These are codes which can correct all patterns of t or fewer errors and no others. There are only a small number of perfect codes which have been found and consequently the equality in (1-3) is almost never achieved.

At the encoder we envision a process by which a k-symbol information sequence is mapped into an n-symbol code sequence. Although the terminology is usually restricted to the so-called linear codes (to be discussed), we shall refer to any such mapping as an (n, k) code. Since the k-symbol sequence can take on 2^k distinct values, inequality (1-3) can be written

$$2^k \leq 2^n \bigg/ \left[1 + n + \binom{n}{2} + \cdots + \binom{n}{t}\right] \tag{1-4}$$

A measure of the efficiency implied by a particular code choice is given by the ratio

$$R = k/n \tag{1-5}$$

where R is defined as the *code rate*. The fraction of transmitted symbols that are redundant is $1 - R$.

The mapping implied by the encoder can be described by a look-up table. For example, the four-word code discussed previously is described in Table 1-2. The portion of the code sequence contained between the dashed lines is identical to the input sequence. Thus, each code sequence is easily and uniquely related to the input. Not all block codes exhibit this property. Those which do are referred to as *systematic* codes. For systematic codes, the concept of redundant digits becomes very clear and in Table 1-2 consists of the digits in positions 1, 4, and 5. Conversely, codes which do not exhibit this property are called *nonsystematic* codes.

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