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**UNITED STATES DISTRICT COURT**  
**CENTRAL DISTRICT OF CALIFORNIA**

THE CALIFORNIA INSTITUTE OF  
TECHNOLOGY,

Plaintiff and Counter-Defendant,

vs.

HUGHES COMMUNICATIONS INC.,  
HUGHES NETWORK SYSTEMS LLC,  
DISH NETWORK CORPORATION,  
DISH NETWORK LLC, and DISHNET  
SATELLITE BROADBAND LLC,

Defendants and Counter-Plaintiffs.

Case No. 2:13-cv-07245-MRP-JEM

**EXPERT REPORT OF DR.  
BRENDAN FREY REGARDING  
INVALIDITY OF PATENTS-IN-  
SUIT**





1 of the claimed invention. In my opinion, these documents do not demonstrate  
2 conception for the reasons stated below.

3 6. I have also been asked for my opinion regarding whether three references  
4 (two by Luby et al. and one by Richardson et al.) were material to the claimed  
5 invention. In my opinion, as explained below, these three references, none of  
6 which were before the patent office during prosecution of the asserted patents,  
7 were material to the claimed invention.

### 8 **BACKGROUND**

#### 9 **A. Qualifications and Experience**

10 7. I received a B.Sc. with Honors in Electrical Engineering from the University  
11 of Calgary in 1990, a M.Sc. in Electrical and Computer Engineering from the  
12 University of Manitoba in 1993, and a Ph.D. in Electrical and Computer  
13 Engineering from the University of Toronto in 1997. Since July 2001, I have been  
14 at the University of Toronto, where I am a Professor of Electrical and Computer  
15 Engineering and Computer Science.

16 8. During my career I have conducted research in the areas of graphical models,  
17 error-correcting coding, machine learning, genome biology and computer vision. I  
18 have authored more than 200 publications and am named as an inventor on nine  
19 patents issued by the U.S. Patent and Trademark Office.

20 9. I have received a number of honors and awards for the research I have  
21 conducted. In 2008, I was named a Fellow of the Institute for Electrical and  
22 Electronic Engineers (IEEE), an honor given to a person with an “extraordinary  
23 record or accomplishments” in the field of electrical engineering. In 2009, I was  
24 named a Fellow of the American Association for the Advancement of Science  
(AAAS), an honor that recognizes “efforts on behalf of the advancement of science  
or its applications which are scientifically or socially distinguished.”



1 10. In 2009, I was awarded a Steacie Fellowship for my work on the theory and  
2 implementation of artificial and natural mechanisms for inferring patterns from  
3 data. The Steacie Fellowship is awarded by the Natural Sciences and Engineering  
4 Research Council of Canada (NSERC) to “outstanding and highly promising  
5 scientists and engineers” who are faculty members of Canadian universities. In  
6 2011, I received the NSERC’s John C. Polanyi Award, in recognition of my  
7 research on inferring genetic codes embedded in DNA that direct activities within  
8 cells.

8 11. Throughout my career I have received funding from various governmental  
9 agencies to support my research, including the Natural Sciences and Engineering  
10 Research Council of Canada, the Canadian Institutes of Health Research, and the  
11 Canadian Institute for Advanced Research.

12 12. A copy of my *curriculum vitae* is attached to this report as Exhibit A.

### 13 **B. Understanding of the Law**

14 13. I am not an attorney. For the purposes of this report, I have been informed  
15 about certain aspects of the law that are relevant to my analysis and opinions. My  
16 understanding of the law is as follows:

#### 17 i) Invalidity in General

18 14. A patent is presumed valid, and a challenger to the validity of a patent must  
19 show invalidity of the patent by clear and convincing evidence. Clear and  
20 convincing evidence is evidence that makes a fact highly probable.

#### 21 ii) Anticipation

22 15. A patent claim is invalid if it is “anticipated” by prior art. For the claim to  
23 be invalid because it is anticipated, all of its requirements must have existed in a  
24 single device or method that predates the claimed invention, or must have been  
described in a single publication or patent that predates the claimed invention.

1 16. The description in a written reference does not have to be in the same words  
2 as the claim, but all of the requirements of the claim must be there, either stated or  
3 necessarily implied, so that someone of ordinary skill in the art, looking at that one  
4 reference would be able to make and use the claimed invention.

5 17. A patent claim is also anticipated if there is clear and convincing proof that,  
6 more than one year before the filing date of the patent, the claimed invention was:  
7 in public use or on sale in the United States; patented anywhere in the world; or  
8 described in a printed publication anywhere in the world. This is called a statutory  
bar.

9 iii) Obviousness

10 18. A patent claim is invalid if the claimed invention would have been obvious  
11 to a person of ordinary skill in the art at the time the application was filed. This  
12 means that even if all of the requirements of a claim cannot be found in a single  
13 prior art reference that would anticipate the claim or constitute a statutory bar to  
14 that claim, the claim is invalid if it would have been obvious to a person of  
15 ordinary skill who knew about the prior art.

16 19. The determination of whether a claim is obvious should be based upon  
17 several factors, including:

- 18 • the level of ordinary skill in the art that someone would have had at the time  
the claimed invention was made;
- 19 • the scope and content of the prior art;
- 20 • what difference, if any, existed between the claimed invention and the prior  
art.

21 20. In considering the question of obviousness, it is also appropriate to consider  
22 any secondary considerations of obviousness or non-obviousness that may be  
23 shown. These include:

- 24 • commercial success of a product due to the merits of the claimed invention;



- 1 • a long felt need for the solution provided by the claimed invention;
- 2 • unsuccessful attempts by others to find the solution provided by the claimed  
invention;
- 3 • copying of the claimed invention by others;
- 4 • unexpected and superior results from the claimed invention;
- 5 • acceptance by others of the claimed invention as shown by praise from  
others in the field or from the licensing of the claimed invention; and
- 6 • independent invention of the claimed invention by others before or at about  
7 the same time as the named inventor thought of it.

8 21. A patent claim composed of several elements is not proved obvious merely  
9 by demonstrating that each of its elements was independently known in the prior  
10 art. In evaluating whether such a claim would have been obvious, it is relevant to  
11 consider if there would have been a reason that would have prompted a person of  
12 ordinary skill in the field to combine the elements or concepts from the prior art in  
13 the same way as in the claimed invention. For example, market forces or other  
design incentives may be what produced a change, rather than true inventiveness.

14 It is also appropriate to consider:

- 15 • whether the change was merely the predictable result of using prior art  
16 elements according to their known functions, or whether it was the result of  
true inventiveness;
- 17 • whether there is some teaching or suggestion in the prior art to make the  
modification or combination of elements claimed in the patent;
- 18 • whether the innovation applies a known technique that had been used to  
19 improve a similar device or method in a similar way; or
- 20 • whether the claimed invention would have been obvious to try, meaning that  
21 the claimed innovation was one of a relatively small number of possible  
approaches to the problem with a reasonable expectation of success by those  
of ordinary skill in the art.

22 22. In considering obviousness, it is important to be careful not to determine  
23 obviousness using the benefit of hindsight; many true inventions might seem  
24 obvious after the fact.



1 23. A single reference can alone render a patent claim obvious, if any  
2 differences between that reference and the claims would have been obvious to a  
3 person of ordinary skill in the art at the time of the alleged invention – that is, if the  
4 person of ordinary skill could readily adapt the reference to meet the claims of the  
5 patent, by applying known concepts to achieve expected results in the adaptation of  
6 the reference.

6 iv) The “Written Description” Requirement

7 24. A patent claim is invalid if the patent specification does not contain a written  
8 description of the invention to which the claim is directed. To satisfy the written  
9 description requirement, a patent specification must describe the claimed invention  
10 in sufficient detail that one of ordinary skill in the art can reasonably conclude that  
11 the inventor had possession of the claimed invention.

12 25. An applicant shows possession of the claimed invention by describing the  
13 claimed invention with all of its limitations using such descriptive means as words,  
14 structures, figures, diagrams, and formulas that fully set forth the claimed  
15 invention. A description that merely renders the invention obvious does not satisfy  
16 the written description requirement.

16 v) Inequitable Conduct and Materiality

17 26. I have been informed that during prosecution, inventors have a duty to  
18 disclose to the Patent Office all information known to the inventors that is material  
19 to the patentability of the claims being examined.

20 27. Information is deemed to be material to patentability when it is not  
21 cumulative to information already before the Patent Office, and when: (1) it  
22 establishes, by itself or in combination with other information, that a claim was  
23 unpatentable; or (2) it refutes, or is inconsistent with, a position the applicant takes  
24



1 in (a) opposing an argument of unpatentability relied on by the Patent Office, or (b)  
2 asserting an argument of patentability.

3 **C. Materials Reviewed**

4 28. Among the materials I have reviewed in forming my opinions are:

- 5 • The '710, '032, '781, and '833 patents;
- 6 • The prosecution histories of the '710, '032, '781, and '833 patents;
- 7 • The prior art of record that was available to the patent examiner;
- 8 • The prior art references discussed herein;
- 9 • Claim Construction Order dated August 6, 2014 (Dkt. No. 105);
- 10 • Declaration of Stephen B. Wicker, dated Oct. 6, 2014 (Dkt. No. 130-10);
- 11 • Transcript of the October 14, 2014 deposition of Stephen B. Wicker;
- 12 • IPR Petition No. IPR2015-00067 and accompanying exhibits, including the  
13 declaration of Henry D. Pfister;
- 14 • IPR Petition No. IPR2015-00068 and accompanying exhibits, including the  
15 declaration of Henry D. Pfister;
- 16 • IPR Petition No. IPR2015-00060 and accompanying exhibits, including the  
17 declaration of Henry D. Pfister;
- 18 • IPR Petition No. IPR2015-00059 and accompanying exhibits, including the  
19 declaration of Henry D. Pfister;
- 20 • IPR Petition No. IPR2015-00061 and accompanying exhibits, including the  
21 declaration of Henry D. Pfister;
- 22 • IPR Petition No. IPR2015-00081 and accompanying exhibits, including the  
23 declaration of Henry D. Pfister;
- 24 • Transcript of the December 11, 2014 deposition of inventor Aamod  
Khandekar;
- Transcript of the January 7, 2015 deposition of inventor Hui Jin;
- Transcript of the Jan 15, 2015 deposition of Dariush Divsalar;
- Laboratory Notebook of Robert McEliece (CALTECH000004472-603);
- Caltech's Supplemental Responses to Defendants' First Set of  
Interrogatories, Nos. 3-5, Jan. 11, 2015;



- 1 • Caltech's Second Supplemental Responses to Interrogatories 1-5 and  
2 Caltech's First Supplemental Responses to Interrogatories 6-11;
- 3 • Email from Brendan Frey to Dariush Divsalar dated Dec. 8, 1999  
4 (CALTECH000024021);
- 5 • Khandekar, Aamod ("Capacity Achieving Codes on the Binary Erasure  
6 Channel") (CALTECH000007321-7349).
- 7 • Khandekar, Aamod, "Graph-based Codes and Iterative Decoding," thesis  
8 dated June 10, 2002.
- 9 • McEliece Email dated March 7, 2000 (CALTECH000008667)
- 10 • Luby, M. et al., "Practical Loss-Resilient Codes," *STOC '97* (1997)
- 11 • Luby, M. et al., "Analysis of Low Density Codes and Improved Designs  
12 Using Irregular Graphs," *STOC '98*, p. 249-259 (1998)
- 13 • Richardson, T. et al. "Design of provably good low-density parity check  
14 codes," *IEEE Transactions on Information Theory* (1999) (preprint)

15 29. Level of Ordinary Skill in the Art

16 30. In my opinion, based on the materials and information I have reviewed, and  
17 on my extensive experience working with people in the technical areas relevant to  
18 the patents-in-suit (*i.e.* in the field of code design), a person of ordinary skill in the  
19 art is a person with a Ph.D. in electrical or computer engineering with emphasis in  
20 signal processing, communications, or coding, or a master's degree in the above  
21 area with at least three years of work experience this field at the time of the alleged  
22 invention.<sup>1</sup> I understand that Caltech has agreed with this definition of the level of  
23 ordinary skill in this case.<sup>23</sup>

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24 <sup>1</sup> I was asked to use a similar qualification for a "person of ordinary skill in the art" for purposes  
of a declaration that I understand was filed in connection with petitions for *Inter Partes Review*  
of the asserted patents. See Declaration of Brendan Frey dated October 14, 2014, at ¶2.

<sup>2</sup> Reporter's Transcript of Claim Construction and Motion Hearing of July 9, 2014, Ex. 1026, at  
98.

<sup>3</sup> This is also consistent with testimony given by, *e.g.*, Dr. Dariush Divsalar, an author of one of  
the prior art references discussed in this report (*see* Divsalar Dep. at 55-56).



**D. Claim Constructions Used in This Report**

31. I understand that the parties have agreed on the following claim constructions:

<b>Claim Term</b>	<b>Agreed-Upon Construction</b>
“irregularly” (’710 and ’032 patents)	“a different number of times”
“interleaving” / “interleaver” / “scramble” (’710 patent)	“changing the order of data elements” / “module that changes the order of data elements”
“sums of bits in subsets of the information bits” / “summing of bits in a subset of the information bits” / “adding additional subsets of information bits” (’781 patent)	“the result(s) of adding together two or more information bits from a subset of information bits” / “adding together two or more information bits from a subset of information bits”
“wherein two or more memory locations of the first set of memory locations are read by the permutation module different times from one another” (’833 patent)	“where two or more memory locations of the first set of memory locations are read by the permutation module a different number of times from one another”
“permutation module” (’833 patent)	“a module that changes the order of data elements”

32. I further understand that the Court in this case has issued a claim construction order construing certain disputed claim terms as follows:

<b>Claim Term</b>	<b>Court’s Construction</b>
“transmitting” / “transmission” (’032 patent)	“sending over a channel”

1	“codeword” (’781 patent)	“a discrete encoded sequence of data elements”
2	“repeat” (’710 and ’032 patents)	plain meaning <sup>4</sup>
3	“combine” / “combining” (’833 patent)	“perform logical operations on”
4	Equation in claim 1 of the ’032 patent (’032 patent)	“the parity bit $x_j$ is the sum of (a) the parity bit $x_{j-1}$ and (b) the sum of a number, ‘a,’ of randomly chosen irregular repeats of the message bits”
5	Tanner Graph term in claims 11 and 18 of ’032 patent (’032 patent)	“a graph representing an IRA code as a set of parity checks where every message bit is repeated, at least two different subsets of message bits are repeated a different number of times, and check nodes, randomly connected to the repeated message bits, enforce constraints that determine the parity bits”

13 33. For the purposes of this report, I have used the constructions given in the  
14 two tables above. For all other claim terms, I have used the plain and ordinary  
15 meaning the term would have to one of ordinary skill in the art.

## 16 **II. OVERVIEW OF THE TECHNOLOGY**

17 34. The four patents-in-suit, which share a common specification, relate to the  
18 field of error-correcting codes. Below I provide a brief introduction to channel  
19 coding and error-correcting codes, and highlight a few of the developments in the  
20 field that are relevant to the asserted patents. Also, attached as Appendix A is a  
21 mathematical description of some properties of error-correcting codes.

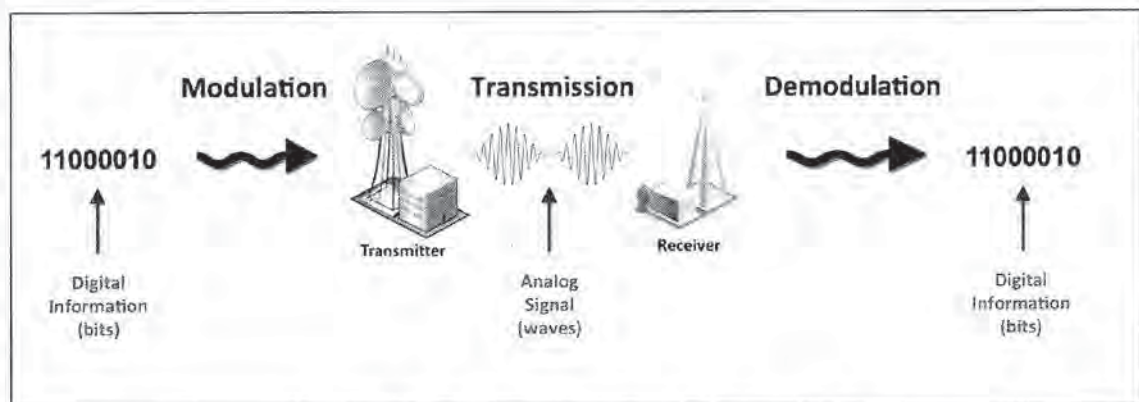
22 <sup>4</sup> The Claim Construction Order dated August 6, 2014 expounded on the plain meaning of  
23 “repeat.” For example, the order said the “plain meaning of ‘repeat’ requires the creation of new  
24 bits corresponding to or reflecting the value of the original bits. In other words, repeating a bit  
with the value 0 will produce another bit with the value 0. The Court will refer to this concept as  
duplication” (Claim Construction Order dated August 6, 2014, p. 10).



## A. Error-Correcting Codes in General

35. Most computing devices and other digital electronics use bits to represent information. A bit is a binary unit of information that may have one of two values: 1 or 0. Any type of information, including, *e.g.*, text, music, images and video information, can be represented digitally as a collection of bits.

36. When transmitting binary information over an analog communication channel, the data bits representing the information to be communicated (also called “information bits” or “source bits”) are converted into an analog signal that can be transmitted over the channel. This process is called *modulation*. The transmitted signal is then received by a receiving device and converted back into binary form. This process, in which a received analog waveform is converted into bits, is called *demodulation*. The steps of modulation and demodulation are shown in the figure below:



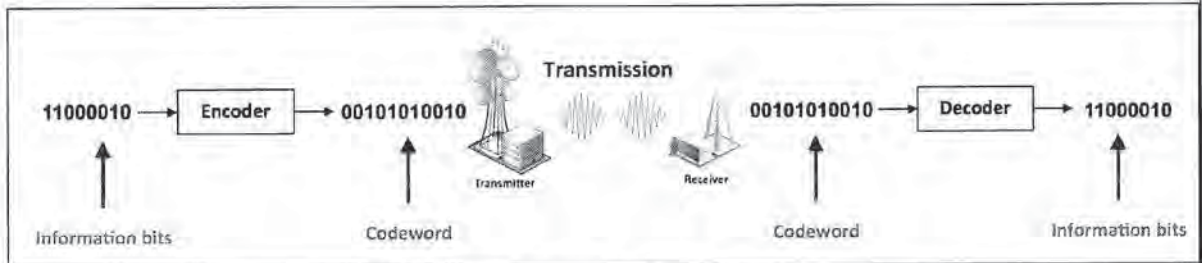
Modulation, Transmission, and Demodulation

37. Transmission over physical channels is never 100% reliable. The transmitted signal can be corrupted during transmission by “noise” caused by, *e.g.*, obstacles obstructing the signal path, interference from other signals, or electrical/magnetic disturbances. Noise can cause bits to “flip” during transmission: for example, because of noise, a bit that was transmitted as a 1 can be corrupted during transmission and demodulated as 0, and vice versa.

1 38. Error-correcting codes were developed to combat such transmission errors.  
2 Using the bits representing the information to be communicated (called  
3 “information bits”, “data bits” or “source bits”) an error-correcting code generates  
4 “parity bits” that allow the receiver to verify that the bits were transmitted correctly,  
5 and to correct transmission errors that may have occurred.

6 39. Bits are encoded by an *encoder*, which receives a sequence of information  
7 bits as input, generates parity bits based on the information bits according to a  
8 particular encoding algorithm, and outputs a sequence of encoded bits (or data  
9 elements) called a *codeword*. The codeword produced by the encoder is then  
10 modulated and transmitted as an analog signal.

11 40. At the receiver the signal is received, demodulated and passed to the *decoder*,  
12 which uses a decoding algorithm to recover the original codeword and the original  
13 information bits.



17 **Encoding and Decoding**

18 41. Error-correcting codes work by adding redundant information to the original  
19 message. Due to redundancy, the information represented by a given information  
20 bit is spread across multiple bits of the codeword. Thus, even if one of those bits is  
21 flipped during transmission, the original information bit can still be recovered from  
22 the others.

23 42. As a simple example, consider an encoding scheme, which I will call  
24 “repeat-three,” that outputs three copies of each information bit. In this scheme,  
the information bits “1 0 1” would be encoded as “111 000 111.” Upon receipt,



1 the decoder converts instances of “111” into “1” and instances of “000” into “0” to  
2 produce the decoded bits “1 0 1,” which match the original information bits.

3 43. Suppose a bit is flipped during transmission, changing “000” to “010.” The  
4 decoder will be able to detect that there was a transmission error, because “010” is  
5 not a valid “repeat-three” codeword. Using a “majority vote” rule, the decoder can  
6 infer that the original information bit was a 0, correcting the transmission error.

7 Thus, due to the redundancy incorporated into the codeword, no information was  
8 lost due to the transmission error.

9 44. Error-correcting codes may be either *systematic* or *non-systematic*. In a  
10 systematic code, both the parity bits and the original information bits are included  
11 in the codeword. In a non-systematic code, the encoded data only includes the  
12 parity bits.

13 45. Systematic and non-systematic codes had been known in the art for decades  
14 prior to May 18, 2000, the claimed priority date of the patents-in-suit (*see, e.g.,*  
15 Wicker Dep. at 77:15-20; *see also, e.g.,* Divsalar Dep. at pp. 66-67).

### 16 **B. Coding Rate**

17 46. Many error-correcting codes encode information bits in groups, or *blocks* of  
18 fixed length  $n$ . An encoder receives an  $k$ -bit block of information bits as input, and  
19 produces a corresponding  $n$ -bit codeword. The ratio  $k/n$  is called the *rate* of the  
20 code. Because the codeword generally includes redundant information,  $n$  is  
21 generally greater than  $k$ , and the rate  $k/n$  of an error-correcting code is generally  
22 less than one.

### 23 **C. Performance of Error-Correcting Codes**

24 47. The effectiveness of an error-correcting code may be measured using a  
variety of metrics.



1 48. One tool used to assess the performance of a code is its *bit-error rate* (BER).  
2 The BER is defined as the number of corrupted information bits divided by the  
3 total number of information bits during a particular time interval. For example, if a  
4 decoder outputs 1000 bits in a given time period, and 10 of those bits are corrupted  
5 (*i.e.*, they differ from the information bits originally received by the encoder), then  
6 the BER of the code during that time period is (10 bit errors) / (1000 total bits) =  
0.01 or 1%.<sup>5</sup>

7 49. The BER of a coded transmission depends on the amount of noise that is  
8 present in the communication channel, the strength of the transmitted signal (*i.e.*,  
9 the power that is used to transmit the modulated waveform), and the performance  
10 of the error-correcting code. An increase in noise tends to increase the error rate  
11 and an increase in signal strength tends to decrease the error rate. The ratio of the  
12 signal strength to the noise, called the “signal-to-noise ratio,” is often used to  
13 characterize the channel over which the encoded signal is transmitted. The signal-  
14 to-noise ratio can be expressed mathematically as  $E_b/N_0$ , in which  $E_b$  is the amount  
15 of energy used to transmit each bit of the signal, and  $N_0$  is the density of the noise  
16 on the channel.<sup>6</sup> The BER of an error-correcting code is often measured for  
17 multiple values of  $E_b/N_0$  to determine how the code performs under various  
channel conditions.

18 50. Error-correcting codes may also be assessed based on their computational  
19 complexity. The complexity of a code is a rough estimate of how many  
20 calculations are required for the encoder to generate the encoded parity bits and  
21 how many calculations are required for the decoder to reconstruct the information

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22 <sup>5</sup> Note that as used herein, BER refers to the *information* BER, which measures the percentage of  
bits that remain incorrect after decoding. This is not to be confused with the *transmission* BER,  
which measures the percentage of bits that are incorrect when they are received by the decoder.

23 <sup>6</sup> More precisely,  $E_b/N_0$  is the *normalized* signal-to-noise ratio. It is a dimensionless quantity that  
24 does not depend on the particular units used to measure the strength of the signal and the  
quantity of noise on the channel.



1 bits from the parity bits. If a code is too complex, it may be impractical to build  
2 encoders/decoders that are fast enough to use it.

3 **D. LDPC Codes, Convolutional Codes, Turbocodes, and Repeat-**  
4 **Accumulate codes**

5 51. In 1963, Robert Gallager described a set of error correcting codes called  
6 Low Density Parity Check (“LDPC”) codes. Gallager described how LDPC codes  
7 provide one method of generating parity bits from information bits using a matrix  
8 populated with mostly 0s and relatively few 1s, and he described how decoding  
9 could be performed using an iterative “message passing” decoding algorithm, as  
described below.<sup>7</sup>

10 52. Gallager’s work was largely ignored over the following decades, as  
11 researchers continued to discover other algorithms for calculating parity bits. These  
12 algorithms included, for example, convolutional encoding (see below) with Viterbi  
13 decoding and cyclic code encoding with bounded distance decoding. In many  
14 cases these new codes could be decoded using low-complexity decoding  
algorithms.

15 53. In 1993, researchers discovered “turbocodes,” a class of error-correcting  
16 codes capable of transmitting information at a rate close to the Shannon Limit – the  
17 maximum rate at which information can be transmitted over a channel.  
18 Turbocodes make use of “convolutional codes”, which were described in the  
19 1960’s and were widely used in telephone modems in the 1980’s and 1990’s. A  
20 convolutional code is a type of error-correcting code that generates parity bits by  
21 processing the information bits in order. The convolutional code contains a  
22 “memory bank” in the form of a short sequence of bits, e.g., 4 bits. When an  
23 information bit  $d_k$  is processed, the memory bits  $s_1, s_2, s_3, s_4$  are combined with the  
information bit to produce a new memory bit and the remaining memory bits are

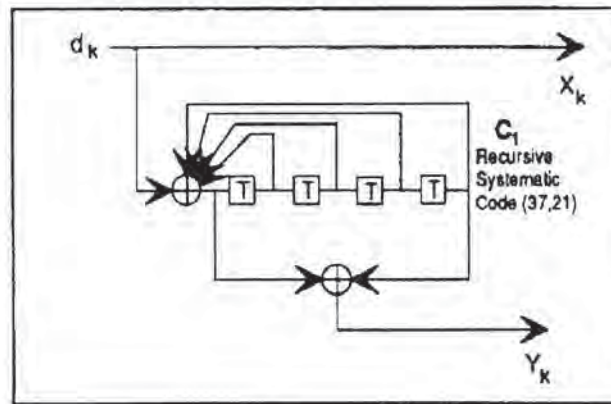
24 <sup>7</sup> Gallager, R., *Low-Density Parity-Check Codes* (Monograph, M.I.T. Press, 1963).



1 “shifted”, so that the last memory bit is discarded. For example, the new memory  
2 bit  $s_1'$  could be computed by  $s_1' = d_k + s_1 + s_2 + s_3 + s_4 \text{ modulo } 2$ , and the other  
3 memory bits would be  $s_2' = s_1$ ,  $s_3' = s_2$ , and  $s_4' = s_3$ . What does “modulo 2” mean?  
4 If the sum of the bits is even, then the sum modulo 2 is zero, whereas if the sum of  
5 the bits is odd, then the sum modulo 2 is one. Note that  $s_4$  has been discarded.  
6 When an information bit is being processed, a parity bit is also generated. The  
7 parity bit  $y_k$  is a combination of the new memory bit and the entire set of current  
8 memory bits, for example,  $y_k = s_1' + s_4 \text{ modulo } 2$ . The combinations used to  
9 determine the new memory bit and the parity bit need not include all of the bits,  
10 e.g., the above example uses all bits to compute the new memory bit, but only  $s_1'$   
11 and  $s_4$  when computing the parity bit. If a particular bit is used in a combination,  
12 we say there is a “tap” connected to that bit. In the example, the parity bit is  
13 connected by a tap to  $s_1'$  and another tap to  $s_4$ . The set of taps for the memory bit  
14 and the set of taps for the parity bit are fixed when processing information bits and  
15 they completely characterize the convolutional code. In a “systematic”  
16 convolutional code, the information bits are also transmitted across the channel, in  
17 addition to the parity bits. Some parity bits and/or some information bits may be  
18 punctured so as to adjust the rate of the convolutional code (the number of  
19 information bits processed divided by the number of bits transmitted). If the new  
20 memory bit doesn't have any taps to any memory bits, the code is called “non-  
21 recursive” and otherwise it is called “recursive”, alluding to the fact that the new  
22 memory bit depends on the bits in the old memory. Using the above example, the  
23 figure below shows how a recursive convolutional code is depicted, where a circle  
24 with a plus inside indicates summation *modulo 2* and a box with a T inside  
indicates a memory location (figure modified from <sup>8</sup>).

<sup>8</sup> Claude Berrou et al., *Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes*, 2 IEEE International Conference on Communications, ICC '93 Geneva. Technical





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54. Convolutional codes are usually decoded using the “Viterbi algorithm” or the “BCJR algorithm”. These algorithms can be viewed as iterative “message passing” decoding algorithms, if we represent the convolutional code using a “Tanner graph” or a “factor graph”, as described below.

55. The main drawback of convolutional codes is that they only produce local redundancy in the output stream. They do not perform well when the channel introduces errors that are nearby. Turbocodes overcome this deficiency by encoding the input bits twice. The input bits are fed to a convolutional encoder in their normal order, and they are also reordered by an interleaver and the reordered bits are encoded by a second convolutional encoder. Using a turbocode, a small number of errors will not result in loss of information unless the errors happen to fall close together in both the original data stream and in the permuted data stream, which is unlikely.

56. A standard turbocoder encodes a sequence of information bits using two convolutional coders. The information bits are passed to the first convolutional coder in their original order. At the same time, a copy of the information bits

1 permuted by an interleaver is passed to the second convolutional coder. The figure  
 2 below shows the structure of a typical turbocoder.<sup>9</sup>

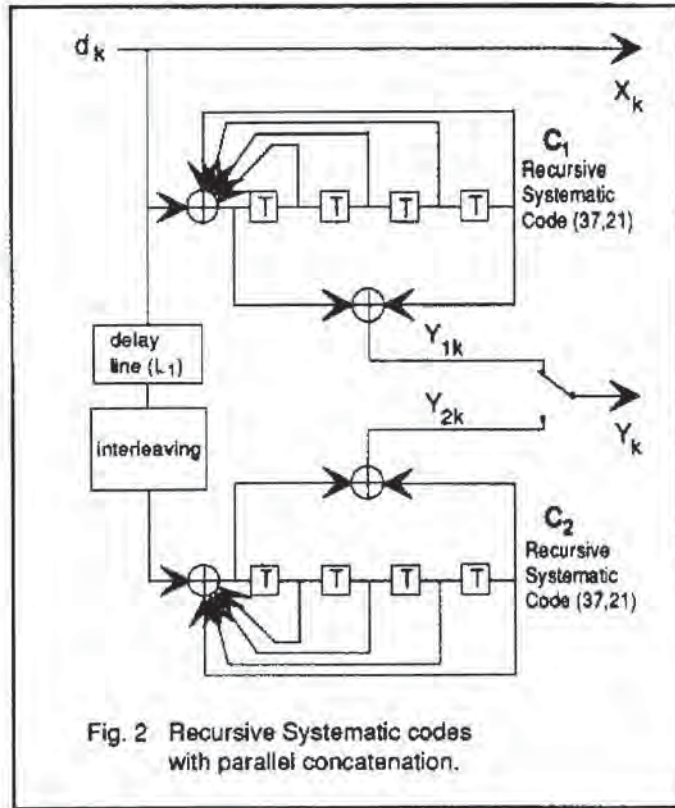


Fig. 2 Recursive Systematic codes with parallel concatenation.

15 57. In 1995, David J. C. MacKay rediscovered Gallager's work from 1963  
 16 relating to low-density parity-check (LDPC) codes and demonstrated that they  
 17 have performance comparable to that of turbocodes.<sup>10</sup> Turbocodes and LDPC  
 18 codes have some common characteristics: both codes use pseudo-random  
 19 permutations to spread out redundancy, and both use iterative "message passing"  
 20 decoding algorithms.

22 <sup>9</sup> Claude Berrou et al., *Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes*, 2 IEEE International Conference on Communications, ICC '93 Geneva. Technical Program, Conference Record 1064 (1993); '032 patent, 1:29-56.

23 <sup>10</sup> MacKay, D. J. C., and Neal, R. M. "Near Shannon Limit Performance of Low Density Parity Check Codes," *Electronics Letters*, vol. 32, pp. 1645-1646 (1996).



1 58. In 1995 and 1996, researchers began to explore “concatenated”  
2 convolutional codes.<sup>11</sup> While turbocodes use two convolutional coders connected  
3 in parallel, concatenated convolutional codes use two convolutional coders  
4 connected in series: the information bits are encoded by a first encoder, the output  
5 of the first encoder is interleaved, and the interleaved sequence is encoded by a  
6 second convolutional code. In such codes, the first and second encoders are often  
7 called the “outer coder” and the “inner coder,” respectively.

8 59. In 1998, researchers developed “repeat-accumulate,” or “RA codes” by  
9 simplifying the principles underlying turbocodes.<sup>12</sup> In RA codes, the information  
10 bits are first passed to a repeater that repeats (*i.e.*, duplicates) the information bits  
11 and outputs a stream of repeated bits (the encoder described above in the context of  
12 the “repeat three” coding scheme is one example of a repeater). The repeated bits  
13 are then passed through an interleaver, which scrambles their order, and then to an  
14 accumulator, where they are “accumulated” to form the parity bits, which are  
15 transmitted across the channel.

16 60. The accumulation operation is a running sum process whereby each input bit  
17 is added to the previous input bits to produce a sequence of running sums, each of  
18 which represents the sum of all input bits yet received. More formally, if an  
19 accumulator receives a sequence of input bits  $i_1, i_2, i_3, \dots, i_n$ , it will produce output  
20 bits  $o_1, o_2, o_3, \dots, o_m$  such that:<sup>13</sup>

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21  
22 <sup>11</sup> Benedetto, S. et al., *Serial Concatenation of Block and Convolutional Codes*, 32.10  
23 Electronics Letters 887-888 (1996).

24 <sup>12</sup> Divsalar, D. et al., “Coding Theorems for Turbo-like Codes,” *Proc. 36th Allerton Conf. on  
Comm., Control and Computing*, 201 (Sept. 1998).

<sup>13</sup> Here I use the  $\oplus$  symbol to denote modulo-2 addition.

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$$\begin{aligned}o_1 &= i_1 \\o_2 &= i_1 \oplus i_2 \\o_3 &= i_1 \oplus i_2 \oplus i_3 \\&\vdots \\o_n &= i_1 \oplus i_2 \oplus i_3 \oplus \dots \oplus i_n\end{aligned}$$

61. The accumulation operation can also be described as a recursive operation in which each output bit is the sum of the previous output bit and the current input bit:

$$\begin{aligned}o_1 &= i_1 \\o_2 &= o_1 \oplus i_2 \\o_3 &= o_2 \oplus i_3 \\&\vdots \\o_n &= o_{n-1} \oplus i_n\end{aligned}$$

62. As this recursive formulation shows, each accumulated bit can be calculated by performing a single modulo-2 addition operation. This relatively low computational complexity is one of the benefits of accumulate codes. In particular, it allows accumulate codes to be encoded quickly and cheaply.

63. Repetition and accumulation were well known in the art by May 18, 2000 and by March 7, 2000, the claimed priority date and the claimed conception date, respectively, of the patents-in-suit (*see, e.g.*, Wicker Dep. at 66:18-67:11, Jin Dep. at 67:8-23, 122:7-13).

**E. Irregularity**

64. A *regular code* is a systematic code that corresponds to a Tanner graph in which each information node is connected to the same number of check nodes, or a nonsystematic code that corresponds to a Tanner graph in which each parity node



1 is connected to the same number of check nodes.<sup>14</sup> By contrast, an *irregular code*  
2 is a systematic code that corresponds to a Tanner graph in which some information  
3 nodes are connected to more check nodes than others, or a nonsystematic code that  
4 corresponds to a Tanner graph in which some parity nodes are connected to more  
5 check nodes than others. The concepts of *regular* and *irregular* need not be  
6 expressed with reference to Tanner graphs, but it is convenient to do so.

65. Irregular LDPC codes were first introduced in a 1997 paper by Luby et al.<sup>15</sup>  
7 The paper showed that irregular codes perform better than regular ones on certain  
8 types of noisy channels. At the time, this paper was widely read by coding  
9 theorists, and gave rise to several lines of research into irregular error-correcting  
10 codes. For example, in my own paper titled “Irregular Turbocodes,” presented at  
11 the 1999 Allerton Conference on Communications, Control, and Computing, I  
12 applied the concept of irregularity to turbocodes by explaining how to construct  
13 irregular turbocodes in which some information bits connect to more check nodes  
14 than others. My experimental results demonstrated that these irregular turbocodes  
15 perform better than the regular turbocodes that were known in the art.

66. By May 18, 2000 and by March 7, 2000, the claimed priority date and the  
16 claimed conception date, respectively, of the patents-in-suit, it was known to those  
17 with ordinary skill in the art that the performance of any type of error-correcting  
18 code could be improved by adding irregularity (*see, e.g.*, Wicker Dep. at 232:6-  
19 233:8). For example, on Dec. 8, 1999, I wrote to Dr. Divsalar, the lead author on  
20 the paper “Coding Theorems for ‘Turbo-Like’ Codes” discussed in this report,  
21 suggesting that the RA codes that he and Dr. Robert McEliece had been working  
22 on should be made irregular (*see* CALTECH000024021).

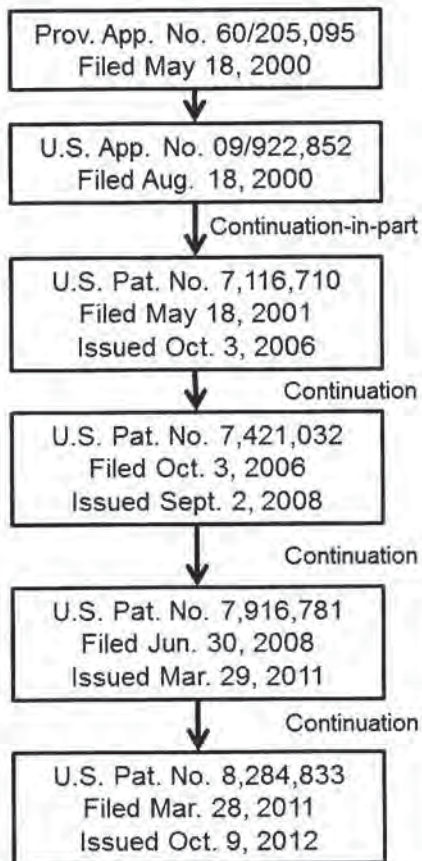
23  
24 <sup>14</sup> For a more complete discussion of Tanner graphs, *see generally* Appendix A.

<sup>15</sup> Luby, M. et al., “Practical Loss-Resilient Codes,” *STOC '97* (1997).

1 **III. THE PATENTS-IN-SUIT**

2 **A. Summary of the Specification.**

3 67. I have been informed that the patents-in-suit share a common specification  
4 and that they were filed as a sequence of continuation applications as shown in the  
5 diagram below.



18 68. The specification, which is common to the four patents-in-suit, is generally  
19 directed to irregular RA codes (or "IRA" codes). Figure 2 of the specification,  
20 reproduced below, shows the structure of an IRA encoder:  
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22  
23  
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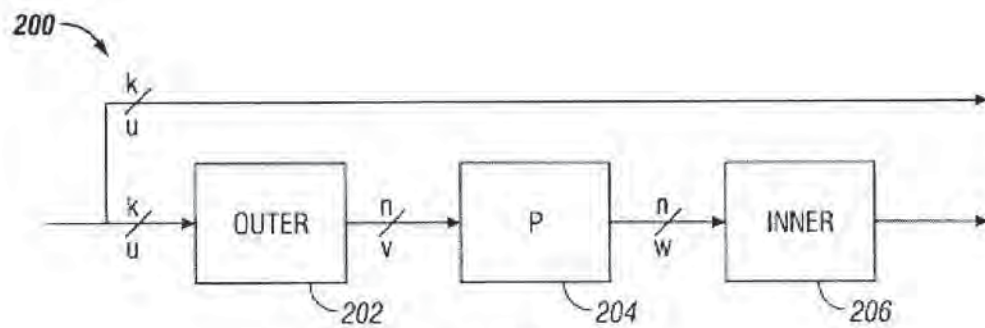


FIG. 2

69. Explaining this figure, the patents describe encoding data using an outer coder 202 connected to an inner coder 206 via an interleaver 204 (labeled “P”) (’710 patent at 2:33-40).

70. Outer coder 202 receives a block of information bits and duplicates each of the bits in the block a given number of times, producing a sequence of repeated bits at its output (*id.* at 2:50-52). The outer coder repeats bits irregularly – *i.e.*, it outputs more duplicates of some information bits than others (*id.* at 2:48-50).

71. The repeated bits are passed to an interleaver 204, where they are scrambled (*id.* at 3:18-22). The scrambled bits are then passed to the inner coder 206, where they are accumulated to form parity bits (*id.* at 2:65-67; 2:33-38). According to the specification:

Such an accumulator may be considered a block coder whose input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by the formula

$$\begin{aligned}
 y_1 &= x_1 \\
 y_2 &= x_1 \oplus x_2 \\
 y_3 &= x_1 \oplus x_2 \oplus x_3 \\
 y_n &= x_1 \oplus x_2 \oplus x_3 \oplus \dots \oplus x_n
 \end{aligned}$$

(*id.* at 3:2-10).

72. The patent specification teaches both systematic and non-systematic codes. In a systematic code, the encoder outputs a copy of the information bits in addition

1 to the parity bits output by inner coder 206 (the systematic output is represented in  
2 Fig. 2. as an arrow running toward the right along the top of the figure).

3 73. I discuss each of the patents individually below. However, I note here that  
4 Caltech has characterized all four of the asserted patents as being directed to IRA  
5 codes.<sup>16</sup>

6 **B. '710 Patent**

7 i) Claims

8 74. The '710 patent includes 33 claims, of which claims 1, 11, 15, and 25 are  
9 independent. Independent claims 1 and 11 are directed to methods of encoding a  
10 signal that include “first encoding” and “second encoding” steps. Independent  
11 claim 15 is directed to a “coder” for encoding bits that includes a “first coder” and  
12 a “second coder.” Claim 25 is directed to a “coding system” that also encodes bits  
13 using a first and second coder, and further includes a decoder for decoding the  
14 encoded bits. I understand that Caltech asserts claims 1, 4, 6, 15, 20, and 22 in this  
15 case.

16 ii) Prosecution History

17 a) First Office Action: September 3, 2004

18 75. The patent office issued a first office action rejecting some of the claims  
19 under 35 U.S.C. § 102 as anticipated by U.S. Patent No. 6,014,411 (to Wang) and  
20 under 35 U.S.C. § 103 as obvious over Wang in view of Wiberg et al., “Codes and  
21 Iterative Decoding on General Graphs,” *1995 Intl. Symposium on Information  
22 Theory*, Sep. 1995, p. 506.

23  
24 <sup>16</sup> See, e.g., Plaintiff’s Technology Tutorial (Dkt. No. 85), p. 1 (which states that “[a]ll of the  
patents in suit relate to a novel error correction technique known as IRA codes”).



1                    *b)      Response: November 24, 2004*

2    76.    In response, the applicant argued that the rejected claims are not anticipated  
3 or obvious over the cited art because they all require that bits be repeated  
4 “irregularly” or “a different number of times” during the first encoding step, while  
5 Wang teaches repeating bits “the same number of times, i.e., regularly” (Response  
6 dated Nov. 24, 2004 at 11).

7                    *c)      Second Office Action: March 4, 2005*

8    77.    The patent office issued a second office action allowing some claims and  
9 rejecting others. In particular, the examiner allowed claim 1 in response to the  
10 applicant’s arguments. The examiner also rejected independent claims 15 and 24,  
11 under 35 U.S.C. § 102 as anticipated by U.S. Patent No. 6,396,423 (to Laumen et  
12 al.). The patent office also rejected several dependent claims under 35 U.S.C. §  
13 103 as obvious over Laumen alone.

14                    *d)      Response: May 5, 2005*

15    78.    In response, the applicant attempted to overcome the examiner’s rejections  
16 by amending claims 15 and 24 to require that the second coder encode bits at a rate  
17 “within 50% of one” (previously, the claims had recited a rate “close to one”)  
18 (Response dated May 5, 2005 at 7-8). In the same amendment, the applicant added  
19 new claims 32-35.

20                    *e)      Third Office Action: July 21, 2005*

21    79.    The patent office issued a third office action maintaining its previous  
22 rejections over Laumen, noting that Laumen teaches a transmission rate of 1/2, and  
23 1/2 is “within 50% of one” (Office Action dated Jul. 21, 2005 at 4).

24                    *f)      Response: October 21, 2005*

80.    To overcome the examiner’s rejection, the applicant canceled claims 32 and  
34 and incorporated their subject matter into claims 15 and 24, respectively. As

1 amended, claims 15 and 24 require that the second coder encode bits at a rate  
2 “within 10% of one” (Response dated Oct. 21, 2005 at 9).

3 **C. '032 Patent**

4 i) Claims

5 81. The '032 patent includes 23 claims, of which claims 1, 11, and 18 are  
6 independent. Independent claim 1 is directed to a method that comprises  
7 generating a sequence of parity bits from a collection of message bits in  
8 accordance with particular mathematical formulae, and making the parity bits  
9 available for transmission. Independent claim 11 is directed to an encoder that  
10 generates a sequence of parity bits from a collection of message bits in accordance  
11 with a particular Tanner Graph. Independent claim 18 is directed to a device for  
12 decoding a data stream that has been encoded in accordance with the same Tanner  
Graph. I understand that Caltech asserts claims 1, 18, 19, and 22 in this case.

13 ii) Prosecution History

14 a) First Office Action: September 6, 2007

15 82. The patent examiner initially allowed pending claims 1-17 and rejected  
16 independent claim 18 and dependent claims 19-24 under 35 U.S.C. § 103 as  
17 obvious over U.S. Patent No. 5,530,707 (to Lin) in view of U.S. Patent No.  
18 6,859,906 (to Hammons et al.).

19 b) Response: Feb 4, 2008

20 83. To overcome the examiner's rejection, the applicant canceled claim 20 and  
21 incorporated its subject matter into independent claim 18. The amendment further  
22 limited claim 18 to require that the message passing decoder of claim 18 be  
23 configured to decode a data stream that has been encoded in accordance with a  
particular Tanner graph.



1           **D. '781 Patent**

2           i)     Claims

3     84.    The '781 patent includes 22 claims, of which claims 1, 13, 19, 20, and 21 are  
4     independent. Independent claim 1 is directed to a two-step process for encoding a  
5     signal, where the first encoding step involves a linear transform operation and the  
6     second involves an accumulation operation. Independent claims 13 and 19 are  
7     directed to methods of encoding a signal that generate codewords by summing  
8     information bits and accumulating the resulting sums. Independent claims 20 and  
9     21 are directed to methods that involve summing information bits and parity bits to  
10    generate a portion of an encoded signal. I understand that Caltech asserts claims  
11    16 and 19 in this case.

11          ii)    Prosecution History

12           a)     First Office Action: October 28, 2010

13    85.    The patent examiner issued a first office action allowing some claims but  
14    rejecting claims 13-17 and 20 as anticipated by U.S. Patent 5,181,207 (to Chapman,  
15    et. al.) and requiring applicants to clarify the term “irregular,” as it appeared in  
16    claims 9 and 23.

17           b)     Response: January 27, 2011

18    86.    To overcome the examiner’s rejection, the applicant canceled claim 21 and  
19    incorporated its subject matter into independent claim 13. As amended, claim 13  
20    requires that “the information bits appear in a variable number of subsets”  
21    (Response dated Jan. 27, 2011 at 4).

22    87.    In accompanying remarks, applicants disagreed with the examiner’s  
23    statement that the term “irregular” was unclear, stating that “[i]t is believed that the  
24    meaning of the term "irregular" in the claims is clear and is well known in the art of  
   computer coding technology” (*id.* at 7) (emphasis added). However, to overcome

1 the examiner's rejection, the applicant amended claims 9 and 23 to remove the  
2 word "irregular," replacing it with the requirement that the information bits appear  
3 "in a variable number of subsets" (*id.* at 3, 6).

#### 4 E. '833 Patent

##### 5 i) Claims

6 88. The '833 patent includes 14 claims, of which claims 1 and 8 are independent.  
7 Independent claims 1 and 8 are directed to an apparatus and a method, respectively,  
8 for encoding information bits that are stored in a first set of memory locations by  
9 combining information bits with parity bits that are stored in a second set of  
10 memory locations, and accumulating the bits in the second set of memory locations.  
11 Both claims require that at least two of the first set of memory locations be read  
12 "different times from one another."<sup>17</sup> I understand that Caltech asserts claims 1, 2,  
13 4, and 8 in this case.

##### 13 ii) Prosecution History

14 89. After the examiner had allowed all pending claims in the application, the  
15 applicant attempted to amend claims 1 and 8 as follows: "wherein a ~~total number~~  
16 ~~of indices~~ two or more memory locations of the first set of memory locations are  
17 read by the permutation module different times from one another ~~represents a~~  
18 ~~variable number~~" (Amendment dated May 7, 2012).

19 90. The examiner did not enter these amendments after allowance because they  
20 changed the scope of the claims that had already been allowed. The applicant  
21 subsequently filed a request for continued examination, after which the examiner  
22 allowed the claims as amended.

23 \_\_\_\_\_  
24 <sup>17</sup> As noted above, the parties have agreed that this claim term requires memory locations to be  
read a different number of times from one another.

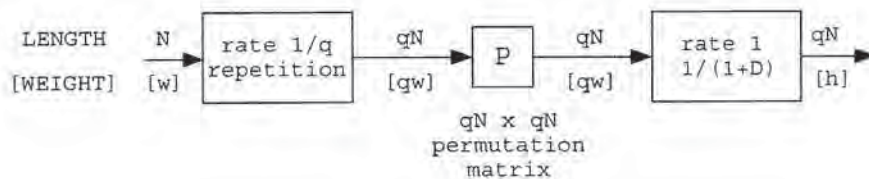


1 **IV. SUMMARY OF THE PRIOR ART**

2 **A. Divsalar**

3 91. D. Divsalar, H. Jin, and R. J. McEliece, "Coding theorems for "turbo-like"  
4 codes," *Proc. 36th Allerton Conf. on Comm., Control and Computing*, Allerton,  
5 Illinois, pp. 201-210 ("Divsalar") was published in Sept. 1998, about 1.5 years  
6 before the filing of the provisional application to which the patents-in-suit claim  
7 priority, and I have been informed that Divsalar qualifies as prior art to all four of  
8 the patents-in-suit.

9 92. Divsalar teaches "repeat and accumulate" codes, which it describes as "a  
10 simple class of rate  $1/q$  serially concatenated codes where the outer code is a  $q$ -fold  
11 repetition code and the inner code is a rate 1 convolutional code with transfer  
12 function  $1/(1 + D)$ " (Divsalar at 1). Fig. 3 of Divsalar, reproduced below, shows  
13 an encoder for a repeat-accumulate code with rate  $N/qN$ :



14  
15  
16 **Figure 3.** Encoder for a  $(qN, N)$  repeat and accumulate  
17 code. The numbers above the input-output lines  
18 indicate the length of the corresponding block, and  
19 those below the lines indicate the weight of the block.

20 93. A block of  $N$  information bits enters the coder at the left side of the figure  
21 and is provided to the repeater (labeled "rate  $1/q$  repetition") (*see id.* at 5). The  
22 repeater duplicates each of the  $N$  information bits  $q$  times and outputs the resulting  
23  $N \times q$  repeated bits, which are then "scrambled by an interleaver of size  $qN$ " (*id.*,  
24 referring to the box labeled "P"). The scrambled bits are "then encoded by a rate 1  
accumulator" (*id.*, emphasis in original; *see also* Divsalar Tr. at pp. 59-63, 68-69).

94. Divsalar describes the accumulator as follows:

1 [W]e prefer to think of [the accumulator] as a block coder whose  
2 input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by  
the formula

$$y_1 = x_1$$

$$y_2 = x_1 + x_2$$

$$y_3 = x_1 + x_2 + x_3$$

$$y_n = x_1 + x_2 + x_3 + \dots + x_n$$

3  
4  
5 (*id.* at 5). The plus signs (“+”) in Divsalar’s formula represent modulo-2, or  
6 exclusive-OR, addition (*see id.*; *see also* Divsalar Tr. 69:10-16).

7 95. Divsalar uses repeat-accumulate codes to prove a conjecture regarding the  
8 interleaver gain exponent (IGE), which is a numerical parameter that estimates the  
9 rate at which the word error rate decreases as the block length increases.

10 96. Divsalar further shows that RA codes have “very good” performance and  
11 that they can be efficiently decoded using a “message passing decoding algorithm”  
12 (*id.* at 9-10).

13 97. Divsalar teaches that turbocodes, serially concatenated convolutional codes  
14 and RA codes can all be viewed as “turbo-like” codes: “We call these systems  
15 “turbo-like” codes and they include as special cases both the classical turbo codes  
16 and the serial concatenation of interleaved convolutional codes” (Divsalar  
17 Abstract) and “In Section 5, we define a special class of turbo-like codes, the  
18 repeat-and-accumulate codes, and prove the IGE conjecture for them” (Divsalar at  
19 1). More specifically, RA codes can be viewed as turbocodes, in which the  
20 information bits are punctured, or truncated, none of the parity bits are punctured,  
21 and the convolutional code is an accumulator. “The accumulator can be viewed as  
22 a truncated rate-1 recursive convolutional encoder with transfer function  $1/(1 + D)$ ”  
23 (Divsalar at 5). Divsalar also makes use of the fact that RA codes can be viewed as  
24 turbocodes to explain the decoder: “But an important feature of turbo-like codes is  
the availability of a simple iterative, message passing decoding algorithm that  
approximates ML decoding. We wrote a computer program to implement this



1 ‘turbo-like’ decoding for RA codes with  $q = 3$  (rate 1/3) and  $q = 4$  (rate 1/4), and  
2 the results are shown in Figure 5” (Divsalar at 9).

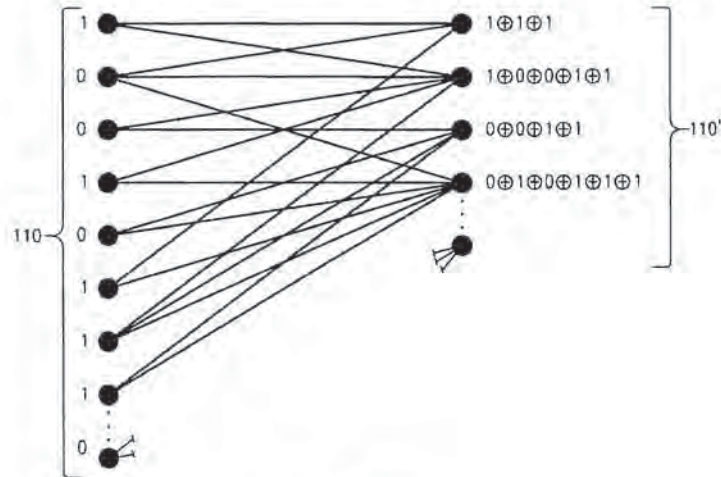
3 98. As explained further below, Divsalar teaches all but one aspect of an IRA  
4 code: irregularity (the “I” in Irregular Repeat-Accumulate). That is, Divsalar  
5 teaches regular repeat-accumulate (RA) codes rather than irregular repeat-  
6 accumulate codes. A single modification to Divsalar – *i.e.*, changing the repeat to  
7 being irregular instead of regular – would result in the IRA codes that Caltech  
8 claims to have invented. I also explain below why it would have been obvious to  
9 one of ordinary skill before the Caltech patents were filed (and before Caltech’s  
10 claimed conception date) to add irregularity to the repeat-accumulate codes of  
11 Divsalar, resulting in the irregular repeat-accumulate codes to which the patents-in-  
12 suit are directed.

### 12 **B. Luby**

13 99. U.S. Patent No. 6,081,909 to Luby et al. (“Luby”), titled “Irregularly  
14 graphed encoding technique,” was filed Nov. 6, 1997, about 2.5 years before the  
15 filing of the provisional application to which the patents-in-suit claim priority, and  
16 I have been informed that Luby qualifies as prior art to all four of the patents-in-  
17 suit.

18 100. The Luby patent mirrors the teachings of Luby’s seminal paper that I  
19 described above, in which the concept of irregular error-correcting codes was first  
20 introduced. Specifically, Luby teaches “a technique for creating loss resilient and  
21 error correcting codes having irregular graphing between the message data and the  
22 redundant data” (Luby at 1:5-10). “Irregular graphing” refers to codes with Tanner  
23 graphs in which some information nodes are connected to more check nodes than  
24 others (*see, e.g., id.* at 3:27-29, stating that “different numbers of first edges are  
associated with the data items”).

1 101. A Tanner graph corresponding to an irregular code is shown in Fig. 17 of  
2 Luby, reproduced below:



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10 *FIG. 17*

11 102. In this figure, the circles on the left represent information bits to be encoded  
12 and the circles on the right represent parity checks computed for these information  
13 bits. Each parity check on the right is computed by summing together (modulo 2)  
14 all of the information bits connected to that parity check by an edge in the graph  
15 (*see id.* at 17:64-67).<sup>18</sup>

16 103. As the figure shows, some information nodes on the left contribute to three  
17 parity checks on the right, while others contribute to two (*i.e.*, all nodes on the left  
18 which are connected to two lines, such as the top node, contribute to two parity  
19 checks and all nodes on the left which are connected to three lines, such as the  
20 second node from the top, contribute to three parity checks). An encoding scheme  
21 with a Tanner graph in which some information nodes are connected to more check  
22 nodes than others is the defining characteristic of an irregular code.

23  
24 <sup>18</sup> I explain what an “edge” is in this context in Appendix A, below.



1           **C. MacKay**

2 104. D. J. C. MacKay, S. T. Wilson, and M. C. Davey, "Comparison of  
3 constructions of irregular Gallager codes," *IEEE Trans. Commun.*, Vol. 47, No. 10,  
4 pp. 1449-1454 ("MacKay") was published in Oct. 1999, about six months before  
5 the filing of the provisional application to which the patents-in-suit claim priority,  
6 and I have been informed that MacKay qualifies as prior art to all four of the  
7 patents-in-suit.

8 105. MacKay is motivated by "[t]he excellent performance of irregular Gallager  
9 codes," and explores "ways of further enhancing these codes" (MacKay at 1459).  
10 In particular, MacKay investigates the constructions of both regular and irregular  
11 Gallager codes with encoding algorithms that have low computational complexity.

12           **D. Ping**

13 106. L. Ping, W. K. Leung, N. Phamdo, "Low Density Parity Check Codes with  
14 Semi-random Parity Check Matrix." *Electron. Letters*, Vol. 35, No. 1, pp. 38-39  
15 ("Ping") was published in Jan. 1999, more than a year before the filing of the  
16 provisional application to which the patents-in-suit claim priority, and I have been  
17 informed that Ping qualifies as prior art to all four of the patents-in-suit.

18 107. Ping teaches constructing LDPC codes that can be encoded in two stages. In  
19 the first encoding stage, a generator matrix is applied to a sequence of information  
20 bits to produce sums of information bits. In the second stage, the sums of  
21 information bits are accumulated recursively to generate the parity bits (*see* Ping at  
22 38).

23 108. Ping's code can be described as an LDPC code with two components: an  
24 outer coder that is an LDGM coder followed by an inner coder that is an

1 accumulator. Thus Ping teaches LDPC codes that are also accumulate codes.<sup>19</sup> I  
2 understand that the codes Caltech has accused of infringement, *i.e.*, the DVB-S2  
3 codes, can also be encoded using LDPC + accumulate coders. One difference  
4 between Ping and the accused codes is that Ping's LDPC code is regular whereas  
5 in the accused DVB-S2 codes, the LDPC code is irregular. As explained below, it  
6 was obvious before Caltech's alleged invention to make codes irregular, *e.g.*,  
7 because it was known that doing so would improve their performance. In  
8 particular, it was obvious before Caltech's alleged invention to make Ping's LDPC  
9 code irregular. Therefore, if Caltech establishes that its claims cover the accused  
10 DVB-S2 codes, then those claims would be invalid in view of Ping and the art that  
11 rendered it obvious to make Ping's LDPC code irregular, *e.g.* Luby, MacKay and  
12 Frey99.

#### 11 E. Frey99

12 109. Frey, B. J. and MacKay, D. J. C., "Irregular Turbocodes," *Proc. 37th*  
13 *Allerton Conf. on Comm., Control and Computing*, Monticello, Illinois ("Frey99")  
14 was published on or before March 20, 2000, which is before the filing of the  
15 provisional application to which the patents-in-suit claim priority, and I have been  
16 informed that Frey99 qualifies as prior art to all four of the patents-in-suit.

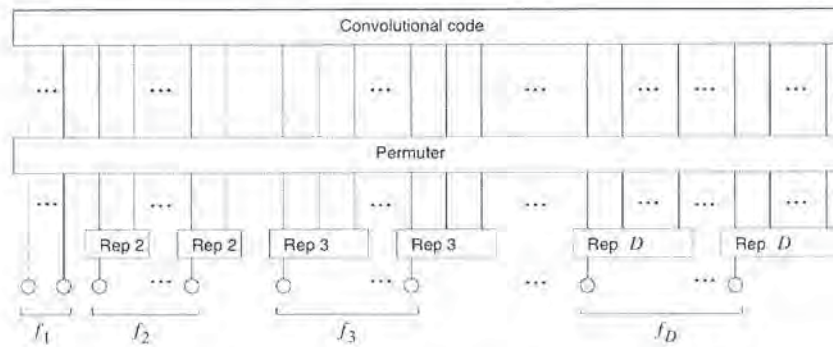
17 110. Frey99 is a paper that I wrote in collaboration with David MacKay. In  
18 Frey99, David MacKay and I applied the concept of irregularity to turbocodes by  
19 explaining how to construct irregular turbocodes, *i.e.*, turbocodes with Tanner  
20 graphs in which some information nodes are connected to more check nodes than  
21 others. Our experimental results demonstrated that these irregular turbocodes  
22 perform better than the regular turbocodes that were known in the art.

23  
24 <sup>19</sup> Below I refer to these codes as "LDPC + accumulate" codes.



1 111. As I explain in Frey99, “an *irregular turbocode* has the form shown in Fig. 2,  
 2 which is a type ‘trellis-constrained code’ as described in [7]. We specify a *degree*  
 3 *profile*,  $f_d \in [0, 1]$ ,  $d \in \{1, 2, \dots, D\}$ .  $f_d$  is the fraction of codeword bits that have  
 4 degree  $d$  and  $D$  is the maximum degree. Each codeword bit with degree  $d^{20}$  is  
 5 repeated  $d$  times before being fed into the permuter. Several classes of permuter  
 6 lead to linear-time encodable codes. In particular, if the bits in the convolutional  
 7 code are partitioned into ‘systematic bits’ and ‘parity bits’, then by connecting each  
 8 parity bit to a degree 1 codeword bit, we can encode in linear time” (Frey99 at 2).

8 112. As this passage explains, the irregular turbocodes I described in Frey99  
 9 operate by irregularly repeating the information bits, interleaving the repeated bits  
 10 using a “permute” (*i.e.*, an interleaver), and encoding the permuted bits using a  
 11 convolutional code. Figure 2 of Frey99, reproduced below, illustrates such an  
 12 irregular turbocode:



18 Figure 2: A general *irregular turbocode*. For  $d = 1, \dots, D$ , fraction  $f_d$  of the codeword  
 19 bits are repeated  $d$  times, permuted and connected to a convolutional code.

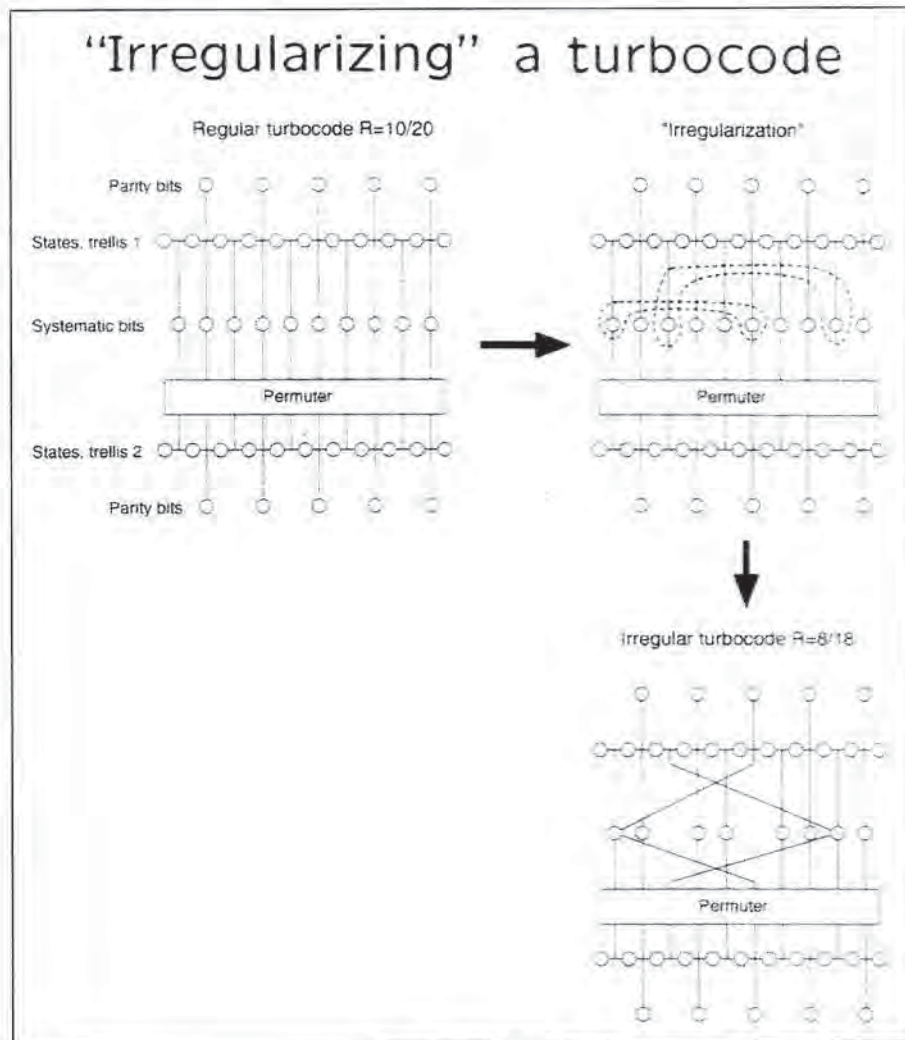
20 113. In this figure, bits in the subset  $f_1$  are not repeated, bits in the subset  $f_2$  are  
 21 repeated twice, bits in the subset  $f_3$  are repeated three times, and bits in the subset  
 22  $f_D$  are repeated  $D$  times.

23 \_\_\_\_\_  
 24 <sup>20</sup> A bit with “degree  $d$ ” is a bit that contributes to  $d$  parity check bits. In Frey99, bits of degree  $d$   
 are repeated  $d$  times prior to permutation.

1 **F. Frey Slides**

2 114. I prepared the Frey Slides (titled “Irregular Turbo-Like Codes”) in  
3 collaboration with David MacKay and presented them at the Allerton Conference  
4 in September, 1999. The Frey Slides contain the material upon which the Frey99  
5 paper, published in the Allerton 1999 conference proceedings, is based.

6 115. In particular, the Frey Slides describe how irregularity can improve code  
7 performance and introduce the concept of irregular turbocodes. Using the same  
8 procedure described in the Frey99 paper, the Frey Slides show how known, regular  
9 turbocodes can be “irregularized,” step by step:

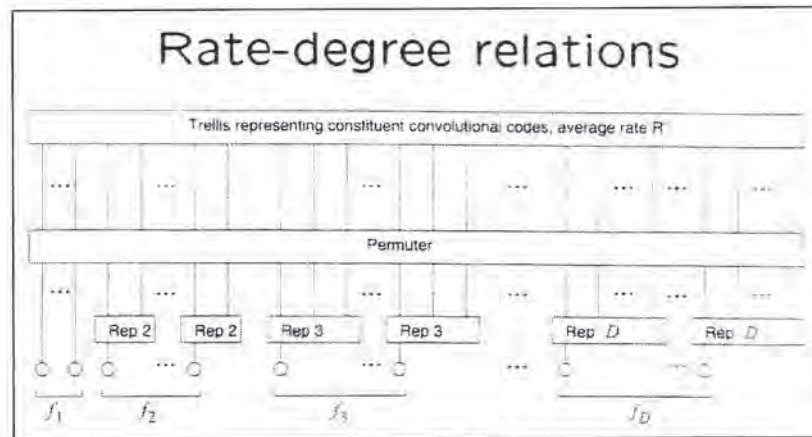


Frey Slides at 4



1 In the figure above, a regular turbocode (upper left) is “irregularized” by tying  
2 information nodes together (upper right), thereby raising their degree, resulting in  
3 an irregular turbocode (lower right).

4 116. Also, using a diagram identical to Figure 2 of Frey99 (described above) the  
5 Frey Slides show how irregular turbocodes can be implemented via irregular  
6 repetition:



14 **Frey Slides at 5**

15 117. The Frey Slides also describe selection of degree profiles (*see id.* at 6) and  
16 provide details regarding the rate of the resulting convolutional coder and the  
17 overall rate of the irregular turbocode (*id.* at 5-8, 13).

18 118. I understand that Caltech has alleged a date of invention of March 7, 2000. I  
19 further understand that Caltech may argue that the Frey99 paper was not published  
20 until after its alleged invention date. In the event that the Court finds that the  
21 patents-in-suit are entitled to a date of invention that predates the publication of  
22 Frey99, and the Frey99 paper is deemed not to be prior art to the patents-in-suit,  
23 then the Frey Slides may be substituted for the Frey99 paper in all of the positions  
24 explained below. For the purposes of the invalidity opinions set forth in this report,  
the teachings of Frey99 and the Frey Slides are interchangeable. To illustrate how  
the Frey Slides may be substituted for Frey99, wherever I cite to Frey99 in the

1 report below, I have also included citations to the corresponding teachings in the  
2 Frey Slides.

3 **G. RA.c**

4 119. Source code file "RA.c," dated September 28, 1998, was written by David  
5 MacKay at the University of California at San Francisco.

6 120. The RA.c source code implements a "[r]epeat-accumulate code simulator."  
7 The file includes a function called "RA\_encode" that performs a repeat-accumulate  
8 encoding operation.

9 121. The operation of RA.c is described in a comment at the beginning of the  
10 source code file:

```
11      /*  
12         RA.c  
13             (c) DJCM 98 09 28  
14  
15         Repeat-accumulate code simulator  
16  
17         read in code definition  
18         loop {  
19             encode source string  
20             add noise  
21             decode  
22         }  
23  
24         Code definition: (stored in "alist")  
25  
26             Use of alist allows arbitrary numbers of repetitions  
27             of each bit.  
28  
29             K           source block length  
30             n_1 n_2 ... n_K   number of repetitions of each source bit  
31             N = sum n_k  
32             alist defines    permutation of N encoded bits  
33                             note, an additional permutation of the N accumulated  
34                             bits may be a good idea. (for non-memoryless channels)  
35  
36             transmitted bits are integral of encoded bits  
37  
38             Future plans:  
39             clump source bits into clumps. Have multiple parallel accumulated streams.  
40             Have little sub-matrices (like GF(q) ) defining response of accumulator to  
41             clumps.  
42  
43         */  
44         (RA.c at 1) (emphasis added).
```

22 122. As shown by the highlighted passages above, the comment at the top of  
23 RA.c explicitly refers to repeat-accumulate codes in which different information  
24 bits are repeated different numbers of times. Therefore, this comment, written



1 more than 1.5 years before the alleged conception date of the patents-in-suit,  
2 explicitly teaches irregular repeat-accumulate codes.

### 3 **H. '999 Patent**

4 123. U.S. Patent No. 4,623,999 to Patterson et al. (hereinafter, the "'999 patent"),  
5 was filed on June 4, 1984, more than 15 years before the filing date of the  
6 provisional application to which the patents-in-suit claim priority, and I have been  
7 informed that the '999 patent qualifies as prior art to all four of the patents-in-suit.

8 124. The '999 patent teaches an encoder for encoding information bits using a  
9 linear error-correcting code. The encoder taught by the '999 patent uses a plurality  
10 of memories that store values used during the encoding process ('999 patent at  
11 Abstract, describing "[a]n efficient look-up table encoder for encoding  $k$  bit  
12 information words with linear error correcting block codes is provided comprising  
13 a plurality of read-only **memories** ...") (emphasis added). The teachings of  
14 the '999 patent illustrate that the use of memories to implement error-correcting  
15 coders was known in the art for decades prior to the claimed priority date of the  
16 patents-in-suit.

### 16 **I. Accused Hughes Products**

17 125. As I explain below, the earliest priority date to which the claims of the '833  
18 patent could be entitled is March 28, 2011, the date those claims were first filed.

19 126. I have been informed that a number of the accused products in this case were  
20 sold by Defendants prior to March 28, 2011. If the claims of the '833 patent are  
21 entitled to a priority date of March 28, 2011, these accused products would qualify  
22 as prior art to the claims of the '833 patent.

## 22 **V. SUMMARY OF ANTICIPATION AND OBVIOUSNESS OPINIONS**

23 127. As I explain in detail below, the asserted claims are either anticipated by or  
24 obvious over the prior art references described above. Broadly speaking, the



1 claimed codes represent the combination of RA codes, which were generally  
2 known by those of ordinary skill in the art by March 7, 2000, with irregularity,  
3 which had been shown years before to improve the performance of codes like RA  
4 codes.

5 128. One of ordinary skill in the art would have been motivated to combine these  
6 two ideas. RA codes are described in detail in Divsalar, published more than a  
7 year before the alleged conception date of the patents-in-suit. The concept of  
8 irregularity had been introduced by Luby in 1997, and by March 7, 2000 had been  
9 thoroughly explored in a number of papers and publications, including Frey99,  
10 MacKay, and the Luby '909 patent, discussed below (in particular, Frey99 teaches  
11 irregular *repetition*, which is specifically required by some of the asserted claims).  
12 By March 7, 2000, both RA codes and irregularity would have been common  
13 knowledge to one of ordinary skill in the art.

14 129. Indeed, prior to March 7, I myself suggested incorporating irregularity into  
15 RA codes. In particular, as described below, I suggested in an email to Dariush  
16 Divsalar that he make his RA codes irregular (*See* Email from Brendan Frey to  
17 Dariush Divsalar dated Dec. 8, 1999 (CALTECH000024021)). Consistent with the  
18 email I sent to Dr. Divsalar, making RA codes irregular was merely an obvious  
19 application of my earlier work on irregular turbocodes, which I presented at the  
20 Allerton 1999 conference, 6 months before Caltech's alleged conception date of  
21 March 7, 2000, and which is described in Frey99 and the Frey Slides.

22 130. I explain these opinions in further detail below, with reference to each  
23 limitation of the various claims that have been asserted by Caltech.

## 24 **VI. THE ASSERTED CLAIMS OF THE '710 PATENT ARE INVALID**

131. As I explain below, asserted claims 1, 4, 6, 15, 20, and 22 of the '710 patent  
are invalid. I also explain why claims 3, 5, and 21, from which claims 4, 6, and 22



1 depend, respectively, are invalid. A summary of the opinions set forth in this  
 2 section is given in the table below:

3 '710 Claim	Frey99 (or Frey slides)	Frey99 (or Frey slides) + Divsalar	Divsalar + Luby	Divsalar + MacKay
4 1	Anticipated	Anticipated by Frey or Obvious	Obvious	Obvious
5 3	Anticipated	Anticipated by Frey or Obvious	Obvious	Obvious
6 4		Obvious	Obvious	Obvious
7 5		Obvious	Obvious	Obvious
8 6		Obvious	Obvious	Obvious
9 15		Obvious	Obvious	Obvious
10 20		Obvious	Obvious	Obvious
11 21		Obvious	Obvious	Obvious
12 22		Obvious	Obvious	Obvious

11 **A. Claim 1 of the '710 Patent is Invalid**

12 132. Claim 1 of the '710 patent reads:

- 13 1. A method of encoding a signal, comprising:
- 14 obtaining a block of data in the signal to be encoded;
- 15 partitioning said data block into a plurality of sub-blocks, each sub-block including a plurality of data elements;
- 16 first encoding the data block to form<sup>21</sup> a first encoded data block, said first encoding including repeating the data elements in different sub-blocks a different number of times;
- 17 interleaving the repeated data elements in the first encoded data block; and
- 18 second encoding said first encoded data block using an encoder that has a rate close to one.

19 i) Claim 1 of the '710 Patent is Anticipated by Frey99

20 21 133. I explain below, one limitation at a time, why claim 1 is anticipated by  
 22 Frey99.

23 <sup>21</sup> I note that the word “from” here should be “form.” That is, this limitation is about forming “a  
 24 first encoded data block.” Notwithstanding that typographical error, I have reproduced the claim as it is printed in the patent.



1                   a)     “A method of encoding a signal”

2 134. Even if the preamble limits the claim, it is taught by Frey99. As I explain  
3 above, Frey99 deals with the construction of irregular turbocodes. The purpose of  
4 the disclosed irregular turbocode is for the encoding and decoding of signals (*see*  
5 *also*, Frey Slides at 4). Frey99 explicitly discloses decoding signals that had been  
6 encoded using the disclosed irregular turbocode. *See, e.g.*, Frey99 at 4 (“***After***  
7 ***receiving the channel output, the decoder computes*** the channel output log-  
8 likelihood ratios ...”) (emphasis added); 4 (“In our simulations, after each iteration,  
9 we check to see if the current decision gives a codeword. If it does, the iterations  
10 terminate and otherwise, ***the decoder iterates further ...***”) (emphasis added); 6  
11 (“Fig. 4 shows the simulated BER- $E_b/N_0$  curves for the original block length  $N$ -  
12 131,072 regular turbocode (dashed line) and its irregular cousin (solid line), using  
13 profile  $e = 10$ ,  $f_e = 0.05$ ”); *see also*, Frey Slides at 2 (“making decoding easier”);  
14 Frey Slides at 9, 11, 12 (showing BER- $E_b/N_0$  curves).

13                   b)     “obtaining a block of data in the signal to be encoded”

14 135. Frey99 deals exclusively with block codes. For example, Frey99 includes  
15 experimental results comparing a regular code and an irregular code, both having  
16 “block length  $N = 131,082$ ” (Frey99 at 6; *see also* Frey Slides at 13, teaching “long  
17 block lengths,” and “short block lengths”). Frey99’s use and discussion of that  
18 block length means that Frey99 takes bits in blocks of 131,082 and encodes them,  
19 just as is required by this claim limitation. Similarly, Frey99 also includes other  
20 discussion of obtaining data in blocks for encoding. For example, Frey99  
21 describes experimental results relating to, *e.g.*, the “block length” of irregular  
22 turbocodes. In selecting a coding profile, Frey99 teaches “making small changes  
23 to a block length  $N = 10,000$  version of the original rate  $R = 1/2$  turbocode  
24 proposed by Berrou *et al.*” (Frey99 at 5) (emphasis added). Also, Frey99 uses the  
“BER” or “block error rate” to compare the performance of various codes (*see, e.g.*

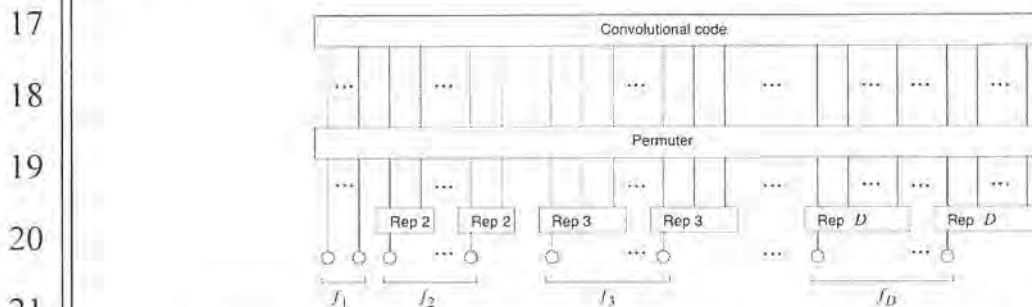


1 Frey99 at Figure 4). Frey99's reference to "block error rate" means that Frey99  
2 obtains data in blocks for encoding.

3 c) "partitioning said data block into a plurality of sub-blocks, each  
4 sub-block including a plurality of data elements"

5 136. Frey99 teaches this limitation. Frey99 describes irregular turbocodes as  
6 follows: "an *irregular turbocode* has the form shown in Fig. 2, which is a type  
7 'trellis-constrained code' as described in [7]. We specify a *degree profile*,  $f_d \in [0,$   
8  $1], d \in \{1, 2, \dots, D\}$ .  $f_d$  *is the fraction of codeword bits that have degree  $d$  and  $D$*   
9 *is the maximum degree. Each codeword bit with degree  $d$  is repeated  $d$  times*  
10 *before being fed into the permuter.* Several classes of permuter lead to linear-time  
11 encodable codes. In particular, if the bits in the convolutional code are partitioned  
12 into 'systematic bits' and 'parity bits', then by connecting each parity bit to a  
13 degree 1 codeword bit, we can encode in linear time." Frey99 at 2 (emphasis  
14 added).

15 137. As described above, Frey99 partitions the information bits into groups,  
16 where the bits in each group all have the same degree (*i.e.*, they are all repeated the  
17 same number of times). Frey99 also illustrates this operation graphically in Figure  
18 2, reproduced below:



22 Figure 2: A general *irregular turbocode*. For  $d = 1, \dots, D$ , fraction  $f_d$  of the codeword  
23 bits are repeated  $d$  times, permuted and connected to a convolutional code.

24

1 138. In Figure 2 of Frey99, the circles at the bottom represent information bits.  
2 The groups of information bits labeled  $f_2, f_3, \dots, f_D$  represent sub-blocks into which  
3 the data block is partitioned (*see also* Frey Slides at 5).

4 139. Thus, the bits that are repeated twice (the bits labeled  $f_2$ ) constitute one sub-  
5 block, the bits that are repeated three times (the bits labeled  $f_3$ ) constitute a second  
6 sub-block, and so on. As shown in Figure 2 of Frey99, each of these sub-blocks  
7 contains a plurality of bits (or “data elements”), as required by claim 1 of the ’710  
8 patent.

9 d) “first encoding the data block to from [sic] a first encoded data  
10 block, said first encoding including repeating the data elements  
11 in different sub-blocks a different number of times”

12 140. Frey99 teaches repeating the data elements in different sub-blocks a  
13 different number of times (which is commonly known as “irregular repetition” to  
14 those of ordinary skill in in the art).

15 141. For example, Figure 2 of Frey99, reproduced above, shows that the data  
16 elements in each sub-block are repeated a different number of times. In Figure 2  
17 of Frey99, the circles at the bottom represent information bits in the data block.  
18 The groups of information bits labeled  $f_2, f_3, \dots, f_D$  represent sub-blocks into which  
19 the data block is partitioned. The blocks labeled “Rep 2,” “Rep 3,” and “Rep D”  
20 represent the step of repetition. For example, an information bit that is connected  
21 to a box labeled “Rep 2” is repeated twice, a bit connected to a box labeled “Rep 3”  
22 is repeated three times, etc.. In the figure above, the repeated bits are represented  
23 by the vertical lines connecting the “Rep  $n$ ” boxes to the box labeled “Permuter”  
24 (*see also* Frey Slides at 5).



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e) “interleaving the repeated data elements in the first encoded data block”

142. Frey99 teaches this limitation. As I explain above, Frey99 teaches codes in which “Each codeword bit with degree  $d$  is repeated  $d$  times before being fed into the permuter” (Frey99 at 2; *see also* Frey99, Figures 1 and 2). Figure 1 of Frey99 illustrates how “a turbocode can be viewed as a code that copies the systematic bits, permutes both sets of these bits and then feeds them into a convolutional code” (Frey99 at 3) (emphasis added). *See also* Frey Slides at 4, 5 (showing copies of systematic bits fed into a “Permuter” block).

143. “Permuting” means “interleaving,” and a “permuter” is an “interleaver,” as both parties have agreed in their Joint Claim Construction Statement (construing both “interleaver” and “permutation module” to mean “module that changes the order of data elements”). Permuting/interleaving bits means changing the order of the bits. The permuter in Figure 2 of Frey99 receives the repeated bits (produced by the blocks labeled “Rep 1,” “Rep 2,” ..., and “Rep  $D$ ”) and interleaves them (*see also* Frey Slides at 5; *see also* Divsalar Tr. at 278:2-23).

f) “second encoding said first encoded data block using an encoder that has a rate close to one”

144. Frey99 teaches this limitation. The “second encoding” taught by Frey99 is a convolutional encoder, which accepts irregularly repeated and permuted bits as input and encodes these bits to produce parity bits, as shown in Figure 2, reproduced below (*see also* Frey Slides at 5):

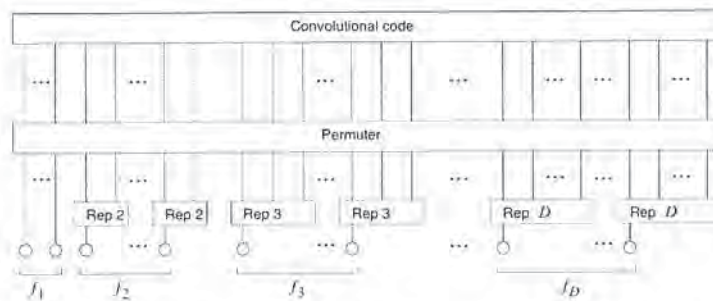


Figure 2: A general *irregular turbo code*. For  $d = 1, \dots, D$ , fraction  $f_d$  of the codeword bits are repeated  $d$  times, permuted and connected to a convolutional code.

145. Frey99 teaches a convolutional coder “with the required convolutional code rate of  $R' = 2/3$ ” (Frey99 at 5). A code with a rate of  $2/3$  has a rate “close to one,” as required by this limitation. Indeed, during prosecution of the '710 patent, the Applicant attempted to overcome a prior art rejection by replacing “a rate close to one” with “a rate within 50% of one,” in issued claim 15 (Response dated May 5, 2005 at 7-8). A code with rate  $2/3$  clearly has a rate “within 50% of one,” and Applicant’s amendment suggests that “a rate close to one” is even broader.

146. However, Frey99 also teaches second encoders with a rate even closer to one. Repetition increases the number of bits input to the convolutional encoder, but the number of bits sent across the channel can be decreased by “puncturing.” Frey99 teaches “it is clear that when the average degree is increased, the rate of the convolutional code must also be increased to keep the overall rate at  $1/2$ ” (Frey99 at 5). Recall from above that the rate of an encoder is equal to the ratio between the number of bits input to the encoder and the total number of bits output by the encoder. A convolutional coder with rate  $2/3$  outputs three bits for every two bits of input. Puncturing the convolutional code lowers the number of output bits, reducing the denominator of the ratio and thus raising the rate of the code.

147. Specifically, Frey99 teaches puncturing the convolutional code to obtain a convolutional code with rate:



$$R' = 1 - \frac{1 - R}{d} = 1 - \frac{1/2}{1/2 + 2(1/2 - f_e) + e f_e}$$

(Frey99 at 5).

148. This equation includes two variables,  $e$ , and  $f_e$ . Frey99 presents results that “show that for  $e = 10, f_e = 0.05$  is a good fraction, and that for  $f_e = 0.05, e = 10$  is a good degree” (Frey99 at 5). Plugging the values  $e = 10$  and  $f_e = 0.05$  into the equation above, we obtain:

$$R' = 1 - \frac{\frac{1}{2}}{\frac{1}{2} + 2\left(\frac{1}{2} - f_e\right) + e \cdot f_e} = \frac{1.4}{1.9} \approx 0.74$$

149. One of ordinary skill in the art would recognize that a rate of 0.74 is a rate “close to one.” See also Frey Slides at 6 (showing equation for convolutional code rate); Frey Slides at 7 (showing “ $d_e = 10$ ”); Frey Slides at 8 (showing “ $f_e = .05$ ”), leading to the same rate  $R' = 0.74$ .

g) Summary

150. As explained above, Frey99 teaches every limitation of claim 1 and therefore anticipates claim 1.

ii) Claim 1 of the '710 Patent is Obvious Over Frey99 In View of Divsalar

151. As explained above, in my opinion, Frey99 teaches every limitation of, and therefore anticipates, claim 1 of the '710 patent. However, in the event Frey99 is found not to teach the “rate close to one” limitation of claim 1, then claim 1 is obvious over the combination of Frey99 and Divsalar.

152. Specifically, I explain later in this section that Divsalar teaches a second encoding step using an encoder with a rate “close to one.” Also, as I explain below, one of ordinary skill in the art would have been motivated to combine Divsalar and

1 Frey99 in general, and would specifically have been motivated to use the  
 2 accumulator of Divsalar in the irregular turbo codes of Frey99. Finally, I explain  
 3 why such a combination would represent a minor modification to the teachings of  
 4 Frey99, and would not fundamentally change its principle of operation.

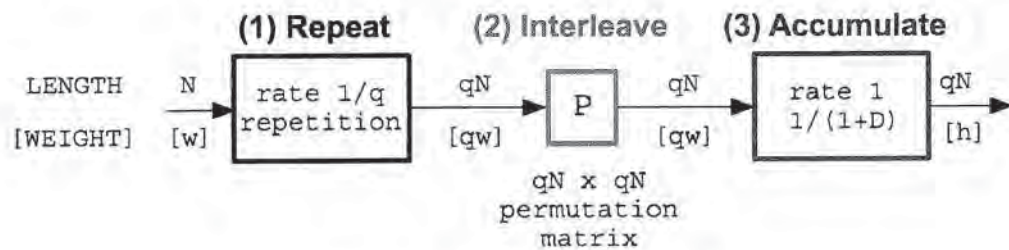
5 a) The accumulator of Divsalar has "a rate close to one"

6 153. If the rate of the second encoder taught by Frey99 were found to not be  
 7 "close to one," as required by the final limitation of claim 1, it would have been  
 8 obvious to substitute Divsalar's rate-1 accumulator for Frey's convolutional code.  
 9 In such a combination, the code rate of the second encoder would be exactly one,  
 10 which would satisfy the "close to one" requirement.

11 154. As explained above, Divsalar teaches an RA code that uses three steps:

- 12 (1) **repeat** bits  $q$  times;
- 13 (2) **interleave** the repeated bits (with the block labeled "P" in Figure 3); and
- 14 (3) **accumulate** the repeated-interleaved bits with the *rate 1* accumulator.

15 Each of these steps is represented by a block in Figure 3, reproduced below.



16 Divsalar, Figure 3 (annotated)

17 155. Divsalar explains the accumulate step as follows:

18 [W]e prefer to think of [the accumulator] as a block coder whose  
 19 input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by  
 20 the formula

$$\begin{aligned}
 21 \quad & y_1 = x_1 \\
 22 \quad & y_2 = x_1 + x_2 \\
 23 \quad & y_3 = x_1 + x_2 + x_3 \\
 24 \quad & y_n = x_1 + x_2 + x_3 + \dots + x_n
 \end{aligned}$$



1 (Divsalar at 5)

2 156. An encoder that outputs  $n$  bits (*i.e.*, “output block  $[y_1, \dots, y_n]$ ”) for every  $n$   
3 bits of input (*i.e.*, “input block  $[x_1, \dots, x_n]$ ”) has a rate of  $n/n = 1$ . Thus, the “second  
4 encoder” taught by Divsalar has a rate of exactly 1 (and it is described in Fig. 3 of  
5 Divsalar as a “rate 1” encoder). Divsalar’s accumulator therefore teaches exactly  
6 the second encoding step of claim 1 of the ’710 patent.<sup>22</sup>

7 *b) Motivations to combine the teachings of Frey99 with those of*  
8 *Divsalar, generally*

9 157. Frey99 and Divsalar are both directed to the same field, namely, the field of  
10 error-correcting codes. Further, Frey99 and Divsalar are both related to variations  
11 on turbo codes. Frey99 is directed to irregular turbo codes (*see, e.g.*, Frey99 at 2,  
12 “[i]n this paper, we show that by tweaking a turbocode so that it is irregular, we  
13 obtain a coding gain ...”; *see also* Frey Slides at 4, titled “Irregularizing” a  
14 turbocode). Divsalar is related to “turbo-like codes” (*see, e.g.*, Divsalar at 2, “In  
15 Section 3 we define the class of ‘*turbo-like*’ codes .... In Section 4 we state a  
16 conjecture ... about the ML decoder performance of *turbo-like* codes. In Section 5,  
17 we define a special class of *turbo-like* codes ...”) (emphasis added).<sup>23</sup> Also, as  
18 explained above, Divsalar teaches that the accumulator is a “truncated rate-1  
19 recursive convolutional encoder with transfer function  $1/(1 + D)$ ” (Divsalar at 5).  
20 Therefore, one of ordinary skill would have been aware of both references and  
21 would have considered them to disclose components that could be substituted for  
22 one another.

22 <sup>22</sup> As confirmed by the testimony of Hui Jin, one of the inventors listed on the patents-in-suit (*see*  
23 Jin Tr. at 122).

24 <sup>23</sup> The “turbo-like” codes described by Divsalar include both classical turbo codes and  
concatenated codes (*see* Divsalar at Abstract). Thus, every turbo code is a “turbo-like code,” as  
that term is used in Divsalar.



1                   c)     Specific motivations to use Divsalar's accumulator in Frey99

2     158. Frey99's second encoder is implemented using a convolutional code.  
3     Divsalar's second encoder is implemented using an accumulator. Accumulation is  
4     a particular type of convolutional code that is simpler than the convolutional code  
5     used in Frey99 (*see, e.g.*, Divsalar Tr. at 279-280). Therefore, one of ordinary skill  
6     would have been motivated to substitute Divsalar's accumulator for Frey99's  
7     convolutional code at least for the following reasons.

8     159. First, using Divsalar's accumulator in place of Frey99's convolutional code  
9     would result in an encoder that was easier to implement in hardware, used fewer  
10    transistors and required fewer computations to produce the encoded codewords.  
11    As explained above, accumulation allows calculating each successive parity bit  
12    using a single modulo-2 addition operation. One of ordinary skill would have thus  
13    been motivated to simplify Frey99's code by replacing the convolutional coder  
14    with Divsalar's accumulator – an even simpler convolutional coder.

15    160. Second, converting Frey99's convolutional code into Divsalar's accumulator  
16    would result in a simpler code that would have been easier to analyze analytically.  
17    Divsalar's original motivation for producing the RA code was to produce a code  
18    that would be easy to analyze analytically. For example, the section of Divsalar  
19    that introduces RA codes begins: “[i]n this section we will introduce a class of  
20    turbo-like codes which are simple enough so that we can prove the IGE conjecture.  
21    We call these codes repeat and accumulate (RA) codes” (Divsalar at 5) (emphasis  
22    added). Indeed, Divsalar attempted to prove the IGE conjecture for more  
23    complicated coding schemes (*see id.* at 1, “[u]nfortunately, the difficulty of the  
24    first step ... has kept us from full success, except for **some very simple coding**  
25    **systems, which we call repeat and accumulate codes**”) (emphasis added). One of  
26    ordinary skill would have been similarly motivated to simplify other codes in order  
27    to make them easier to study; one such simplification that would have been



1 obvious to one of ordinary skill in the art would be replacing Frey99's  
2 convolutional coder with Divsalar's relatively less complex accumulator.

3 161. Also, convolutional coders and accumulators are related. That is,  
4 accumulation is a simple form of convolutional coding.<sup>24</sup> One of ordinary skill  
5 would recognize an accumulator as a simple form of convolutional coder. Divsalar  
6 teaches: "The accumulator can be viewed as a truncated rate-1 recursive  
7 convolutional encoder with transfer function  $1/(1 + D)$ " (Divsalar at 5). Thus, if  
8 one of ordinary skill wanted to simplify the convolutional coder taught in Frey99,  
9 e.g., for the reasons given above, an accumulator would have been a logical choice  
10 because it would be a simple form of the convolutional coder explicitly disclosed  
11 in Frey99.

12 162. Further, using Divsalar's accumulator in place of the convolutional encoder  
13 explicitly taught in Frey99 would have been a routine substitution of one  
14 component for another and the resulting combination would have performed as  
15 expected.

16 163. Finally, my own presentation at the Allerton Conference in September 1999  
17 taught that making a turbocode irregular would improve its performance. Below, I  
18 provide additional evidence that it would be obvious to a person of ordinary skill in  
19 the art that the RA code of Divsalar is a simple convolutional code and that it could  
20 be made irregular. The email below was sent to Dariush Divsalar by myself on  
21 December 8, 1999, nearly three months before the claimed date of conception of  
22 the patents-in-suit. The email mentions my paper on irregular turbocodes (Frey99)  
23 and Dariush Divsalar and Robert McEliece's work on RA codes (Divsalar), and  
24 further goes on to mention combining the two pieces of work.

---

<sup>24</sup> Divsalar described his accumulator as a convolutional code (Divsalar at 1 ("...and the inner code is a rate 1 convolutional code...")).

1 CALTECH000024021

2 From: Brendan Frey  
3 Sent: Wed 12/08/1999  
4 To: <Dariush.Divsalar@jpl.nasa.gov>  
5 Cc: <frey@dendrite.uwaterloo.ca>  
6 Bcc:  
7 Subject:

8 Hi, Dariush.

9 I'd like to get back to work on the irregular turbocodes and win some  
10 world records. Have you had a chance to look through the Allerton  
11 paper? Do you think JPL would be interested in irregular turbocodes.  
12 Have you heard back from Fabrizio about the possibility of me doing  
13 some consulting work at JPL?

14 Regardless, it would be interesting to extend the work that you and Bob  
15 have done to the case of irregular turbocodes.

16 On another subject, are you planning to submit a paper to the IEEE  
17 trans IT special issue, "Codes on Graphs and Iterative Algorithms"?

18 Brendan.

19 PS: What's the latest on what went wrong with the Mars lander? I hope  
20 it isn't being blamed on the communication system...

21 **Email from Dr. Frey to Dr. Divsalar (CALTECH000024021)**

22 164. Other similarities between Divsalar and Frey99 further motivate the  
23 combination

24 165. As I explain in this section, Divsalar teaches not only the "rate close to one"  
limitation, but also most of the remaining limitations of claim 1 of the '710 patent.  
The similarity and combinability of Frey99 and Divsalar is evidenced by the  
number of claim limitations they both teach.

i. "A method of encoding a signal"

166. To the extent that the preamble is determined to be a limitation of the claim,  
it is taught by Divsalar. Divsalar describes a "turbo-like" code called a repeat-  
accumulate code. The purpose of the disclosed repeat-accumulate code is for the  
encoding and decoding of signals. Divsalar explicitly discloses decoding signals  
that had been encoded using the disclosed repeat-accumulate code. *See, e.g.,*  
Divsalar at 2 ("Finally, in Section 6 we present performance curves for some RA  
codes, using an iterative, turbo-like, decoding algorithm. This performance is seen



1 to be remarkably good, despite the simplicity of the codes and the suboptimality of  
2 the decoding algorithm”); 9 (“Figure 4. Comparing the RA code ‘cutoff threshold’  
3 to the cutoff rate of random codes using both the classical union bound and the  
4 Viterbi-Viterbi improved union bound.”).

5 ii. “obtaining a block of data in the signal to be encoded”

6 167. Divsalar deals exclusively with block codes. The repeat-accumulate codes  
7 introduced by Divsalar are encoded by receiving an “input block” or “information  
8 block of length  $N$ ” and passing the block to the repeater (Divsalar at 5). *See also,*  
9 for example, Figure 3, reproduced above.

10 iii. “first encoding the data block to from a first encoded data  
11 block, said first encoding including repeating the data  
12 elements in different sub-blocks”

13 168. Divsalar teaches a first encoding step that includes repeating information  
14 bits, as shown in Figure 3, reproduced above.

15 169. A block of  $N$  information bits enters the coder at the left side of the figure  
16 and is provided to the repeater (labeled “rate  $1/q$  repetition”) (Divsalar at 5). The  
17 repeater duplicates each of the  $N$  information bits  $q$  times and outputs the resulting  
18  $N \times q$  repeated bits (*id.*).

19 170. While Divsalar does not teach partitioning the data block into a plurality of  
20 sub-blocks and repeating information bits in in different sub-blocks “a different  
21 number of times” (*i.e.*, irregular repetition), these limitations are taught by Frey99,  
22 as explained above.<sup>25</sup>

23 <sup>25</sup> Note that the “partitioning” and the “different number of times” limitations of claim 1 are  
24 related. Any coding scheme that repeats different information bits different numbers of times  
(such as that taught in Frey99) will *de facto* partition information bits into sub-blocks (*i.e.*, with  
bits in one sub-block being repeated one number of times and with bits in another sub-block  
being repeated a different number of times).



1                   iv.    “interleaving the repeated data elements in the first encoded  
2                                    data block”

3 171. Divsalar teaches this limitation. Figure 3 of Divsalar, reproduced above,  
4 shows a “permutation matrix” (the box labeled “P”). After the repeater duplicates  
5 each of the  $N$  information bits  $q$  times and outputs  $N \times q$  repeated bits, the repeated  
6 bits are “scrambled by an interleaver of size  $qN$ ” (Divsalar at 5).

7                   v.    Summary

8 172. As explained above, claim 1 of the '710 patent is obvious in view of the  
9 combination of Frey99 and Divsalar. In particular, it would have been obvious to  
10 use Divsalar’s accumulator in place of the convolutional encoder disclosed in  
11 Frey99.

12                   iii)   Claim 1 of the '710 Patent is Obvious Over Divsalar in View of One  
13                                    of MacKay or Luby

14 173. I explain below, limitation by limitation, why claim 1 is rendered obvious by  
15 a combination of Divsalar and either MacKay or Luby. As noted above, Divsalar  
16 teaches all but one feature of the IRA codes that Caltech claims to have invented.  
17 That is, Divsalar teaches *regular* repeat-accumulate codes instead of *irregular*  
18 repeat-accumulate codes. Adding one feature, irregularity, to Divsalar results in  
19 the claimed IRA codes. As explained below, it would have been obvious to  
20 combine the teachings of Divsalar with the irregularity taught in either of Luby or  
21 MacKay.

22 174. In this section I describe how Divsalar teaches the remaining limitations of  
23 claim 1 of the '710 patent (*i.e.*, the limitations unrelated to irregularity). I also  
24 explain that any limitation not taught by Divsalar is taught by both Luby and  
MacKay. Also, as I explain below, one of ordinary skill in the art would have been  
motivated to combine Divsalar and Luby or MacKay in general, and would  
specifically have been motivated to incorporate the one necessary feature from



1 Luby or MacKay – irregularity – forming a combination that meets every  
2 limitation of claim 1 of the ‘710 patent. Finally, I explain why such a combination  
3 would only represent a minor modification to the teachings of Divsalar, and would  
4 not fundamentally change its principle of operation or purpose.

5 a) Divsalar teaches every limitation of Claim 1 except irregularity

6 175. As I explain above (with respect to the combination of Frey99 and Divsalar),  
7 Divsalar teaches:

- 8 • “A method of encoding a signal;”
- 9 • “obtaining a block of data in the signal to be encoded;”
- 10 • “first encoding the data block to form a first encoded data block, said first  
11 encoding including repeating;”
- 12 • “interleaving the repeated data elements in the first encoded data block;” and
- 13 • “second encoding said first encoded data block using an encoder that has a  
14 rate close to one.”

15 176. The only portions of the claim that Divsalar fails to teach are: “partitioning  
16 said data block into a plurality of sub-blocks, each sub-block including a plurality  
17 of data elements” (I will call this the “partitioning” limitation); and “said first  
18 encoding including repeating the data elements in different sub-blocks a different  
19 number of times” (which I will call the “irregularity” limitation).

20 b) Both Luby and MacKay Teach the Partitioning and Irregularity  
21 Limitations

22 177. One of ordinary skill would have needed to incorporate only one feature  
23 from Luby or MacKay into Divsalar – irregularity – to form a combination that  
24 meets every limitation of claim 1 of the ‘710 patent. Below, I explain why one of  
ordinary skill would have been motivated to combine the teachings of Divsalar  
with the irregularity taught in both Luby and MacKay.

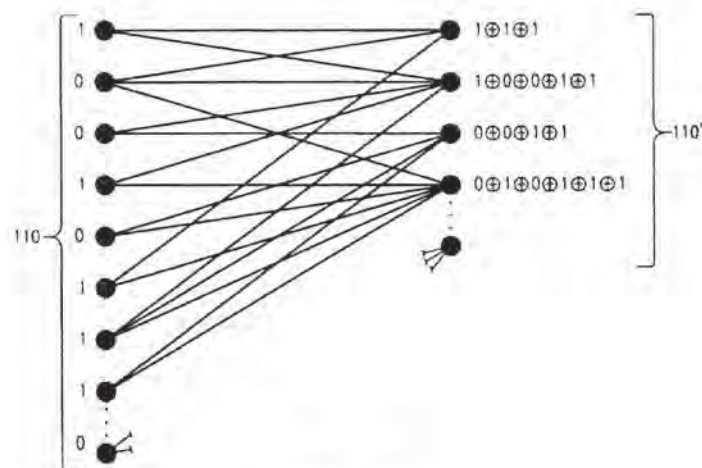
1 178. As I explain above, the partitioning limitation and the irregularity limitation  
2 are related: by repeating different information bits different numbers of times, a  
3 coding scheme *de facto* partitions information bits into sub-blocks. However, for  
4 the sake of clarity, I will discuss both limitations individually in the remainder of  
5 this subsection.

- 6 i. “partitioning said data block into a plurality of sub-blocks,  
7 each sub-block including a plurality of data elements”

8 179. Because Divsalar’s repetition is regular instead of irregular, Divsalar does  
9 not partition the blocks into sub-blocks for purposes of repetition. However, as  
10 part of their general teaching of irregularity, both Luby and MacKay teach  
11 partitioning the blocks into sub-blocks.

12 180. Luby teaches that sparse graph codes can be improved by using “irregular  
13 graphing” (*see, e.g.*, Luby at 11:23-49). The “irregular graphing” encoder used by  
14 Luby “partition[s] said data block into a plurality of sub-blocks, each sub-block  
15 including a plurality of elements.”

16 181. This process is represented graphically in Figure 17 of Luby, reproduced  
17 below:



21 FIG. 17



1 182. In this figure, the filled circles on the left represent information bits to be  
 2 encoded, and the filled circles on the right represent parity checks computed for  
 3 these information bits. In the above figure, each parity check on the right is  
 4 computed by summing together (modulo 2) all of the information bits connected to  
 5 that parity check by an edge in the graph (*see, e.g.*, Luby at 17:64-67, “[t]he  
 6 redundant data items associated with nodes at the layer 110’ are computed by an  
 7 exclusive-or operation of the message bits to which they are connected”).

8 183. As shown above, some information bits are connected to two parity checks  
 9 (*i.e.*, have a degree of two) and other information bits are connected to three parity  
 10 checks (*i.e.*, have a degree of three). Luby “partition[s] said data block into a  
 11 plurality of sub-blocks, each sub-block including a plurality of elements” by  
 12 assigning a first group of two or more input bits a first degree (*e.g.*, two) and a  
 13 second group of input bits a second degree (*e.g.*, three). In this scheme, the input  
 14 bits with a degree of two constitute one sub-block (shown below in green) and the  
 15 input bits with a degree of three constitute a second sub-block (shown below in  
 16 red):

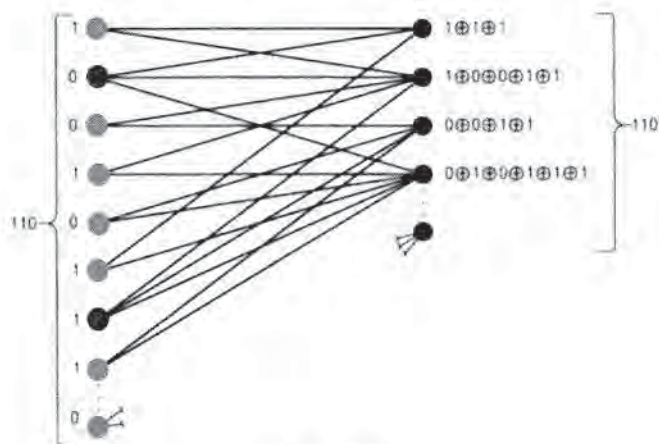


FIG. 17

1 184. Each of the sub-blocks includes at least two input bits, as shown in the  
2 colored figure above.

3 185. MacKay also teaches this limitation. MacKay builds on the earlier work of  
4 Luby to examine the properties of certain irregular Gallager codes (*see, e.g.,*  
5 MacKay at 1449). Like Luby, MacKay describes assigning different degrees to  
6 different bits: “We can define an irregular Gallager code in two steps. First, we  
7 select a *profile* that describes the desired number of columns of each weight and  
8 the desired number of rows of each weight. The parity check matrix of a code can  
9 be viewed as defining a bipartite graph with ‘bit’ vertices corresponding to the  
10 columns and ‘check’ vertices corresponding to the rows. Each nonzero entry in the  
11 matrix corresponds to an edge connecting a bit to a check. The profile specifies  
the degrees of the vertices in this graph.” (MacKay at 1449-1450).

12 186. As the passage above explains, in the parity-check matrices taught by  
13 MacKay, each information bit corresponds to a particular column, where the  
14 weight of that column (*i.e.*, the number of 1s contained in that column of the  
15 parity-check matrix) represents the degree of the information bit.<sup>26</sup> MacKay also  
16 teaches systematic codes that use constructions of parity check matrices where  
17 some columns correspond to information bits and other columns correspond to  
parity bits.

18 187. As shown in Table 1 of MacKay, reproduced below, the irregular code with  
19 “Profile 93” partitions the information blocks into two sub-blocks: one sub-block  
20 having degree 3 and the other having degree 9:

---

21  
22 <sup>26</sup> In general, depending on how the matrix is represented, a particular information bit can  
23 correspond to either a row or a column of the generator matrix. That is, if the vector of  
24 information bits is multiplied by the generator matrix on the right (denoted  $\mathbf{v}G$ ), then each  
information bit will correspond to a row of the generator matrix. Conversely, if the vector of  
information bits is multiplied by the generator matrix on the left (denoted  $G\mathbf{v}$ ), then each  
information bit will correspond to a column of the generator matrix.



TABLE I  
THE TWO PROFILES STUDIED IN THIS PAPER

Profile 3	Column weight	Fraction of columns	Row weight	Fraction
Profile 93	3	1	6	1
	3	11/12	7	1
	9	1/12		

(MacKay at 1451)

As the table above indicates,  $1/12^{\text{th}}$  of the information bits in “Profile 93” have degree 9, and the remaining  $11/12^{\text{th}}$ s of information bits have degree 3. Because the parity check matrices taught by MacKay have many more than 12 columns (*see id.*, showing “blocklength about  $N = 10\,000$ ”), each of these sub-blocks contains a plurality of bits, or data elements.

ii. “... repeating the data elements in different sub-blocks a different number of times”

188. As I explain above, Divsalar teaches repeating information bits. As shown in Figure 3, a block of  $N$  information bits enters the coder at the left side of the figure and is provided to the repeater (labeled “rate  $1/q$  repetition”) (Divsalar at 5). The repeater duplicates each of the  $N$  information bits  $q$  times and outputs the resulting  $N \times q$  repeated bits (*id.*).

189. While Divsalar does not teach repeating data elements in different sub-blocks a different number of times (*i.e.*, “irregular” repetition), one of ordinary skill in the art would have known to combine the repetition of Divsalar with the irregularity of Luby or MacKay.

190. As I explain above, Luby teaches that sparse graph codes can be improved by using “irregular graphing” (*see, e.g.*, Luby at 11:23-49). “Irregular graphing” refers to codes with Tanner graphs in which some information nodes are connected to more check nodes than others (*see, e.g., id.* at 3:27-29, stating that “different numbers of first edges are associated with the data items”). Thus, combining the



1 “irregular” encoder taught by Luby with the repetition taught by Divsalar would  
2 result in an encoder that “repeat[s] the data elements in different sub-blocks a  
3 different number of times,” as required by claim 1.

4 191. The irregularity taught by Luby is represented graphically in Figure 17. In  
5 particular, the version of Luby’s Fig. 17 reproduced above with green and red  
6 highlighting shows that some information bits (colored red) contribute to three  
7 parity checks whereas other information bits (colored green) contribute to only two  
8 parity checks.

9 192. While Luby does not explicitly teach repetition, Fig. 17 shows that  
10 information bits are *used* a different number of times. Reuse is not, in general,  
11 repetition; it is possible to reuse bits without repeating them.<sup>27</sup> However, the  
12 irregular reuse taught by Luby can be implemented using the repetition of Divsalar,  
13 as I explain in more detail below. In other words, one way to incorporate Luby’s  
14 irregularity into Divsalar was to make Divsalar’s repetition irregular.

15 193. MacKay also teaches this limitation. MacKay builds on the earlier work of  
16 Luby to examine the properties of certain irregular Gallager codes (*see, e.g.*,  
17 MacKay at 1449). Like Luby, MacKay describes assigning different degrees to  
18 different information bits: “[w]e can define an irregular Gallager code in two steps.  
19 First, we select a profile that describes the desired number of columns of each  
20 weight and the desired number of rows of each weight. The parity check matrix of  
21 a code can be viewed as defining a bipartite graph with ‘bit’ vertices corresponding  
22 to the columns and ‘check’ vertices corresponding to the rows. Each nonzero entry  
23 in the matrix corresponds to an edge connecting a bit to a check. The profile  
24 specifies the degrees of the vertices in this graph” (MacKay at 1449-1450).

---

<sup>27</sup> I understand that the Plaintiff attempted to argue that the two terms are synonymous, but the Court was correctly not persuaded by this argument. *See* Claim Construction Order (Dkt. No. 105) at 11 (“Caltech argues that “repeat” can also refer to the re-use of a bit, but the patent’s claims and specification support the Court’s construction”).



1 194. As the passage above explains, in the parity-check matrices taught by  
2 MacKay, each information bit corresponds to a particular column, where the  
3 weight of that column (*i.e.*, the number of 1s contained in that column of the  
4 parity-check matrix) represents the degree of the information bit.

5 195. As I explain above with reference to Table 1 of MacKay, reproduced above,  
6 the irregular code with “Profile 93” taught by MacKay effectively partitions the  
7 information blocks into two sub-blocks: one sub-block having degree three and the  
8 other having degree nine. The information bits in the first sub-block contribute to  
9 three parity checks, while the information bits in the second sub-block contribute

10 196. Like the scheme taught by Luby, the codes taught by MacKay involve  
11 irregular *reuse* of information bits. As I explain above, reuse is not, in general,  
12 repetition. However, as with Luby, the irregularity taught by MacKay can be  
13 implemented using the repetition of Divsalar (and one way to incorporate  
14 MacKay’s irregularity into Divsalar was to make the repetition irregular), as I  
15 explain in more detail below.

16 c) *Motivations to combine the teachings of Divsalar with those of  
Luby or MacKay, generally*

17 197. Divsalar, Luby and MacKay are directed to the same field, namely, the field  
18 of error-correcting codes. Further, all three references are related to variations and  
19 improvements on linear error-correcting codes, and in particular to error-correcting  
20 codes that can be encoded quickly. *See, e.g.*, Divsalar at 1 (referring to “practical  
21 encoding and decoding algorithms”) (emphasis added); *see also* Luby at 2:51-55  
22 (“it is an objective of the present invention to provide a technique for creating loss  
23 resilient and error correcting codes which substantially reduce the time required to  
24 encode and decode messages”) (emphasis added); *see also* MacKay at 1449  
 (“whereas Gallager codes normally take  $N^2$  time to encode, we investigate



1 constructions of regular and irregular Gallager codes that allow more rapid  
2 encoding and have smaller memory requirements in the encoder”) (emphasis  
3 added). Accordingly, one of ordinary skill would have been aware of all the  
4 references and further would have understood that the teaching of one reference  
5 would inform that of the others. That is, one of ordinary skill would have expected  
6 to apply the teachings of the references to each other.

7 d) Motivations to Incorporate the irregularity of Luby or MacKay  
8 into the RA codes of Divsalar

9 198. Both Luby and MacKay are related to modifying known regular codes by  
10 introducing irregularity. MacKay notes that “[t]he best known binary Gallager  
11 codes are irregular codes” (MacKay at 1449), explaining that “[t]he excellent  
12 performance of irregular Gallager codes is the motivation for this paper, in which  
13 we explore ways of further enhancing these codes” (*id.*). Similarly, Luby shows  
14 that incorporating irregularity into known regular codes can improve performance  
15 (*see, e.g.*, Luby at 21:52-55, stating that “the failure rate using the [irregular]  
16 techniques described above provide a much lower failure rate than those obtainable  
17 with regular graphing of the left and right nodes utilized in conventional error  
18 correction encoding”). In view of the fact that both Luby and MacKay teach how  
19 regular codes can be improved by the introduction of irregularity, one of ordinary  
20 skill in the art would have been motivated to incorporate irregularity into the  
21 regular repeat-accumulate codes of Divsalar.

22 199. Luby’s work on irregularity is fundamental to the field of coding,  
23 representing a major advance in coding theory with broad applicability across  
24 various types of codes. By the time the patents-in-suit were filed, a person of  
ordinary skill in the art would know that regular codes could be improved by the  
addition of irregularity.



1 200. Consistent with this view, Aamod Khandekar, one of the inventors named on  
2 the patents-in-suit, wrote in his Ph.D. thesis that “Luby et al. also introduced the  
3 concept of irregularity, which seems to provide hope of operating arbitrarily close  
4 to channel capacity in a practical manner, on a wide class of channel models”  
5 (Khandekar Thesis at 2).<sup>28</sup> Khandekar hails “the introduction of irregular LDPC  
6 codes by Luby et al.” as a “major breakthrough” (*id.* at 46) and states that IRA  
7 codes were merely an application of Luby’s “concept of irregularity to the  
ensemble of RA codes” as described in Divsalar (*id.* at 47; *see also id.* at 51).

8 201. For at least these reasons, it would have been obvious to one of ordinary  
9 skill in the art to incorporate irregularity into the RA codes of Divsalar.

10 e) *Incorporating the irregularity of Luby or MacKay into the RA*  
11 *codes of Divsalar would not have been difficult*

12 202. Incorporating irregularity into the RA codes of Divsalar would have been  
13 simple for one of ordinary skill; *i.e.*, one of ordinary skill would have converted the  
14 regular repeater shown in Figure 3 of Divsalar (reproduced above) into an  
15 *irregular* repeater. This modification would allow the other two components of the  
encoder – the interleaver and the accumulator – to remain unchanged.<sup>29</sup>

16 203. Divsalar teaches repeating each information bit  $q$  times (*see* Divsalar at 5).  
17 Using an irregular repeater that repeats some information bits more than others,  
18 and then interleaving and accumulating the irregularly repeated bits, would  
19 naturally result in an irregular code.

20 <sup>28</sup> When Khandekar refers to “Luby et al.,” he is referring to Michael G. Luby and several of his  
21 colleagues. Michael G. Luby is the first-named inventor on the Luby reference. Khandekar cites  
22 four separate academic articles by Luby in his graduate thesis, which deals with the subject of  
IRA codes (*see* Khandekar Thesis at 103).

23 <sup>29</sup> Minor modifications could be made to the interleaver to account for interleaving a different  
24 number of bits. However, such modifications would not strictly be necessary. For example, if  
Divsalar’s “repeat every bit  $q$  times” strategy were changed such that one bit was repeated  $q+1$   
times and another bit were repeated  $q-1$  times, the repeat would be irregular and the interleaver  
would still deal with the same number of bits per block.



1 204. Nor would this modification have been challenging from a technological  
2 standpoint. Repeaters – whether regular or irregular – are conventional  
3 components that have been used for decades in a wide variety of digital  
4 electronics.<sup>30</sup> Modifying an existing encoder to replace a regular repeater with an  
5 irregular one would be a simple matter for one of ordinary skill in the art. Also,  
6 modifying the message passing decoder would be a simple matter for one of  
7 ordinary skill in the art, since the rules of deriving the decoder from the Tanner  
graph were broadly understood at the time.

8 205. Further, such a modification would preserve the simplicity of the RA codes  
9 taught by Divsalar. As I explain above, Divsalar introduced RA codes specifically  
10 because they are simple enough to analyze mathematically. IRA codes do not  
11 significantly add to the complexity of RA codes in this respect.

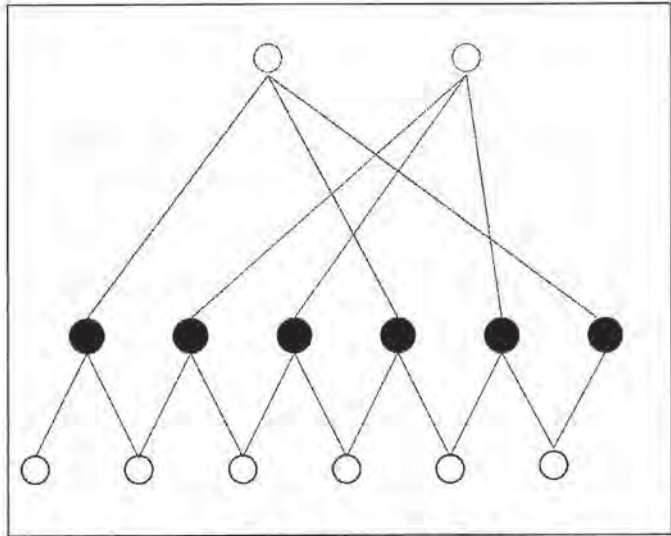
12 206. Indeed, IRA codes are so similar to RA codes that the Tanner graph  
13 representing any RA code can be modified to represent an IRA code by the  
14 addition of a single edge. For example, the figure below, taken from a presentation  
15 delivered by Aamod Khandekar, one of the inventors of the patents-in-suit,  
16 corresponds to an RA code in which each information node (the hollow circles at  
the top) contributes to three parity checks (represented by filled circles):

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24 <sup>30</sup> As confirmed by, *e.g.*, the testimony of Stephen Wicker (*see* Wicker Tr. at 67).

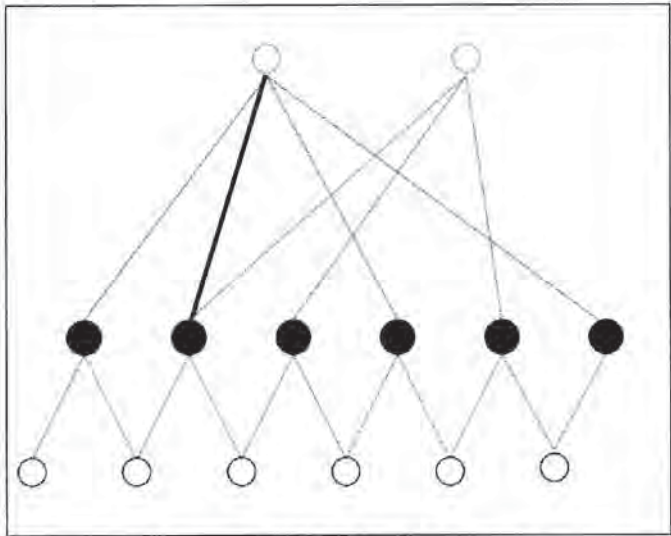


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Tanner Graph of an RA Code (CALTECH000007326)

207. This RA code can be turned into an IRA code by adding a single edge (shown in red below). Addition of that single edge (shown in red) makes the bit be repeated four times instead of three.<sup>31</sup>



Tanner Graph of an IRA Code

208. It would have been obvious to one of ordinary skill in the art to incorporate irregularity into Divsalar's RA code by making the repeater irregular. Divsalar's RA code repeater is a simple, obvious, and straightforward component to which to

<sup>31</sup> As confirmed by, e.g., the testimony of Dariush Divsalar (see Divsalar Tr. at 248-249).

1 apply irregularity. As shown by the two Tanner graphs above, making the repeat  
2 irregular is exceedingly simple and does not overly complicate the code  
3 analytically. Choosing to make the repeater irregular is one of a finite number of  
4 identified, predictable ways to improve the performance of the code (which is the  
5 purpose of making a code irregular as taught by Frey99, MacKay, and Luby).<sup>32</sup>

6 209. Finally, as I explain in the remainder of this section, Luby and MacKay  
7 teach not only the partitioning and irregular repetition limitations, but several of  
8 the other limitations of claim 1 of the '710 patent as well. The similarity and  
9 combinability of Divsalar and Luby or MacKay is evidenced by the number of  
claim limitations they all teach.

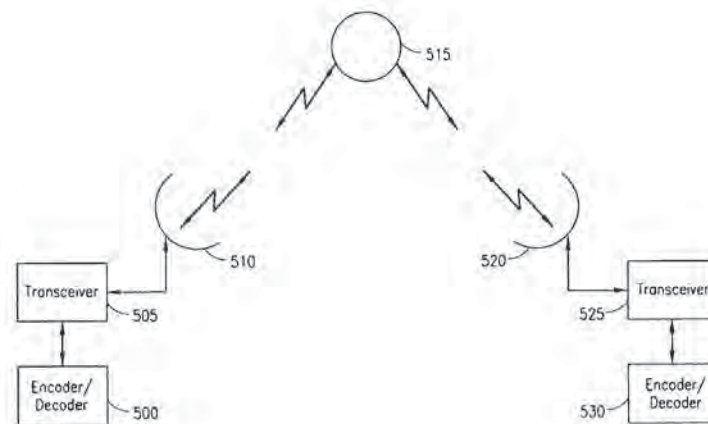
10 i. “A method of encoding a signal”

11 210. The preamble is taught by MacKay and Luby. As I explain above, MacKay  
12 describes both regular and irregular Gallager codes. The purpose of the disclosed  
13 Gallager codes is for the encoding and decoding of signals. MacKay explicitly  
14 discloses decoding signals that had been encoded using the disclosed Gallager  
15 codes. *See, e.g.*, MacKay at 1451 (“In the experiments presented here, we study  
16 binary codes with rate 1/2 and blocklength about  $N = 10\,000$ . We simulate an  
17 additive white Gaussian noise channel in the usual way [2] and examine the block  
18 error probability as a function of the signal-to-noise ratio. The error bars we show  
19 are one standard deviation error bars on the estimate of the logarithm of the block  
20 error probability  $p$  defined”); *id.* (“Fig. 3 (a) Comparison of one representative of  
each of the constructions ... (b) Representatives of all six constructions in Fig. 2”).

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23 <sup>32</sup> There are of course many options for making the repeat irregular (*e.g.*, repeat one bit one more  
24 times than the others, or use degree profiles suggested by Luby or MacKay) and a person of  
ordinary skill would have been motivated to design a particular code that had good performance.  
However, the decision to incorporate irregularity itself into the repeater was an easy one.



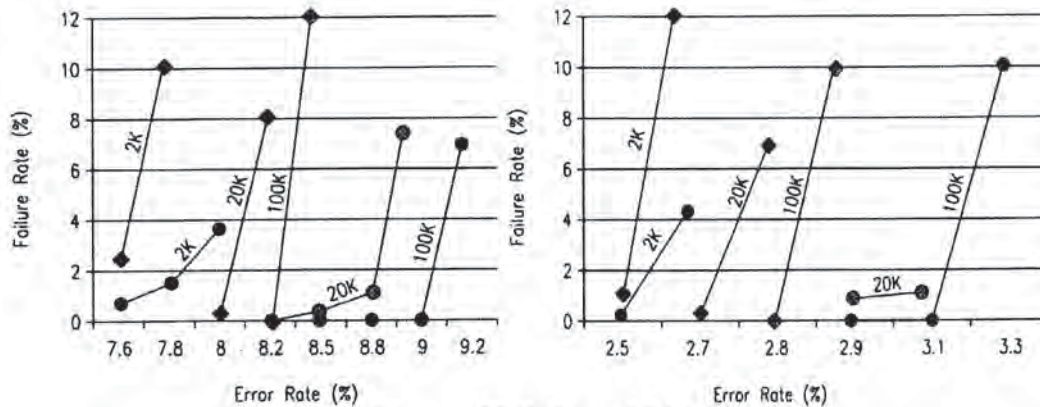
1 211. The preamble is also taught by Luby. As I explain above, Luby introduces  
2 irregularity to codes. The purpose of the disclosed irregular codes is for the  
3 encoding and decoding of signals. Luby explicitly discloses decoding signals that  
4 had been encoded using the disclosed codes. *See, e.g.*, Luby at Figs. 23, 24  
5 (showing percent failure rate vs. percent error rate for various codes); *see also id.*  
6 at Fig. 25 (reproduced below, showing a signal being encoded, modulated,  
7 transmitted, received, and decoded):



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14 **Luby, Fig. 25**

15 ii. “obtaining a block of data in the signal to be encoded”

16 212. Luby deals exclusively with block codes. For example, Figures 23 and 24 of  
17 Luby show experimental results comparing codes of various block lengths (*i.e.*,  
18 block lengths of 2K, 20K, or 100K bits):  
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Luby, Figures 23 (left) and 24 (right)

213. Mackay also deals with block codes. For example, Figure 1 of MacKay, reproduced below, shows experimental results relating to codes of various block lengths (*see, e.g.*, the block lengths of 24,000 and 65,536 bits identified in the caption of Figure 1):

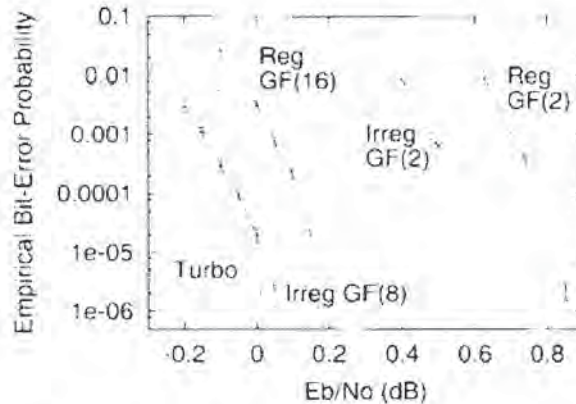


Fig. 1. Empirical results for Gaussian channel. rate 1/4 left-right: irregular LDPC,  $GF(8)$  blocklength 24000 bits; JPL Turbo, blocklength 65536 bits; regular LDPC,  $GF(16)$ , blocklength 24448 bits; irregular LDPC,  $GF(2)$ , blocklength 64000 bits; regular LDPC,  $GF(2)$ , blocklength 40000 bits. (Reproduced from [1].)

MacKay, Figure 1

214. As the figures above indicate, each of the codes taught by Luby and MacKay are associated with a “block length.” The “block length” of a code is the number of bits, or “data elements,” contained in the group of information bits that is encoded as a unit.



1           **B. Claim 3 of the '710 Patent is Invalid**

2 215. Claim 3 of the '710 patent reads:

3                   3. The method of claim 1, wherein said first encoding is carried out  
4                   by a first coder with a variable rate less than one, and said second  
5                   encoding is carried out by a second coder with a rate substantially  
6                   close to one.

7           i)       Claim 3 of the '710 Patent is Anticipated by Frey99

8 216. As I explain above, Frey99 teaches every limitation of claim 1. Frey also  
9 teaches the limitations added by claim 3, namely that the “first encoding is carried  
10 out by a first coder with a variable rate less than one” and that the “second  
11 encoding is carried out by a second coder with a rate substantially close to one.”

12 217. Frey99’s first coder is the collection of blocks labeled “Rep2,” “Rep 3,” to  
13 “Rep D” in Figure 2. The rate of that encoder is a “variable rate less than one.”  
14 Because the number of times bits are repeated varies from 1 to  $D$  (*see, e.g.*, Frey99  
15 at Figure 2; *see also* Frey Slides at 5), the rate of the first encoder varies within a  
16 block between 1 and  $1/D$ , where  $D$  may be set as high as desired. Also, because in  
17 an irregular turbocode some bits are duplicated at least once, the rate of the first  
18 encoder is always less than or equal to 1, and so the first encoder always has a rate  
19 less than one.

20 218. In the paragraph immediately above, I interpreted “variable rate” to refer to  
21 an encoder with a rate that varies within a block. As I explain above, under this  
22 interpretation of “variable rate,” the encoder taught by Frey99 is a “variable rate”  
23 encoder. However, if “variable rate” were construed to mean that the rate of the  
24 encoder varies from block to block, then this claim would have been obvious in  
view of Frey99 because changing the rate of a code over time would have been  
easy for one of ordinary skill. “Variable rate,” however, should not be construed to  
mean that the rate of the encoder varies from block to block because such an  
interpretation is not supported by the specification of the patents.



1 219. Frey99 also teaches a second coder with a rate “substantially close to one.”  
2 As described above, Frey99 teaches a convolutional coder with a rate  $R' \approx 0.74$ .  
3 This is a rate “substantially close to one.”

4 220. In summary, Frey99 teaches each and every limitation of claim 3 and  
5 therefore anticipates it.

6 ii) Claim 3 of the '710 Patent is Obvious Over Frey99 in View of  
7 Divsalar and Over the Frey Slides in View of Divsalar

8 221. As I explain above, the combination of Frey99 and Divsalar teaches every  
9 limitation of claim 1.

10 222. Even if Frey99 is found not to teach a second encoder with a rate  
11 “substantially close to one,” this limitation is taught by Divsalar. As explained  
12 above, the “second coder” of Divsalar is an accumulator with a rate exactly equal  
13 to 1.<sup>33</sup> It would have been obvious to one of ordinary skill in the art to combine the  
14 teachings of Frey99 with those of Divsalar, also for the reasons given above.

15 223. Therefore, if Frey99 is found to not teach a second coder with a rate  
16 substantially close to one, then claim 3 is obvious over the combination of Frey99  
17 and Divsalar.

18 iii) Claim 3 of the '710 Patent is Obvious Over Divsalar in View of One  
19 of Luby or MacKay

20 224. As I explain above, Divsalar combined with either Luby or MacKay renders  
21 claim 1 of the '710 patent obvious.

22 225. Divsalar in combination with either of Luby or MacKay also teaches a “first  
23 coder with a variable rate less than one.” The “first encoding” step taught by these  
24 combinations of references is “irregular repetition,” in which different information

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24 <sup>33</sup> As confirmed by the testimony of Hui Jin, one of the inventors listed on the patents-in-suit (*see*  
Jin Tr. at 122).



1 bits are repeated different numbers of times (*i.e.*, Divsalar’s repeater modified by  
2 the irregular teaching of Luby or MacKay).

3 226. The rate of the first encoder taught by these combinations of references is  
4 less than one. Because the first encoder is based on the principle of repetition, it  
5 always outputs more bits than it accepts as input, because it outputs multiple  
6 duplicates of each information bit. As explained above, the “rate” of an encoder is  
7 the ratio between the number of input bits and the number of output bits, so the rate  
8 of a repetition-based encoder is always less than one.

9 227. The rate of the first encoder taught by these combinations of references is  
10 also “variable.” By combining the repetition of Divsalar with the irregularity of  
11 Luby or MacKay, we obtain an encoder that repeats different information bits  
12 different numbers of times. Therefore, depending on the particular information bit  
13 being encoded, the ratio of input bits to output bits – *i.e.*, the rate of the first  
14 encoder – varies.

15 228. Further, as explained above, Divsalar also teaches a second coder with a rate  
16 equal to one, and thus, a rate “substantially close to one” as required by claim 3 of  
17 the ’710 patent.<sup>34</sup>

18 229. As I explain in detail above, it would have been obvious to incorporate the  
19 irregularity of Luby or MacKay (*i.e.*, a “variable rate encoder”) with the repeat-  
20 accumulate codes taught by Divsalar. Thus, the combination of Divsalar with  
21 either Luby or MacKay renders claim 3 of the ’710 patent obvious.

22 230. Finally, as explained above, although “variable rate” should not be construed  
23 to mean that the rate of the encoder varies from block to block, the claim would  
24 still be obvious over Divsalar in view of either MacKay or Luby because changing  
the rate of a code over time would have been easy for one of ordinary skill.

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<sup>34</sup> As confirmed by, *e.g.*, the testimony of Hui Jin (*see* Jin Tr. at 122).

1           **C. Claim 4 of the '710 Patent is Invalid**

2 231. Claim 4 of the '710 patent reads:

3                   4. The method of claim 3, wherein the second coder comprises an  
4 accumulator.

5           i)     Claim 4 of the '710 Patent is Obvious Over Frey99 in View of  
6 Divsalar

7 232. As I explain above, the combination of Frey99 and Divsalar teaches every  
8 limitation of claim 3. Claim 4 adds to claim 3 that “the second coder comprises an  
9 accumulator.” As explained above, Divsalar teaches that the second coder is an  
10 accumulator and it would have been obvious to use Divsalar’s accumulator in  
11 Frey99. Claim 4 is therefore obvious over the combination of Frey99 and Divsalar.

12           ii)    Claim 4 of the '710 Patent is Obvious Over Divsalar in View of One  
13 of Luby or MacKay

14 233. As I explain above, the combination of Divsalar with one of Luby or  
15 MacKay teaches every limitation of claim 3. I have also explained that the  
16 “second coder” of Divsalar is an “accumulator,” as required by claim 4.

17           **D. Claim 5 of the '710 Patent is Invalid**

18 234. Claim 5 of the '710 patent reads:

19                   5. The method of claim 4, wherein the data elements comprises  
20 bits.

21           i)     Claim 5 of the '710 Patent is Obvious Over Frey99 in View of  
22 Divsalar

23 235. As I explain above, the combination of Frey99 and Divsalar teaches every  
24 limitation of claim 4. Claim 5 adds to claim 4 that “the data elements comprise  
bits.” Both Frey99 and Divsalar teach methods of encoding signals in which the  
“data elements” comprise bits.



1 236. For example, Frey99 teaches codes in which “Each codeword *bit* with  
2 degree  $d$  is repeated  $d$  times before being fed into the permuter” (Frey99 at 2)  
3 (emphasis added) (*see also* Frey Slides at 4, showing “parity bits” and “systematic  
4 bits”, and at 5 in which open circles also represent bits). Divsalar teaches a  
5 “*binary linear* ( $n, k$ ) block code” (Divsalar at 2) (emphasis added). One of  
6 ordinary skill in the art would understand Divsalar’s “binary” block code is a code  
in which the input data elements are “binary digits” or “bits.”

7 ii) Claim 5 of the '710 Patent is Obvious Over Divsalar in View of One  
8 of Luby or MacKay

9 237. As I explain above, the combination of Divsalar with one of Luby or  
10 MacKay teaches every limitation of claim 4. I have also explained that Divsalar  
11 teaches methods of encoding signals in which the “data elements” comprise bits.

12 238. Further, both Luby and MacKay teach encoding systems and methods that  
13 operate on bits. *See, e.g.*, Luby at 3:17-20 (“a method is provided for encoding a  
14 message having a plurality of data items, e.g. message packets or data bits”); *see*  
also, e.g., MacKay at Figure 1.

15 **E. Claim 6 of the '710 Patent is Invalid**

16 239. Claim 6 of the '710 patent reads:

17 6. The method of claim 5, wherein the first coder comprises a  
18 repeater operable to repeat different sub-blocks a different number  
of times in response to a selected degree profile.

19 i) Claim 6 of the '710 Patent is Obvious Over Frey99 in View of  
20 Divsalar

21 240. As I explain above, the combination of Frey99 and Divsalar teaches every  
22 limitation of claim 5. Further, Frey99 teaches the limitation added by claim 6, *i.e.*,  
23 repeating “different sub-blocks a different number of times in response to a  
24 selected degree profile.” As I explain in Frey99, “an *irregular turbocode* has the  
form shown in Fig. 2, which is a type ‘trellis-constrained code’ as described in [7].



1 We specify a *degree profile*,  $f_d \in [0, 1]$ ,  $d \in \{1, 2, \dots, D\}$ .  $f_d$  is the fraction of  
2 codeword bits that have degree  $d$  and  $D$  is the maximum degree. Each codeword  
3 bit with degree  $d$  is repeated  $d$  times before being fed into the permuter” (Frey99 at  
4 2) (emphasis in original) (*see also* Frey Slides at 5, titled “Rate-degree relations  
5 and 6, titled “Simplified degree profiles”).

6 241. The “degree profile” described in the above passage from Frey99 determines  
7 what fraction of information bits are repeated  $d$  times, for all relevant values of  $d$   
8 (*see also, e.g.*, Frey Slides at 6, “Degree  $d_e$ : Fraction  $f_e$  ‘elite’ bits have degree  $d_e$ ”).

9 ii) Claim 6 of the '710 Patent is Obvious Over Divsalar in View of One  
10 of Luby or MacKay

11 242. As I explain above, the combination of Divsalar with one of Luby or  
12 MacKay teaches every limitation of claim 5. Further, these combinations also  
13 teach the limitation added by claim 6.

14 243. MacKay teaches constructing an irregular Gallager code by selecting a  
15 degree profile: “We can define an irregular Gallager code in two steps. First, we  
16 select a *profile* that describes the desired number of columns of each weight and  
17 the desired number of rows of each weight. The parity check matrix of a code can  
18 be viewed as defining a bipartite graph with ‘bit’ vertices corresponding to the  
19 columns and ‘check’ vertices corresponding to the rows. Each nonzero entry in the  
20 matrix corresponds to an edge connecting a bit to a check. The profile specifies  
21 the degrees of the vertices in this graph” (MacKay at 1449-1450) (emphasis in  
22 original).

23 244. Luby also teaches constructing an irregular code by selecting a degree  
24 profile. This process is represented graphically in Figure 17 of Luby, reproduced  
below:



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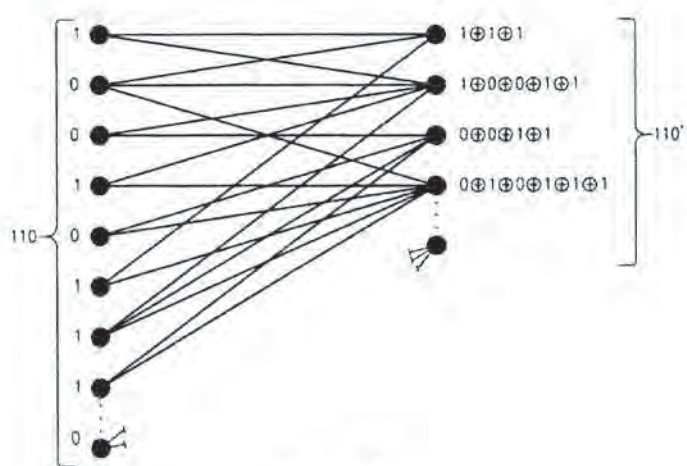


FIG. 17

245. This figure represents a degree profile for an irregular error-correcting code. As shown above, some information bits are connected to two parity checks (*i.e.*, have a degree of two) and other information bits are connected to three parity checks (*i.e.*, have a degree of three). A person of ordinary skill in the art would understand that assigning a first group of two or more input bits a first degree (*e.g.*, two) and a second group of input bits a second degree (*e.g.*, three) is selection of a “degree profile” for the code.<sup>35</sup>

246. By combining the repetition of Divsalar with the degree profiles taught by either Luby or MacKay, one of ordinary skill in the art would obtain “a repeater operable to repeat different sub-blocks a different number of times in response to a selected degree profile,” as required by claim 6 of the ’710 patent.

247. Further, as I explain in detail above, it would have been obvious to incorporate the irregular degree profiles of Luby or MacKay with the repeat-accumulate codes taught by Divsalar. Thus, the combination of Divsalar with either Luby or MacKay renders claim 6 of the ’710 patent obvious.

<sup>35</sup> This is consistent with the testimony of Dariush Divsalar (*see, e.g.*, Divsalar Tr. at 143-146).

1           **F. Claim 15 of the '710 Patent is Invalid**

2 248. Claim 15 of the '710 patent reads as follows:

3           15. A coder comprising:

4           a first coder having an input configured to receive a stream of bits,  
5           said first coder operative to repeat said stream of bits irregularly  
6           and scramble the repeated bits; and

7           a second coder operative to further encode bits output from the  
8           first coder at a rate within 10% of one.

9           i)    Claim 15 of the '710 Patent is Obvious Over Frey99 in View of  
10           Divsalar

11 249. I explain below that Frey99 teaches every limitation of claim 15 of the '710  
12 patent except the requirement that the second coder encode bits “at a rate within 10%  
13 of one.” Also, as explained above with respect to claim 1, Divsalar teaches a  
14 second coder, *i.e.*, an accumulator, that has a rate of exactly one, and it would have  
15 been obvious to use Divsalar’s accumulator in Frey99. Therefore, claim 1 is  
16 obvious in view of the combination of Frey99 and Divsalar.

17           a)    Frey99 teaches every limitation of Claim 1 except “a rate within  
18           10% of one”

19           i.    “A coder comprising ...”

20 250. Even if the preamble limits the claim, it is taught by Frey99. As I explain  
21 above, Frey99 deals with the construction of irregular turbocodes. A person of  
22 ordinary skill in the art would recognize that these turbocodes encode information  
23 bits using “a coder.” Further, in the experimental results disclosed in Frey99 (and  
24 the Frey Slides) (*e.g.*, as identified above with respect to the preamble of claim 1),  
the encoded bits were produced by a coder.



- 1                   ii. “a first coder having an input configured to receive a stream  
2 of bits, said first coder operative to repeat said stream of bits  
3 irregularly and scramble the repeated bits”

4 251. Frey99 teaches this limitation. As explained above in the context of claim 1  
5 of the '710 patent, Frey99 teaches a first coder that irregularly repeats bits. Frey99  
6 further teaches that the irregularly repeated bits are passed as input to a permuter,  
7 which scrambles the repeated bits.

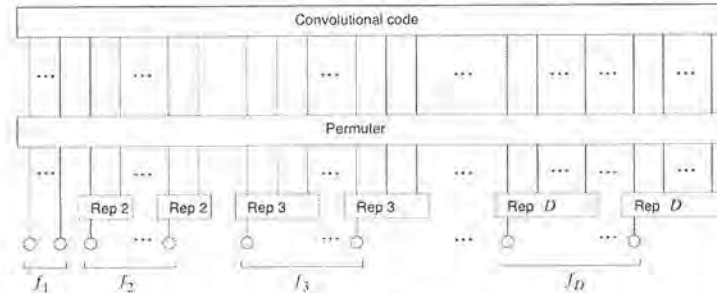
8 252. A “stream” of bits, as that term is used by those of ordinary skill in the art, is  
9 merely a sequence of bits. Block encoders like the encoders taught by Frey99, and  
10 the ones described in the specification of the patents-in-suit, receive a “stream” of  
11 bits and partition that stream into blocks of bits. Each block of bits is then encoded  
12 by a first encoder, the encoded bits are then interleaved, and the interleaved bits are  
13 encoded by a second encoder, producing a codeword. One of ordinary skill in the  
14 art would thus understand that the methods and systems taught in Frey99 operate  
15 on a stream of bits.<sup>36</sup>

- 16                   iii. “a second coder operative to further encode bits output from  
17 the first coder”

18 253. Frey99 teaches “a second coder operative to further encode bits output from  
19 the first encoder.” The “second coder” taught by Frey99 is a convolutional  
20 encoder, which accepts irregularly repeated and permuted bits as input and encodes  
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22 <sup>36</sup> I understand that Caltech has accused DVB-S2 LDPC encoders of infringement. The DVB-  
23 S2 LDPC encoder is a block encoder that operates on fixed size blocks. If by “stream,” Caltech  
24 meant an un-partitioned continuous set of bits, then the “stream” limitation could not be  
infringed. I therefore understand “stream” in the asserted claims to mean a sequence of bits.  
Even in the absence of considerations of DVB-S2, “sequence of bits” is the meaning one of  
ordinary skill would assign to “stream” in the asserted patents. I note that in the *Inter Partes*  
*Review*, Prof. Pfister considered the alternate interpretation of “stream,” i.e., an un-partitioned  
continuous set of bits. Even under that interpretation, the claims using the “stream” limitation  
would be obvious in view of the references addressed herein. However, for the reasons  
explained herein, Caltech must interpret “stream” to cover block codes to preserve its  
infringement case and therefore under Caltech’s application of the claims, references that  
describe block codes meet the “stream” limitation.

1 these bits to produce parity bits, as shown in Figure 2, reproduced below (*see also*  
2 Frey Slides at 5):



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Figure 2: A general *irregular turbocode*. For  $d = 1, \dots, D$ , fraction  $f_d$  of the codeword bits are repeated  $d$  times, permuted and connected to a convolutional code.

b) “at a rate within 10% of one”

254. As I explain above, the accumulator of Divsalar is a “second encoder” with a rate that is exactly equal to 1.

c) One of ordinary skill in the art would have been motivated to combine Divsalar’s accumulator with the irregular turbocodes of Frey99

255. As I explain above, one of ordinary skill in the art would have been motivated to combine Divsalar and Frey99 in general, and would specifically have been motivated to use the accumulator of Divsalar in Frey99.

d) The combinability of Frey99 and Divsalar is further demonstrated by Divsalar’s teaching of other limitations of claim 15

256. As I explain in this section, Divsalar teaches not only a “rate within 10% of one,” but also most of the remaining limitations of claim 15 of the ‘710 patent. The similarity and combinability of Frey99 and Divsalar is evidenced by the number of claim limitations they both teach.



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i. “A coder comprising ...”

257. Divsalar teaches the preamble. As I explain above, Divsalar describes a “turbo-like” code called a repeat-accumulate code. A “coder” capable of encoding information bits using a repeat-accumulate code is shown in Figure 3 of Divsalar, reproduced above.

ii. “a first coder having an input configured to receive a stream of bits, said first coder operative to repeat said stream of bits”

258. As explained above with reference to claim 1 of the '710 patent, Divsalar teaches a first coder that repeats bits. While Divsalar does not teach repeating the bits “irregularly,” it would have been obvious to one of ordinary skill in the art to combine the repetition of Divsalar with the irregular repetition of Frey99, as I explained above with reference to claim 1 of the '710 patent.

iii. “and scramble the repeated bits”

259. Divsalar teaches this limitation. Figure 3 of Divsalar, reproduced above, shows a “permutation matrix” (the box labeled “P”). As I explain in detail above, after the repeater duplicates each of the  $N$  information bits  $q$  times and outputs  $N \times q$  repeated bits, the repeated bits are “scrambled by an interleaver of size  $qN$ ” (Divsalar at 5).

ii) Claim 15 of the '710 Patent is Obvious Over Divsalar in View of One of Luby or MacKay

260. Claim 15 is rendered obvious by a combination of Divsalar and either MacKay or Luby. As noted above, Divsalar teaches all but one feature of the IRA codes that Caltech claims to have invented. That is, Divsalar teaches *regular* repeat-accumulate codes instead of *irregular* repeat-accumulate codes. Adding one feature, irregularity, to Divsalar results in the claimed IRA codes. As explained in detail above, with reference to claim 1 of the '710 patent, it would

1 have been obvious to combine the teachings of Divsalar with the irregularity taught  
2 in either of Luby or MacKay.

3 a) Divsalar teaches every limitation of Claim 15 except irregularity

4 261. As I explain above, Divsalar teaches:

- 5
- 6 • “A coder comprising: ...”
  - 7 • “a first coder having an input configured to receive a stream of bits, said first  
8 coder operative to repeat said stream of bits ...”
  - 9 • “... and scramble the repeated bits;” and
  - 10 • “a second coder operative to further encode bits output from the first coder  
11 at a rate within 10% of one”

12 262. The only portion of the claim that Divsalar fails to teach is: “... repeat[ing]  
13 said stream of bits *irregularly* ....” (the “irregularity” limitation).<sup>37</sup>

14 b) Both Luby and MacKay teach the irregularity limitation

15 263. As explained above with reference to claim 1, Luby and MacKay each teach  
16 irregularity and one of ordinary skill would have been motivated to incorporate  
17 that irregularity into Divsalar. Doing so results in a combination that teaches all  
18 limitations of claim 15.

### 19 **G. Claim 20 of the '710 Patent is Invalid**

20 264. Claim 20 of the '710 patent reads as follows:

21 20. The coder of claim 15, wherein the first coder comprises a low-  
22 density generator matrix coder.

23 i) Claim 20 of the '710 Patent is Obvious Over Frey99 in View of  
24 Divsalar

25 265. As I explained above, the combination of Frey99 with Divsalar teaches  
26 every limitation of claim 15. Both Frey99 and Divsalar also teach the limitation

27 \_\_\_\_\_  
28 <sup>37</sup> To be clear, Divsalar does teach “repeating said stream of bits,” but does not teach “repeating  
29 said stream of bits *irregularly*.”



1 added by claim 20, *i.e.*, that the “first coder comprises a low-density generator  
2 matrix coder.”

3 266. As explained in Appendix A, a generator matrix is a mathematical  
4 representation of an encoder that represents how information bits are transformed  
5 into encoded bits. A generator matrix is a two-dimensional array of 1s and 0s. A  
6 “low-density” generator matrix is a matrix with a relatively small number of 1s  
7 compared to the number of 0s.<sup>38</sup>

8 267. The generator matrix associated with a “repeat” encoder (whether regular, as  
9 taught by Divsalar, or irregular, as taught by Frey99) is a low-density generator  
10 matrix. For example, the following is a generator matrix that can be used to repeat  
11 each information bit three times:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

16 268. In this matrix, the rows correspond to bits input to the LDGM encoder: the  
17 first row corresponds to a first bit input to the encoder, the second row corresponds  
18 to a second input bit, the third to the third, and so on. Because each column of this  
19 matrix contains only a single “1,” each parity bit produced by this matrix will be a  
20 duplicate (or “repeat”) of one of the input (or “information”) bits. If a column  
21 contained more than a single “1,” then the corresponding parity bit would be a

22 \_\_\_\_\_  
23 <sup>38</sup> Caltech agrees with this interpretation of “low-density.” As Caltech explains in its *Markman*  
24 tutorial, “[m]atrices with contain mostly zeroes and very few ones are called sparse matrices or  
low-density matrices” (Dkt. No. 85 at 14:13-14; *see also* Wicker Tr. at 60; *see also, e.g.*, Jin Tr.  
at 174).



1 combination of (or sum of) information bits, but this matrix contains no such  
2 columns.

3 269. The number of repeated bits generated by this encoder is defined by the  
4 number of “1s” appearing in each input bit’s row. Using the generator matrix  
5 above, encoding a stream of input bits beginning with “101...” would result in an  
6 encoded sequence of bits that begins “111000111...”

7 270. As the example above shows, a generator matrix corresponding to a repeat  
8 encoder has exactly one “1” per column. Thus, a  $k \times n$  repeater matrix, with  $k$   
9 rows and  $n$  columns, contains a total of  $n$  1s, for a total density of  $n/(kn) = 1/k$ .

10 One of ordinary skill in the art would understand that a matrix having only a single  
11 1 per column is a “low-density” matrix.<sup>39</sup>

12 271. Summarizing, the repeaters taught in both Frey99 and Divsalar correspond  
13 to an LDGM coder that uses a generator matrix of the form illustrated above.

14 Because that matrix is “low-density,” both Frey99 and Divsalar teach the limitation  
15 added by claim 20.

16 ii) Claim 20 of the '710 Patent is Obvious Over Divsalar in View of  
17 One of Luby or MacKay

18 272. As I explained above, the combination of Divsalar with one of Luby or  
19 MacKay teaches every limitation of claim 15. Also, as I explain above, Divsalar  
20 teaches a first coder, *i.e.*, a repeater, that is a low-density generator matrix coder.  
Even when Divsalar’s repeater is made into an irregular repeater by incorporating  
Luby’s or MacKay’s teaching of irregularity, the repeater remains a low-density

---

21 <sup>39</sup> A generator matrix for repeating a very small block size may not be low density. For example,  
22 of the block size is two and each bit is repeated twice, the matrix would have four elements, two  
23 ones and two zeroes. With half of the elements being non-zero, the matrix would not be low  
24 density. However, such degenerate cases do not detract from the point that in general generator  
matrices for repeat codes are low density. Once the block size is increased sufficiently, the  
matrix will become low density. For example, a generator matrix for the block sizes explicitly  
contemplated in Frey99, Divsalar, Luby or MacKay would all be low density.



1 generator matrix coder. Therefore, claim 20 is obvious over Divsalar in view of  
2 one of Luby or MacKay.

3 **H. Claim 21 of the '710 Patent is Invalid**

4 273. Claim 21 of the '710 patent reads:

5 21. The coder of claim 15, wherein the second coder comprises a  
6 rate 1 linear encoder.

7 i) Claim 21 of the '710 Patent is Obvious Over Frey99 in View of  
8 Divsalar

9 274. As I explained above, the combination of Frey99 and Divsalar teaches every  
10 limitation of claim 15.

11 275. Also, as explained above with reference to claim 1 of the '710 patent,  
12 Divsalar teaches a second coder that is an accumulator having a rate equal to 1.

13 Also, an accumulator is a linear encoder. The generator matrix for an accumulator  
14 has the form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \dots \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \dots \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & \dots \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & \dots \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

15 276. As explained in Appendix A, a generator matrix represents a linear  
16 transformation, and any code (such as this one) that can be represented using a  
17 generator matrix is a linear code.  
18  
19  
20

1 277. Claim 22 of the '710 patent underscores the fact that an accumulator is a  
2 linear encoder. It recites “[t]he coder of claim 21, wherein the second coder  
3 comprises an accumulator.” It logically follows that Divsalar’s accumulator is a  
4 particular example of a “rate 1 linear encoder,” as required by claim 21 (*see also*,  
5 *e.g.*, Jin Dep. at 122:7-13).

6 ii) Claim 21 of the '710 Patent is Obvious Over Divsalar in View of  
7 One of Luby or MacKay

8 278. As I explained above, the combination of Divsalar and either Luby or  
9 MacKay renders claim 15 obvious. I also explained above how Divsalar teaches a  
10 second encoder comprising a rate 1 linear encoder. Therefore, claim 21 is also  
11 obvious over Divsalar in view of one of Luby or MacKay.

12 **I. Claim 22 of the '710 Patent is Invalid**

13 279. Claim 22 of the '710 patent reads:

14 22. The coder of claim 21, wherein the second coder comprises an  
15 accumulator.

16 i) Claim 22 of the '710 Patent is Obvious Over Frey99 in View of  
17 Divsalar

18 280. Above I explain how Divsalar and Frey99 render obvious claim 21, and  
19 further how Divsalar teaches a “second encoder” that comprises an “accumulator.”  
20 Therefore, claim 22 is also obvious over the combination of Divsalar and Frey99.

21 ii) Claim 22 of the '710 Patent is Obvious Over Divsalar in View of  
22 One of Luby or MacKay

23 281. Above I explain how Divsalar combined with either Luby or MacKay render  
24 obvious claim 21, and further how Divsalar teaches a “second encoder” that  
comprises an “accumulator.” Therefore, claim 22 is also obvious over Divsalar  
combined with either Luby or MacKay.



1 **VII. THE ASSERTED CLAIMS OF THE '032 PATENT ARE INVALID**

2 282. As I explain below, asserted claims 1, 18, 19, and 22 of the '032 patent are  
 3 invalid. A summary of the opinions set forth in this section is given in the table  
 4 below:

'032 Claim	Ping + Frey99 (or Frey slides)	Ping + MacKay or Luby	Divsalar + Frey99 (or Frey slides), Luby, or MacKay	Divsalar + Frey99 (or Frey slides), Luby, or MacKay + Ping
1	Obvious	Obvious (under Caltech's construction of "repeat")	Obvious	
18			Obvious	Obvious (Ping not necessary)
19			Obvious	Obvious
22			Obvious	Obvious (Ping not necessary)

12 **A. Claim 1 of the '032 Patent is Invalid**

13 283. Claim 1 of the '032 patent reads:

14 1. A method comprising:  
 15 receiving a collection of message bits having a first sequence in a  
 16 source data stream;  
 17 generating a sequence of parity bits, wherein each parity bit "x<sub>j</sub>" in  
 18 the sequence is in accordance with the formula

$$x_j = x_{j-1} + \sum_{i=1}^a v_{(j-1)a+i}$$

19 where "x<sub>j-1</sub>" is the value of a parity bit "j-1," and

$$\sum_{i=1}^a v_{(j-1)a+i}$$

20 is the value of a sum of "a" randomly chosen irregular repeats of  
 21 the message bits; and  
 22 making the sequence of parity bits available for transmission in a  
 23 transmission data stream.  
 24

1           i) Claim 1 of the '032 patent is Obvious over Ping In View of Frey99  
2           (or Frey Slides)

3 284. I explain below, one limitation at a time, why claim 1 is rendered obvious  
4 by Ping in view of Frey99 (or Frey Slides).

5           a) “receiving a collection of message bits having a first sequence in  
6           a source data stream”

7 285. Ping teaches “receiving a collection of message bits having a first sequence  
8 in a source data stream.”

9 286. Ping refers to the collection of information bits to be encoded using the  
10 vector variable name **d**. Ping states: “[d]ecompose the codeword **c** as **c = [p, d]**,  
11 where **p** and **d** contain the parity and information bits, respectively” (Ping at 38).  
12 Ping goes on to provide equations from which “**p** = { $p_i$ } can easily be calculated  
13 from a given **d** = { $d_i$ }” (*id.*).

14 287. The term “message bits” is synonymous with “information bits.” One of  
15 ordinary skill in the art would understand that the information bits **d**, as taught by  
16 Ping, is a “collection of message bits having a first sequence.”

17 288. Further, as I explain above, under Caltech’s application of the claims, a  
18 “data stream,” is merely a sequence of bits. Block encoders like the ones taught by  
19 Ping, and the ones described in the specification of the patents-in-suit, receive a  
20 “collection of message bits having a first sequence in a source data stream.”  
21  
22  
23  
24



1           b) "generating a sequence of parity bits, wherein each parity bit " $x_j$ "  
2           in the sequence is in accordance with the formula

$$3 \qquad x_j = x_{j-1} + \sum_{i=1}^a v_{(j-1)a+i} "$$

4 289. This limitation means that each parity bit  $x_j$  in the claimed sequence of parity  
5 bits is equal to the sum of the previous parity bit  $x_{j-1}$  and the sum of " $a$ "  
6 information bits,  $\sum_{i=1}^a v_{(j-1)a+i}$ .

7 290. This is precisely the coding method taught by Ping. Specifically, Ping  
8 teaches an encoding operation that calculates the parity bits  $\{p_i\}$  using the  
9 information bits  $\{d_i\}$  as an input as follows:

$$10 \qquad p_1 = \sum_j h_{1j}^d d_j$$

$$11 \qquad p_i = p_{i-1} + \sum_j h_{ij}^d d_j$$

12  
13  
14  
15           (Ping at 38) (Eq. 4)

16 291. In Ping, the parity bits, referenced in the claim as  $x_j$ , are denoted using the  
17 letter  $p$  (e.g., in Ping, the  $i^{\text{th}}$  parity bit is denoted  $p_i$ ).

18 292. As required by claim 1 of the '032 patent, the first parity bit of Ping,  $p_1$ , is  
19 calculated as the sum of a subset of information bits and, as shown below, each  
20 subsequent parity bit  $p_i$  is calculated by adding together the previous parity bit  $p_{i-1}$   
21 (the green box) and a sum of bits in a subset of information bits (the red box).<sup>40</sup>

22 <sup>40</sup> As was well understood by those of ordinary skill, the " $\Sigma$ " symbol denotes a summation. For  
23 example,  $\sum_j h_{ij}^d d_j$  means  $h_{i1}^d d_1 + h_{i2}^d d_2 + \dots + h_{ij}^d d_j$ , where  $J$  is an integer. If the " $h$ " values  
24 are all either zero or one, as they are in Ping, and the " $d$ " values are information bits, then this  
equation produces a sum of information bits.

$$p_i = p_{i-1} + \sum_j h_{ij}^d d_j$$

Ping, Eq. 4

c) "where " $x_{j-1}$ " is the value of a parity bit " $j-1$ ," and

$$\sum_{i=1}^a v_{(j-1)a+i}$$

is the value of a sum of " $a$ " randomly chosen irregular repeats of the message bits"

293. Ping teaches an encoding method in which each parity bit is the sum of the previous parity bit plus the sum of a number of randomly chosen collections of the message bits. The expression  $\sum_{i=1}^a v_{(j-1)a+i}$ , as it appears in the claim, is given in Ping as  $\sum_j h_{ij}^d d_j$  (Ping at 38).

294. The variable  $h_{ij}^d$  represents the value at the  $i^{\text{th}}$  row and the  $j^{\text{th}}$  column of the parity check matrix  $\mathbf{H}^d$  (see *id.*). The variable  $d_j$  represents the value of the  $j^{\text{th}}$  information bit. Thus,

$$\sum_j h_{ij}^d d_j$$

represents the sum of the bits in a subset of information bits (specifically, the subset of information bits  $d_j$  where  $h_{ij}^d = 1$ ). As Ping explains, the matrix  $\mathbf{H}^d$  is randomized.  $\mathbf{H}^d$  is comprised of  $t$  sub-blocks  $\mathbf{H}^{d1}, \dots, \mathbf{H}^{dt}$  as follows:

$$\mathbf{H}^d = \begin{pmatrix} \mathbf{H}^{d1} \\ \vdots \\ \mathbf{H}^{dt} \end{pmatrix}$$

(Ping at 38)



1 295. Ping further states: “[i]n each sub-block  $\mathbf{H}^{di}$ ,  $I = 1, 2 \dots t$ , we *randomly*  
2 create exactly one element 1 per column and  $kt/(n-k)$  1s per row” (*id.*) (emphasis  
3 added). Thus, the particular information bits summed by the expression  $\sum_j h_{ij}^d d_j$   
4 are “randomly chosen,” as required by claim 1 of the ’032 patent.<sup>41</sup>

5 296. Ping therefore teaches everything in this limitation except the “irregular  
6 repeats” limitation. While Ping does not teach “irregular repeats” of the message  
7 bits, Frey99 (and the Frey Slides) teaches irregular repetition, as I explained above.

8 297. For the reasons given below, it would have been obvious to one of ordinary  
9 skill in the art to combine the accumulation-based encoding of Ping with the  
10 irregular repetition of Frey99 (or the Frey Slides).

11 *d) “making the sequence of parity bits available for transmission in*  
*a transmission data stream”*

12 298. As explained above, the codeword taught by Ping comprises parity bits,  
13 (denoted with the boldface letter **p**). Specifically, Ping teaches “[d]ecompos[ing]  
14 the codeword **c** as **c** = [**p**, **d**], where **p** and **d** contain the parity and information bits,  
15 respectively” (Ping at 38).

16 299. Ping also analyzes the performance of the codes it describes, graphing the  
17 BER of various LDPC-accumulate coders against various values of  $E_b/N_0$ :  
18  
19  
20  
21

---

22 <sup>41</sup> I understand that Caltech has argued that the asserted claims cover the DVB-S2 algorithm. A  
23 DVB-S2 encoder merely implements previously defined deterministic (non-random) operations.  
24 Therefore, under Caltech’s application of the claims, “randomly chosen” must refer to random  
choices made while defining the coding algorithm itself (as opposed to making random choices  
during the encoding itself).

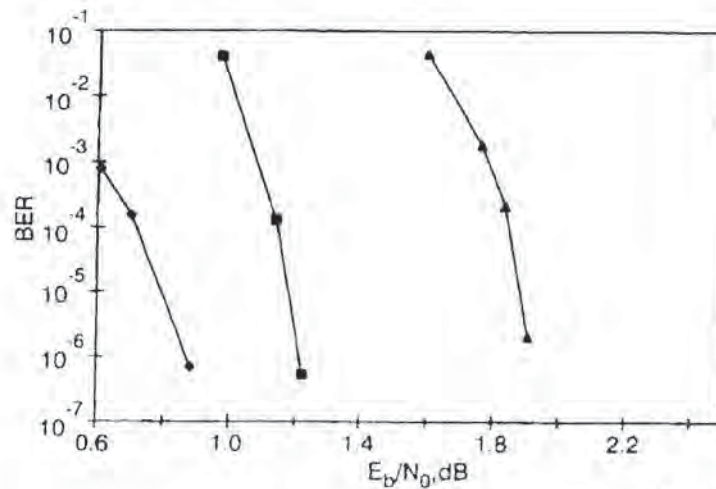


Fig. 1 Performances of LDPC codes generated by semi-random parity check matrixes with  $k = 30000$

- ◆  $R = 1/3$
- $R = 1/2$
- ▲  $R = 2/3$

Ping, Fig. 1

300. The concepts of “BER” and  $E_b/N_0$  only make sense in the context of generating codewords (including the parity bits  $\mathbf{p}$ ), transmitting them over a noisy channel, and decoding them at the other end. Thus, Ping teaches “making the sequence of parity bits available for transmission in a transmission data stream,” as required by claim 1 of the ’032 patent.

e) Summary

301. As explained above, the combination of Ping and Frey99 (or the Frey Slides) teaches every limitation of claim 1 of the ’032 patent.

f) Motivations to Combine the teachings of Ping with those of Frey99 (or the Frey Slides)

302. Ping and Frey99 (or the Frey Slides) are both directed to the same field, namely the field of error-correcting codes. In particular, both Ping and Frey99 relate to linear error-correcting codes and enhancements thereto: Frey99, titled “Irregular Turbocodes,” teaches modifying known coding techniques to include



1 irregularity; Ping, titled “Low Density Parity Check Codes with Semirandom  
2 Parity Check Matrix,” teaches constructing LDPC codes that can be encoded  
3 efficiently and have good BER vs.  $E_b/N_0$  performance (*see* Ping at 39). Given that  
4 both references relate to improvements to error-correcting codes, one of ordinary  
5 skill in the art would have been motivated to combine their teachings.

6 303. Further, as explained above, Luby and MacKay taught that performance of a  
7 code could be improved by making the code irregular and that teaching was well  
8 known in the art prior to Caltech’s claimed conception date or its filing date. That  
9 well-known teaching would have further motivated one of ordinary skill to  
10 incorporate Frey’s irregular repetition into Ping’s coding algorithm.

11 304. Further, combining irregular repetition as taught by Frey99 (and the Frey  
12 Slides) with accumulation as taught by Ping would have been a simple matter for  
13 one of ordinary skill in the art, as described above with reference to the asserted  
14 claims of the ’710 patent. Such a combination would involve a routine substitution  
15 of one component for another and the resulting combination would have performed  
16 as expected.

17 ii) Claim 1 of the ’032 patent is Obvious over Ping In View of MacKay  
18 or Luby

19 305. I understand that the Plaintiff attempted to argue that “repeat” and “reuse”  
20 are synonymous, but the Court was correctly not persuaded by this argument. *See*  
21 Claim Construction Order (Dkt. No. 105) at 11 (“Caltech argues that ‘repeat’ can  
22 also refer to the re-use of a bit, but the patent’s claims and specification support the  
23 Court’s construction”). Unless stated otherwise, my invalidity opinions in this  
24 report are based on the Court’s construction that “repeat” should be given its plain  
25 meaning, which is “duplication.” Claim Construction Order dated August 6, 2014,  
26 p. 10.



1 306. Plaintiff's infringement arguments, however, still appear to be based on an  
2 interpretation of "repeat" that does not require duplication, but merely reuse.<sup>42</sup>  
3 Under this interpretation, claim 1 of the '032 patent would be rendered obvious by  
4 Ping in view of either MacKay or Luby. Further, repeating bits in order to reuse  
5 them would not have been inventive and instead would have been nothing more  
6 than an implementation detail. Accordingly, even under the proper construction in  
7 which repeat means duplicate, the claims are still obvious over Ping in view of  
8 either MacKay or Luby.

8 307. As I explain above, Ping teaches every limitation of claim 1 except  
9 "irregular repeats." Neither MacKay nor Luby teach "repeats" under the Court's  
10 construction of the term "repeat – that is, they do not teach duplicating bits.  
11 However, MacKay and Luby do teach irregular *reuse* of bits, as I explain above  
12 with reference to the claims of the '710 patent.

13 308. It would have been obvious to one of ordinary skill in the art to combine the  
14 LDPC-accumulate coders of Ping with the irregularity of MacKay or Luby. As  
15 described above, Luby and MacKay are directed to the same field, namely the field  
16 of error correcting codes, and specifically, variations and improvements on linear  
17 error-correcting codes that allow them to be encoded more quickly. Ping is related  
18 to the same field; Ping, titled "Low Density Parity Check Codes with Semirandom  
19 Parity Check Matrix," teaches constructing LDPC codes that can be encoded  
20 efficiently and have good BER vs.  $E_b/N_0$  performance (*see* Ping at 39). Given that  
21 Ping, MacKay, and Luby relate to improvements to error-correcting codes, one of  
22 ordinary skill in the art would have been motivated to combine the teachings of  
23 Ping with those of at least one of Luby or MacKay.

---

23 <sup>42</sup> That is, in DVB-S2, the parity bits are not repeats of the information bits. Rather, in DVB-S2,  
24 each parity bit is the sum of a collection of information bits. Thus, although information bits  
may be reused in DVB-S2's LDPC code, they are not repeated.



1 309. Further, because Luby and MacKay both taught that irregular codes perform  
2 better than regular ones, one of ordinary skill would have been motivated to  
3 incorporate irregularity into Ping. Ping's code is regular because each column in  
4 Ping's  $H^d$  matrix contains the same number of ones, i.e., each of Ping's columns  
5 contains exactly "t" ones (Ping at 38). However, changing Ping's  $H^d$  matrix such  
6 that not all columns had the same weight would have made Ping's code irregular  
7 and would have been an easy way for one of ordinary skill to incorporate  
8 irregularity into Ping. As explained above, MacKay teaches parity-check matrices  
9 in which each information bit corresponds to a column, where the weight of that  
10 column (i.e., the number of 1s contained in that column of the parity-check matrix)  
11 represents the degree of the information bit. MacKay also notes that "[t]he best  
12 known binary Gallager codes are *irregular* codes whose parity check matrices have  
13 *nonuniform* weight per column" (Mackay at 1449) (emphasis in original). Given  
14 these teachings of MacKay, it would have been obvious to one of ordinary skill in  
15 the art to incorporate irregularity into the LDPC-accumulate coders of Ping by  
16 making the column weights of the parity check matrix  $H^d$  nonuniform.

15 310. Summarizing, Ping teaches a code that can be described as a regular LDPC  
16 followed by an accumulate (or a serial concatenated code in which the outer coder  
17 is a regular LDPC coder and the inner coder is an accumulator).<sup>43</sup> Thus, in Ping's  
18 code, every parity bit is the sum of (a) the previous parity bit and (b) a sum of  
19 randomly chosen regular "reuses" of the message bits. One of ordinary skill in the  
20 art would have been motivated by the teachings of Luby and MacKay to replace  
21 Ping's regular LDPC coder with an irregular LDPC coder.

22 \_\_\_\_\_  
23 <sup>43</sup> Ping's equation (4) for  $p_i$ , is of the form  $p_i = p_{i-1} + X$ , which is an accumulate operation and  
24 shows that Ping's outer coder is an accumulator. Further, the summation term in equation (4)  
(denoted by "X" in the prior sentence) provides an LDPC encoding, thus showing that Ping's  
inner coder is an LDPC coder.



1 311. In Ping's code as modified to include irregularity per the teachings of Luby  
2 or MacKay, each parity bit would be the sum of (a) the previous parity bit and (b) a  
3 sum of randomly chosen irregular "reuses" of the message bits.

4 312. Thus, under Caltech's theory that "repeat" means "reuse," claim 1 of  
5 the '032 patent would be rendered obvious by Ping in view of MacKay or Luby.  
6 Also, as noted above, repeating bits in order to reuse them would not have been  
7 inventive and instead would have been nothing more than an obvious  
8 implementation detail. Accordingly, even under the proper construction in which  
9 repeat means duplicate, the claims are still obvious over Ping in view of either  
10 MacKay or Luby.

11 iii) Claim 1 of the '032 Patent is Obvious over Divsalar in view of Luby  
12 or MacKay

13 313. I explain below, one limitation at a time, why claim 1 is rendered obvious by  
14 Divsalar in view of Luby or MacKay.

15 a) "receiving a collection of message bits having a first sequence in  
16 a source data stream"

17 314. As explained above with reference to the claims of the '710 patent, Divsalar  
18 teaches "receiving a collection of message bits having a first sequence." Also for  
19 the reasons explained above, while Divsalar does not explicitly make reference to  
20 an input configured to receive a "data stream," as required by this limitation, one  
21 of ordinary skill in the art would understand that the methods and systems taught in  
22 Divsalar operate on a data stream.

23 b) "generating a sequence of parity bits, wherein each parity bit "x<sub>j</sub>"  
24 in the sequence is in accordance with the formula

$$x_j = x_{j-1} + \sum_{i=1}^a v_{(j-1)a+i}$$



1 315. This limitation means that each parity bit  $x_j$  in the claimed sequence of parity  
2 bits is equal to the sum of the previous parity bit  $x_{j-1}$  and the sum of “ $a$ ”  
3 information bits,  $\sum_{i=1}^a v_{(j-1)a+i}$ .

4 315. This limitation means that each parity bit  $x_j$  in the claimed sequence of parity  
5 bits is equal to the sum of the previous parity bit  $x_{j-1}$  and the sum of “ $a$ ”  
6 information bits,  $\sum_{i=1}^a v_{(j-1)a+i}$ .

7 316. As explained above, the accumulator of Divsalar performs an accumulation  
8 operation as follows:

9 [W]e prefer to think of [the accumulator] as a block coder whose  
input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by  
10 the formula

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= x_1 + x_2 \\ y_3 &= x_1 + x_2 + x_3 \\ y_n &= x_1 + x_2 + x_3 + \dots + x_n \end{aligned}$$

11  
12 (Divsalar at 5)

13 317. This operation can be represented recursively using the equation  $y_i = y_{i-1} + x_i$ .  
14 Using the recursive formulation, one can see that each parity bit  $y_i$  is the sum of the  
15 previous parity bit  $y_{i-1}$  and a single information bit  $x_i$ . Therefore, for the case in  
16 which  $a = 1$ , Divsalar meets this limitation.

17 318. For cases in which “ $a$ ” is greater than one, this limitation would be met by  
18 modifying the teachings of Divsalar so that each parity bit  $y_i$  is the sum of the  
19 previous parity bit  $y_{i-1}$  and *multiple* information bits  $x_{i1}$ ,  $x_{i2}$ , and  $x_{i3}$ . That is,  
20 modifying the teachings of Divsalar so that  $y_i = y_{i-1} + (x_{i1} + x_{i2} + x_{i3})$ .

21 319. It would have been obvious to implement such a code by inserting a step  
22 between the interleaver and the accumulator that sums consecutive groups of  $a$   
23 repeated bits. For example, for  $a = 3$ , this step would receive repeated information  
24 bits  $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, \dots$  and would output the sums of consecutive  
groups of three repeated information bits  $(x_1 + x_2 + x_3), (x_4 + x_5 + x_6), (x_7 + x_8 +$

1  $x_9$ ), .... These sums would then be passed to the accumulator, resulting in a code  
2 where  $y_i = y_{i-1} + (x_{i1} + x_{i2} + x_{i3})$ , which satisfies this limitation of claim 1 of  
3 the '032 patent.

4 320. As explained below with reference to the claims of the '781 patent, this  
5 effect can also be achieved by “puncturing” some of the parity bits  $y_i$  output by  
6 Divsalar. If two out of every three parity bits were punctured, leaving only  $y_1, y_4,$   
7  $y_7$ , etc., then each parity bit (*e.g.*,  $y_4$ ) would be the sum of the previous parity bit  
8 (*i.e.*,  $y_1$ ) and a group of three information bits (*i.e.*,  $x_1 + x_2 + x_3$ ). Thus, this would  
9 also result in a code where  $y_i = y_{i-1} + (x_{i1} + x_{i2} + x_{i3})$ , which satisfies this limitation  
10 of claim 1 of the '032 patent. As explained below, “puncturing” parity bits would  
11 have been well known to one of ordinary skill in the art (*see, e.g.*, Frey99 at 3).

12 321. Modifying the teachings of Divsalar in this way would have been obvious in  
13 view of the teachings of Luby or MacKay. As explained above Luby and MacKay  
14 teach LDPC codes (*see, e.g.*, Luby at 17:58-60, “[f]or example, a low-density  
15 parity check code defined by a graph similar to that used between the other layers  
16 is particularly suitable for this purpose ...”; *see also, e.g.*, MacKay at Fig. 1). It  
17 was well known in the art that LDPC codes (*a.k.a.* “Gallager codes”) are a class of  
18 high-performance error-correcting codes with desirable properties. As explained  
19 above, Gallager codes had been known in the art for decades by the time the  
20 patents-in-suit were filed, and had been the subject of intensive research since 1995,  
21 when they were rediscovered by David J. C. MacKay. Combining the repeat-  
22 accumulate codes taught by Divsalar with the LDPC codes taught by Luby and  
23 MacKay to create an LDPC-accumulate coder would have been obvious to one of  
24 ordinary skill in the art.



1 c) “where “ $x_{j-1}$ ” is the value of a parity bit “ $j-1$ ,” and

$$\sum_{i=1}^a v_{(j-1)a+i}$$

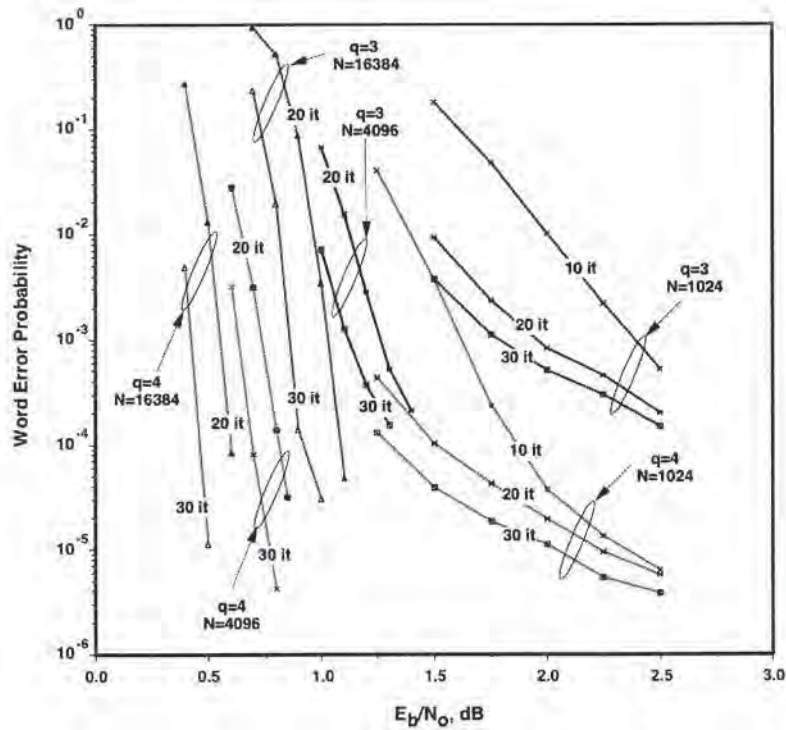
2  
3 is the value of a sum of “ $a$ ” randomly chosen irregular repeats  
4 of the message bits”

5 322. As I explain above, it would have been obvious to modify the teachings of  
6 Divsalar with the LDPC codes taught by Luby or MacKay by inserting a  
7 summation step between the interleaver and the accumulator of Divsalar. The  
8 resulting code would be a code in which  $x_i = x_{i-1} + (v_{i1} + v_{i2} + v_{i3})$  (where  $a = 3$ ).  
9 The quantity  $(v_{i1} + v_{i2} + v_{i3})$  is the value of a sum of “ $a$ ” repeats of the message  
10 bits. Because the bits are permuted by the interleaver prior to this step of  
11 summation, the repeated information bits  $v_{i1}$ ,  $v_{i2}$ ,  $v_{i3}$  are “randomly chosen,” at least  
12 according to Caltech’s interpretation, repeats of the message bits, as required by  
13 the claim.

14 323. Divsalar does not teach “irregular” repeats,” as required by claim 1.  
15 However, as explained above with reference to the claims of the ’710 patent, it  
16 would have been obvious to one of ordinary skill in the art to combine the regular  
17 repetition of Divsalar with the irregularity of MacKay and Luby, resulting in  
18 irregular repetition.

19 d) “making the sequence of parity bits available for transmission in  
20 a transmission data stream”

21 324. Divsalar teaches this limitation. Divsalar analyzes the performance of the  
22 codes it describes, graphing the word error probability of various RA codes against  
23 various values of  $E_b/N_0$ :  
24



Divsalar, Fig. 5

325. The concepts of “BER” and  $E_b/N_0$  only make sense in the context of generating codewords (including parity bits), transmitting them over a noisy channel, and decoding them at the other end. Thus, Divsalar teaches “making the sequence of parity bits available for transmission in a transmission data stream,” as required by claim 1 of the ’032 patent.<sup>44</sup>

e) Summary

326. As explained above, the combination of Divsalar and either Luby or MacKay teaches every limitation of claim 1 of the ’032 patent.

f) Motivations to Combine

327. As explained above with reference to the claims of the ’710 patent, one of ordinary skill in the art would have been motivated to combine the repeat accumulate codes of Divsalar with the irregular LDPC codes taught by MacKay

<sup>44</sup> This is consistent with the testimony of Dariush Divsalar (*see, e.g.*, Divsalar Tr. at 76-77).



1 and Luby, resulting in an LDPC-accumulate coder that satisfies the limitations of  
2 claim 1 of the '032 patent.

3 **B. Claim 18 of the '032 Patent is Invalid**

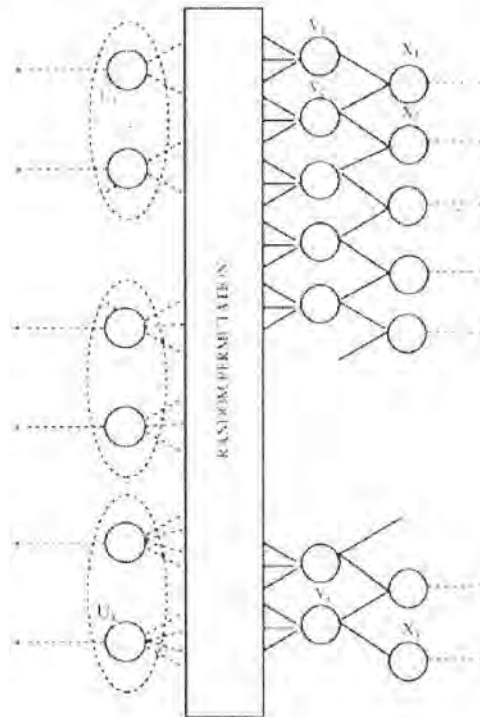
4 328. Claim 18 of the '032 patent reads:

5 18. A device comprising:

6 a message passing decoder configured to decode a received data stream that  
7 includes a collection of parity bits,

8 the message passing decoder comprising two or more check/variable nodes  
9 operating in parallel to receive messages from neighboring checking/variable  
10 nodes and send updated messages to the neighboring variable/check nodes,

11 wherein the message passing decoder is configured to decode the received data  
12 stream that has been encoded in accordance with the following Tanner graph:

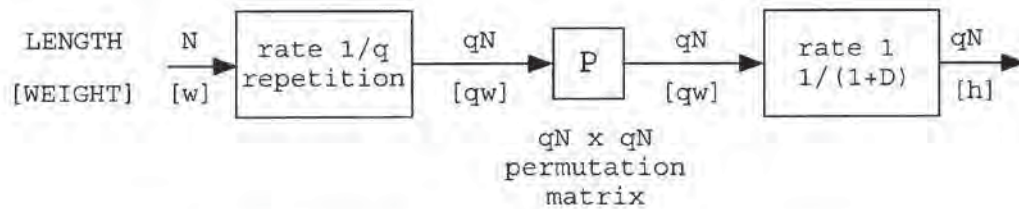


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21 i) Claim 18 of the '032 Patent is Obvious over Divsalar in View of  
22 Frey99, Luby, or MacKay

23 329. I explain below, one limitation at a time, why Claim 18 of the '032 patent is  
24 rendered obvious by Divsalar in view of Frey99, Luby, or MacKay.

a) "a device comprising ..."

330. The encoding methods taught by Divsalar are performed by "a device." A schematic diagram of such a device (*i.e.*, an "encoder") is shown by Divsalar, Fig. 3, reproduced below:



**Figure 3.** Encoder for a  $(qN, N)$  repeat and accumulate code. The numbers above the input-output lines indicate the length of the corresponding block, and those below the lines indicate the weight of the block.

**Divsalar, Fig. 3**

b) "a message passing decoder configured to decode a received data stream that includes a collection of parity bits"

331. Divsalar teaches "a message passing decoder configured to decode a received data stream that includes a collection of parity bits." Divsalar teaches that "an important feature of turbo-like codes is the availability of a simple iterative, **message passing decoding** algorithm that approximates ML decoding. We wrote a computer program to implement this "turbo-like" decoding for RA codes with  $q = 3$  (rate 1/3) and  $q = 4$  (rate 1/4), and the results are shown in Figure 5" (Divsalar at 9) (emphasis added).

332. Message passing decoders are conventional elements that were well known in the prior art.<sup>45</sup> A message passing decoder repeatedly calculates several related mathematical functions, where the functions are called "messages." A message passing decoder includes "variable nodes," which represent information bits, and

<sup>45</sup> See generally, *e.g.*, Judea Pearl, *Reverend Bayes on Inference Engines: A Distributed Hierarchical Approach*, Proceedings of the Second National Conference on Artificial Intelligence Pittsburgh, PA 133-136 (1982) ("Pearl").



1 “check nodes,” which represent mathematical constraints that the information bits  
2 must follow. Using an algorithm for decoding called “message passing decoding,”  
3 messages are passed among the variable and check nodes in order to determine the  
4 original values of the information bits. One of ordinary skill would understand that  
5 this is what is referenced by Divsalar’s use of the term “message passing  
6 decoding.”<sup>46</sup>

7 333. Further, as explained above with reference to the asserted claims of the ’710  
8 patent, the decoding methods taught by Divsalar are intended to be applied to a  
9 “data stream.”

10 c) “the message passing decoder comprising two or more  
11 check/variable nodes operating in parallel to receive messages  
12 from neighboring checking/variable nodes and send updated  
13 messages to the neighboring variable/check nodes”

14 334. This limitation is directed to features that are obvious elements in any  
15 message-passing decoder, including the message-passing decoder taught by  
16 Divsalar. A person of ordinary skill in the art would recognize that conventional  
17 implementations of decoding by “message passing”, decoding by the “sum-product  
18 algorithm”, decoding by “belief propagation”, decoding by “probability  
19 propagation”, decoding a code defined by a “Tanner graph” and decoding a code  
20 defined by a “factor graph” would in particular comprise two or more  
21 check/variable nodes operating in parallel to receive messages from neighboring  
22 check/variable nodes and send updated messages to the neighboring variable/check  
23 nodes. These operations are described in several publications prior to Divsalar,  
24 including in the teachings of R. G. Gallager (“Low Density Parity Check Codes”,  
monograph, M.I.T. Press, 1963), of B. J. Frey and F. R. Kschischang (Presented at  
the Allerton Conference in September 1995, proceedings published in May 1996),

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<sup>46</sup> As confirmed by the testimony of Dariush Divsalar (*see, e.g.*, Divsalar Tr. at 152-153).



1 of B. J. Frey, F. R. Kschischang, H.-A. Loeliger and N. Wiberg (Presented at the  
2 Allerton Conference in September 1996, proceedings published in May 1997) and  
3 of R. J. McEliece, D. J. C. Mackay and J.-F. Cheng (published in the IEEE Journal  
4 on Selected Areas in Communications, February 1998). I discuss message passing  
5 decoding further below with respect to Frey99, but that description of message  
6 passing decoding applies here as well.

7 335. Divsalar teaches that “an important feature of turbo-like codes is the  
8 availability of a simple iterative, *message passing decoding* algorithm that  
9 approximates ML decoding” (Divsalar at 9) (emphasis added). The iterative  
10 message-passing algorithm taught by Divsalar operates according to the principles  
11 common to conventional message-passing decoders (as taught by, *e.g.* Pearl) and  
12 therefore meets this limitation.<sup>47</sup>

13 *d) Tanner Graph*

14 336. The Court has construed this term to require “a graph representing an IRA  
15 code as a set of parity checks where every message bit is repeated, at least two  
16 different subsets of message bits are repeated a different number of times, and  
17 check nodes, randomly connected to the repeated message bits, enforce constraints  
18 that determine the parity bits.”

19 337. As explained above, an IRA code is an “irregular repeat-accumulate” code,  
20 in which information bits are irregularly repeated and optionally interleaved, with  
21 the interleaved bits being passed to an accumulator, which generates the parity bits.

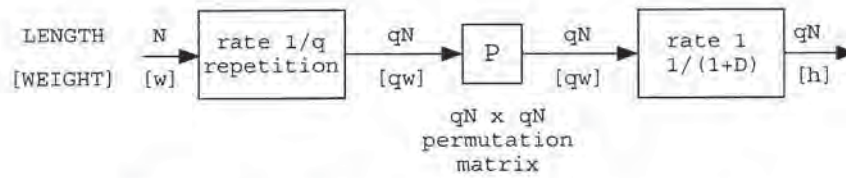
22 *i. Divsalar teaches every requirement of the Tanner Graph*  
23 *limitation except irregularity*

24 338. Divsalar teaches repeating message bits, as required by the Court’s  
construction. This is shown in Figure 3 of Divsalar, reproduced below:

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<sup>47</sup> This is consistent with the testimony of Dariush Divsalar (*see* Divsalar Tr. at 152-153).





**Figure 3.** Encoder for a  $(qN, N)$  repeat and accumulate code. The numbers above the input-output lines indicate the length of the corresponding block, and those below the lines indicate the weight of the block.

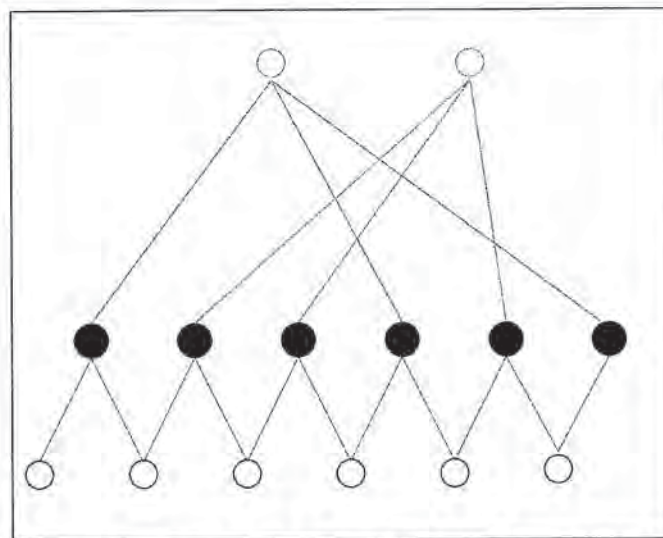
339. As explained above, a block of  $N$  information bits enters the coder at the left side of the figure and is provided to the repeater (labeled “rate  $1/q$  repetition”) (Divsalar at 5). The repeater duplicates each of the  $N$  information bits  $q$  times and outputs the resulting  $N \times q$  repeated bits (*id.*).

340. Divsalar also teaches “an [I]RA code as a set of parity checks ... check nodes, randomly connected to the repeated message bits, enforce constraints that determine the parity bits” required by the Court’s construction of this term. In particular, Divsalar’s interleaver disposed between the repeater and accumulator satisfies the “randomly connected” portion of the Court’s construction.<sup>48</sup> Further, the accumulator taught by Divsalar satisfies the “[I]RA code ... enforce constraints that determine the parity bits.” The ’032 patent itself teaches that an accumulator satisfies those constraints and therefore Divsalar’s accumulator satisfies them as well.

341. As explained above with reference to claim 1 of the ’710 patent, the drawing below represents a Tanner graph for a simple version of the RA code taught in Divsalar. That Tanner graph is “a graph representing an [I]RA code as a set of parity checks where every message bit is repeated, [at least two different subsets of message bits are repeated a different number of times,] and check nodes, randomly

<sup>48</sup> Again, I have interpreted “randomly connected” consistent with Caltech’s apparent application of the claim to DVB-S2 to mean implementation of an algorithm that may have been defined in advance using random operations.

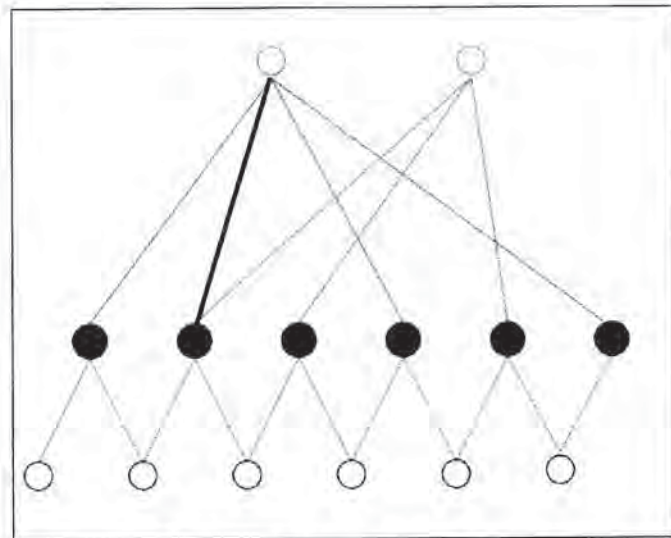
1 connected to the repeated message bits, enforce constraints that determine the  
2 parity bits.” That is, the Tanner graph of Divsalar’s RA code meets all  
3 requirements imposed by the Court’s construction of the Tanner graph term in  
4 claim 18, except that it is regular instead of irregular. In the Tanner graph below,  
5 the message bits are the two open circles at the top. They are both repeated twice.  
6 The check nodes are the black circles in the middle. They are randomly connected  
7 to the message bits. The open circles at the bottom represent parity bits. The  
8 connections between the check nodes and the parity bits enforce constraints that  
9 determine the parity bits. In particular, those constraints require the parity bits to  
10 be the accumulation of the repeated, interleaved message bits.



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**Tanner Graph of an RA Code (CALTECH000007326)**

19 342. Only a single change is required to make the Tanner graph above meet all  
20 requirements imposed by the Court’s construction of the Tanner graph limitation of  
21 claim 18. That is, if the repeat of the message bits is made irregular, e.g., by  
22 inserting one extra edge between one of the message bits and one of the check  
23 nodes (e.g., as shown by the extra red edge in the Tanner graph below) such that  
24 one message bit is repeated four times instead of three, then the Tanner graph  
meets all aspects of the Court’s construction.





**Tanner Graph of an IRA Code**

- ii. The irregularity required by the Tanner graph limitation is taught by Frey99, Luby, and MacKay

343. While Divsalar does not teach a scheme in which “at least two different subsets of message bits are repeated a different number of times” as required by the Court’s construction, this limitation is taught by each of Frey99, Luby, and MacKay.

344. As I have explained above, Frey99 teaches the claimed irregular repetition. Also, as explained above, Luby and MacKay also teach the benefits of irregular codes.

*e) Summary*

345. As explained above, every limitation of claim 18 is taught by Divsalar except the irregularity required by the Court’s construction of the Tanner graph limitation, which is taught by each of Luby, MacKay, and Frey99.

1           ii) One of ordinary skill in the art would have been motivated to  
2           incorporate the irregularity of Frey99, Luby, or MacKay into the RA  
3           codes of Divsalar

4           a) Combining the RA codes of Divsalar with the irregularity of  
5           Frey99, Luby, or MacKay, generally

6           346. For the reasons explained above with reference to the asserted claims of  
7           the '710 patent, it would have been obvious to incorporate irregularity (as  
8           motivated by Luby, MacKay or Frey99) into the repeat-accumulate codes taught  
9           by Divsalar.

10           b) Other similarities between Divsalar and each of Frey99, Luby,  
11           and MacKay further motivate the combination

12           347. The motivations to combine Divsalar with any one of Frey99, Luby, or  
13           MacKay references are strengthened by the fact that all four of these references  
14           teach “message passing” decoders, as required by claim 18.

15           348. As described above, Divsalar teaches a message passing decoder.

16           349. Frey99 teaches a “message passing” decoder as well. The irregular  
17           turbocodes of Frey99 are decoded using an interactive application of the sum-  
18           product algorithm, which is a type of message-passing decoder. Frey99 states:  
19           “[w]e construct irregular turbocodes with systematic bits that participate in varying  
20           number of trellis sections. These codes can be decoded by the iterative application  
21           of the sum-product algorithm (a low-complexity, more general form of the  
22           turbodecoding algorithm)” (Frey99 at 1). A person of ordinary skill in the art  
23           would recognize that an “iterative application of the sum-product algorithm,” as  
24           described by Frey99, describes a “belief propagation” decoder, which is a type of  
            “message passing” decoder (as shown by, e.g., claim 22 of the '032 patent). See



1 also Frey Slides at 3 (showing non-elite and elite bits being “pinned down  
2 SLOWLY” and “pinned down QUICKLY,” respectively) (emphasis in original).<sup>49</sup>  
3 350. Conventional implementations of this kind of decoder consist of  
4 check/variable nodes operating in parallel to receive messages from neighboring  
5 check/variable nodes and send updated messages to the neighboring variable/check  
6 nodes, as described in the teaching of B. J. Frey, F. R. Kschischang, H.-A. Loeliger  
7 and N. Wiberg (presented at the Allerton Conference in September 1996,  
8 proceedings published in May 1997). The message sent from a variable node to a  
9 check node is comprised of one number for every possible value<sup>50</sup> of the variable,  
10 which is computed by taking the product of the corresponding messages received  
11 by the variable from other check nodes. The message sent from a check node to a  
12 variable node is comprised of one number for every possible value of the variable,  
13 which is computed by adding together terms that correspond to configurations of  
14 all other variables connected to the check node such that the parity check is  
15 satisfied, where each term is given by the product of the corresponding messages  
16 received by the check node from those variable nodes. An information bit is  
17 decoded by examining its corresponding variable node and for every possible value  
18 of the variable computing a number by taking the product of the corresponding  
19 messages received by the variable from all check nodes that it is connected to. The  
20 bit is decoded by setting it to the value with the largest number. There are  
21 variations on these operations that involve different ways of scaling the messages,  
22 different ways of scheduling the order in which messages are updated, different

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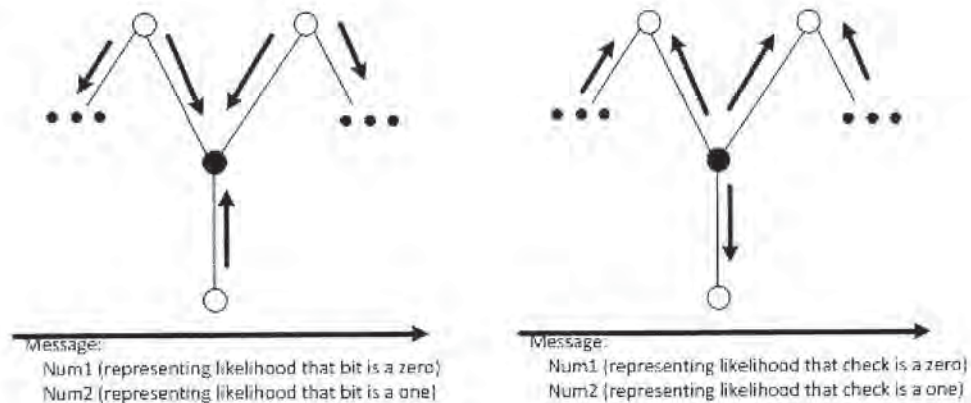
21 <sup>49</sup> One of ordinary skill in the art would have recognized that “pinning down” bits refers to the  
22 operation of a message passing decoder, which “pins down” an information bit by iteratively  
23 applying the sum-product algorithm to the corresponding variable node.

23 <sup>50</sup> In binary coding systems, such as all of the references discussed herein, each variable can have  
24 only one of two possible values, i.e., 0 or 1. Thus, for any given information or parity bit, one  
number is computed representing the likelihood that the bit has value 0 and another number is  
computed representing the likelihood that the bit has value 1.



1 arithmetic operations that may be used, and different ways of more efficiently  
2 storing the numbers comprising the messages, all of which were known to those of  
3 ordinary skill before Caltech's alleged invention.

4 351. The drawing below graphically illustrates the above described operation of  
5 the message passing decoding algorithm using a small portion of a Tanner graph.  
6 In the drawings below, variable nodes corresponding to information and parity bits  
7 are the open circles at the top and bottom of the diagram, respectively, and the  
8 filled circle in the middle is a check node. In one cycle, as shown on the left,  
9 each variable node computes a message and sends its message to the check nodes to  
10 which it is connected. In the next cycle, as shown in the right, each check node  
11 computes a message and sends its message to the variable nodes to which it is  
12 connected. Several such iterations are performed, *e.g.*, until a solution stabilizes or  
13 until a maximum number of iterations has been reached. Then, each information  
14 or parity bit can combine multiple messages from its neighboring check nodes and  
15 use those messages to determine whether its value should be zero or one.



21 352. Luby also teaches message passing decoders, stating that "[t]o properly  
22 decode corrupted bits conventional *belief propagation* is utilized. *Belief*  
23 *propagation* is described in detail in "The Forward-Backward Algorithm" by G.  
24 David Forney, Jr. in Proceedings of the 34th Allerton Conference on



1 Communication, Control, and Computing (October, 1996), pp. 432-446” (Luby  
2 18:29-38) (emphasis added). As explained above (and as confirmed by, *e.g.*, claim  
3 22 of the '032 patent), a “belief propagation” decoder is a particular type of  
4 “message passing” decoder.

5 353. MacKay also teaches message passing decoders, teaching codes that “can be  
6 practically decoded with Gallager’s sum-product algorithm giving near Shannon  
7 limit performance” (MacKay at 1449). As explained above, the “sum-product  
8 algorithm” refers to a particular type of message-passing decoder.

### 9 **C. Claim 19 of the '032 Patent is Invalid**

10 354. Claim 19 of the '032 patent reads:

11 19. The device of claim 18, wherein the message passing decoder  
12 is configured to decode the received data stream that includes the  
13 message bits.

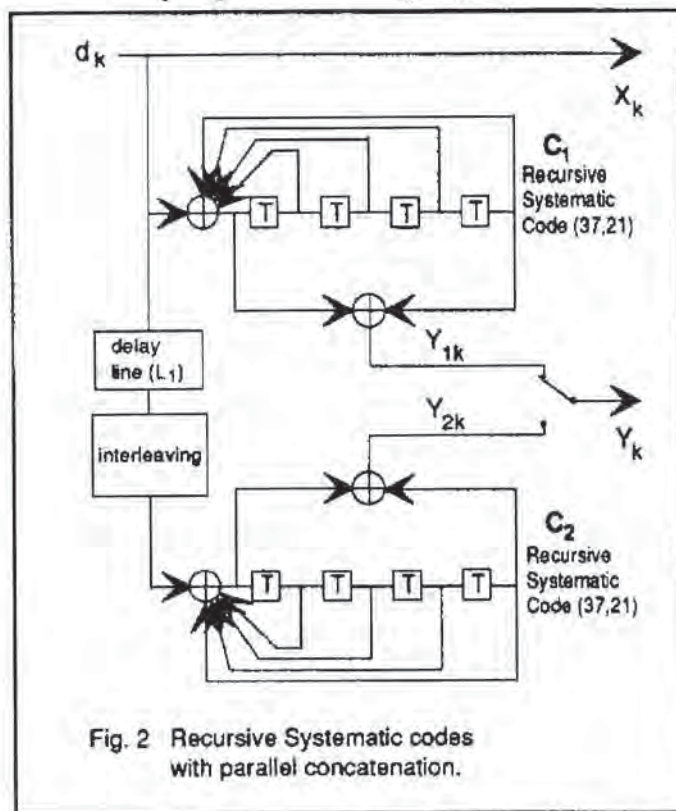
14 355. Claim 19 is rendered obvious by Divsalar in combination with one of Frey99,  
15 Luby, or MacKay, alone or further in combination with Ping ((Divsalar + (Frey99,  
16 Luby or MacKay)) alone or (Divsalar + (Frey99, Luby or MacKay) + Ping)).

17 356. I explain above that Divsalar in combination with one of Frey99, Luby, or  
18 MacKay renders obvious every limitation of Claim 18.

19 357. Frey99 explicitly teaches transmitting the message bits as well as the parity  
20 bits. “In particular, if the bits in the convolutional code are partitioned into  
21 “systematic bits” and “parity bits”, then by connecting each parity bit to a degree 1  
22 codeword bit, we can encode in linear time” (Frey99 at 2). It was widely accepted  
23 at the time that “systematic bits” refers to message bits that are transmitted.

24 MacKay also teaches transmitting message bits (*see* MacKay at Fig. 5, showing  
“[b]its  $t_1 \dots t_k$  defined to be source bits”). Further, both systematic and non-  
systematic codes were known long before Caltech’s alleged invention. As an

1 example, Berrou's original disclosure of turbocodes was of a systematic code.<sup>51</sup> In  
 2 Berrou's Figure 2, copied below, the arrow at the top pointing to the right  
 3 represents transmission of the information bits, making the code a systematic code.



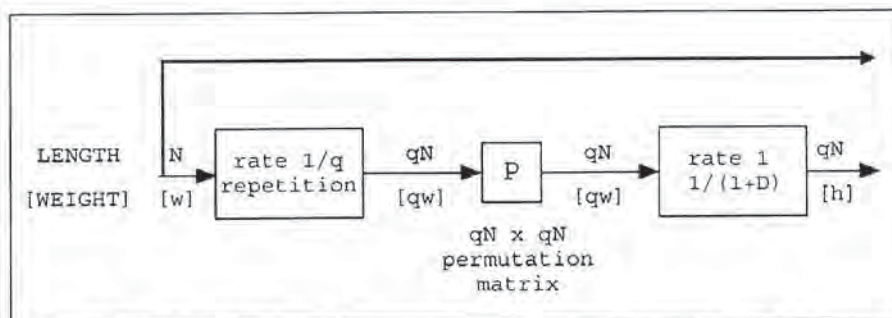
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Berrou, Figure 2

358. One of ordinary skill reading Divsalar or Luby would have understood that  
 17 making the disclosed codes be systematic instead of non-systematic would have  
 18 simple and obvious. Figure 3 of Divsalar is copied again below with an added red  
 19 line to indicate the change to Divsalar that would make its code be systematic  
 20 instead of non-systematic.<sup>52</sup>

<sup>51</sup> Claude Berrou et al., *Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes*, 2 IEEE International Conference on Communications, ICC '93 Geneva. Technical Program, Conference Record 1064 (1993); '032 patent, 1:29-56.

<sup>52</sup> This is consistent with the testimony of Dariush Divsalar (*see* Divsalar Tr. at 67-68).





Divsalar, Figure 3 (modified to show a systematic code)

359. While Divsalar and Luby do not explicitly teach decoding a received data stream that “includes the message bits” (*i.e.*, decoding a *systematic* code) this limitation would have been obvious to one of ordinary skill in view of those references alone. As explained above, Frey99 and MacKay both explicitly teach systematic codes.

360. Moreover, systematic codes are taught by Ping. Ping defines a “codeword  $\mathbf{c}$  as  $\mathbf{c} = [\mathbf{p}, \mathbf{d}]$ , where  $\mathbf{p}$  and  $\mathbf{d}$  contain the parity and information bits, respectively” (Ping at 38). Ping goes on to provide equations from which “ $\mathbf{p} = \{p_i\}$  can easily be calculated from a given  $\mathbf{d} = \{d_i\}$ ” (*id.*). Thus, the codewords of Ping, which collectively comprise the “data stream” received by the decoder, include the information bits  $\mathbf{d}$ .

361. As explained above, Frey99 and MacKay teach systematic codes, but additionally, it would have been obvious to one of ordinary skill in the art to modify any of Divsalar or Luby to make the code be systematic. Moreover, it would have further been obvious to incorporate Ping’s teaching of systematic codes into Divsalar. Systematic codes had been well known in the art for decades prior to the claimed priority date of the patents-in-suit (*see, e.g.*, Wicker Dep. at 77:15-20). It would have been obvious to one of ordinary skill in the art that the techniques taught by Divsalar (as well as those taught by Frey99, Luby, and MacKay) can be applied equally to both systematic and non-systematic codes.



1 362. As described above, Divsalar, Frey99, Luby, and MacKay are directed to the  
2 same field, namely the field of error correcting codes, and specifically, variations  
3 and improvements on linear error-correcting codes that allow them to be encoded  
4 more quickly. Ping is related to the same field; Ping, titled “Low Density Parity  
5 Check Codes with Semirandom Parity Check Matrix,” teaches constructing LDPC  
6 codes that can be encoded efficiently and have good BER vs.  $E_b/N_0$  performance  
7 (*see* Ping at 39). Given that all four of these references relate to improvements to  
8 error-correcting codes, one of ordinary skill in the art would have been motivated  
9 to combine the teachings of Ping with those of Divsalar and at least one of Frey99,  
Luby, or MacKay.

10 363. Therefore, claim 19 is obvious over Divsalar in combination with one of  
11 Frey99, Luby, or MacKay, alone or further in combination with Ping.

12 **D. Claim 22 of the '032 Patent is Invalid**

13 364. Claim 22 of the '032 patent reads:

14 22. The device of claim 18, wherein the message passing decoder  
15 comprises a belief propagation decoder.

16 365. Claim 22 is rendered obvious by Divsalar in combination with one of Frey99,  
Luby, or MacKay.

17 366. I explain above that Divsalar in combination with any one of Frey99, Luby  
18 or MacKay teaches every limitation of claim 18.

19 367. The additional limitation imposed by claim 22 (*i.e.*, that the decoder  
20 comprise “a belief propagation decoder”) is taught by Divsalar. Divsalar teaches  
21 that “an important feature of turbo-like codes is the availability of a simple  
22 *iterative, message passing* decoding algorithm that approximates ML decoding.”  
23 (Divsalar at 9) (emphasis added). A person of ordinary skill in the art would  
24 recognize that the “iterative message passing” algorithm described in the above  
passage refers to a “belief propagation decoder.”



1 368. As explained above with reference to claim 18, Frey99, Luby, and MacKay  
 2 also teach belief propagation decoders, as required by claim 22. For example,  
 3 Luby states that “[t]o properly decode corrupted bits conventional *belief*  
 4 *propagation* is utilized. *Belief propagation* is described in detail in "The Forward-  
 5 Backward Algorithm" by G. David Forney, Jr. in Proceedings of the 34th Allerton  
 6 Conference on Communication, Control, and Computing (October, 1996), pp. 432-  
 446” (Luby 18:29-38) (emphasis added).

7 369. Therefore, claim 22 is obvious over Divsalar in view of any one of Frey99,  
 8 Luby, or MacKay.

9 **VIII. THE ASSERTED CLAIMS OF THE '781 PATENT ARE INVALID**

10 370. As I explain below, asserted claims 16 and 19 of the '781 patent are invalid.  
 11 I also explain why claims 13, 14, and 15, from which claim 16 depends, are invalid.  
 12 A summary of the opinions set forth in this section is given in the table below:

'781 Claim	Divsalar	Ping	Ping + MacKay	Divsalar + Frey99 (or Frey slides), or MacKay	Divsalar + Luby + (Ping, Frey99, MacKay or '999 Patent)
13	Anticipated / Obvious		Obvious	Obvious	Obvious (over Divsalar + Frey99)
14	Anticipated / Obvious		Obvious	Obvious	Obvious (over Divsalar + Frey99)
15	Obvious		Obvious	Obvious	Obvious
16	Obvious		Obvious	Obvious	Obvious
19	Anticipated / Obvious	Anticipated	Anticipated (by Ping)	Anticipated (by Divsalar)	Anticipated / Obvious (by Divsalar and Ping)

19 **A. Claim 13 of the '781 Patent is Invalid**

20 371. Claim 13 of the '781 patent reads:

21 13. A method of encoding a signal, comprising:  
 22 receiving a block of data in the signal to be encoded, the block of  
 data including information bits; and  
 23 performing an encoding operation using the information bits as an  
 input, the encoding operation including an accumulation of mod-2  
 24

1 or exclusive-OR sums of bits in subsets of the information bits, the  
2 encoding operation generating at least a portion of a codeword,  
3 wherein the information bits appear in a variable number of subsets.

4 i) Claim 13 of the '781 Patent is Anticipated by Divsalar

5 372. I explain below, one limitation at a time, why claim 13 is anticipated by  
6 Divsalar.

7 a) "A method of encoding a signal"

8 373. Even if the preamble, "[a] method of encoding a signal," limits the claim, it  
9 is taught by Divsalar, as explained above with reference to the asserted claims of  
10 the '710 patent.

11 b) "receiving a block of data in the signal to be encoded, the block  
12 of data including information bits"

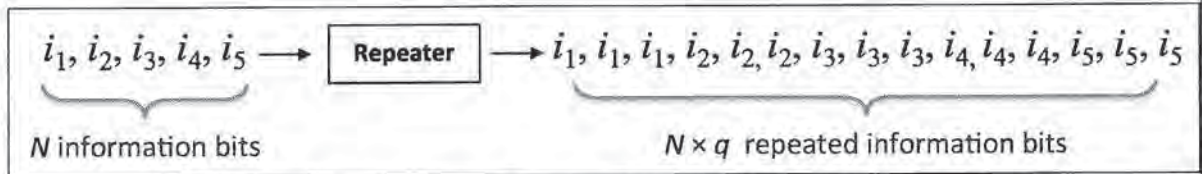
13 374. Divsalar teaches this limitation. As explained above with reference to  
14 the '710 patent, Divsalar deals exclusively with block codes. The repeat-  
15 accumulate codes introduced by Divsalar are encoded by receiving an "information  
16 block of length  $N$ " and passing the block to the repeater. See Divsalar at 5 ("[a]n  
17 information block of length  $N$  is repeated  $q$  times...") and, for example, Figure 3,  
18 reproduced above. Divsalar refers to the input block as an "information block"  
19 because it includes information bits.

20 c) "performing an encoding operation using the information bits as  
21 an input, the encoding operation including an accumulation of  
22 mod-2 or exclusive-OR sums of bits in subsets of the information  
23 bits, the encoding operation generating at least a portion of a  
24 codeword"

25 375. Divsalar teaches this limitation. As explained above, Figure 3 of Divsalar  
26 depicts an encoder that is operable to perform an encoding operation in which an  
27 "information block of length  $N$ " is fed into a repeater, which repeats each of the  $N$

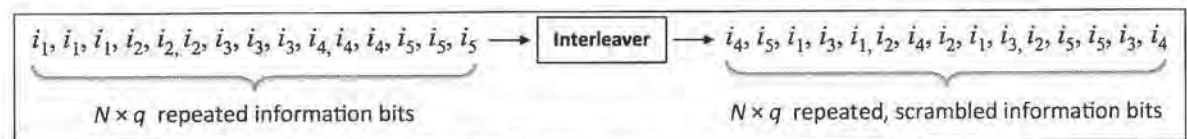


1 information bits  $q$  times, producing a total of  $N \times q$  repeated bits. This process is  
 2 illustrated below for  $N = 5$ , and  $q = 3$ :<sup>53</sup>



6 **Example of Repetition as Described in Divsalar, with  $N=5$  and  $q=3$**

7 376. Figure 3 of Divsalar, reproduced above, shows a “permutation matrix” (the  
 8 box labeled “P”). After the repeater duplicates each of the  $N$  information bits  $q$   
 9 times and outputs  $N \times q$  repeated bits, the repeated bits are “scrambled by an  
 10 interleaver of size  $qN$ ” (Divsalar at 5). Continuing the example above, with  $N = 5$   
 and  $q = 3$ , the interleaving process may be illustrated as follows:



13 **Example of Interleaving as Described in Divsalar, with  $N=5$  and  $q=3$**

14 The example interleaving shown in the figure above illustrates just one of many  
 15 permutations that may be used to scramble the repeated information bits. One of  
 16 ordinary skill in the art would understand that the interleaver shown in Figure 3 of  
 17 Divsalar would be prefabricated or preprogrammed to implement one of the  
 possible permutations.

18 377. As explained above, the repeated, scrambled information bits are then  
 19 *accumulated*. Divsalar explains the accumulate step as follows:

20 [W]e prefer to think of [the accumulator] as a block coder whose  
 21 input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by  
 the formula

$$y_1 = x_1$$

$$y_2 = x_1 + x_2$$

23 <sup>53</sup> Divsalar explicitly discloses, *e.g.*, in Figure 5, several values for  $N$  and  $q$ . Divsalar’s values of  
 24  $N$  are much larger than 5, but I have used  $N = 5$  in this example for ease of exposition. RA codes  
 with  $q = 3$  are explicitly disclosed by Divsalar.

$$y_3 = x_1 + x_2 + x_3$$

$$y_n = x_1 + x_2 + x_3 + \dots + x_n$$

(Divsalar at 5)

378. This accumulation operation operates on “sums of bits in subsets of the information bits.” To explain why this is the case, I will continue the example above (with  $N = 5$  and  $q = 3$ ).

379. In our example,  $N = 5$  and  $q = 3$ , so the accumulator will accept 15  $x$  bits as input, and produce 15  $y$  bits as output. The excerpt from Divsalar above provides explicit equations for  $y_1, y_2,$  and  $y_3,$  and a general equation for each  $y_n$  thereafter. Writing these equations out explicitly for each of the 15  $y$  bits yields the following 15 equations:

$$y_1 = x_1$$

$$y_2 = x_1 + x_2$$

$$y_3 = x_1 + x_2 + x_3$$

$$y_4 = x_1 + x_2 + x_3 + x_4$$

$$y_5 = x_1 + x_2 + x_3 + x_4 + x_5$$

$$y_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$y_7 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$y_8 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

$$y_9 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

$$y_{10} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

$$y_{11} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

$$y_{12} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

$$y_{13} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

$$y_{14} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

$$y_{15} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$$

380. As explained above, the  $x$  bits taught by Divsalar are repeated, interleaved information bits. Each  $x$  bit (such as, *e.g.*,  $x_3$ ) is a duplicate of one of the information bits. For example, continuing our interleaving example above, the correspondence between  $x$  bits and  $i$  bits is as follows:



1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$	$x_{15}$
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$i_4$	$i_5$	$i_1$	$i_3$	$i_1$	$i_2$	$i_4$	$i_2$	$i_1$	$i_3$	$i_2$	$i_5$	$i_5$	$i_3$	$i_4$

**Correspondence between repeated, scrambled information bits  $x$  and information bits  $i$**

As the figure above shows, each  $i$  bit corresponds to exactly 3  $x$  bits (because every information bit is duplicated  $q=3$  times before interleaving). For example, the three duplicates of the 4<sup>th</sup> information bit  $i_4$  are  $x_1$ ,  $x_7$ , and  $x_{15}$ .

381. Continuing our example, we can substitute each of the  $x$  variables in the 15 equations above for the corresponding  $i$  bit, yielding:

- $y_1 = i_4$
- $y_2 = i_4 + i_5$
- $y_3 = i_4 + i_5 + i_1$
- $y_4 = i_4 + i_5 + i_1 + i_3$
- $y_5 = i_4 + i_5 + i_1 + i_3 + i_1$
- $y_6 = i_4 + i_5 + i_1 + i_3 + i_1 + i_2$
- $y_7 = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4$
- $y_8 = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2$
- $y_9 = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1$
- $y_{10} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3$
- $y_{11} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3 + i_2$
- $y_{12} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3 + i_2 + i_5$
- $y_{13} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3 + i_2 + i_5 + i_5$
- $y_{14} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3 + i_2 + i_5 + i_5 + i_3$
- $y_{15} = i_4 + i_5 + i_1 + i_3 + i_1 + i_2 + i_4 + i_2 + i_1 + i_3 + i_2 + i_5 + i_5 + i_3 + i_4$

382. In modulo-2 addition, adding a bit to itself always results in 0; that is, two identical information bits cancel each other out. Therefore, if an information bit appears an even number of times in one of the equations above, it cancels out entirely, and if it appears an odd number of times, it effectively appears only once. Performing these cancellations yields:

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24

Explicit Equation	Recursive Equation
$y_1 = i_4$	$y_1 = i_4$
$y_2 = i_4 + i_5$	$y_2 = y_1 + i_5$
$y_3 = i_4 + i_5 + i_1$	$y_3 = y_2 + i_1$
$y_4 = i_4 + i_5 + i_1 + i_3$	$y_4 = y_3 + i_3$
$y_5 = i_4 + i_5 + i_3$	$y_5 = y_4 + i_1$
$y_6 = i_4 + i_5 + i_3 + i_2$	$y_6 = y_5 + i_2$
$y_7 = i_5 + i_3 + i_2$	$y_7 = y_6 + i_4$
$y_8 = i_5 + i_3$	$y_8 = y_7 + i_2$
$y_9 = i_5 + i_3 + i_1$	$y_9 = y_8 + i_1$
$y_{10} = i_5 + i_1$	$y_{10} = y_9 + i_3$
$y_{11} = i_5 + i_1 + i_2$	$y_{11} = y_{10} + i_2$
$y_{12} = i_1 + i_2$	$y_{12} = y_{11} + i_5$
$y_{13} = i_1 + i_2 + i_5$	$y_{13} = y_{12} + i_5$
$y_{14} = i_1 + i_2 + i_5 + i_3$	$y_{14} = y_{13} + i_3$
$y_{15} = i_1 + i_2 + i_5 + i_3 + i_4$	$y_{15} = y_{14} + i_4$

383. The above table includes explicit and recursive equations for each of the  $y$  bits. The explicit equations on the left show that the  $y$  bits are “sums of bits in subsets of the information bits.” For example,  $y_{14}$  is the sum of bits in the subset of the information bits containing  $i_1, i_2, i_5$  and  $i_3$ . The recursive equations on the right illustrate that the process of computing the  $y$  bits using an “accumulation” operation in which each of the  $y$  bits (except the first) is the sum of the previous  $y$  bit and an  $i$  bit.

384. Therefore, as this example shows, the encoding shown in Figure 3 of Divsalar is an accumulation of “sums of bits in subsets of the information bits” (*i.e.*, the subsets of the  $i$  bits that appear in each of the explicit equations).

385. Because all of the variables in the above equations are bits, the “+” sign in Divsalar represents modulo-2 addition, or “exclusive-OR,” as required by this



1 limitation. Further, because the  $y$  bits are the output of the encoding process,  
2 accumulation is the last step in the encoding process, the  $y$  bits form a codeword.

3 386. Therefore, for the reasons explained above, Divsalar teaches “performing an  
4 encoding operation using the information bits as an input, the encoding operation  
5 including an accumulation of mod-2 or exclusive-OR sums of bits in subsets of the  
6 information bits, the encoding operation generating at least a portion of a  
7 codeword,” as required by claim 13 of the '781 patent.

8 387. Alternatively, a punctured version of the Divsalar code would satisfy the  
9 requirement of claim 13 that “the encoding operation include[e] an accumulation  
10 of mod-2 or exclusive-OR sums of bits in subsets of the information bits, the  
11 encoding operation generating at least a portion of a codeword.” As explained  
12 above, the accumulation of Divsalar can be represented using the following  
13 equations:

$$y_1 = x_1$$

$$y_2 = x_1 + x_2$$

$$y_3 = x_1 + x_2 + x_3$$

$$y_4 = x_1 + x_2 + x_3 + x_4$$

$$y_5 = x_1 + x_2 + x_3 + x_4 + x_5$$

$$y_6 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$y_7 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$y_8 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$$

$$y_9 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

$$y_{10} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

$$y_{11} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11}$$

$$y_{12} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12}$$

$$y_{13} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13}$$

$$y_{14} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14}$$

$$y_{15} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$$

1 Outputting some, but not all of the parity bits  $y_i$  would result in a code in which  
2 each parity bit is the sum of the previous parity bit and the sum of the bits in a  
3 subset of the information bits. For example, if only  $y_1$ ,  $y_7$ ,  $y_9$ , and  $y_{15}$  were output  
4 by the encoder, the resulting code could be described using the following 4  
5 equations:

$$y_1 = x_1$$

$$y_7 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$y_9 = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9$$

$$y_{15} = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} + x_{15}$$

8 Substituting the explicit equations from the table above would yield

$$y_1 = i_4$$

$$y_7 = i_5 + i_3 + i_2$$

$$y_9 = i_5 + i_3 + i_1$$

$$y_{15} = i_1 + i_2 + i_5 + i_3 + i_4$$

12 In other words,

$$y_1 = i_4$$

$$y_7 = y_1 + (i_4 + i_5 + i_3 + i_2)$$

$$y_9 = y_7 + (i_2 + i_1)$$

$$y_{15} = y_9 + (i_4 + i_2)$$

16 Here, each parity bit, (e.g.,  $y_9$ ) is calculated as the accumulation of the previous  
17 parity bit (i.e.,  $y_7$ ) and the sum of a subset of the information bits (i.e.,  $i_2 + i_1$ ).

18 Outputting only some of the parity bits of Divsalar, while omitting others, is a  
19 technique called “puncturing,” that was well known in the art by Caltech’s alleged  
20 conception date (see, e.g., Frey99 at 3, “... some extra parity bits must be  
21 punctured”).

22 d) “wherein the information bits appear in a variable number of  
subsets”

23 388. As explained above, the encoding operation of Divsalar includes an  
24 accumulation of mod-2 sums of bits in subsets of information bits. One example



1 of such an accumulation, for  $N = 5$ ,  $q = 3$  and a particular interleaver, is given by  
2 the explicit equations given in the table above, namely:

3  $y_1 = i_4$

4  $y_2 = i_4 + i_5$

5  $y_3 = i_4 + i_5 + i_1$

6  $y_4 = i_4 + i_5 + i_1 + i_3$

7  $y_5 = i_4 + i_5 + i_3$

8  $y_6 = i_4 + i_5 + i_3 + i_2$

9  $y_7 = i_5 + i_3 + i_2$

10  $y_8 = i_5 + i_3$

11  $y_9 = i_5 + i_3 + i_1$

12  $y_{10} = i_5 + i_1$

13  $y_{11} = i_5 + i_1 + i_2$

14  $y_{12} = i_1 + i_2$

15  $y_{13} = i_1 + i_2 + i_5$

16  $y_{14} = i_1 + i_2 + i_5 + i_3$

17  $y_{15} = i_1 + i_2 + i_5 + i_3 + i_4$

18 Here, each bit  $y$  of the codeword is the sum of a different subset of information bits  
19 (denoted using the letter  $i$ ).

20 389. As these equations show, different information bits appear in different  
21 numbers of subsets. For example, the information bit  $i_4$  appears in seven of the  
22 equations above, while the information bit  $i_1$  appears in nine of the equations.<sup>54</sup>

23 Also, while the equations above result from using an interleaver that scrambles bits  
24 according to one particular permutation, different information bits will appear in  
25 different numbers of subsets no matter how the bits are permuted by the interleaver.

26 In particular, for some of the example values of  $N$  and  $q$  explicitly disclosed in  
27 Divsalar in Figure 5, *e.g.*,  $N = 1024$  and  $q = 3$ , regardless of what interleaver is  
28 used, the information bits will appear in a variable number of subsets. Thus,

29 \_\_\_\_\_  
30 <sup>54</sup> Because the “subsets” are the groups of  $i$  bits that appear on the right-hand side of each of the  
31 equations above, if an  $i$  bit appears in an equation, it is a member of the corresponding subset.

1 Divsalar teaches that the information bits appear in variable numbers of subsets, as  
2 required by the claim.

3 390. This limitation also holds for the “punctured” version of Divsalar discussed  
4 above. For example, it holds for the example given above, where the parity bits  
5 are represented by the equations:

$$6 \quad y_1 = i_4$$

$$7 \quad y_7 = y_1 + (i_4 + i_5 + i_3 + i_2)$$

$$8 \quad y_9 = y_7 + (i_2 + i_1)$$

$$9 \quad y_{15} = y_9 + (i_4 + i_2)$$

10 Here, the information bits appear in a variable number of subsets. The information  
11 bit  $i_4$ , for example, appears in three subsets, while the information bit  $i_5$  appears in  
12 only one.

13 *e) Summary*

14 391. As explained above, Divsalar teaches every limitation of claim 13 and  
15 therefore anticipates claim 13.

16 *ii) Claim 13 of the '781 Patent is Obvious over Divsalar in View any*  
17 *one of Frey99, Luby, or MacKay*

18 392. As explained above, Divsalar teaches every limitation of, and therefore  
19 anticipates, claim 13 of the '781 patent. However, in the event Divsalar is found  
20 not to teach the “wherein the information bits appear in a variable number of  
21 subsets” limitation of claim 13, then claim 13 is obvious over the combination of  
22 Divsalar and any one of Frey99, Luby or MacKay.

23 393. Specifically, if the term “wherein the information bits appear in a variable  
24 number of subsets,” is interpreted to require that the claimed encoding method be



1 *irregular*, then each of Frey99, Luby, and MacKay teaches this limitation.<sup>55</sup> As I  
2 explain above, one of ordinary skill in the art would have been motivated to  
3 combine Divsalar and Frey99, Luby or MacKay in general, and would specifically  
4 have been motivated to use the irregular repetition of Frey99, or the irregularity of  
5 Luby and MacKay, with the RA codes of Divsalar. I also explain why such a  
6 combination would represent a minor modification to the teachings of Divsalar,  
7 and would not fundamentally change its principle of operation.

8 394. For at least the reasons given above, claim 13 is obvious over the  
9 combination of Divsalar and any one of Frey99, Luby and MacKay.

10 iii) Claim 13 of the '781 Patent is Obvious over Ping in View of  
11 MacKay

12 395. I explain below, one limitation at a time, why claim 13 is rendered obvious  
13 by Ping in combination with MacKay.<sup>56</sup>

14 a) "A method of encoding a signal, comprising ..."

15 396. Ping teaches the preamble. As I explain above, Ping teaches constructing  
16 LDPC codes that can be encoded in two stages. In the first encoding stage, a  
17 generator matrix is applied to a sequence of information bits to produce sums of  
18 information bits. In the second stage, the sums of information bits are accumulated  
19 recursively to generate the parity bits (*see* Ping at 38).

20 <sup>55</sup> As noted above, during prosecution of the '781 patent, the applicant edited claims 9 and 23  
21 effectively replacing "irregular" with "variable number of subsets." Response dated January 27,  
22 2011, at 3 and 5-6. In its remarks regarding that amendment, the applicant stated, "It is believed  
23 that the meaning of the term 'irregular' in the claims is clear and is well known in the art...  
24 However, claims have been amended to recite '...wherein the information bits appear in a  
variable number of subsets' to obviate the objections." Response dated January 27, 2011, at 7  
(emphasis original). In view of this file history, Caltech may argue that "variable number of  
subsets" requires irregularity. However, the applicant may have simply been broadening the  
claims when it replaced "irregular" with "variable number of subsets." In any case, the claims  
do not clearly require irregularity. However, for the sake of completeness, I have addressed the  
term under the two possible interpretations herein, one in which irregularity is required and one  
in which it is not.

<sup>56</sup> *See generally* Divsalar Tr. at 61:15-71:11.



1 397. A person of ordinary skill in the art would recognize that encoding/decoding  
2 signals is the purpose of the “LDPC + accumulate” codes taught by Ping.

3 b) “receiving a block of data in the signal to be encoded, the block  
4 of data including information bits”

5 398. Ping teaches this limitation. Ping deals exclusively with block codes, as I  
6 explain above. Specifically, Ping denotes the block of information bits to be  
7 encoded using the vector variable name  $\mathbf{d}$  (see Ping at 38, “[d]ecompose the  
8 codeword  $\mathbf{c}$  as  $\mathbf{c} = [\mathbf{p}, \mathbf{d}]$ , where  $\mathbf{p}$  and  $\mathbf{d}$  contain the parity and information bits,  
9 respectively”). Ping goes on to provide equations from which “ $\mathbf{p} = \{p_i\}$  can easily  
10 be calculated from a given  $\mathbf{d} = \{d_i\}$ .” That is, Ping provides equations that describe  
11 the process of *encoding* a block of information bits  $\mathbf{d}$ , as required by this limitation  
12 (*id.*). Thus, the vector of information bits  $\mathbf{d}$  is a “block of data in the signal to be  
13 encoded, the block of data including information bits.”

14 c) “performing an encoding operation using the information bits as  
15 an input, the encoding operation including an accumulation of  
16 mod-2 or exclusive-OR sums of bits in subsets of the information  
17 bits, the encoding operation generating at least a portion of a  
18 codeword”

19 399. Ping teaches this limitation. Specifically, Ping teaches an encoding  
20 operation that calculates the parity bits  $\{p_i\}$  using the information bits  $\{d_i\}$  as input.  
21 Ping’s encoding scheme is encapsulated by the following equations:

$$22 \quad p_1 = \sum_j h_{1j}^d d_j$$
$$23 \quad p_i = p_{i-1} + \sum_j h_{ij}^d d_j$$

24



(Ping at 38)

400. In these equations, the variable  $h_{ij}^d$  represents the value at the  $i^{\text{th}}$  row and the  $j^{\text{th}}$  column of the parity check matrix  $\mathbf{H}^d$ , and the variable  $d_j$  represents the value of the  $j^{\text{th}}$  information bit (*see id.*). Thus, the summation

$$\sum_j h_{ij}^d d_j$$

represents the sum of the bits in a subset of information bits. Specifically, it represents the sum of the subset of information bits  $d_j$  where  $h_{ij}^d = 1$ . As Ping explains, there are  $kt/(n-k)$  information bits in each row of the parity check matrix, meaning that there  $kt/(n-k)$  bits in each subset of the information bits (*id.*).

401. Further, the encoding taught by Ping includes an accumulation of these sums of bits in subsets of the information bits. The first parity bit of Ping,  $p_1$ , is calculated as the sum of a subset of information bits. As illustrated in the color-coded equation below, each subsequent parity bit  $p_i$  is calculated by adding together the previous parity bit  $p_{i-1}$  (shown in blue) and the sum of bits in a subset of information bits (shown in red):

$$p_i = p_{i-1} + \sum_j h_{ij}^d d_j$$

I explain above that this type of operation, in which each new element is calculated by adding something to the previous element, is called an “accumulation.”

402. Also, the addition taught by Ping is modulo-2 or “mod-2” addition (which is the same operation as “exclusive-OR”). When the addition symbol “+” and the summation symbol “ $\Sigma$ ” have bits as operands, as they do here, one of ordinary skill

1 in the art would understand that these symbols refer to modulo-2 addition and  
2 modulo-2 summation, respectively.

3 403. When complete, this process produces the parity bits  $\{p_i\}$  which, as Ping  
4 explains, is a portion of the codeword " $\mathbf{c} = [\mathbf{p}, \mathbf{d}]$ , where  $\mathbf{p}$  and  $\mathbf{d}$  contain the parity  
5 and information bits, respectively" (Ping at 38).

6 d) "wherein the information bits appear in a variable number of  
7 subsets"

8 404. As explained above, MacKay teaches implementing parity-check matrices in  
9 which every information bit corresponds to a column, where the weight of that  
10 column (*i.e.*, the number of 1s contained in that column of the parity-check matrix)  
11 represents the degree of the information bit.

12 405. As explained above, MacKay teaches parity-check matrices for which each  
13 column corresponds to an information bit or a parity bit, and each row corresponds  
14 to a parity check: "[t]he parity check matrix of a code can be viewed as defining a  
15 bipartite graph with 'bit' vertices corresponding to the columns and 'check'  
16 vertices corresponding to the rows. Each nonzero entry in the matrix corresponds  
17 to an edge connecting a bit to a check. The profile specifies the degrees of the  
18 vertices in this graph" (MacKay at 1449-1450).

19 406. Thus, each row in the parity-check matrices of MacKay corresponds to a  
20 subset of information bits that are summed during the encoding process. In a given  
21 row, if the entry corresponding to an information bit is a 1, that information bit is a  
22 member of the subset. If the entry corresponding to an information bit is 0, the  
23 information bit is not a member of the subset.

24 407. In the parity-check matrices of MacKay, the number of ones in a column  
that corresponds to an information bit (*i.e.*, the column weight) equals to the  
number of times that information bit appears in a subset. MacKay also notes that



1 “[t]he best known binary Gallager codes are *irregular* codes whose parity check  
2 matrices have *nonuniform* weight per column,” meaning that the best codes are  
3 those in which the information bits appear in a variable number of subsets (Mackay  
4 at 1449) (emphasis in original).

5 e) Summary

6 408. As explained above, the combination of Ping and Mackay teaches every  
7 limitation of claim 13 of the '781 patent.

8 f) Motivations to Combine the teachings of Ping with those of  
9 MacKay

10 409. As I explain above, it would have been obvious to one of ordinary skill in  
11 the art to combine the LDPC-accumulate coders of Ping with the irregularity of  
12 MacKay. Specifically, it would have been obvious to one of ordinary skill in the  
13 art to incorporate irregularity into the LDPC-accumulate coders of Ping by making  
14 the column weights of the parity check matrix  $\mathbf{H}^d$  that correspond to information  
15 bits nonuniform, resulting in a code in which “the information bits appear in a  
16 variable number of subsets,” as required by claim 13.

17 410. It would have been obvious to incorporate MacKay’s irregularity into Ping  
18 because the two codes are so similar and it was known that irregularity improves  
19 coding as explained above. Alternatively, it would have been obvious to  
20 incorporate Ping’s accumulate stage into MacKay. MacKay’s irregular LDPC  
21 code teaches all limitations of claim 13 of the '781 patent except the “accumulation”  
22 limitation (i.e., as explained above, MacKay teaches (a) block codes and therefore  
23 teaches the “receiving” limitation, (b) LDPC codes and therefore teaches  
24 computing parity bits that are sums of bits in subsets of the information bits, and (c)  
irregular codes and therefore teaches the “information bits appear in a variable  
number of subsets” limitation). However, both Ping and Divsalar taught the  
benefit of adding an accumulation stage to an outer encoder. Thus, regardless of

1 whether one of ordinary skill incorporated MacKay's irregularity into Ping or  
2 Ping's accumulator into MacKay, the combination of Ping and MacKay renders  
3 claim 13 of the '781 patent obvious.

4 **B. Claim 14 of the '781 Patent is Invalid**

5 411. Claim 14 of the '781 patent reads:

6 14. The method of claim 13, further comprising: outputting the  
7 codeword, wherein the codeword comprises parity bits.

8 a) Claim 14 is Anticipated by Divsalar, and rendered obvious by a  
9 combination of Divsalar and Frey99, Luby, or MacKay

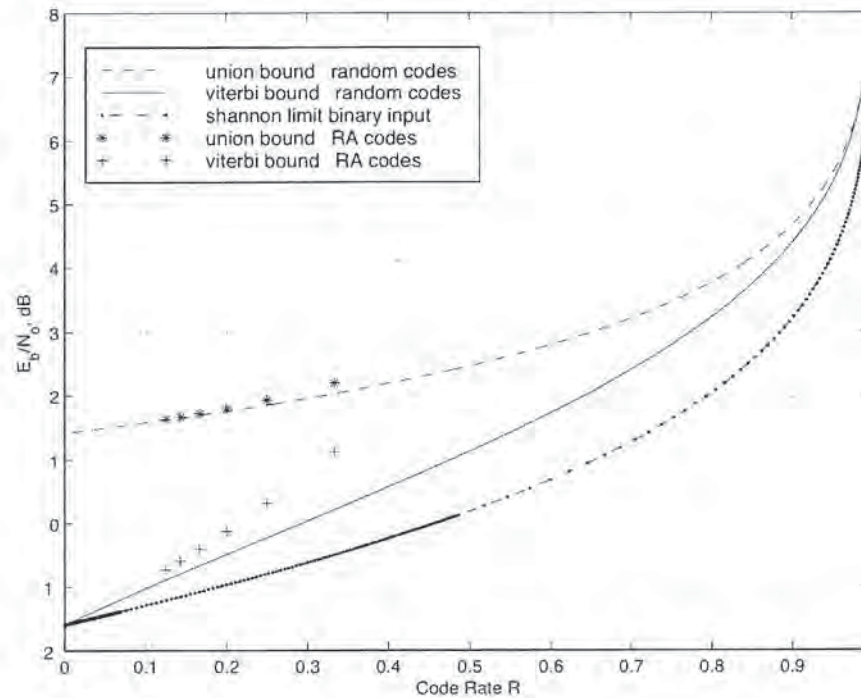
10 412. Claim 14 is anticipated by Divsalar, and is obvious over Divsalar in view of  
11 any one of Frey99, Luby or MacKay. As I explain above, claim 13 of the '781  
12 patent is anticipated by Divsalar, and rendered obvious by a combination of  
13 Divsalar and any one of Frey99, Luby, or MacKay. Claim 14 adds to claim 13  
14 "outputting the codeword, wherein the codeword comprises parity bits." Divsalar,  
15 Frey99, Luby, and MacKay all teach methods of encoding signals that comprise  
16 outputting a codeword that comprises parity bits.

17 413. As explained above, the encoder shown in Figure 3 of Divsalar, reproduced  
18 above, produces an a "**output** block [ $y_1, \dots, y_n$ ]" of parity bits that is included in the  
19 codeword (Divsalar at 5) (emphasis added).

20 414. Divsalar also describes the performance of the RA codes it teaches by  
21 graphing the code rate  $R$  against the normalized signal-to-noise ratio  $E_b/N_0$ :<sup>57</sup>

22  
23  
24 <sup>57</sup> The normalized signal-to-noise ratio  $E_b/N_0$  is described in detail above and in Appendix A.





Divsalar, Figure 4

The concept of  $E_b/N_0$  only makes sense in the context of outputting codewords, transmitting them over a noisy channel, and decoding them at the other end.

415. As described above, the irregular turbocoding techniques taught by Frey99 also involve outputting a codeword that comprises parity bits (*see generally*, Frey99 at 1-4). *See also* Frey Slides at 4-5. Like Divsalar, Frey99 includes experimental results; it includes a plot of BER against  $E_b/N_0$  for various irregular turbocodes (*see* Frey99, Figure 4). *See also* Frey Slides at 9, 11, 12 (showing BER- $E_b/N_0$  curves). As explained above, Luby and MacKay also teach outputting codewords that include parity bits (*see, e.g.*, Luby at 1:46-60, MacKay at Fig. 5).

b) Claim 14 is Rendered Obvious by a combination of Ping and MacKay

416. Claim 14 is rendered obvious by a combination of Ping and MacKay. As I explain above, claim 13 of the '781 patent is rendered obvious by a combination of Ping and MacKay. Claim 14 adds to claim 13 "outputting the codeword, wherein

1 the codeword comprises parity bits.” Luby and MacKay also both teach outputting  
2 a codeword that comprises parity bits (*see, e.g.,* Luby at 1:46-60, MacKay at Fig.  
3 5).

4 417. Ping teaches “outputting” the codeword. Like Divsalar and Frey99, Ping  
5 describes the performance of the codes it discloses using plots of the BER of  
6 various codes against the normalized signal-to-noise ratio  $E_b/N_0$ :

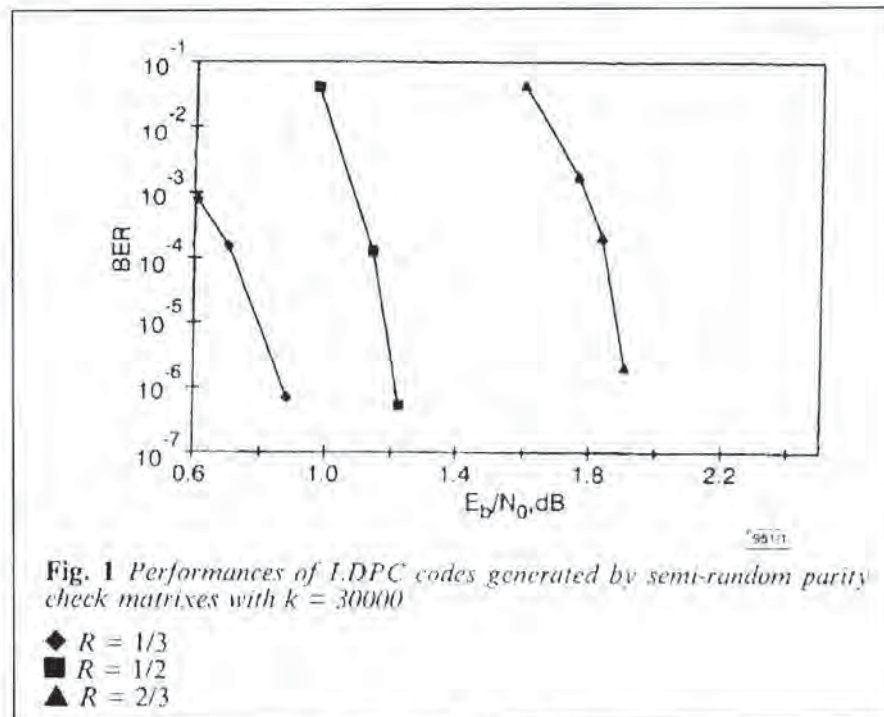


Fig. 1 Performances of LDPC codes generated by semi-random parity check matrixes with  $k = 30000$

◆  $R = 1/3$   
■  $R = 1/2$   
▲  $R = 2/3$

Ping, Figure 1

17 As I explain above, the concepts of “BER” and  $E_b/N_0$  only make sense in the  
18 context of generating codewords, transmitting them over a noisy channel, and  
19 decoding them at the other end.

20 418. Further, as I explain above, Ping teaches a coding scheme in which the  
21 codeword includes both information and parity bits.<sup>58</sup> Specifically, Ping teaches  
22 that “the codeword  $\mathbf{c}$  as  $\mathbf{c} = [\mathbf{p}, \mathbf{d}]$ , where  $\mathbf{p}$  and  $\mathbf{d}$  contain the parity and  
23 information bits, respectively” (Ping at 38).

24 <sup>58</sup> Such codes are called *systematic codes*.



1 419. Therefore, claim 14 is rendered obvious by a combination of Ping and  
2 MacKay.

3 **C. Claim 15 of the '781 Patent is Invalid**

4 420. Claim 15 of the '781 patent reads:

5 15. The method of claim 14, wherein outputting the codeword  
6 comprises: outputting the parity bits; and outputting at least some  
of the information bits.

7 421. Claim 15 is rendered obvious by Divsalar alone or in combination with Ping,  
8 Frey99, MacKay or the '999 patent, and is also obvious over Divsalar in view of  
9 Luby, alone or in further in view of either Ping, Frey99, MacKay or the '999 patent  
10 (i.e., (Divsalar + Luby alone or (Divsalar + Luby + (Ping, Frey99, MacKay or  
11 the '999 patent))). As I explain above, claim 14 of the '781 patent is anticipated by  
12 Divsalar, and obvious over Divsalar in view of any one of Frey99, Luby, or  
13 MacKay. Claim 15 adds to claim 14 “wherein outputting the codeword comprises:  
14 outputting the parity bits; and outputting at least some of the information bits.”  
15 That is, a systematic code would teach the limitations added by claim 15 and as  
16 explained above, systematic codes were known long before Caltech’s alleged  
17 invention. Frey99 and MacKay teach systematic codes (*see* Frey99 at 3; MacKay  
18 at Fig. 5, showing “[b]its  $t_1 \dots t_k$  defined to be source bits”) and it would have  
19 been obvious to make the codes of Divsalar or Luby systematic. Also, the  
additional limitation of claim 15 is taught explicitly by each of Ping and the '999  
patent.

20 422. As explained above, Ping teaches a systematic code wherein “outputting the  
21 codeword comprises: outputting the parity bits; and outputting at least some of the  
22 information bits.”

23 423. Also, as I explain above in the context of the '032 patent, one of  
24 ordinary skill in the art would have been motivated to combine the



1 teachings of Divsalar, Frey99, Luby or MacKay, and Ping. Therefore,  
2 claim 15 is rendered obvious by Divsalar in combination with Ping, and  
3 is also rendered obvious by the following combinations:

- 4 • Divsalar alone (systematic codes being well known) or in combination with  
Ping (Ping teaching the systematic code)
- 5 • Divsalar in combination with Frey99 or MacKay (Frey99 and MacKay each  
6 teaching both irregularity and systematic codes)
- 7 • Divsalar in combination with Luby (Luby teaching irregularity and  
8 systematic codes being well known)
- Divsalar in combination with Luby and Ping (Luby teaching irregularity and  
Ping teaching systematic codes)

9 424. The '999 patent also teaches a systematic code in which "outputting the  
10 codeword comprises: outputting the parity bits; and outputting at least some of the  
11 information bits":

12 Suitable *codewords* for such schemes have been generated in a  
13 variety of ways. For systematic encoding of cyclic codes, one such  
14 method utilizes serial data input/output wherein each information  
15 word is applied to an (n-k)-stage shift register with feedback  
16 connections based on a generator polynomial. *After the  
information bits are shifted into the register and simultaneously  
into the communication channel, the n-k parity bits formed in the  
register are shifted into the channel, thus forming the complete  
codeword.*

( '999 Patent at 1:25-34) (emphasis added).

17 As indicated by the passage above, the '999 patent teaches encoding schemes in  
18 which a "complete codeword" includes both the "information bits" and "the n-k  
19 parity bits."

20 425. Like Divsalar, Frey99, Luby, and MacKay, the '999 patent relates to  
21 methods of improving the performance of linear error-correcting codes. It was  
22 filed in 1984 and granted in 1986, well over a decade before the claimed priority  
23 date of the patents-in-suit. By March 7, 2000, the alleged conception date of the  
24 patents-in-suit, the technology described in the '999 patent would have been well



1 known in the field, and one of ordinary skill in the art at the time would have been  
2 motivated to combine the teachings of Divsalar and Frey99, Luby, or MacKay with  
3 those of the '999 patent. Therefore, claim 15 is rendered obvious by Divsalar in  
4 combination with the '999 patent, and is also rendered obvious by a combination  
5 of Divsalar, Frey99, Luby, or MacKay, and the '999 patent.

6 426. Summarizing, systematic codes were notoriously well known before  
7 Caltech's alleged invention. Ping, Frey99, MacKay and the '999 patent are  
8 examples of references that teach systematic codes. It would have been obvious to  
9 make a code like the ones taught in Divsalar or Luby systematic in view of the  
10 general knowledge of one of ordinary skill, *e.g.*, as exemplified by Ping, Frey99,  
11 MacKay and the '999 patent.

12 427. Claim 15 of the '781 patent is also rendered obvious by a combination of  
13 Ping and MacKay. As I explain above, Ping and MacKay teach every element of  
14 claim 14, and Ping and MacKay also teach the additional limitation imposed by  
15 claim 15. Therefore, claim 15 is obvious over a combination of Ping and MacKay.

#### 16 **D. Claim 16 of the '781 Patent is Invalid**

17 428. Claim 16 of the '781 patent reads:

18 16. The method of claim 15, wherein the parity bits follow the  
19 information bits in the codeword.

20 429. Claim 16 is obvious over Divsalar in view of either Ping, Frey99, MacKay  
21 or the '999 patent, and is also obvious over Divsalar in view of Luby, alone or  
22 further in view of either Ping, Frey99, MacKay or the '999 patent (*i.e.*, (Divsalar +  
23 Luby alone or (Divsalar + Luby + (Ping, Frey99, MacKay or the '999 patent))).

24 430. As I explain above, claim 15 of the '781 patent is rendered obvious by  
Divsalar alone or in combination with references that teach irregularity (Frey99,  
MacKay or Luby), if claim 13 is found to require irregularity, and with references  
that teach systematic codes (Frey99, MacKay, Ping and the '999 patent).



1 431. Claim 16 adds to claim 15 “wherein the parity bits follow the information  
2 bits in the codeword.” That is, claim 16 adds to claim 15 that in the systematic  
3 code the bits must appear in a particular order, with the parity bits following the  
4 information bits. Whether the parity bits precede or follow the systematic bits, or  
5 appear in some other order, is not significant and the limitation added by claim 16  
6 is obvious in view of any teaching of a systematic code, and as stated above  
7 systematic codes were notoriously well known before Caltech’s alleged  
8 invention.<sup>59</sup>

8 432. Further, the specification of the ’781 patent offers no guidance regarding  
9 what it means for the parity bits to “follow” the information bits in the codeword,  
10 and Caltech has argued specifically that a sequence of bits can be a “codeword”  
11 even if that sequence is never transmitted (*see* Dkt. No. 67 at 17-19), so this claim  
12 limitation cannot be interpreted to require that the parity bits be transmitted at a  
13 later time than the information bits, or vice versa.

13 433. I conclude that “the parity bits follow the information bits in the codeword”  
14 requires only that the parity bits and the information bits not be intermingled  
15 within the codeword. A codeword therefore satisfies the requirements of claim 16  
16 if the parity bits are located at one end of the codeword, with the information bits  
17 located at the other.

18 434. This requirement taught by Ping. As explained above, Ping teaches that “the  
19 codeword  $\mathbf{c}$  as  $\mathbf{c} = [\mathbf{p}, \mathbf{d}]$ , where  $\mathbf{p}$  and  $\mathbf{d}$  contain the parity and information bits,  
20 respectively” (Ping at 38). This defines the codeword  $\mathbf{c}$  as a vector, with the parity  
21 bits  $\mathbf{p}$  at one end, and the information bits  $\mathbf{d}$  at the other. Therefore, Ping teaches  
22 the additional limitation imposed by claim 16.

---

23 <sup>59</sup> The testimony of Dr. Dariush Divsalar (the author of the Divsalar reference) confirms my own  
24 opinion that the order of systematic and parity bits within a codeword is not significant (*see*  
Divsalar Dep. at 71:15-73:11).



1 435. Alternatively, to the extent that claim 16 requires that a codeword be  
2 transmitted such that the information bits are transmitted *earlier in time* than the  
3 parity bits, this limitation would have been obvious to a person of ordinary skill in  
4 the art based on the teachings of Ping alone. Ping does not specify any temporal  
5 relationship between the transmission of the parity bits **p** and the information bits **d**,  
6 but the teachings of Ping encompass schemes in which the parity bits are  
7 transmitted at a later time than the information bits.

8 436. The '999 patent also teaches this limitation, under either interpretation  
9 considered above. That is, it teaches that the parity bits are at one end of the  
10 codeword with the information bits at the other, and it also (and more specifically)  
11 teaches methods in which the information bits in a codeword are transmitted  
12 *earlier in time* than the parity bits:

13 Suitable codewords for such schemes have been generated in a  
14 variety of ways. For systematic encoding of cyclic codes, one such  
15 method utilizes serial data input/output wherein each information  
16 word is applied to an (n-k)-stage shift register with feedback  
17 connections based on a generator polynomial. *After the*  
18 *information bits are shifted into the register and simultaneously*  
19 *into the communication channel, the n-k parity bits formed in the*  
20 *register are shifted into the channel, thus forming the complete*  
21 *codeword.*

22 ('999 Patent at 1:25-34) (emphasis added).

23 As the above passage explains, the '999 patent teaches encoding schemes in which  
24 the information bits are shifted onto the communication channel (*i.e.*, transmitted)  
*earlier in time* than the associated "n-k parity bits," which are transmitted later.

437. One of ordinary skill in the art would have been motivated to combine the  
teachings of Divsalar and/or Frey99, Luby, or MacKay with those of either Ping or  
the '999 patent, for the reasons outlined above with reference to claim 15 of  
the '781 patent. Therefore, Claim 16 is rendered obvious by Divsalar alone or in  
combination with Ping or the '999 patent, and is also rendered obvious by a

1 combination of Divsalar, with Frey99, Luby, or MacKay, alone or in further  
2 combination with either Ping or the '999 patent.

3 438. Claim 16 of the '781 patent is also rendered obvious by a combination of  
4 Ping and MacKay. As I explain above, Ping and MacKay teach every element of  
5 claim 15, and Ping and MacKay also teach the additional limitation imposed by  
6 claim 16. Therefore, claim 16 is obvious over a combination of Ping and MacKay.

7 **E. Claim 19 of the '781 Patent is Invalid**

8 439. Claim 19 of the '781 patent reads:

9 19. A method of encoding a signal, comprising:  
10 receiving a block of data in the signal to be encoded, the block of  
11 data including information bits; and  
12 performing an encoding operation using the information bits as an  
13 input, the encoding operation including an accumulation of mod-2  
or exclusive-OR sums of bits in subsets of the information bits, the  
encoding operation generating at least a portion of a codeword,  
wherein at least two of the information bits appear in three subsets  
of the information bits.

14 i) Claim 19 of the '781 Patent is Anticipated by Divsalar

15 440. I explain below, one limitation at a time, why claim 19 is anticipated by  
16 Divsalar.

17 a) "A method of encoding a signal, comprising ..."

18 441. Divsalar teaches the preamble, as I explain above with reference to claim 13  
19 of the '781 patent.

20 b) "receiving a block of data in the signal to be encoded, the block  
of data including information bits"

21 442. Divsalar teaches this limitation, as I explain above with reference to claim  
22 13 of the '781 patent.



1 c) “performing an encoding operation using the information bits as  
2 an input, the encoding operation including an accumulation of  
3 mod-2 or exclusive-OR sums of bits in subsets of the information  
4 bits, the encoding operation generating at least a portion of a  
5 codeword”

6 443. Divsalar teaches this limitation, as I explain above with reference to claim  
7 13 of the '781 patent.

8 d) “wherein at least two of the information bits appear in three  
9 subsets of the information bits”

10 444. As explained above, the encoding operation of Divsalar includes an  
11 accumulation of mod-2 sums of bits in subsets of information bits. One example  
12 of such an accumulation, for  $N = 5$  and  $q = 3$ , is given by the equations:

13  $y_1 = i_4$   
14  $y_2 = i_4 + i_5$   
15  $y_3 = i_4 + i_5 + i_1$   
16  $y_4 = i_4 + i_5 + i_1 + i_3$   
17  $y_5 = i_4 + i_5 + i_3$   
18  $y_6 = i_4 + i_5 + i_3 + i_2$   
19  $y_7 = i_5 + i_3 + i_2$   
20  $y_8 = i_5 + i_3$   
21  $y_9 = i_5 + i_3 + i_1$   
22  $y_{10} = i_5 + i_1$   
23  $y_{11} = i_5 + i_1 + i_2$   
24  $y_{12} = i_1 + i_2$   
 $y_{13} = i_1 + i_2 + i_5$   
 $y_{14} = i_1 + i_2 + i_5 + i_3$   
 $y_{15} = i_1 + i_2 + i_5 + i_3 + i_4$

Here, each bit  $y$  of the codeword is the sum of a different subset of information bits (denoted using the letter  $i$ ).

445. As these equations show, at least two of the information bits appear in at least three subsets of information bits. For example, the information bit  $i_4$  appears

1 in seven of the equations above, and information bit  $i_1$  appears in nine of the  
2 equations.

3 446. I interpret this limitation to be met as long as two information bits appear in  
4 three or more subsets, because information bits that appear in more than three  
5 subsets necessarily “appear in three subsets.” For example, if an information bit  
6 appears in seven subsets (like, *e.g.*,  $i_4$ ) and another information bit appears in nine  
7 subsets (like, *e.g.*,  $i_i$ ) then both information bits appear in “three subsets” of  
8 information bits and this limitation is met.

9 447. While the equations above result from an example in which  $N = 5$  and  $q = 3$ ,  
10 other values of  $N$  and  $q$  will necessarily produce codes in which at least two of the  
11 information bits appear in three subsets of the information bits. For example an  
12 RA code with  $N = 1024$ ,  $q = 3$ , which Divsalar specifically discloses, will produce  
13 a code in which at least two of the information bits appear in three subsets of the  
14 information bits (*see* Divsalar, Figure 5).

15 448. Also, while the equations above result from using an interleaver that  
16 scrambles bits according to one particular permutation, at least two information  
17 bits will necessarily appear in three subsets of the information bits no matter how  
18 the bits are permuted by the interleaver. Thus, Divsalar teaches that “at least two  
19 of the information bits appear in three subsets of the information bits,” as required  
20 by claim 19. At a minimum, Divsalar renders claim 19 obvious because it would  
21 have been easy for one of ordinary skill to construct RA codes according to  
22 Divsalar in which “at least two of the information bits appear in three subsets of  
23 the information bits” as shown by the example above.

24 449. This limitation also holds for the “punctured” version of Divsalar discussed  
above. For example, it holds for the example given above, where the parity bits  
are represented by the equations:



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$$y_1 = i_4$$
$$y_7 = y_1 + (i_4 + i_5 + i_3 + i_2)$$
$$y_9 = y_7 + (i_2 + i_1)$$
$$y_{15} = y_9 + (i_4 + i_2)$$

450. Here, the information bits appear in a variable number of subsets. The information bit  $i_4$ , for example, appears in three subsets, while the information bit  $i_5$  appears in only one.

451.

e) Summary

452. As explained above, Divsalar teaches every limitation of, and therefore anticipates, claim 19.

ii) Claim 19 of the '781 Patent is Anticipated by Ping

453. I explain below, one limitation at a time, why claim 19 is anticipated by Ping.

a) "A method of encoding a signal, comprising ..."

454. Ping teaches this limitation, as I explain above with reference to claim 13 of the '781 patent.

b) "receiving a block of data in the signal to be encoded, the block of data including information bits"

455. Ping teaches this limitation, as I explain above with reference to claim 13 of the '781 patent.

c) "performing an encoding operation using the information bits as an input, the encoding operation including an accumulation of mod-2 or exclusive-OR sums of bits in subsets of the information bits, the encoding operation generating at least a portion of a codeword"

456. Ping teaches this limitation, as I explain above with reference to claim 13 of the '781 patent.

1 d) “wherein at least two of the information bits appear in three  
2 subsets of the information bits”

3 457. Ping teaches this limitation. As I explain above, there are  $kt/(n-k)$  1s in each  
4 row of the parity check matrix, and  $t$  1s in each column (see Ping at 38). Because  
5 there are  $t$  ones in every column, each subset of information bits that is summed  
6 prior to accumulation contains exactly  $t$  bits.

7 458. Ping specifically teaches coding schemes “using  $t=4$ ,” in which each  
8 information bit appears in four distinct subsets of the information bits (Ping at 39).  
9 As I explain above, if two information bits appear in four subsets of the  
10 information bits, both information bits necessarily appear in three subsets of the  
11 information bits, and therefore Ping meets this limitation.

12 e) Summary

13 459. As explained above, Ping teaches every limitation of claim 19 and therefore  
14 anticipates claim 19.

15 **IX. THE ASSERTED CLAIMS OF THE '833 PATENT ARE INVALID**

16 460. As I explain below, asserted claims 1, 2, 4, and 8 of the '833 patent are  
17 invalid. A summary of the arguments set forth in this section is given in the table  
18 below:

'833 Claim	Divsalar + (Frey99 (or Frey slides), Luby or MacKay)	Ping + MacKay	Divsalar + (Frey99 (or Frey slides), Luby or MacKay) + '999 Patent	Ping + MacKay + '999 Patent
1	Obvious	Obvious	Obvious	Obvious
2	Obvious	Obvious	Obvious	Obvious
4	Obvious	Obvious	Obvious	Obvious
8	Obvious	Obvious	Obvious	Obvious

19 **A. Claim 1 of the '833 Patent is Invalid**

20 461. Claim 1 of the '833 patent reads:



1           1. An apparatus for performing encoding operations, the apparatus  
2           comprising:

3           a first set of memory locations to store information bits;

4           a second set of memory locations to store parity bits;

5           a permutation module to read a bit from the first set of memory  
6           locations and combine the read bit to a bit in the second set of  
7           memory locations based on a corresponding index of the first set of  
8           memory locations and a corresponding index of the second set of  
9           memory locations; and

10          an accumulator to perform accumulation operations on the bits  
11          stored in the second set of memory locations,

12          wherein two or more memory locations of the first set of memory  
13          locations are read by the permutation module different times from  
14          one another.

15          i)    Claim 1 of the '833 Patent is obvious over Divsalar in view of  
16                Frey99, Luby or MacKay

17       462. I explain below, one limitation at a time, why claim 1 is obvious over  
18       Divsalar in view of any one of Frey99, Luby or MacKay (*i.e.*, Divsalar + (Frey99,  
19       Luby or MacKay)).

20           a)    "an apparatus for performing encoding operations"

21       463. Divsalar teaches the preamble. As I explain above, Divsalar describes a  
22       "turbo-like" code called a repeat-accumulate code. A "coder" capable of encoding  
23       information bits using a repeat-accumulate code is shown in Figure 3 of Divsalar,  
24       reproduced above.

          b)    "a first set of memory locations to store information bits"

          464. Divsalar teaches this limitation. A person of ordinary skill in the art would  
          recognize that the input block comprising information bits would be stored in a set  
          of memory locations (*i.e.*, the block of  $N$  bits input to the repeater as shown in  
          Figure 3 would have been stored in memory locations).

          465. It would have been understood by one of ordinary skill in the art that the  
          encoder of Divsalar may be implemented on a general-purpose computer, where



1 the information bits would be stored in a buffer comprising a set of memory  
2 locations. Indeed, one of ordinary skill in the art would have understood that  
3 Divsalar *himself* implemented an RA encoder using a computer program. Divsalar  
4 states “[w]e wrote a *computer program* to implement this “turbo-like” decoding  
5 for RA codes with  $q = 3$  (rate 1/3) and  $q = 4$  (rate 1/4), and the results are shown in  
6 Figure 5” (Divsalar at 9) (emphasis added). Divsalar ran this decoding program,  
7 using sample encoded data as input, and measured the resulting coding  
8 performance, which is plotted in Figure 5. One of ordinary skill in the art would  
9 have understood Divsalar to have implemented an *encoder* program, in order to  
10 generate the sample encoded data provided to the decoder.

11 466. Even if the RA codes of Divsalar were implemented using special-purpose  
12 hardware components, an obvious implementation would have been to store the  
13 information bits in a first set of memory locations within a memory buffer.

14 467. Divsalar itself is silent regarding the first set of memory locations, but so is  
15 the specification of the '833 patent. If one of ordinary skill would have understood  
16 the claimed first set of memory locations and their use from the '833 specification,  
17 then they would have understood it from Divsalar too.<sup>60</sup>

18 *c) “a second set of memory locations to store parity bits”*

19 468. Divsalar teaches this limitation. A person of ordinary skill in the art would  
20 recognize that the “output block  $[y_1, \dots, y_n]$ ” comprising parity bits would be  
21 stored in a set of memory locations. *See* Divisalar at 5.

22 469. It would have been understood by one of ordinary skill in the art that the  
23 encoder of Divsalar may be implemented on a general-purpose computer, where  
24 the parity bits would be stored in a buffer comprising a set of memory locations.

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<sup>60</sup> See Wicker Tr. at 109-11.



1 As explained above, Divsalar himself implemented an encoder program, in order to  
2 generate the sample encoded data provided to the decoder.

3 470. Even if the RA codes of Divsalar were implemented using special-purpose  
4 hardware components, an obvious implementation would have been to store the  
5 parity bits in a second set of memory locations within a memory buffer.

6 471. Divsalar itself is silent regarding the second set of memory locations, but so  
7 is the specification of the '833 patent. If one of ordinary skill would have  
8 understood the claimed second set of memory locations and their use from the '833  
9 specification, then they would have understood it from Divsalar too.

10 d) "a permutation module to read a bit from the first set of memory  
11 locations and combine the read bit to a bit in the second set of  
12 memory locations based on a corresponding index of the first set  
13 of memory locations and a corresponding index of the second set  
14 of memory locations"

15 472. As explained above, the RA encoder shown in Fig. 3 of Divsalar comprises  
16 three stages: repeat, interleave, and accumulate. In an implementation of the repeat  
17 and interleave stages that would have been obvious to one of ordinary skill in the  
18 art, the repeat stage of the encoder reads each of the  $N$  information bits stored in  
19 the first set of memory locations  $q$  times, and sequentially writes the resulting  
20 duplicated bits to a set of  $N \times q$  memory locations (*i.e.*, the "second set of memory  
21 locations). Interleaving is accomplished by writing the bits into the second set of  
22 memory locations in one order and then reading them out of the second set of  
23 memory locations in a different order (*e.g.*, writing the bits into the second set of  
24 memory locations in a pseudo-random order and then reading the bits out in  
sequential order).



1 473. The act of writing a bit to one of the second set of memory locations  
2 constitutes “combining” the scrambled bit with the destination value.<sup>61</sup> One of  
3 ordinary skill in the art would have recognized that memory buffers are generally  
4 initialized at the start of the encoding process, setting the contents of every  
5 memory location in the buffer to zero. When the permutation module writes a bit *b*  
6 into one of the second set memory locations after it has been initialized, it has the  
7 effect of “combining” *b* with the value already stored in the memory location (*i.e.*,  
8 a 0) using an XOR operation.<sup>62</sup>

8 474. Collectively, the repeat and interleaving stages of the encoder in this  
9 implementation constitute the claimed “permutation module.”<sup>63</sup> The repeat stage  
10 reads each information bit multiple times from the first set of memory locations,  
11 and the interleaving stage changes the order of the repeated bits by writing them  
12 into the second set of memory locations in one order and reading them out in a  
13 different order.<sup>64</sup>

13 475. Divsalar is silent regarding how to perform the interleaving using the first  
14 and second sets of memory locations, but so is the specification of the ’833 patent.  
15 If one of ordinary skill would have understood, from the ’833 specification, how to  
16

17  
18 <sup>61</sup> Here I interpret the word “combine” according to the Court’s construction of that term, which  
19 is to “perform logical operations on” (Claim Construction Order dated August 6, 2014, p. 18).  
20 An XOR operation is a “logical operation” that falls within the scope of this construction (*see*,  
21 *e.g.*, ’833 patent at claim 2, which reads “[t]he apparatus of claim 1, wherein the permutation  
22 module is configured to perform the combine operation to include performing mod-2 or  
23 exclusive-OR sum”).

21 <sup>62</sup> Use of such combinatorial logic feeding the input to memories is a common and obvious  
22 technique.

22 <sup>63</sup> Here I interpret the term “permutation module” to mean “a module that changes the order of  
23 data elements,” as both parties have agreed in their Joint Claim Construction Statement.

23 <sup>64</sup> Strictly speaking, reading each bit multiple times from the first set of memory locations is not  
24 required. That is, a repeat can be accomplished by reading a bit once and writing it multiple  
times to different memory locations. However, it would not be inventive to read the bit multiple  
times, *i.e.*, once for every time the bit is repeated.



1 perform the claimed interleaving using the memory locations, then they would  
2 have understood it from Divsalar too.

3 e) "an accumulator to perform accumulation operations on the bits  
4 stored in the second set of memory locations"

5 476. Divsalar teaches this limitation. As I explain above, the final stage of the  
6 encoder shown in Fig. 3 of Divsalar is an accumulator, which performs  
7 accumulation operations on the interleaved, repeated information bits (Divsalar at 5).  
8 In a software implementation of the RA encoder (such as the "computer program"  
9 that Divsalar himself used to measure the performance of RA codes) one of  
10 ordinary skill in the art would have recognized that an obvious way to implement  
11 the accumulation stage of the encoder would be to accumulate the scrambled bits  
12 *in place*. That is, the "output block  $[y_1, \dots, y_n]$ " would overwrite the "input block  
13  $[x_1, \dots, x_n]$ " (Divsalar at 5). Because in-place accumulation uses the same set of  
14 memory locations for both its input and output, it has the benefit of not requiring  
15 any additional memory.

16 477. The two tables below illustrate how the accumulation is performed *in place*  
17 in the second set of memory locations. Initially, the bit  $x_1$  is stored in the first  
18 location,  $x_2$  is stored in the second location, and so on. For the accumulation  
19 operation, the contents of the first location need not change. But, the second  
20 location is replaced with the sum of the current value of the second location and  
21 the prior location, *i.e.*,  $x_1 + x_2$ . That operation effectively removed the value  $x_2$  as a  
22 standalone quantity from the memory. But, that quantity is not needed for any of  
23 the future operations. Rather than  $x_2$ , the sum  $x_1 + x_2$  is what is needed for the next  
24 operation, and that is exactly what is now stored in the second location. The next  
step is to add that quantity,  $x_1 + x_2$ , to the current value at the third location and that  
sum,  $x_1 + x_2 + x_3$ , is then stored at the third location. That process then continues



1 for the remainder of the bits. Because each value that is overwritten is no longer  
 2 needed for future computations, the accumulation can be performed in place.

<b>Index</b>	1	2	3	4	...
<b>Stored Value</b>	$x_1$	$x_2$	$x_3$	$x_4$	...

3  
4 **2<sup>nd</sup> set of memory locations before accumulation**

<b>Index</b>	1	2	3	4	...
<b>Stored Value</b>	$x_1$	$x_1+x_2$	$x_1+x_2+x_3$	$x_1+x_2+x_3+x_4$	...

5  
6  
7 **2<sup>nd</sup> set of memory locations after accumulation**

8 478. At the end of the accumulation process, the “output block  $[y_1, \dots, y_n]$ ” (i.e.,  
 9 the parity bits) would be stored in the second set of memory locations (see Divsalar  
 10 at 5).

11 479. Divsalar is silent as to how the accumulation would be performed within the  
 12 second set of memory locations, but so is the specification of the '833 patent. If  
 13 one of ordinary skill would have understood how to perform the claimed  
 14 accumulation within the second set of memory locations from the '833  
 15 specification, then they would have understood it from Divsalar too.

16 f) “wherein two or more memory locations of the first set of  
 17 memory locations are read by the permutation module different  
 18 times from one another”

19 480. The repeat stage of Divsalar is regular, and so the permutation module reads  
 20 every memory location in the first set of memory locations the same number of  
 21 times. However, Frey99 teaches irregular repetition, and Luby and MacKay both  
 22 teach the benefits of irregular coding, as explained above. Incorporating the  
 23 irregular repetition of Frey99, or the irregularity of Luby or MacKay, into the  
 24 implementation of the Divsalar encoder described above would result in an



1 irregular repeater which reads some of the first set of  $N$  memory locations more  
2 times than it reads others, as required by this limitation.<sup>65</sup>

3 481. Divsalar does not explicitly explain that the repeat is accomplished by  
4 reading the same bit out of memory more than once. However, neither does the  
5 specification of the '833 patent. If one of ordinary skill would have understood,  
6 from the '833 specification, how to perform repeating by reading bits more than  
7 once from memory, then they would have understood it from Divsalar too.

8 g) Summary

9 482. As explained above, the combination of Divsalar and Frey99, Luby or  
10 MacKay teaches every limitation of claim 1 of the '833 patent.

11 h) Motivations to combine the irregular repeater of Frey99, or the  
12 irregularity of Luby or MacKay, with the RA codes of Divsalar

13 483. As I explain above, one of ordinary skill in the art would have been  
14 motivated to combine Divsalar and Frey99, Luby or MacKay in general, and  
15 would specifically have been motivated to use the irregular repetition of Frey99, or  
16 the irregularity of Luby or MacKay, with the RA codes of Divsalar. Briefly,  
17 Frey99, Luby and MacKay all taught the benefits of irregular coding and one of  
18 ordinary skill would have understood that Divsalar's RA could would have  
19 benefited from making it irregular. I also explain above why such a combination  
20 would require only a minor modification to the teachings of Divsalar, and would  
21 not fundamentally change its principle of operation.

22 ii) Claim 1 of the '833 Patent is obvious over Ping in view of MacKay

23 484. I explain below, one limitation at a time, why claim 1 is obvious over Ping  
24 in view of MacKay.

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<sup>65</sup> Here I interpret "different times from one another" to mean "a different number of times from one another," as both parties have agreed in their Joint Claim Construction Statement.



1                   a)     “an apparatus for performing encoding operations”

2 485. Ping teaches the preamble. As I explain above, Ping teaches constructing  
3 LDPC codes that can be encoded in two stages. In the first encoding stage, a  
4 generator matrix is applied to a sequence of  $k$  information bits to produce sums of  
5 information bits. In the second stage, the sums of information bits are accumulated  
6 recursively to generate  $n-k$  parity bits (*see* Ping at 38). One of ordinary skill in the  
7 art would have understood that the encoding process taught by Ping would be  
8 implemented by “an apparatus for performing encoding operations.”

8                   b)     “a first set of memory locations to store information bits”

9 486. Ping teaches this limitation. A person of ordinary skill in the art would  
10 recognize that an implementation of Ping would store information bits in a set of  
11 memory locations. Specifically, it would have been obvious to one of ordinary  
12 skill in the art to implement the encoding processes disclosed by Ping using  
13 hardware (*e.g.*, a general-purpose computer or special-purpose electronic  
14 components) that comprises a first set of memory locations for storing information  
15 bits.

16 487. Like Divsalar, Ping is silent regarding the first set of memory locations, but  
17 so is the specification of the '833 patent. If one of ordinary skill would have  
18 understood the claimed first set of memory locations and their use from the '833  
19 specification, then they would have understood it from Ping too.

19                   c)     “a second set of memory locations to store parity bits”

20 488. Ping teaches this limitation. A person of ordinary skill in the art would  
21 recognize that an implementation of Ping would store parity bits in a set of  
22 memory locations. Specifically, it would have been obvious to one of ordinary  
23 skill in the art to implement the encoding processes disclosed by Ping using  
24 hardware (*e.g.*, a general-purpose computer or special-purpose electronic



1 components) that comprises a second set of memory locations for storing parity  
2 bits.

3 489. Like Divsalar, Ping is silent regarding the second set of memory locations,  
4 but so is the specification of the '833 patent. If one of ordinary skill would have  
5 understood the claimed second set of memory locations and their use from the '833  
6 specification, then they would have understood it from Ping too.

7 *d) "a permutation module to read a bit from the first set of memory*  
8 *locations and combine the read bit to a bit in the second set of*  
9 *memory locations based on a corresponding index of the first set*  
*of memory locations and a corresponding index of the second set*  
*of memory locations"*

10 490. Ping teaches this limitation under Caltech's apparent interpretation of  
11 "permutation module." As explained above, Ping teaches constructing LDPC  
12 codes that can be encoded in two stages. In the first encoding stage, sums of  
13 subsets of the information bits are computed by reading each of the  $k$  information  
14 bits from the first set of memory locations and, and combining, using an XOR  
15 operation, the read bit with the bit stored in one of a second set of  $n-k$  memory  
16 locations. Eventually, each of the second set of memory locations will contain the  
17 sum of a subset of the information bits, which Ping denotes:

$$\sum_j h_{ij}^d d_j$$

18  
19  
20 (Ping at 38).

21 491. The first encoding stage of Ping does not "change the order of data elements,"  
22 as required by the Court's construction of "permutation module." Rather, in Ping,  
23 the second set of memory locations stores sums of bits rather than reordered  
24 versions of the bits themselves. Plaintiff's infringement arguments, however, still

1 appear to be based on an interpretation of “permutation module” that does not  
2 require changing the order of bits themselves. Under Caltech’s interpretation, the  
3 first encoding stage of Ping would constitute a “permutation module,” as required  
4 by this limitation.<sup>66</sup>

5 492. Like Divsalar, Ping is silent regarding how to perform the interleaving using  
6 the first and second sets of memory locations. But so is the specification of  
7 the ’833 patent. If one of ordinary skill would have understood, from the ’833  
8 specification, how to perform the claimed interleaving using the memory locations,  
9 then they would have understood it from Ping too.

10 e) “an accumulator to perform accumulation operations on the bits  
11 stored in the second set of memory locations”

12 493. Ping teaches this limitation. As I explain above, the final stage of the  
13 encoding process disclosed in Ping is an accumulation, in which the parity bits  $\mathbf{p} =$   
14  $\{p_i\}$  are computed by accumulating the sums calculated during the first stage,  
15 defined recursively as follows:

$$16 \quad p_1 = \sum_j h_{1j}^d d_j$$

$$17 \quad p_i = p_{i-1} + \sum_j h_{ij}^d d_j$$

18  
19  
20 (Ping at 38)

21 494. In a software implementation of the encoding processes taught by Ping, one  
22 of ordinary skill in the art would have recognized that an obvious way to  
23 implement the accumulation stage would be to accumulate the scrambled bits *in*

24 <sup>66</sup> That is, in the LDPC codes of DVB-S2, the parity bits are all sums of two or more information bits. But there is no need to reorder the information bits to construct such parity bits.



1 *place*, as was explained above with respect to Divsalar. Because in-place  
2 accumulation uses the same set of memory locations for both its input and output,  
3 it has the benefit of not requiring any additional memory.

4 495. At the end of the accumulation process, the parity bits  $\mathbf{p} = \{p_i\}$  would be  
5 stored in the second set of  $n-k$  memory locations.

6 496. Like Divsalar, Ping is silent as to how the accumulation would be performed  
7 within the second set of memory locations, but so is the specification of the '833  
8 patent. If one of ordinary skill would have understood how to perform the claimed  
9 accumulation within the second set of memory locations from the '833  
specification, then they would have understood it from Ping too.

10 *f) "wherein two or more memory locations of the first set of*  
11 *memory locations are read by the permutation module different*  
12 *times from one another"*

13 497. In the LDPC codes disclosed by Ping, the parity-check matrix  $\mathbf{H}$  has a  
14 column weight of  $t$ , so the permutation module would read each information bit  $t$   
15 times, combining the information bit into  $t$  different locations in the second set of  
memory locations (*see* Ping at 38).

16 498. However, as explained above, MacKay teaches parity check matrices with  
17 nonuniform column weights. Implementing the LDPC-accumulate coders  
18 disclosed by Ping using the irregular parity check matrices disclosed by MacKay  
19 would result in a permutation module that reads some of the first set of memory  
20 locations more times than it reads others, as required by this limitation.<sup>67</sup>

21 499. Like Divsalar, Ping and MacKay do not explicitly explain reading the same  
22 bit out of memory more than once. However, neither does the specification of

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23 <sup>67</sup> As noted above with respect to Divsalar, strictly speaking the bits need not be read multiple  
24 times to repeat them, i.e., each bit could be read once and then written multiple times. However,  
implementing the repeat by reading each bit multiple times would have been obvious.



1 the '833 patent. If one of ordinary skill would have understood, from the '833  
2 specification, reading bits more than once from memory, then they would have  
3 understood it from both Ping and MacKay too.

4 g) Summary

5 500. As explained above, the combination of Ping and MacKay teaches every  
6 limitation of claim 1 of the '833 patent.

7 h) Motivations to combine the irregularity of MacKay with the  
8 LDPC-accumulate coders of Ping

9 501. As I explain above, one of ordinary skill in the art would have been  
10 motivated to combine Ping and MacKay in general, and would specifically have  
11 been motivated to combine the LDPC-accumulate coders disclosed by Ping with  
12 the irregular parity check matrices disclosed by MacKay. I also explain why such  
13 a combination would require only a minor modification to the teachings of Ping,  
14 and would not fundamentally change its principle of operation.

15 502. For at least the reasons given above, claim 1 of the '833 patent is obvious  
16 over the combination of Ping and MacKay.

17 iii) Claim 1 of the '833 Patent is obvious over Divsalar in view of one of  
18 Frey99, Luby or MacKay and further in view of the '999 Patent

19 503. As explained above, Divsalar and Frey99, Luby or MacKay together teach  
20 every limitation of, and render obvious, claim 1 of the '833 patent. However, in  
21 the event Divsalar and Frey99, Luby or MacKay are found not to teach the  
22 "memory locations" recited in claim 1, then claim 1 is obvious over the  
23 combination of Divsalar, Frey99, Luby or MacKay and the '999 patent (*i.e.*,  
24 Divsalar + (Frey99, Luby or MacKay) + '999 patent).

504. At a high level, use of memories for implementing codes was notoriously  
well known in the art long before Caltech's alleged invention. One of ordinary



1 skill reading Divsalar, Frey99, Luby or MacKay would have understood that their  
2 codes are implemented using memories as described above. The '999 patent is  
3 merely an example of a reference showing generally how use of memories was  
4 known.

5 505. As I explain above, the '999 patent teaches an encoder for encoding  
6 information bits using a linear error-correcting code. The encoder taught by  
7 the '999 patent uses a plurality of memories that store values used during the  
8 encoding process ('999 patent at Abstract). While the memories taught by the '999  
9 patent are read-only memories, one of ordinary skill in the art would have  
10 recognized that writable memories may also be used to implement the encoding  
11 process (*see* '999 patent at Abstract). I also explain above why one of ordinary  
12 skill in the art would have been motivated to combine Divsalar, Frey99, Luby or  
13 MacKay and the '999 patent.

14 506. Therefore, for at least the reasons given above, claim 1 is obvious over the  
15 combination of Divsalar, Frey99, Luby, or MacKay and the '999 patent.

16 iv) Claim 1 of the '833 Patent is obvious over Ping in view of MacKay  
17 and the '999 Patent

18 507. As explained above, Ping and MacKay together teach every limitation of,  
19 and render obvious, claim 1 of the '833 patent (under Caltech's apparent  
20 interpretation of "permutation module"). However, in the event Ping and MacKay  
21 are found not to teach the "memory locations" recited in claim 1, then claim 1 is  
22 obvious over the combination of Ping, MacKay, and the '999 patent.

23 508. As noted above, at a high level, use of memories for implementing codes  
24 was notoriously well known in the art long before Caltech's alleged invention.  
One of ordinary skill reading Ping or MacKay would have understood that their  
codes are implemented using memories as described above. The '999 patent is



1 merely an example of a reference showing generally how use of memories was  
2 known.

3 509. As I explain above, the '999 patent teaches an encoder for encoding  
4 information bits using a linear error-correcting code using memories that store  
5 values used during the encoding process ('999 patent at Abstract).

6 510. Further, one of ordinary skill in the art would have been motivated to  
7 combine Ping and MacKay with the '999 patent. Like Ping and MacKay, the '999  
8 patent relates to methods of improving the performance of linear error-correcting  
9 codes. It was filed in 1984 and granted in 1986, well over a decade before the  
10 claimed priority date of the patents-in-suit. By March 7, 2000, the alleged  
11 conception date of the patents-in-suit, the technology described in the '999 patent  
12 would have been well known in the field, and one of ordinary skill in the art at the  
13 time would have been motivated to combine the teachings of Ping and MacKay  
14 with those of the '999 patent.

15 511. Therefore, for at least the reasons given above, claim 1 is obvious over the  
16 combination of Ping, MacKay, and the '999 patent.

17 **B. Claim 2 of the '833 Patent is Invalid**

18 512. Claim 2 of the '833 patent reads:

19 2. The apparatus of claim 1, wherein the permutation module is  
20 configured to perform the combine operation to include performing  
21 mod-2 or exclusive-OR sum.

22 513. Claim 2 is rendered obvious by a combination of Divsalar and Frey99, Luby  
23 or MacKay, and by a combination of Ping and MacKay. Claim 2 is also rendered  
24 obvious by each of these combinations considered in view of the '999 patent.

514. As I explained above, claim 1 is rendered obvious by each of these  
combinations. Claim 2 adds to claim 1 "wherein the permutation module is  
configured to perform the combine operation to include performing mod-2 or



1 exclusive-OR sum.” This additional limitation of claim 2 is taught by each of  
2 Divsalar and Ping, as explained above.

3 515. Therefore, claim 2 is rendered obvious by a combination of Divsalar and  
4 Frey99, and by a combination of Ping and MacKay, and is also rendered obvious  
5 by each of these combinations in view of the '999 patent.

6 **C. Claim 4 of the '833 Patent is Invalid**

7 516. Claim 4 of the '833 patent reads:

8 4. The apparatus of claim 1, wherein the accumulator is configured  
9 to perform the accumulation operation to include a mod-2 or  
10 exclusive-OR sum of the bit stored in a prior index to a bit stored  
11 in a current index based on a corresponding index of the second set  
12 of memory locations.

13 517. Claim 4 is rendered obvious by a combination of Divsalar and Frey99, Luby  
14 or MacKay, and by a combination of Ping and MacKay. Claim 4 is also rendered  
15 obvious by each of these combinations, considered in view of the '999 patent.

16 518. As I explain above, claim 1 is rendered obvious by each of these  
17 combinations. Claim 4 adds to claim 1 “wherein the accumulator is configured to  
18 perform the accumulation operation to include a mod-2 or exclusive-OR sum of  
19 the bit stored in a prior index to a bit stored in a current index based on a  
20 corresponding index of the second set of memory locations.”

21 519. This additional limitation of claim 4 is taught by each of Divsalar and Ping.  
22 For example, the accumulator of Divsalar calculates the parity bits  $y_1, \dots, y_n$  as  
23 follows:

24 [W]e prefer to think of [the accumulator] as a block coder whose  
input block  $[x_1, \dots, x_n]$  and output block  $[y_1, \dots, y_n]$  are related by  
the formula

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= x_1 + x_2 \\ y_3 &= x_1 + x_2 + x_3 \\ y_n &= x_1 + x_2 + x_3 + \dots + x_n. \end{aligned}$$

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(Divsalar at 5)

520. From the above passage, one can see that all of the parity bits  $y_i$  (except the first parity bit  $y_1$ ) can be defined by the recursive formula  $y_i = y_{i-1} + x_i$ . Thus, calculating  $y_i$  involves taking the mod-2 sum of the bit stored in a prior index (*i.e.*, parity bit  $y_{i-1}$ ) and a bit stored in a current index (*i.e.*, repeated information bit  $x_i$ ).

521. Similarly, Ping defines each parity bit  $p_i$  (except the first parity bit  $p_1$ ) recursively as follows:

$$p_i = p_{i-1} + \sum_j h_{i,j}^d d_j$$

(Ping at 38)

522. Thus, calculating  $p_i$  involves taking the mod-2 sum of the bit stored in a prior index (*i.e.*, parity bit  $p_{i-1}$ ) and a bit stored in a current index (*i.e.*, the sum of the  $i^{\text{th}}$  subset of information bits  $\sum_j h_{i,j}^d d_j$ ).

523. Therefore, claim 4 is rendered obvious by a combination of Divsalar and Frey99, and by a combination of Ping and MacKay, and is also rendered obvious by each of these combinations in view of the '999 patent.

**D. Claim 8 of the '833 Patent is Invalid**

524. Claim 8 of the '833 patent reads:

- 8. A method of performing encoding operations, the method comprising:
  - receiving a sequence of information bits from a first set of memory locations;
  - performing an encoding operation using the received sequence of information bits as an input, said encoding operation comprising:
    - reading a bit from the received sequence of information bits, and
    - combining the read bit to a bit in a second set of memory locations based on a corresponding index of the first set of memory locations



1 for the received sequence of information bits and a corresponding  
2 index of the second set of memory locations; and

3 accumulating the bits in the second set of memory locations,  
4 wherein two or more memory locations of the first set of memory  
5 locations are read by the permutation module different times from  
6 one another.

7 i) Claim 8 of the '833 Patent is obvious over Divsalar in view of  
8 Frey99, Luby or MacKay

9 525. I explain below, one limitation at a time, why claim 8 is obvious over  
10 Divsalar in view of Frey99, Luby or MacKay.

11 a) "a method of performing encoding operations"

12 526. Divsalar teaches the preamble, as I explain above with reference to claim 1  
13 of the '833 patent.

14 b) "receiving a sequence of information bits from a first set of  
15 memory locations"

16 527. Divsalar teaches this limitation, as I explain above with reference to claim 1  
17 of the '833 patent.

18 c) "performing an encoding operation using the received sequence  
19 of information bits as an input"

20 528. Divsalar teaches this limitation, as I explain above with reference to claim 1  
21 of the '833 patent.

22 d) "said encoding operation comprising: reading a bit from the  
23 received sequence of information bits, and combining the read  
24 bit to a bit in a second set of memory locations based on a  
corresponding index of the first set of memory locations for the  
received sequence of information bits and a corresponding index  
of the second set of memory locations"

529. Divsalar teaches this limitation, as I explain above with reference to claim 1  
of the '833 patent.

1 e) "accumulating the bits in the second set of memory locations,  
2 wherein two or more memory locations of the first set of memory  
3 locations are read by the permutation module different times  
4 from one another"

5 530. Frey99, Luby and MacKay each teach this limitation, as I explain above  
6 with reference to claim 1 of the '833 patent.

7 f) Summary

8 531. As explained above, the combination of Divsalar and Frey99, Luby or  
9 MacKay teaches every limitation of claim 8 of the '833 patent.

10 g) Motivations to combine the irregular repeater of Frey99, Luby  
11 or MacKay with the RA codes of Divsalar

12 532. As I explain above, one of ordinary skill in the art would have been  
13 motivated to combine Divsalar and Frey99, Luby or MacKay in general, and  
14 would specifically have been motivated to use the irregular repetition of Frey99, or  
15 the irregularity of Luby or MacKay, with the RA codes of Divsalar. I also explain  
16 why such a combination would require only a minor modification to the teachings  
17 of Divsalar, and would not fundamentally change its principle of operation.

18 533. For at least the reasons given above, claim 8 of the '833 patent is obvious  
19 over Divsalar in view of Frey99, Luby or MacKay.

20 ii) Claim 8 of the '833 Patent is obvious over Ping in view of MacKay

21 534. I explain below, one limitation at a time, why claim 8 is obvious over Ping  
22 in view of MacKay.

23 a) "a method of performing encoding operations"

24 535. Ping teaches the preamble, as I explain above with reference to claim 1 of  
the '833 patent.



1           b) “receiving a sequence of information bits from a first set of  
2           memory locations”

3 536. Ping teaches this limitation, as I explain above with reference to claim 1 of  
4 the '833 patent.

5           c) “performing an encoding operation using the received sequence  
6           of information bits as an input”

7 537. Ping teaches this limitation, as I explain above with reference to claim 1 of  
8 the '833 patent.

9           d) “said encoding operation comprising: reading a bit from the  
10           received sequence of information bits, and combining the read  
11           bit to a bit in a second set of memory locations based on a  
12           corresponding index of the first set of memory locations for the  
13           received sequence of information bits and a corresponding index  
14           of the second set of memory locations”

15 538. Ping teaches this limitation, as I explain above with reference to claim 1 of  
16 the '833 patent.

17           e) “accumulating the bits in the second set of memory locations,  
18           wherein two or more memory locations of the first set of memory  
19           locations are read by the permutation module different times  
20           from one another”

21 539. MacKay teaches this limitation, as I explain above with reference to claim 1  
22 of the '833 patent.

23           f) Summary

24 540. As explained above, the combination of Ping and MacKay teaches every  
limitation of claim 8 of the '833 patent.

          g) Motivations to combine the irregularity of MacKay with the  
LDPC-accumulate coders of Ping

541. As I explain above, one of ordinary skill in the art would have been  
motivated to combine Ping and MacKay in general, and would specifically have

1 been motivated to combine the LDPC-accumulate coders disclosed by Ping with  
2 the irregular parity check matrices disclosed by MacKay. I also explain why such  
3 a combination would require only a minor modification to the teachings of Ping,  
4 and would not fundamentally change its principle of operation.

5 542. For at least the reasons given above, claim 8 of the '833 patent is obvious  
6 over Ping in view of MacKay.

7 iii) Claim 8 of the '833 Patent is obvious over Divsalar in view of one of  
8 Frey99, Luby or MacKay and further in view of the '999 Patent

9 543. As explained above, Divsalar and Frey99, Luby or MacKay together teach  
10 every limitation of, and render obvious, claim 8 of the '833 patent. However, in  
11 the event Divsalar and Frey, Luby or MacKay are found not to teach the "memory  
12 locations" recited in claim 8, then claim 8 is obvious over the combination of  
13 Divsalar, Frey99, Luby or MacKay and the '999 patent.

14 544. As I explain above, the '999 patent teaches an encoder for encoding  
15 information bits using a linear error-correcting code. The encoder taught by  
16 the '999 patent uses a plurality of memories that store values used during the  
17 encoding process ('999 patent at Abstract).

18 545. Further, I also explain above why one of ordinary skill in the art would have  
19 been motivated to combine Divsalar, Frey99, Luby or MacKay and the '999 patent.  
20 Therefore, for at least the reasons given above, claim 8 is obvious over the  
21 combination of Divsalar, Frey99, Luby or MacKay and the '999 patent.

22 iv) Claim 8 of the '833 Patent is obvious over Ping in view of MacKay  
23 and the '999 Patent

24 546. As explained above, Ping and MacKay together teach every limitation of,  
and render obvious, claim 8 of the '833 patent. However, in the event Ping and



1 MacKay are found not to teach the “memory locations” recited in claim 8, then  
2 claim 1 is obvious over the combination of Ping, MacKay, and the ’999 patent.

3 547. As I explain above, the ’999 patent teaches an encoder for encoding  
4 information bits using a linear error-correcting code using memories that store  
5 values used during the encoding process (’999 patent at Abstract).

6 548. Further, as I explain above, one of ordinary skill in the art would have been  
7 motivated to combine Ping and MacKay with the ’999 patent. Therefore, for at  
8 least the reasons given above, claim 8 is obvious over the combination of Ping,  
9 MacKay, and the ’999 patent.

10 **E. Claims 1, 2, 4, and 8 are Invalid for Lack of Written Description.**

11 549. Independent claims 1 and 8 recite “memory locations.” Specifically, these  
12 claims recite “a first set of memory locations” for storing information bits, and “a  
13 second set of memory locations” for storing parity bits. They further require  
14 reading an information bit from one of the first set of memory locations and  
15 combining the read bit with a bit in the second set of memory locations based on a  
16 “corresponding index of the first set of memory locations.” Dependent claims 2  
17 and 4 inherit these limitations from independent claim 1, from which they depend.

18 550. However, the first reference to “memory locations,” sets of “memory  
19 locations,” or indices pointing to “memory locations” in the prosecution histories  
20 of the patents-in-suit appears in the claims of the ’833 patent, filed on March 28,  
21 2011. For this reason, it is my opinion that the claims of the ’833 patent are invalid  
22 because the disclosure lacks sufficient written description of the claimed invention.

23 551. In the alternative, in the event that one or more claims of the ’833 patent are  
24 found not to be invalid under the written description requirement, the earliest  
priority date to which those claims could be entitled is March 28, 2011, the date  
those claims were first filed.



1 552. The claims of the '833 patent, and in particular their use of memory  
2 locations and indexes may have been obvious to one of ordinary skill in the art in  
3 view of the '833 specification. As noted above, use of memories for coding was  
4 well known before Caltech's alleged invention. However, I understand that the  
5 test for written description is not whether the invention would have been obvious  
6 but whether the words or figures of the specification show the inventors were in  
7 possession of the invention. The specification of the '833 patent does not  
8 communicate to one of ordinary skill in the art that the inventors were in  
9 possession of the alleged invention.

9 **F. All asserted claims are invalid over Hughes' products**

10 553. As noted above, I have been told that a number of the accused products in  
11 this case were sold by Defendants prior to March 28, 2011. I have further been  
12 informed that Caltech has not varied its infringement contentions for any of the  
13 products, i.e., it has treated all accused products the same. I have not studied the  
14 accused products. However, if Caltech succeeds in demonstrating infringement of  
15 any claims of the '833 patent, then those claims would be invalid over the accused  
16 products that were sold prior to March 28, 2011. That is, I have been informed of  
17 the axiom of patent law, "that which infringes if later, anticipates if earlier." If  
18 Caltech proves that its '833 claims cover the accused products, then those same  
19 products that preceded the '833 claims would invalidate those claims.

19 **X. SECONDARY CONSIDERATIONS**

20 554. I have reviewed Caltech's Second Supplemental Responses to Defendants'  
21 First Set of Interrogatories, which relate in part to secondary considerations of non-  
22 obviousness (*see* Caltech's First and Second Supplemental Responses to  
23 Interrogatory Number 5). It is my opinion that the supposed indicia of  
24 nonobviousness identified by Caltech in these responses are not relevant to the



1 claims of the patents-in-suit, and fail to provide support for Caltech's position that  
2 the asserted claims are not obvious.

3 555. Caltech contends that the claimed invention was not obvious because it  
4 enjoyed commercial success, but to support this contention Caltech's responses  
5 merely present evidence that "irregular repeat accumulate (IRA) codes" have been  
6 commercially successful. While the patents-in-suit relate generally to IRA codes,  
7 the asserted claims do not cover all possible implementations of IRA codes. A  
8 product may use "IRA codes" without using the claimed invention. Therefore,  
9 pointing to the supposed commercial success of products that use "IRA codes"  
10 does not demonstrate that the claimed invention itself was not obvious. This  
11 objection is not limited to Caltech's evidence of "commercial success," but also  
12 applies to the other supposed indicia of non-obviousness that Caltech identifies.  
13 Even if it were true (and it is not) that IRA codes have been "widely praised by the  
14 industry," have "overcome skepticism from experts," or have achieved  
15 "unexpected results," these facts would not demonstrate the non-obviousness of the  
16 particular class of codes that is covered by the asserted claims. Also, as part of its  
17 allegation of commercial success, Caltech has pointed to sales of the accused  
18 products. However, I understand that the accused products have not been shown to  
19 infringe and, without such a showing, sales of those products do not demonstrate  
20 non-obviousness of the claims. Further, Caltech has not provided any evidence  
21 that any commercial success of the accused products was based on the features of  
22 those products.

23 556. Further, Caltech contends that the asserted claims are not obvious because  
24 they have "overcome skepticism from experts," and that high-performance, low  
complexity codes had "long eluded the telecommunications industry." However,  
these contentions are incorrect. In fact, experts were not skeptical about the  
feasibility of implementing codes characterized by both good error correcting



1 performance and computational efficiency. Rather, prior to Caltech's alleged  
2 invention, it was well known that codes could be designed that have these  
3 properties. For example, Ping states that "[t]he new method can achieve  
4 essentially the same performance as the standard LDPC encoding method with  
5 significantly reduced complexity" (Ping at 39). Similarly, MacKay describes a  
6 class of "fast encoding" Gallager codes that allow for reduced encoding  
7 complexity while demonstrating "equally good performance" (MacKay at 1449;  
8 *see also* MacKay at 1452, "Decoding Times: Not only do these irregular codes  
9 outperform the regular codes, they require fewer iterations ..."). Indeed, months  
10 before the alleged conception date of the patents-in-suit, I myself had suggested  
11 making repeat-accumulate codes irregular (*see* CALTECH000024021).

12 557. Caltech has also failed to demonstrate that the claimed codes achieved  
13 unexpected results. Prior to Caltech's alleged invention, it was well understood  
14 that making a code irregular would improve its performance (*see generally, e.g.,*  
15 Luby, MacKay, Frey99, Frey Slides, Luby97, Luby98, Richardson). For example,  
16 Frey99 demonstrates that irregular turbocodes outperform the corresponding  
17 regular turbo codes, and the improvement in performance is consistent with what  
18 was shown previously by Luby and MacKay. Based on these results, it was  
19 expected that adding irregularity to RA codes would also improve performance,  
20 which was later found to be the case. In conclusion, the supposed indicia of non-  
21 obviousness identified by Caltech fail to show that the claimed invention was  
22 obvious.

23 558. Also, I understand that simultaneous invention by others is evidence of  
24 obviousness. As noted above, I myself suggested to Dariush Divsalar that he make  
his RA codes irregular. *See* Email from Brendan Frey to Dariush Divsalar dated  
Dec. 8, 1999 (CALTECH000024021). If turning RA codes into IRA codes was  
inventive, I made that invention myself before Caltech claims to have done so.



1 Also, as noted above, David MacKay also produced IRA codes with his RA.c  
2 software before Caltech claims to have developed its alleged invention. My work  
3 and that of David MacKay shows simultaneous development by others, and further  
4 evidences the obviousness of Caltech's claims.

5 **XI. THE MARCH 7 2000 MCELIECE EMAIL**

6 559. I have been informed that Caltech argues that an email dated March 7, 2000  
7 sent by Robert McEliece, one of the inventors of the patents-in-suit, is evidence of  
8 the conception of the inventions to which the asserted claims are directed.

9 560. In its entirety, the email reads as follows:

10 From: rjm (Robert J. McEliece)  
11 Sent: Tue 3/07/2000 4:12 PM (GMT-08:00)  
12 To: <aamod>, <hui>, <mas>  
13 Cc:  
14 Subject: A thought

15 Hi all,

16 It just occurred to me that our "generalized" RA codes are just  
low-density GENERATOR matrix codes, followed by an accumulator.  
For example, ordinary RA codes are  $S(1,q)$  LDGM codes + accumulator.

So what we want to consider is whether irregular LDGM outer codes  
gain us anything.

(Incidentally, Tommy Cheng considered LDGM codes in his thesis.)

--Bob

CALTECH000008667

17 561. The emails suggests trying to incorporate irregularity into a class of known  
18 codes, but does not indicate whether the resulting irregular code would result in  
19 any improvement over the state of the art. The first paragraph describes a set of  
20 "generalized" RA codes that are "just low-density generator matrix codes,  
21 followed by an accumulator." These codes were known in the art, and described in,  
22 e.g., the Divsalar reference, as I explain above.

23 562. The sole mention of irregularity in the email appears in the second paragraph,  
24 which contains only the sentence "[s]o what we want to consider is whether



1 irregular LDGM outer codes gain us anything.” This sentence characterizes  
2 incorporating irregularity into LDGM-accumulate codes as an idea for further  
3 consideration, and not as a fully conceived invention.

4 563. CALTECH000008667 does not explain how to design or implement the  
5 “irregular LDGM outer codes” that are referenced in the second paragraph. In  
6 particular, irregular codes depend crucially on a feature known as a “degree profile”  
7 (as described in claim 6 of the ’710 patent), but CALTECH000008667 nowhere  
8 mentions which degree profile to use or how the degree profile should be selected.  
9 In fact, CALTECH000008667 does not explicitly state that irregular *repetition*  
10 should be used for the LDGM outer code, which is required by many of the  
11 asserted claims of the patents-in-suit, as I explain above. Given the lack of a  
12 concrete suggestion as to *how* to incorporate irregularity into LDGM codes, the  
13 email above does not show that the inventors, at the time of the email, had made  
14 the invention claimed in the patents.

15 564. In my opinion, CALTECH000008667 at most expresses McEliece’s *hope*  
16 that adding irregularity to LDGM-accumulate codes would produce desirable  
17 results (*see id.*, “so what we want to *consider* is *whether* irregular LDGM outer  
18 codes gain us anything”) (emphasis added).

19 565. Finally, CALTECH000008667 only suggests adding irregularity to LDGM  
20 codes, but the claimed invention purports to be applicable to a broader class of  
21 codes than LDGM codes. For example, in the ’710 claims, only dependent claims  
22 7, 13, and 20 recite a first-encoding step involving a “low-density generator matrix.”  
23 This implies that independent claims 1, 11, and 15 (from which claims 7, 13, and  
24 20 depend, respectively) are intended to cover a broader class of codes than  
irregular LDGM-accumulate codes. Also, in the ’710 claims, only dependent  
claims 4, 5 and 7 recite a second-encoding step involving an “accumulator.” This  
implies that independent claims 1, 2 and 3 (from which claims 4, 5 and 7 depend)



1 are intended to cover a broader class of codes than irregular LDGM-accumulate  
2 codes. However, the email reproduced above mentions irregularity only in the  
3 context of LDGM-accumulate codes, and does not suggest incorporating  
4 irregularity into a broader class of codes.

5 566. Further, the attached email is silent about limitations in the claims of the  
6 asserted patents (e.g., “obtaining a block of data” of the ’710 claims; the equations  
7 of claim 1 of the ’032 patent; the message passing decoder or Tanner graph of  
8 claim 18 of the ’032 patent; or the memory locations or indices of the ’833 claims).

9 567. Further, none of the other documents identified by Caltech as evidence of  
10 Conception predate the provisional applications to which the patents-in-suit claim  
11 priority.

## 12 **XII. INVENTORSHIP**

13 568. Divsalar’s 1998 paper disclosed everything in ’781 claim 19. I have been  
14 informed that Divsalar worked jointly in collaboration with the named  
15 inventors. Divsalar should have been named an inventor on that patent. All of the  
16 other asserted claims rely on the repeat-accumulate code disclosed by  
17 Divsalar. Divsalar should also have been named an inventor on the other patents.

## 18 **XIII. MATERIALITY**

19 569. I have been asked for my opinion on whether the following three references  
20 were material to the patentability of the claimed invention:

- 21 • Luby, M. et al., “Practical Loss-Resilient Codes,” *STOC '97* (1997)  
22 (hereinafter, “Luby97”)
- 23 • Luby, M. et al., “Analysis of Low Density Codes and Improved Designs  
24 Using Irregular Graphs,” *STOC '98*, p. 249-259 (1998) (hereinafter,  
“Luby98”)



- 1       • Richardson, T. et al. “Design of provably good low-density parity check  
2       codes,” *IEEE Transactions on Information Theory* (1999) (preprint)  
3       (hereinafter, “Richardson99”)

3 570. For the reasons set forth in detail below, each of these references was  
4 material to the patentability of the claims of the patents-in-suit. In particular, each  
5 reference teaches irregularity, a concept that is central to the claimed invention but  
6 which was not taught by any of the references that were before the Patent Office  
7 during prosecution.

7 571. For each of these three references, I rely upon the entire disclosure of the  
8 reference in forming my opinions with respect to materiality. Without limiting that  
9 basis in any way, certain aspects of each reference demonstrating their materiality  
10 are briefly discussed below. Additional aspects are set forth in the claim charts  
11 attached as Exhibits F, G, and H.

12       **A. Luby97**

13 572. Luby97 is material to the patentability of all asserted claims of the patents-  
14 in-suit because it teaches the concept of irregularity. (*See* Luby 97, *passim*; *see*  
15 *also* Khandekar Thesis at CALTECH000003301). In particular, Luby 97 teaches  
16 that making a regular code irregular will improve that code’s performance. For  
17 example, Luby 97 teaches: “In contrast with many applications of random graphs  
18 in computer science, our graphs are not regular. Indeed, the analysis in Section 6  
19 shows that it is not possible to approach channel capacity with regular graphs.”  
20 (Luby 97 at 153.) Indeed, Luby 97 teaches that because regular graphs “cannot  
21 yield codes that are close to optimal,” “irregular graphs are a necessary component  
22 of our design.” (*Id.* at 151-52.) Thus, Luby 97 concludes, “irregular degree  
23 sequences are better than regular degree sequences.” (*Id.* at 158.) This concept  
24 was not taught by any of the references that were before the Patent Office during  
prosecution.



1 573. Luby 97 describes the performance gain from converting a regular code into  
2 an irregular code. Luby 97 states, for example: “In this paper, we present codes  
3 that can be encoded and decoded in linear time while providing near optimal loss  
4 protection.” (*Id.* at 151.) Irregular codes, Luby 97 explains, “can transmit over  
5 lossy channels at rates extremely close to capacity.” (*Id.* at 150.)

6 574. Luby 97 also teaches how to convert a regular code into an irregular code.  
7 This “requires the careful choice of a random *irregular* bipartite graph, where the  
8 structure of the *irregular* graph is extremely important” (Luby97 at Abstract)  
(emphasis added). Luby 97 goes on to explain:

9 “Our encoding and decoding algorithms are almost symmetrical.  
10 Both are extremely simple, computing exactly one exclusive-or  
11 operation for each edge in a randomly chosen bipartite graph. As  
12 in many similar applications, the graph is chosen to be sparse,  
13 which immediately implies that the encoding and decoding  
14 algorithms are fast. Unlike many similar applications, the graph is  
15 not regular; instead it is quite irregular with a carefully chosen  
16 degree sequence.”

17 575. (*Id.* at 151-52.) Note that the term “degree sequence” is equivalent to  
18 the term “degree profile”, as referred to in the patents-in-suit, in Frey99 and  
19 in MacKay. Luby 97 goes on to provide the “tools” for how “to *design* good  
20 irregular degree sequences.” (*Id.* at 152 (emphasis in original); *see also id.*  
21 at 153-59.)

22 576. Importantly, Luby 97 teaches that one way of encoding an irregular code is  
23 by using an irregular low-density generator matrix (“LDGM”):

24 “The  $\beta n$  check bits of the code  $C(B)$  described in Section 2 can be  
computed by multiplying the vector of  $n$  message bits by the  $\beta n \times n$   
matrix,  $M(B)$ , whose  $(i, j)$ -th entry is 1 if there is an edge in  $B$   
between left node  $i$  and right node  $j$  and is 0 otherwise (the  
multiplication is over the field of two elements). We choose our  
graphs  $B$  to be sparse, so that the resulting matrix  $M(B)$  is sparse  
and the multiplication can be performed quickly.”



1 (*Id.* at 157; *see also id.* (teaching that “the average number of 1s per row in  $M(B)$  is  
2  $n \ln(1/\epsilon)$ ; so, the Gaussian elimination can be performed in time  $O(n \ln(1/\epsilon))$ ”);  
3 Divsalar Tr. (232:18-25, 238:20-241:8 (“what we’ve got here is a – in Luby ’97, an  
4 irregular low-density generator matrix”).)

5 577. Irregular LDGM codes are central to the claimed invention of the patents-in-  
6 suit. (*See, e.g.*, ’710 patent, col. 3:54-55 and Fig. 4.) The inventors discussed  
7 among themselves that IRA codes “are just low-density GENERATOR matrix  
8 codes, followed by an accumulator.” (*See* CALTECH000008667.) Each of the  
9 patents states that the outer coder of the claimed IRA codes “may be a low-density  
10 generator matrix (LDGM) coder that performs an irregular repeat of the  $k$  bits in  
11 the block, as shown in Fig.4.” (*See, e.g.*, ’710 patent, col. 3:51-54.) And  
12 dependent claims of the patents recite this embodiment explicitly. (*See, e.g.* ’710  
13 patent, claims 7 and 20; ’032 patent, claim 6; ’781 patent, claim 5.) Luby 97’s  
14 disclosure of an irregular LDGM in the prior art would thus have been material to a  
15 Patent Examiner considering the patentability of the claims of the patents-in-suit.

16 578. In his doctoral thesis, Dr. Khandekar acknowledges that “Luby et al. also  
17 introduced the concept of irregularity” in error correction codes, which was a  
18 “major breakthrough” in 1997, and that IRA codes are merely an application of  
19 Luby’s “concept of irregularity to the ensemble of RA codes” described in  
20 Divsalar. (CALTECH000003345, 3346; *see also* CALTECH000003293 (IRA  
21 codes “are adapted from the previously known class of repeat-accumulate (RA)  
22 codes”); CALTECH000003350 (“Having reviewed the basic properties of irregular  
23 LDPC codes, let us now apply the concept of irregularity to the ensemble of RA  
24 codes defined in Section 1.2.6 to get the ensemble of irregular RA codes.”). Dr.  
Khandekar’s thesis even shows the Tanner graph of Luby 97’s irregular LDPC  
code (Fig. 3.1), the Tanner graph of Divsalar’s RA code (Fig. 1.6), and how when  
combined these produce the Tanner graph of an IRA code (Fig. 3.2).



1 (CALTECH000003315, 3347, and 3350.) Showing similar awareness of Luby's  
2 materiality, Dr. Jin wrote to a Caltech colleague on May 4, 2000 – just fourteen  
3 days before filing his provisional application with the Patent Office – that “the  
4 papers on codes achieving BEC capacity are most written by Luby,” that Luby's  
5 subject is “irregular low density parity check codes,” and that Dr. Jin's “group is  
6 also working on that subject, ... but that hasn't been disclosed yet.”

7 (CALTECH000008875 (emphasis added).) The Patent Examiner had a copy of  
8 Divsalar during prosecution of the patents-in-suit, but was never provided a copy  
9 of Luby 97 or informed that the claimed IRA codes were merely an application of  
10 Luby's “concept of irregularity” to Divsalar's RA codes. Nor was the Patent  
11 Examiner informed of the fact that, as Dr. Jin testified to during his deposition, the  
12 accumulator of the patents-in-suit is identical to the accumulator disclosed in  
13 Divsalar. (Jin Tr. at 122:7-13, 129:5-15, 134:12-18; *see also* Wicker Tr. at 87:2-9,  
14 95:15-20, 109:9-20; Khandekar Tr. at 306:6-17.) Nor was the Patent Examiner  
15 told what Dr. Jin freely admitted to his Caltech colleague – that Luby 97 disclosed  
16 the same “subject” as his and his named co-inventor's work. It is more likely than  
17 not that the claims would not have issued in their present form had this information  
18 been disclosed to the Patent Office.

19 579. The applicants also made affirmative representations to the Patent Office  
20 regarding the patentability of their then-pending patent claims that they could not  
21 have made had they disclosed Luby 97. In an office action dated September 3,  
22 2004, the Patent Examiner rejected then pending claims of the '710 patent as  
23 invalid in light of U.S. Patent No. 6,014,411 to Wang (hereinafter, “Wang”).

24 (CALTECH000000117-118, 120-124.) The applicants responded on November 22,  
2004 by arguing that their claims were patentable over Wang:

580. The encoding arrangement shown in Figure 5 of Wang uses a fixed  
repetition rate “r” ...



1 ...  
2 There is no indication in Wang that the rate  $r$  is irregular. Rather,  
3 all bits are repeated the same number of times, i.e., regularly.

4 Each of independent claims 11, 15, and 24 recites that in a first  
5 encoding, bits are repeated “irregularly” or “a different number of  
6 times”. Accordingly, Applicant submits that claims 11, 15, and 24,  
7 and their dependencies, are allowable.

8 (CALTECH000000110-111.) The applicants’ sole argument for the patentability  
9 of their claims over Wang was thus that “[t]here is no indication in Wang that the  
10 rate  $r$  is irregular,” and “[r]ather, [in Wang] all bits are repeated the same number  
11 of times, i.e., regularly.” (*Id.*) Because Luby 97 teaches that replacing a regular  
12 code with an irregular code produces substantially improved performance, as  
13 discussed above, this reference supplies the precise element that the applicants  
14 claimed was missing from Wang. Had Luby 97’s prior art teaching to improve the  
15 performance of regular codes by making them irregular been disclosed to the  
16 Patent Office, the argument would have been significantly weakened, and the  
17 claims would not have issued in their present form (because, *e.g.*, the Examiner  
18 would have been equipped to respond by pointing out that making a regular code  
19 irregular – the basis for Caltech’s distinction – was already well known in the art).  
20 For at least the reasons given above, Luby97 is material to the patentability of the  
21 claimed invention.

## 22 **B. Luby98 and Richardson99**

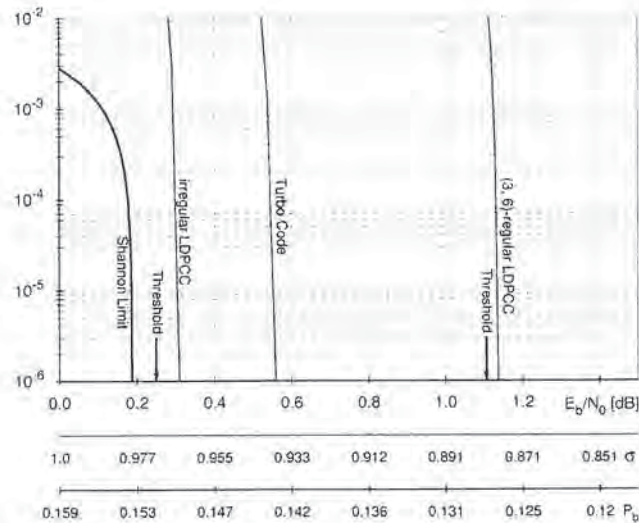
23 581. Luby98 and Richardson99 are also material to the patentability of the claims  
24 of the patents in suit because they teach irregular LDPC codes, which are not  
taught by any of the references that were before the Patent Office during  
prosecution.

582. Irregular LDPC codes are the primary focus of the Richardson99 paper  
(Richardson99 at 1) (“In this paper we present *irregular* low-density parity check



1 codes (LDPCs) which exhibit performance extremely close to the best possible as  
 2 determined by the Shannon capacity formula”) (emphasis in original).

3 Richardson99 includes experimental data indicating that irregular LDPC codes  
 4 exhibit significantly lower error rates than both regular LDPC codes and regular  
 5 turbocodes, as shown in Figure 2, reproduced below:



13 **Richardson, Fig. 2, comparing performance of various codes**

14 583. Acknowledging its materiality, Dr. Jin testified that Richardson99 is “very  
 15 relevant to [the] patent,” and that it “represents the best codes in irregular LDPC  
 16 code.” (Jin Tr. at 199:9-16.) After testifying that Richardson99 was material,  
 17 however, Dr. Jin testified that he chose to instead disclose a non-prior art 2001  
 18 version of the paper to the Patent Office. (Jin Tr. at 211:7-14 (“Q. The question is:  
 19 The version of Richardson and Urbanke that’s actually disclosed here on the  
 20 patents themselves is the 2001 version, correct? A. This is correct. We had their  
 21 original preprint and I think that after their publication become official, that we  
 22 changing [sic] to the official version of that paper.”); *see also*  
 23 CALTECH000023593 (showing publication of Richardson99 in April 1999).)

24 584. Similarly, Luby98 describes “error-correcting codes based on random  
*irregular* bipartite graphs, which we call *irregular codes*” (Luby98 at 249)

1 (emphasis added). In particular, Luby98 describes irregular Gallager codes (*i.e.*,  
2 LDPC codes), and teaches that adding irregularity to known regular Gallager codes,  
3 can significantly enhance decoding performance (*see id.*).

4 585. The idea of irregular codes is central to the claimed invention, and is not  
5 taught by any of the references that were before the Patent Office during  
6 prosecution of the patents-in-suit. As I explain above, the importance of  
7 irregularity to the claimed invention is underscored by the applicant's own  
8 statements about the Wang reference during prosecution of the '710 patent and  
9 their disclosure of the 2001 non-prior art version of Richardson.

10 586. For at least these reasons, the Luby98 and Richardson99 references are  
11 material to the patentability of the claimed invention.

#### 12 **XIV. CLAIM CHARTS**

13 587. Attached hereto as exhibits B-E are claim charts that summarize the  
14 invalidity analysis presented herein. Attached hereto as exhibits F-H are claim  
15 charts that summarize the materiality analysis presented herein. The evidence  
16 presented in these charts is intended as a representative sample of the evidence  
17 relied upon in this report; it is not an exhaustive list of evidence upon which I rely.

#### 18 **XV. TRIAL EXHIBITS**

19 588. I may rely on visual aids and demonstrative exhibits that demonstrate the  
20 bases of my opinions. Examples of these visual aids and demonstrative exhibits  
21 may include, for example, claim charts, patent drawings, excerpts from patent  
22 specifications, file histories, interrogatory responses, deposition testimony and  
23 deposition exhibits, as well as charts, diagrams, videos and animated or computer-  
24 generated video.



1 589. Other than as referred to in this report, I have not yet prepared any exhibits  
2 for use at trial as a summary or support for the opinions expressed in this report,  
3 but I expect to do so in accordance with the Court's scheduling order.

4 **XVI. COMPENSATION**

5 590. I am being paid at my ordinary and customary hourly rate of \$600, plus  
6 expenses, for my time spent working on this matter. My compensation does not  
7 depend on the outcome of this case.

8 **XVII. SUPPLEMENTATION OF OPINIONS**

9 591. I reserve the right to supplement my opinions after I have and the  
10 opportunity to review expert reports or other materials from Plaintiff or other  
11 additional documents or materials that are brought to my attention.

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## APPENDIX A

### **Mathematical Representations of Error-Correcting Codes**

592. Coding theorists often think of error-correcting codes in linear-algebraic terms. Linear algebra is the branch of mathematics that deals with vectors and the linear transformations that can be applied to vectors.<sup>68</sup>

593. In linear-algebraic terms, a  $k$ -bit block of information bits is a  $k$ -dimensional vector of bits and an  $n$ -bit codeword is an  $n$ -dimensional vector of bits. The encoding process, which converts blocks of information bits into codewords, is a linear transformation that maps  $k$ -dimensional bit vectors to  $n$ -dimensional bit vectors. This transformation is represented by a  $k \times n$  matrix  $G$  called a *generator matrix*. For an information vector  $\mathbf{u} = [u_1, u_2, u_3, \dots, u_k]$  the codeword  $\mathbf{x} = [x_1, x_2, x_3, \dots, x_n]$  is given by:  $\mathbf{x} = \mathbf{u}G$ , where:

$$\mathbf{x} = \mathbf{u}G = \begin{bmatrix} \sum_{i=1}^k G_{i,1} u_i \\ \sum_{i=1}^k G_{i,2} u_i \\ \sum_{i=1}^k G_{i,3} u_i \\ \vdots \\ \sum_{i=1}^k G_{i,n} u_i \end{bmatrix}$$

594. The image of  $G$ , denoted  $\text{Im}(G)$ , represents the set of  $n$ -dimensional vectors that are valid codewords. Because  $k < n$ ,  $G$  is not surjective, meaning that not all  $n$ -dimensional vectors are valid codewords. A  $(n-k) \times n$  matrix  $H$ , called a *parity check matrix*, can be used to determine whether a particular  $n$ -dimensional vector

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<sup>68</sup> A linear transformation is a mathematical function that preserves addition and scalar multiplication. More formally, a function  $f$  is linear if and only if, for all  $x, y$ , and  $\alpha$ :  $\alpha f(x + y) = f(\alpha x) + f(\alpha y)$ . Every matrix represents a linear transformation.



1 is a valid codeword. In particular, for an  $n$ -dimensional vector  $\mathbf{x}$ ,  $\mathbf{x}$  is a valid  
2 codeword if and only if  $H\mathbf{x} = \mathbf{0}$ . In linear-algebraic terms, the image of  $G$  is equal  
3 to the kernel of  $H$ .

4 595. Each of the  $n - k$  rows of the parity-check matrix  $H$  represents an equation  
5 that a valid codeword must satisfy. For example, consider a codeword  $\mathbf{x}$  and a  
6 parity check matrix  $H$  given as follows:

$$7 \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

11 596. If  $\mathbf{x}$  is a valid codeword, the product  $H\mathbf{x}$  must be equal to  $\mathbf{0}$ , so we have:

$$12 \quad H\mathbf{x} = \begin{bmatrix} x_3 + x_4 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathbf{0}$$

15 As this equation shows, the first row of  $H$  represents the constraint that  $x_3 + x_4 = 0$ ,  
16 and the second row of  $H$  represents the constraint that  $x_1 + x_2 = 0$ . If the vector  $\mathbf{x}$   
17 satisfies both of these constraints, it is a valid codeword.

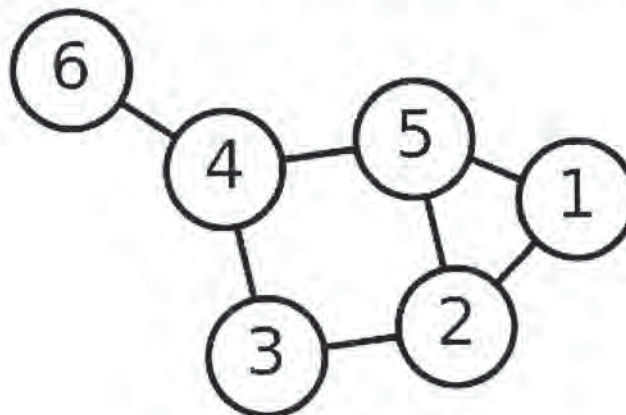
18 597. In practice, parity-check matrices have hundreds or thousands of rows, each  
19 of which represents an equation of the form  $x_a + x_b + \dots + x_z = 0$ , similar to those  
20 shown in the above example. These equations are called *parity-check* equations.

21 598. Another popular mathematical representation of error-correcting codes is the  
22 "Tanner Graph." Tanner graphs were named after R. Michael Tanner, who  
23 described the concept in a 1981 publication titled "A Recursive Approach to Low

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1 Complexity Codes.”<sup>69</sup> A Tanner graph is a graphical depiction of the parity matrix  
2  $H$ .

3 599. A “graph” in this context is a group of objects, or *nodes*, that may be linked  
4 together by connections called *edges*. A simple graph is shown below:



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11 A Simple Graph

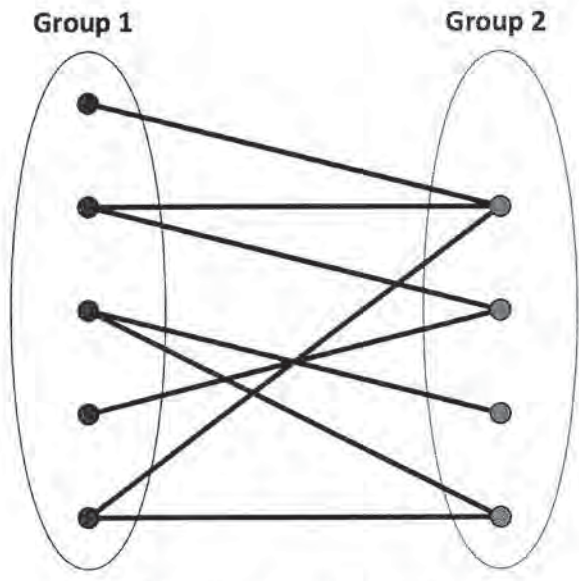
12 600. The nodes in the graph above are represented by the circles labeled 1  
13 through 6, and the edges are represented as lines connecting the nodes (*e.g.*, the  
14 straight line connecting nodes 6 and 4 is an edge). When two nodes are connected  
15 by an edge, we say that the nodes are *adjacent*.

16 601. Tanner graphs are part of a class of graphs called *bipartite* graphs. A  
17 bipartite graph is a graph whose nodes can be divided into two groups, such that  
18 every edge connects a node from one group to a node from the other (*i.e.*, no two  
19 nodes in the same group are adjacent). A bipartite graph is shown below:  
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24 <sup>69</sup> Tanner, R. M., “A recursive approach to low complexity codes,” *IEEE Transactions on Information Theory*, vol. 27, pp. 533–547 (September 1981).



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A Simple Bipartite Graph

The two groups of nodes in the bipartite graph above are labeled “Group 1” and “Group 2.” Note that no node in Group 1 is connected to any other node in Group 1, and no node in Group 2 is connected to any other node in Group 2.

602. Tanner graphs are bipartite graphs that represent error-correcting codes. A Tanner graph includes one group of nodes called *variable nodes* that correspond to the information and parity bits, and a second group of nodes *check nodes* that represent the relationship between the parity and information bits. The variable nodes include two types of nodes: *information nodes* that correspond to information bits input to the code, and *parity nodes*, that correspond to parity bits. Generally, a Tanner graph will have  $n$  variable nodes and  $n - k$  check nodes, where  $n$  is the number of bits in the codeword, and  $k$  is the number of information bits per block. Variable nodes are not connected to other variable nodes, and check nodes are not connected to other check nodes (this is what makes Tanner graphs bipartite).

603. Intuitively, one can think of a Tanner graph as a representation of the interrelationships among the bits of a codeword. Each variable node  $v_i$  corresponds to a bit  $b_i$  in the codeword. Each check node represents a mathematical



1 relationship among the bits to which it is connected. Specifically, when a check  
2 node is connected to variable nodes  $v_1, v_2, v_3, \dots, v_r$ , it means that the corresponding  
3 bits of the codeword must add up to 0. That is:  $b_1 \oplus b_2 \oplus b_3 \oplus \dots \oplus b_r = 0$ . Each  
4 check node of the Tanner graph represents a different group of encoded bits that  
5 must sum to 0.

6 604. As I mentioned above, a Tanner graph for a particular code is a graphical  
7 depiction of that code's parity-matrix  $H$ . In a Tanner graph, each of the variable  
8 nodes  $v_1 \dots v_n$  represents a bit in the codeword, and each of the check nodes  $c_1 \dots$   
9  $c_{n-k}$  represents a parity-check equation. As I explained earlier, each column of  $H$   
10 represents a bit of the codeword, and each row of  $H$  represents a parity-check  
11 equation that a valid codeword must satisfy. Thus, the variable nodes and check  
12 nodes correspond to the columns and rows of  $H$ , respectively. The edges of the  
13 Tanner graph correspond to the 1s in the parity-check matrix: if there is a 1 at the  
14  $i^{\text{th}}$  row and the  $j^{\text{th}}$  column of  $H$  (i.e., if  $H_{i,j} = 1$ ), then there is an edge connecting the  
15  $i^{\text{th}}$  check node to the  $j^{\text{th}}$  variable node. Conversely, if  $H_{i,j} = 0$ , the  $i^{\text{th}}$  check node  
16 and the  $j^{\text{th}}$  variable node are not connected.

17 605. Matrices and Tanner graphs are two equivalent ways of describing error-  
18 correcting cods. Every linear code has a matrix representation and a Tanner graph  
19 representation.

20 606. A related graphical representation of codes is the factor graph (Kschischang  
21 et al, IEEE Trans Inform Theory Vol 47, No 2, pp 498-519, February 2001). A  
22 factor graph is more general than a Tanner graph in two ways: (a) Some of the  
23 variable nodes may be unobserved, i.e., in the context of coding some variables  
24 may not correspond to information bits or parity bits; (b) The check nodes can  
implement more general functional relationships between the variables and can  
even represent continuous relationships such as those found in probability theory.

Convolutional codes can be represented by factor graphs, where there are three

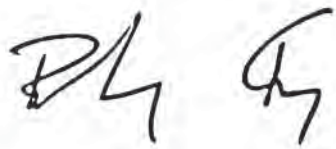


1 types of variable: (a) variables corresponding to information bits; (b) variables  
2 corresponding to parity bits; and (c) variables that correspond to the memory of the  
3 convolutional code. The latter variables are usually not transmitted over the  
4 channel. In a truncated convolutional code, the information bits themselves may  
5 not be transmitted, resulting in a non-systematic code. Or, some parity bits may be  
6 punctured. In all of these scenarios, an iterative sum-product decoding algorithm  
7 can be used to determine the codeword and the information bits, given the output  
8 of the channel.

8 607. It is widely recognized that Tanner graphs can be modified slightly to allow  
9 for variables that are not transmitted across the channel, such as variables  
10 corresponding to the memory of a convolutional code. Consequently, within the  
11 context of coding, Tanner graphs can be used to represent codes with such  
12 “unobserved variables”, and in this report I frequently refer to Tanner graphs with  
13 this additional functionality.  
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Date: January 28, 2015



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Dr. Brendan Frey, PhD.